Fading wiretap channel with no CSI anywhere

Pritam Mukherjee, Şennur Ulukuş

University of Maryland, College Park

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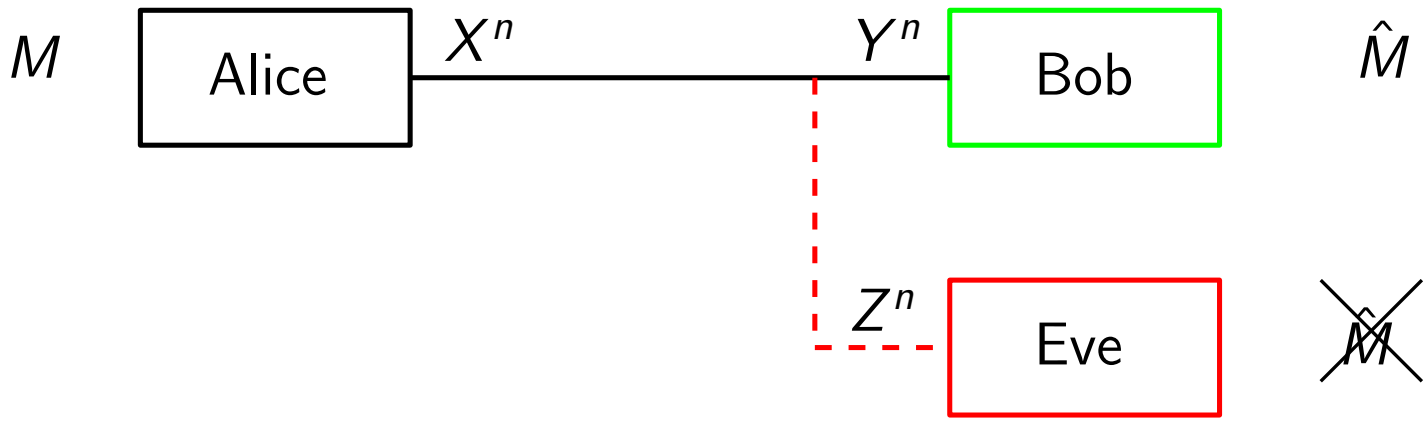
The Wiretap channel

- Introduced by Wyner in 1975



$$\text{Reliability: } \mathbb{P}(\hat{M} \neq M) \leq \epsilon$$

$$\text{Security: } \frac{1}{n} I(M; Z^n) \leq \epsilon$$

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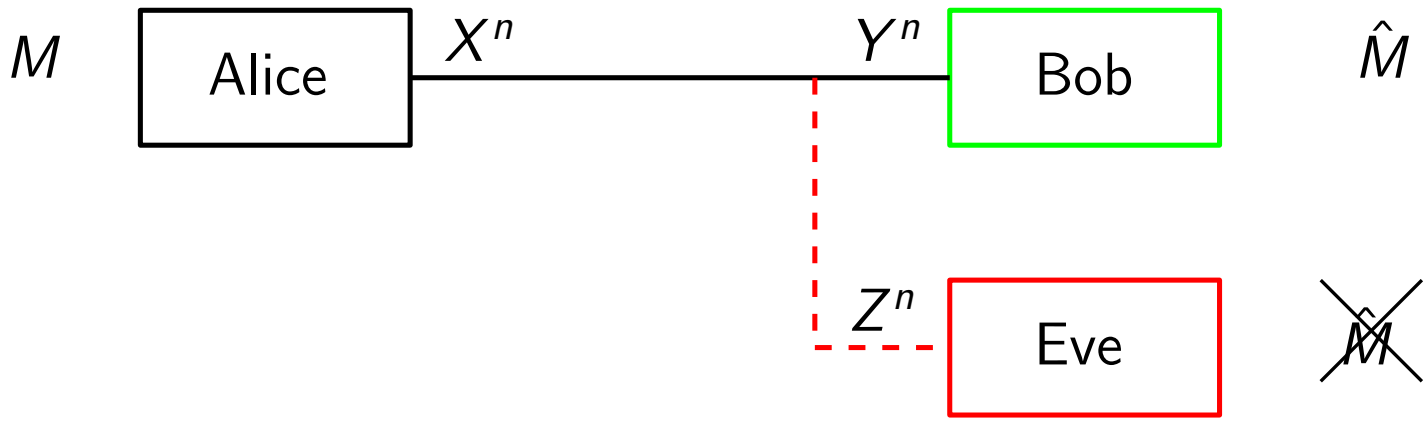

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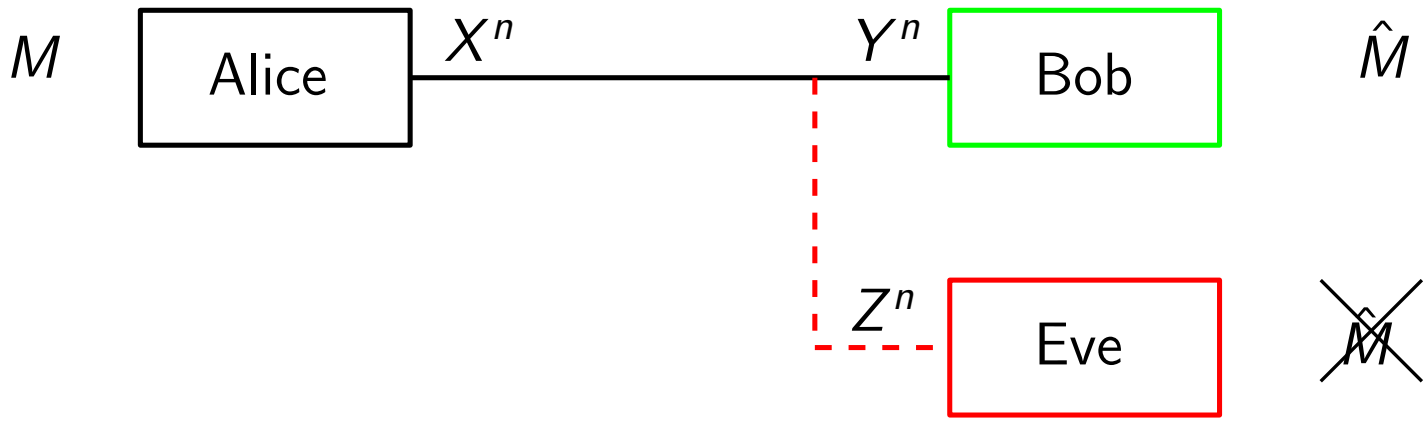

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- *Secrecy capacity* given by,

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General wiretap channel

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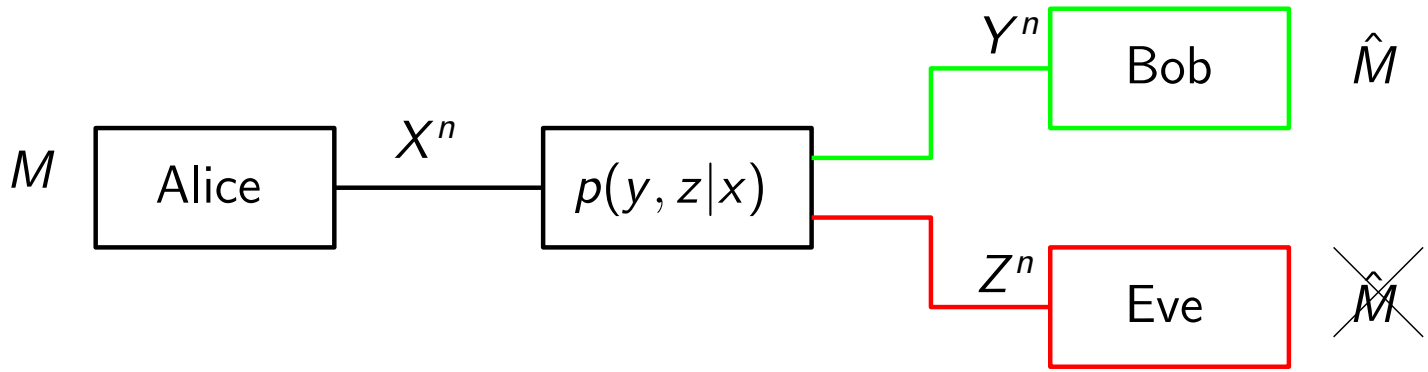

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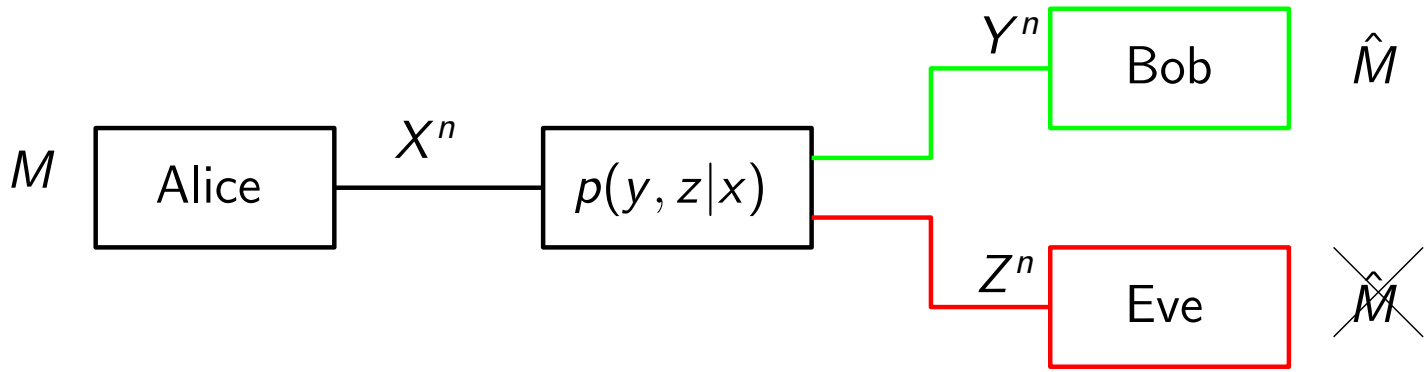

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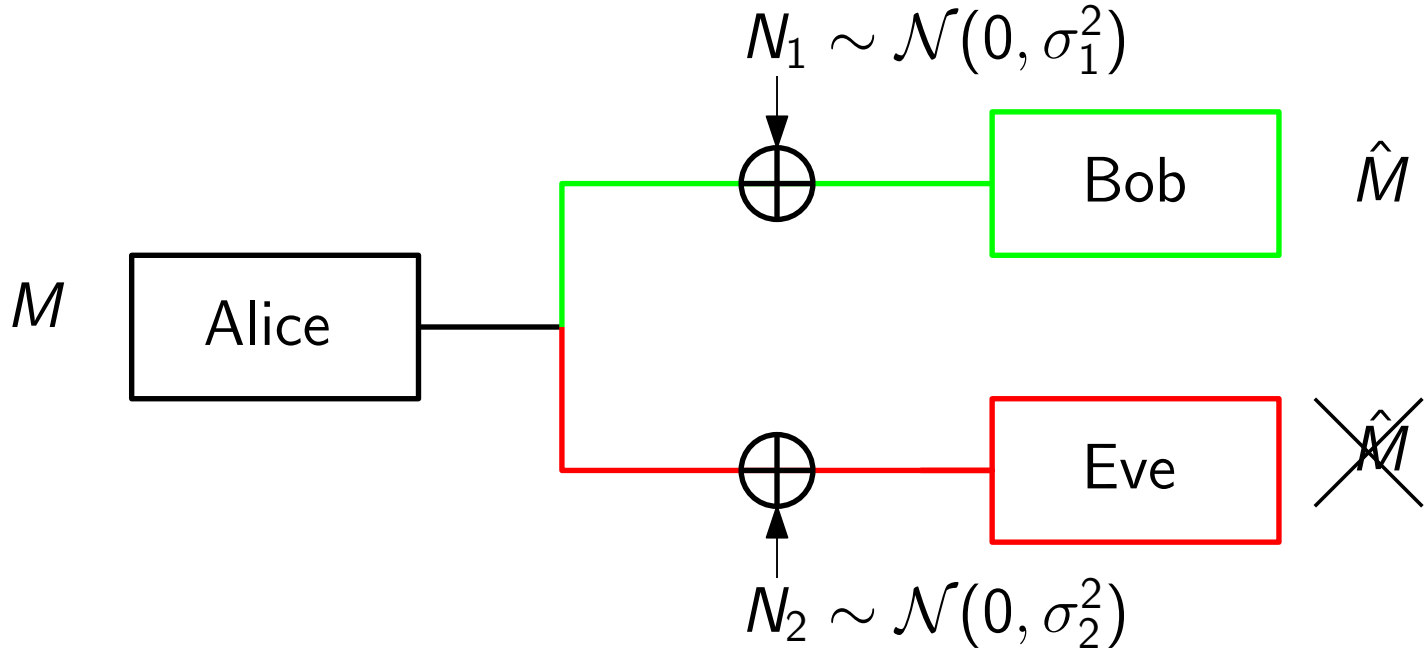
The Gaussian wiretap channel

- Leung-Yan-Cheong and Hellman in 1978.



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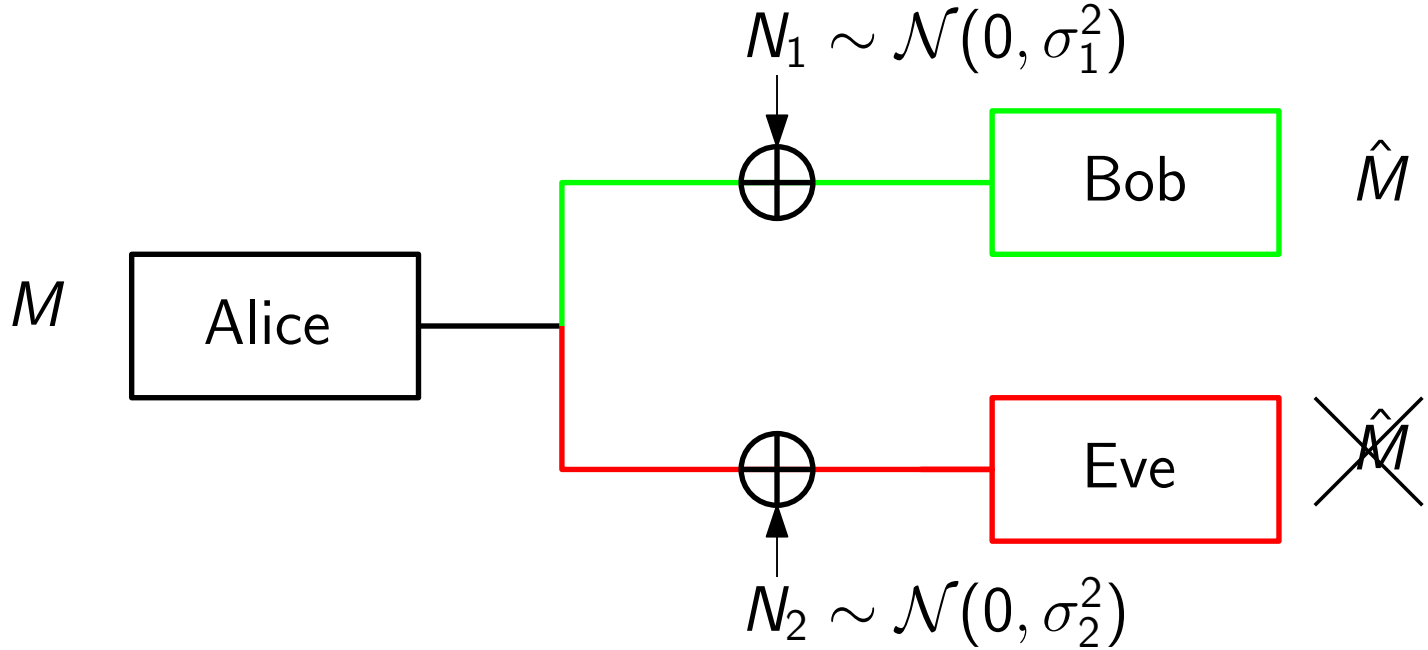
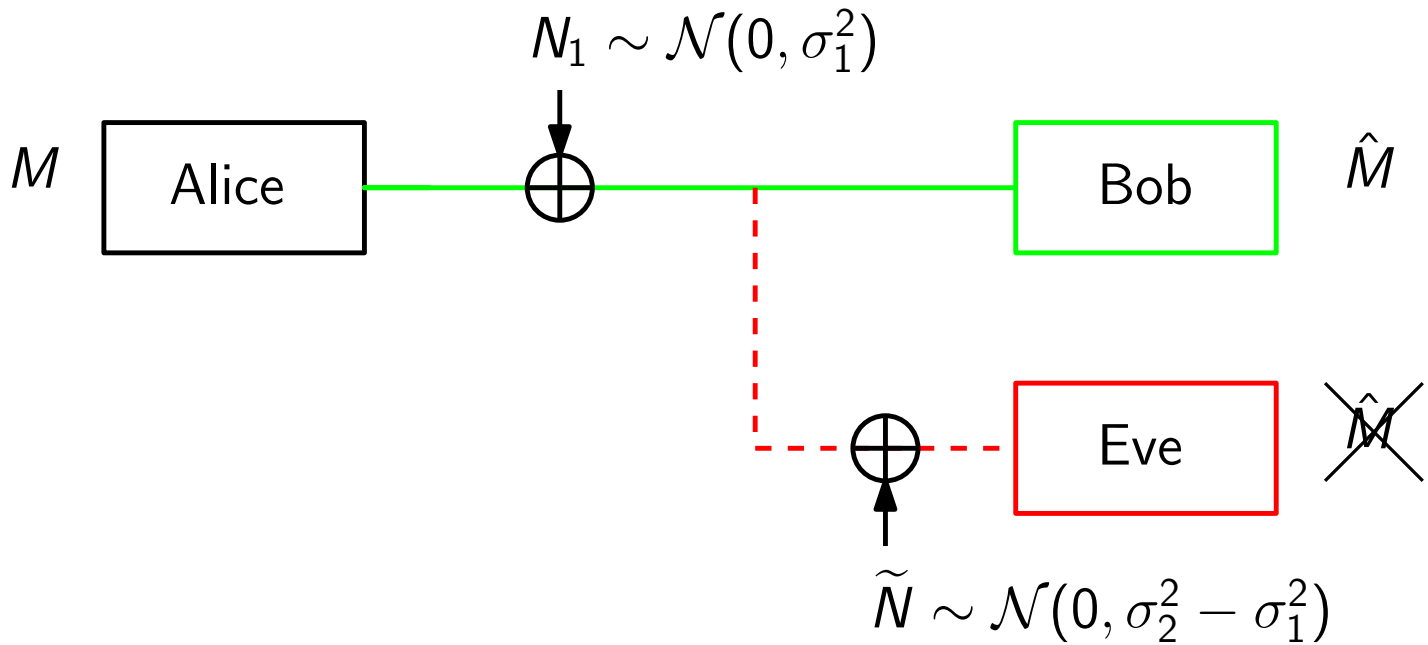


- *Secrecy capacity* is given by,

$$C_s = \frac{1}{2}\log\left(1 + \frac{P}{\sigma_1^2}\right) - \frac{1}{2}\log\left(1 + \frac{P}{\sigma_2^2}\right)$$

The Gaussian wiretap channel (contd.)

- Secrecy capacity depends on $p(x, y)$ and $p(x, z)$, **not** $p(x, y, z)$.



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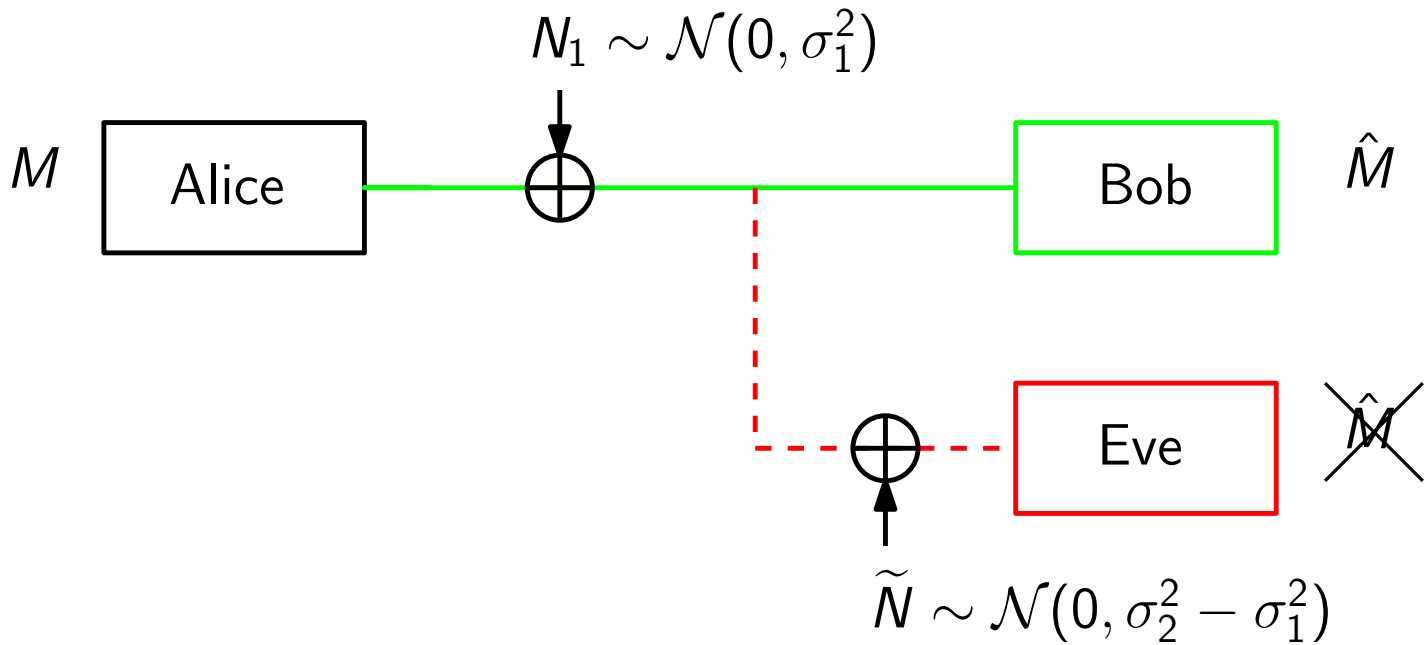

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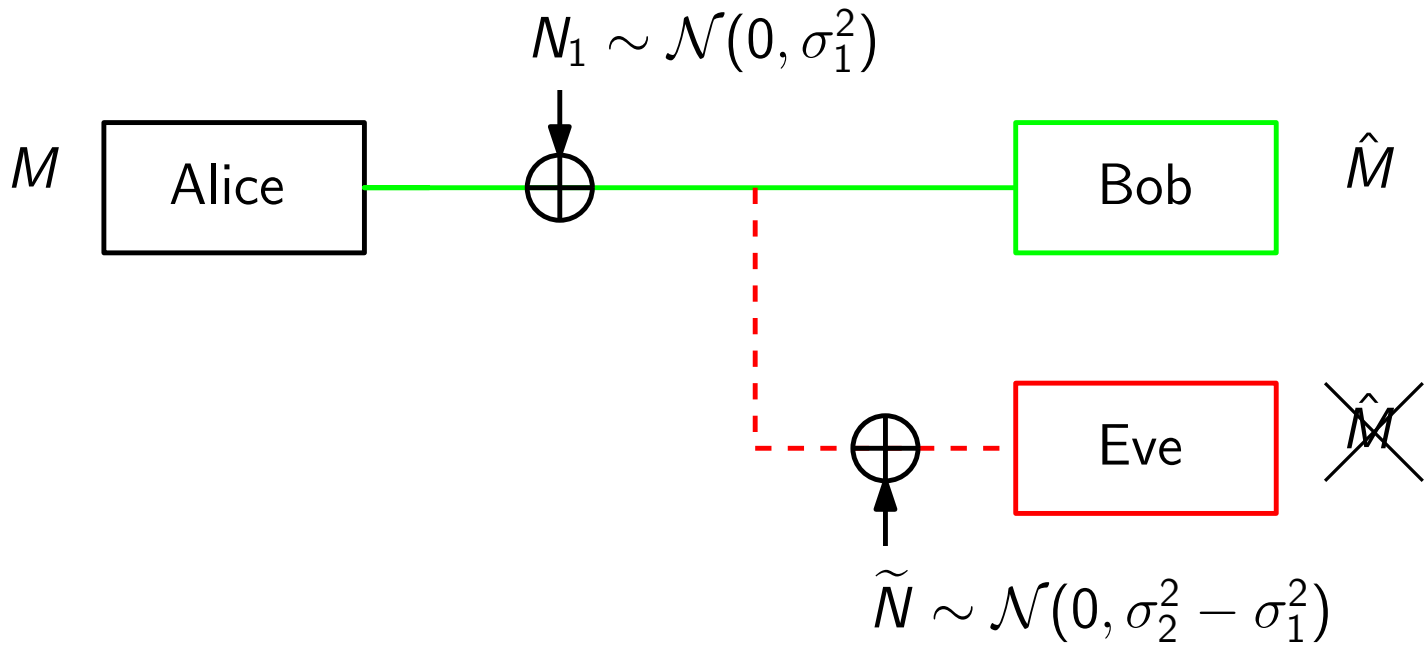
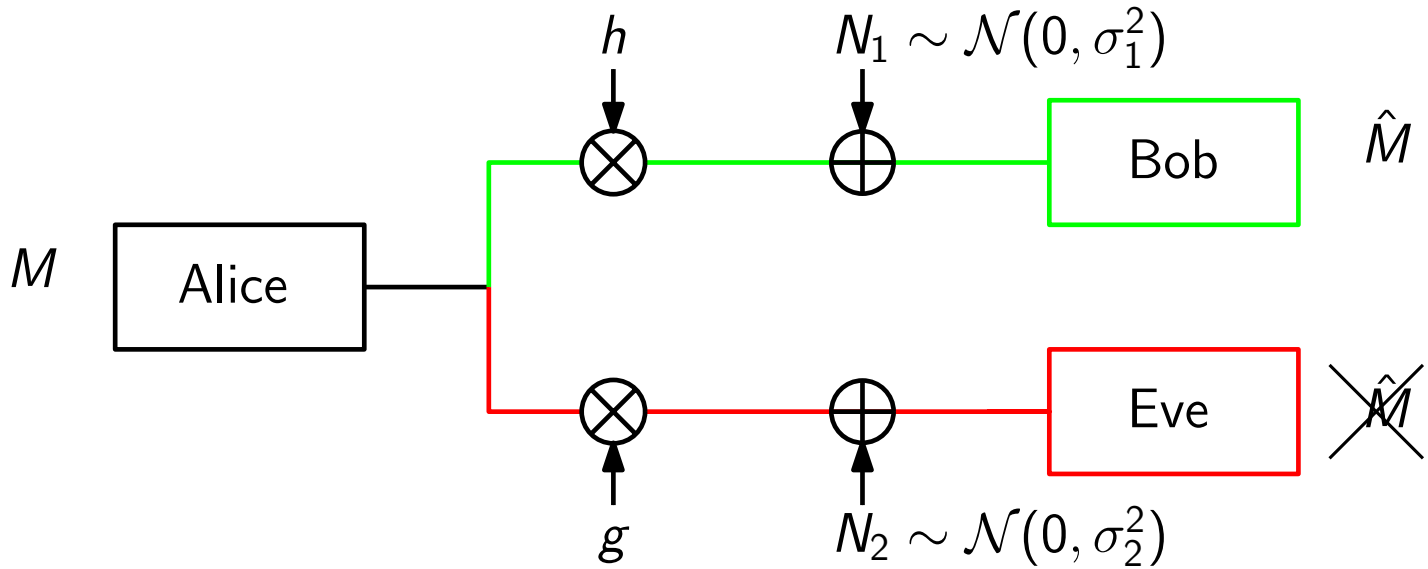


- The degraded version has same secrecy capacity.
- Apply Wyner's result on the degraded wiretap channel.

The wireless fading wiretap channel

- A wireless scenario with Rayleigh fading.



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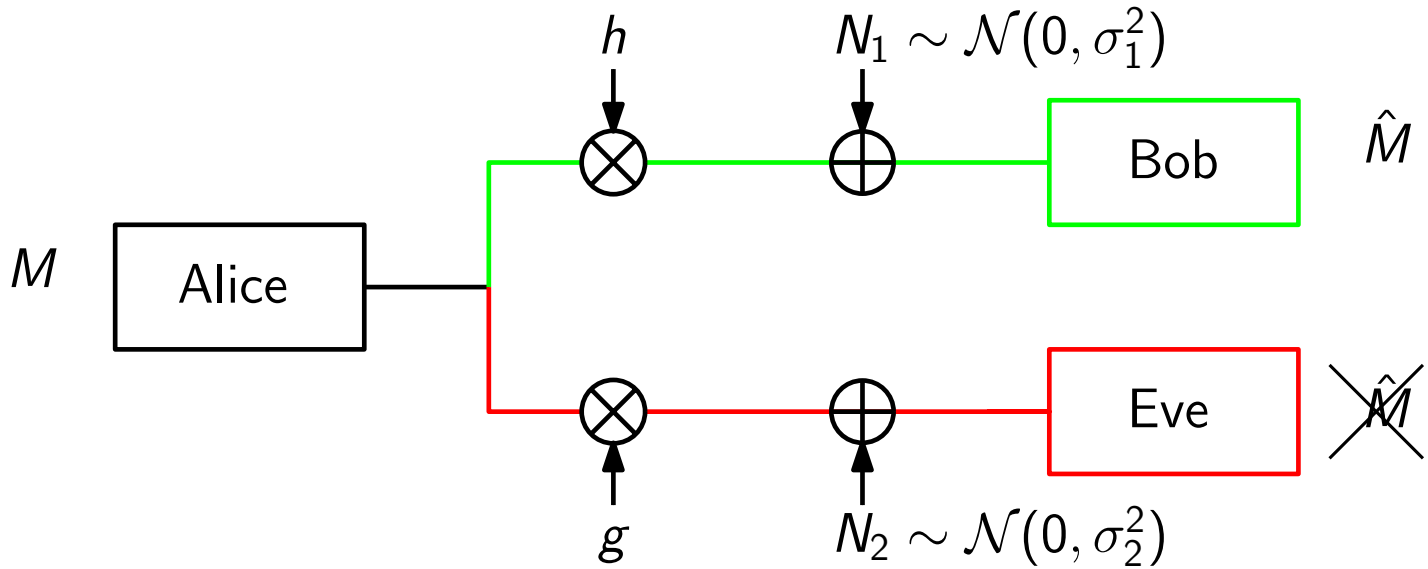

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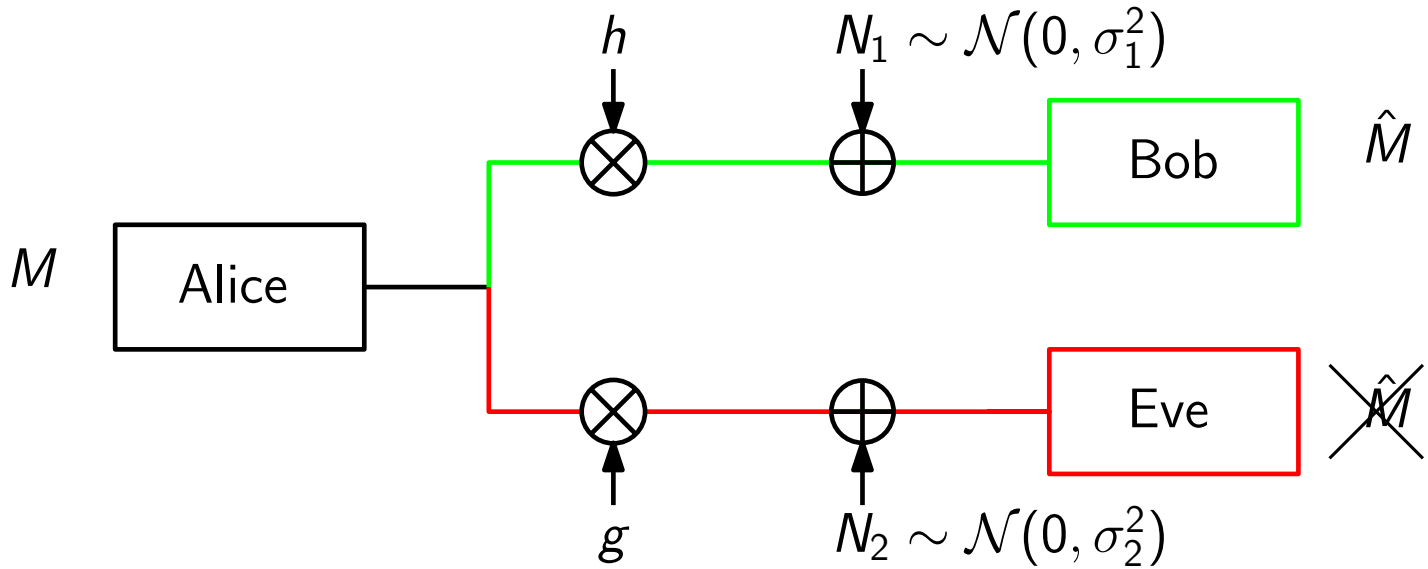
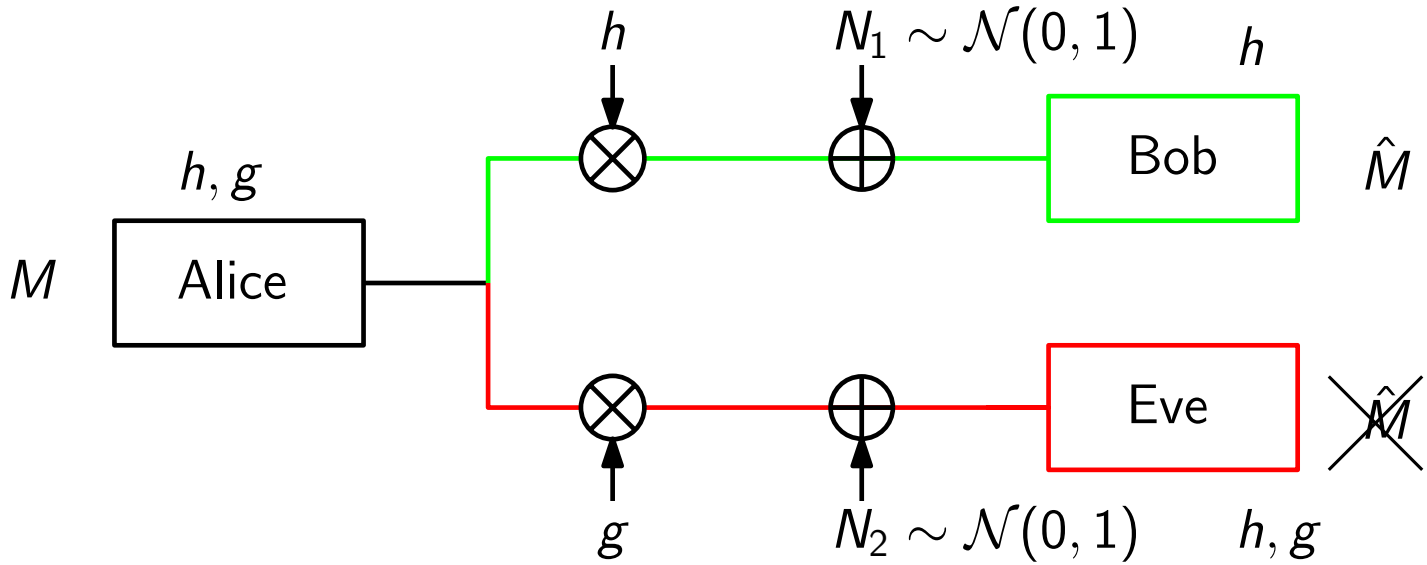


- h, g correspond to Rayleigh fading model.
- Secrecy capacity will depend on availablility of CSI.

Fading wiretap channel with full CSIT, CSIR

- Liang-Poor-Shamai, Li-Yates-Trappe,Gopala-Lai-El Gamal.



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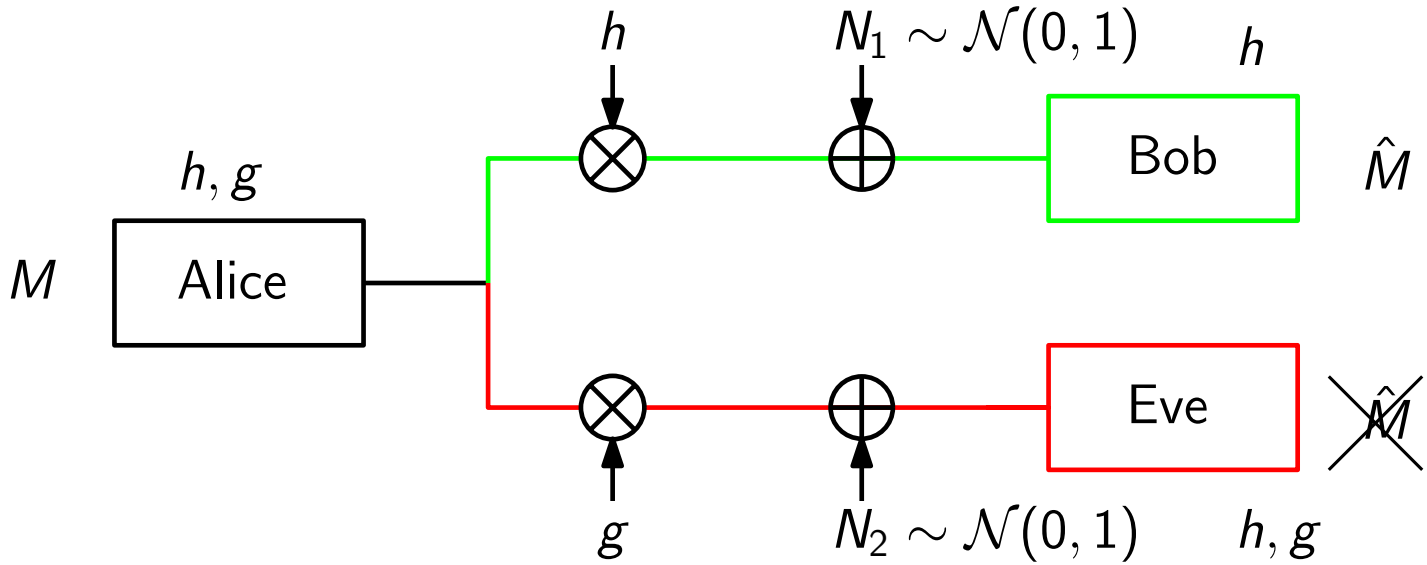

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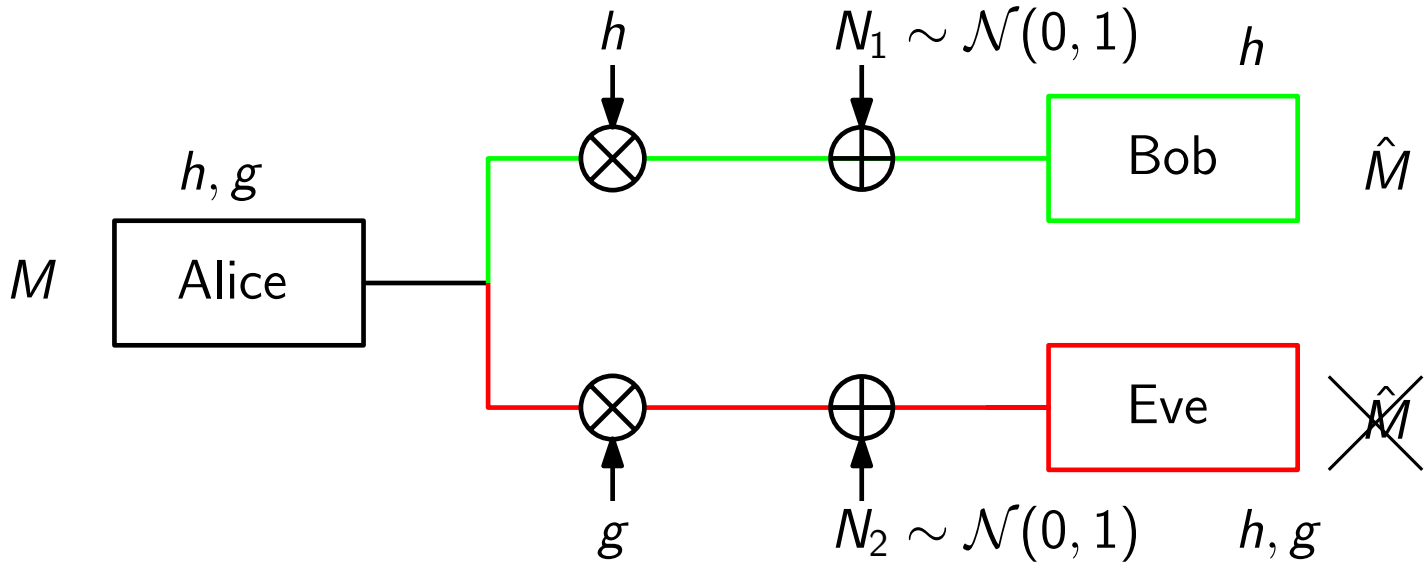

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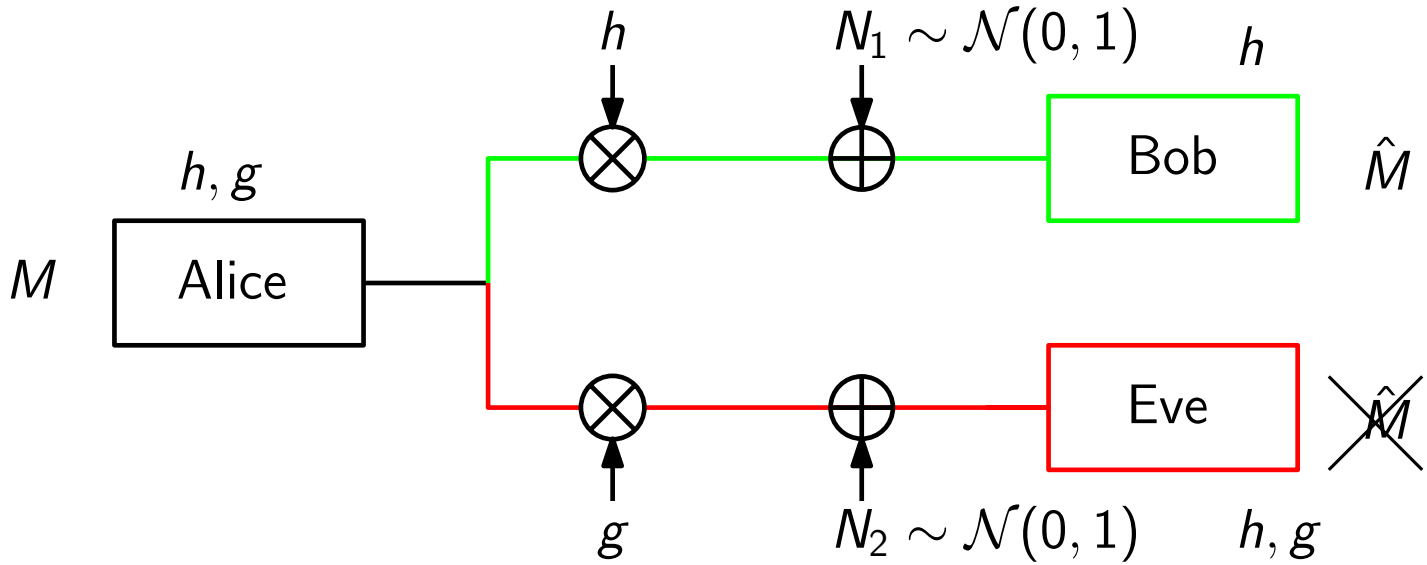
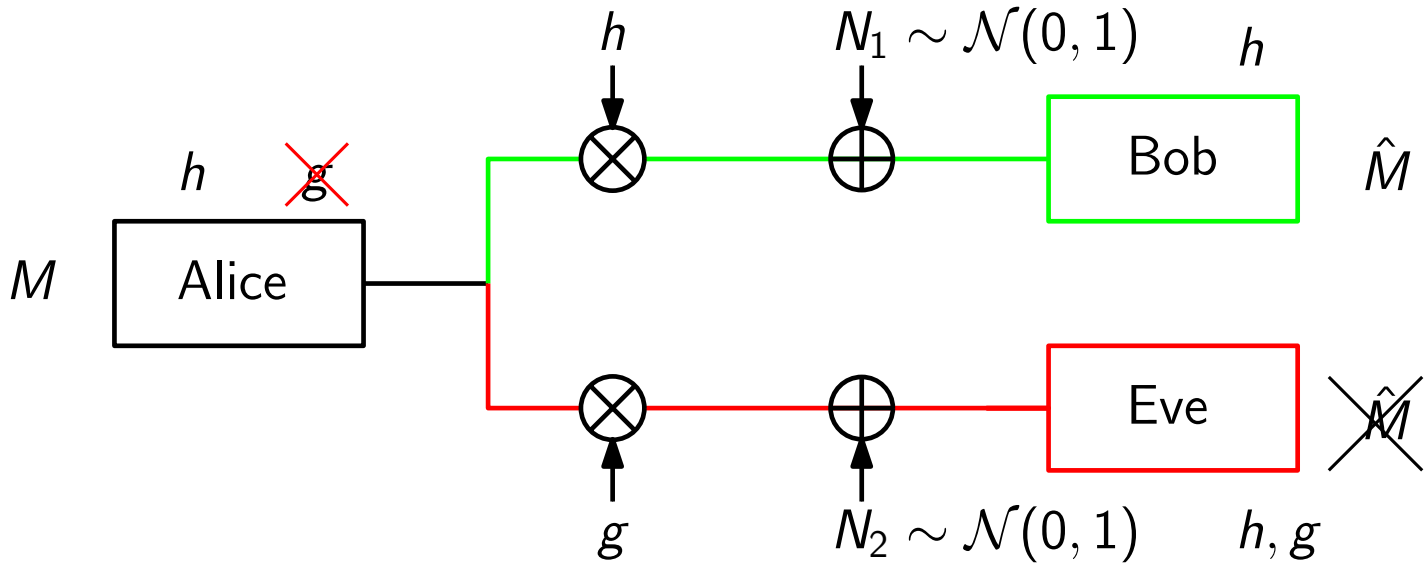


- Equivalent to independent parallel channels.
- Independent Gaussian signaling is optimal.
- Power allocation using water-filling.

Fading wiretap channel with main channel CSIT

- Gopala, Lai, El Gamal, 2008.



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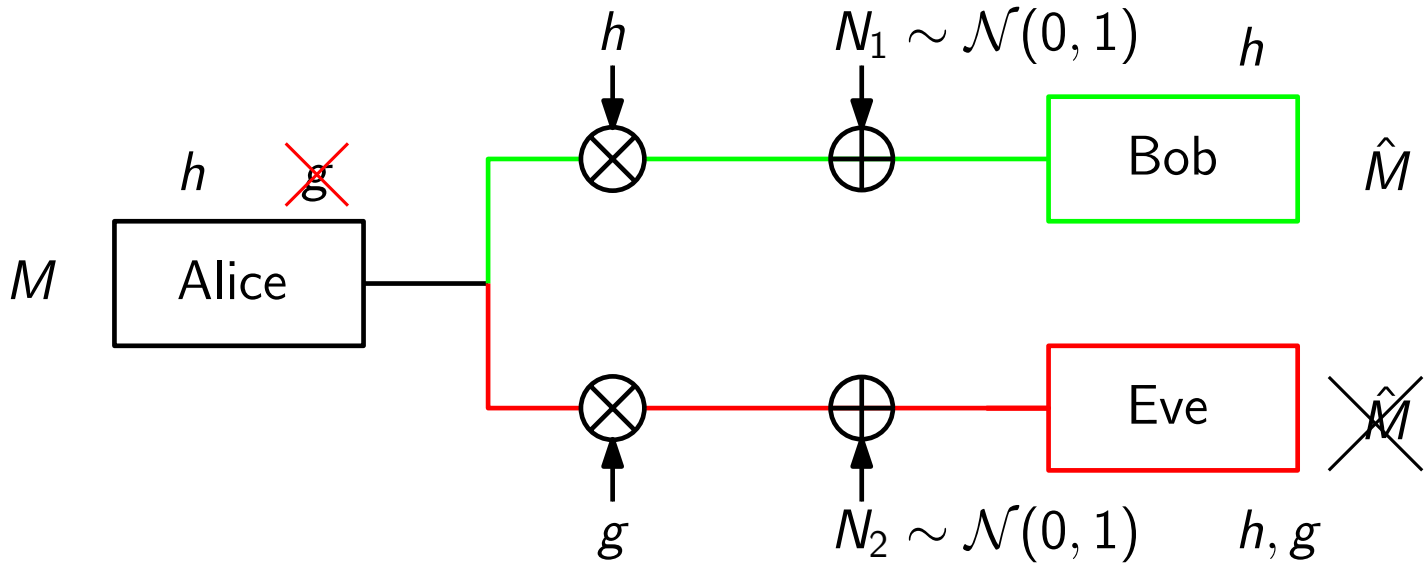

- Codelength $\gg$ Coherence time $\gg 1$

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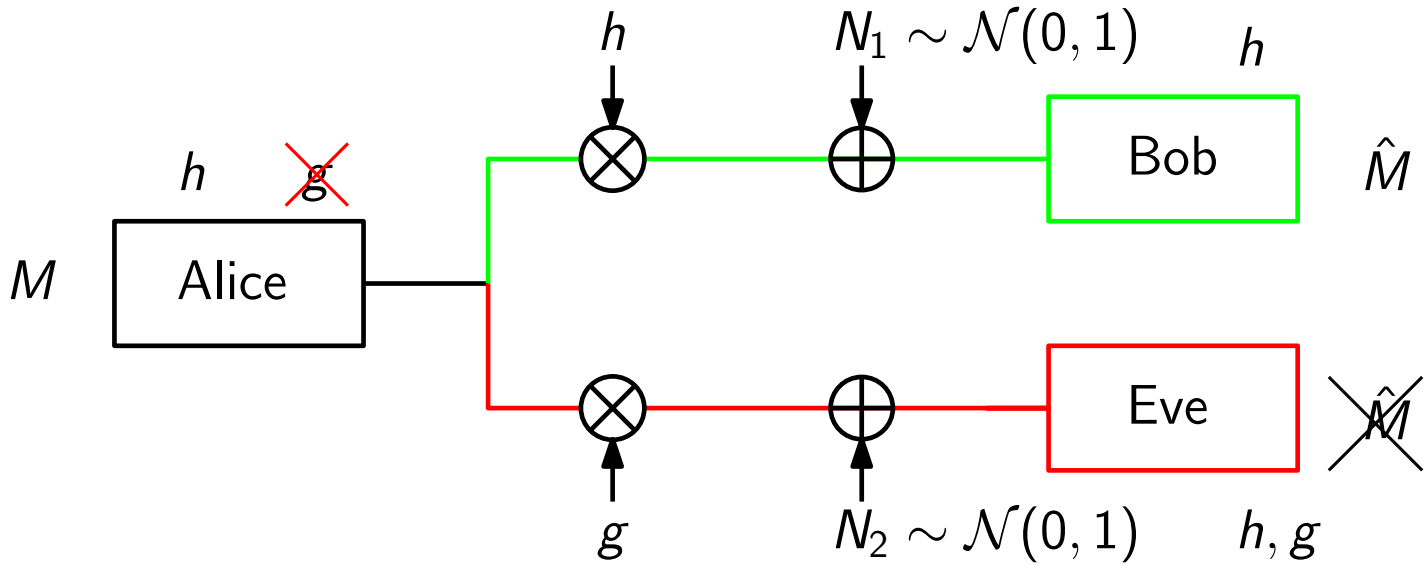

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- **Only** main channel CSI available at Alice.

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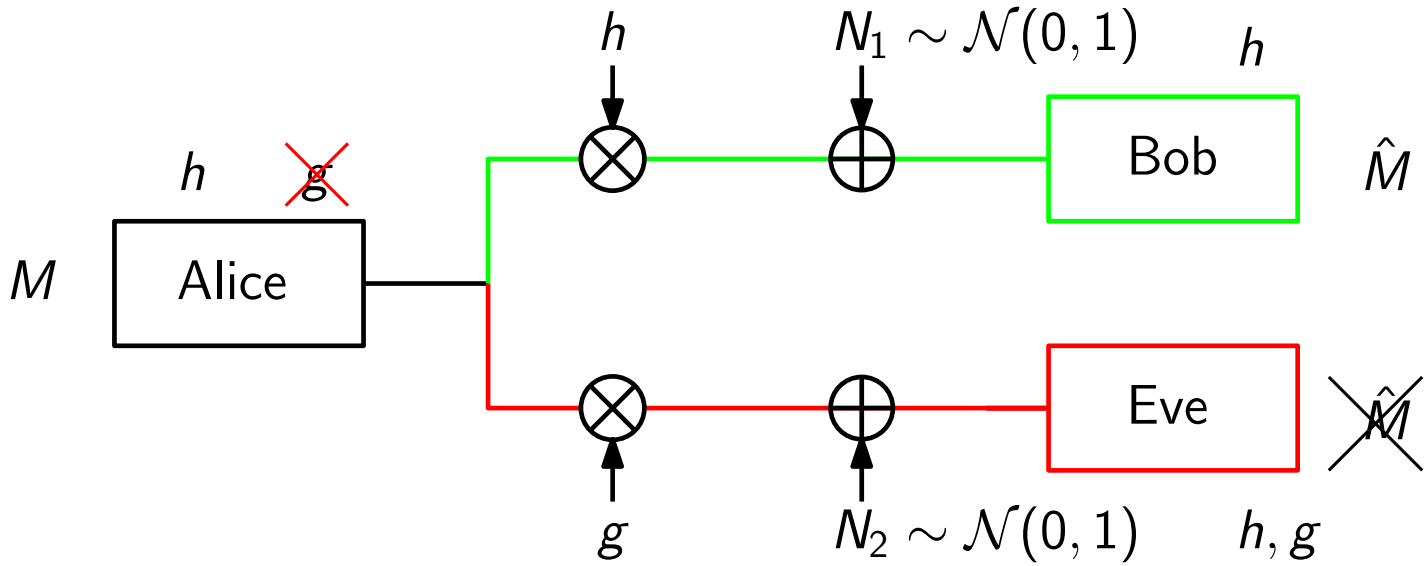
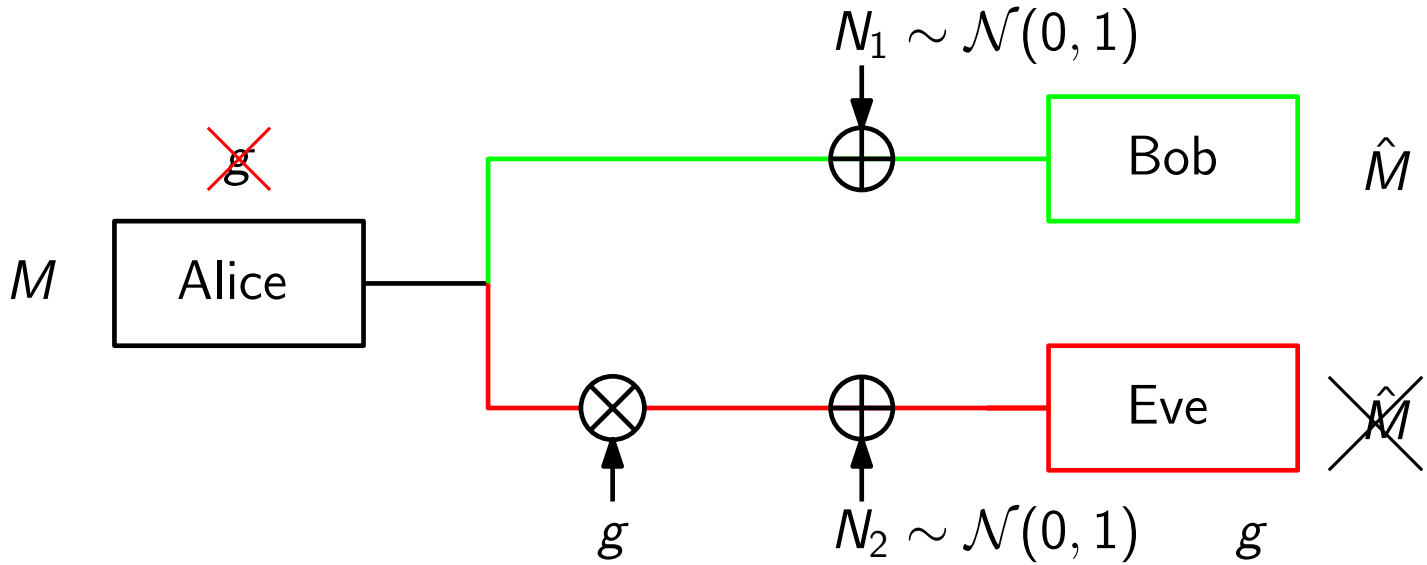


- Codelength $\gg$ Coherence time $\gg 1$
- **Only** main channel CSI available at Alice.
- **Variable-rate** transmission scheme with Gaussian signaling is optimal.

Fading wiretap channel with main channel CSIT

- Li, Yates, Trappe, 2010.



- **Fixed** main channel, **fading** eavesdropper channel.

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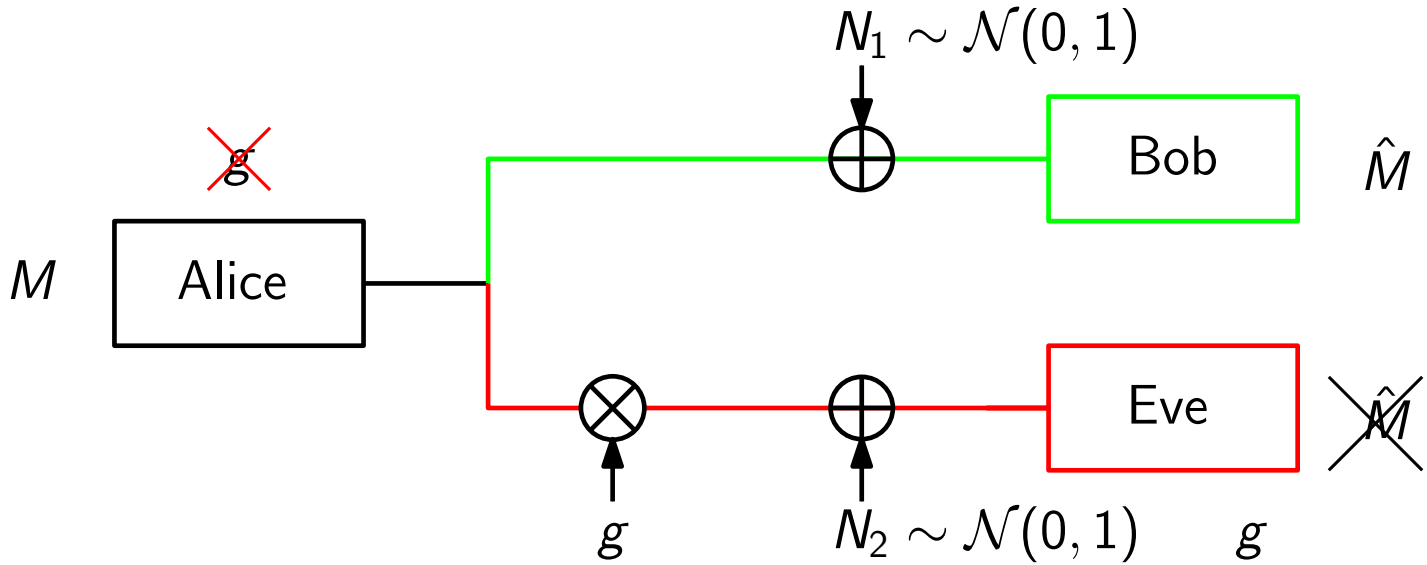

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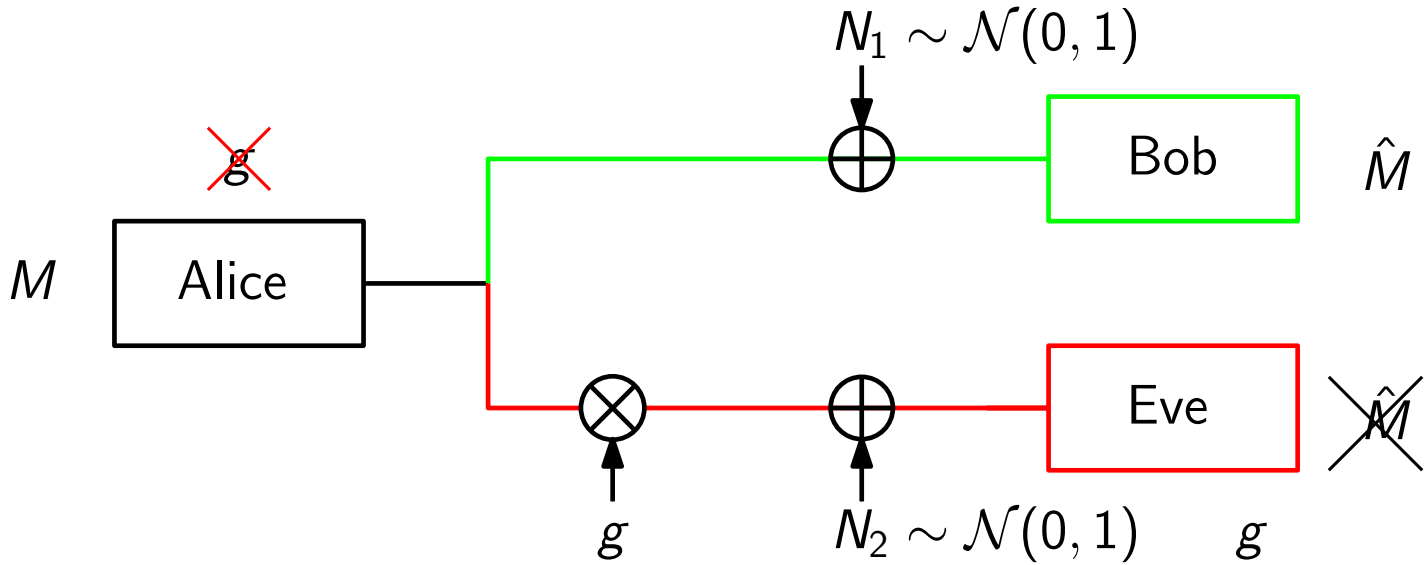

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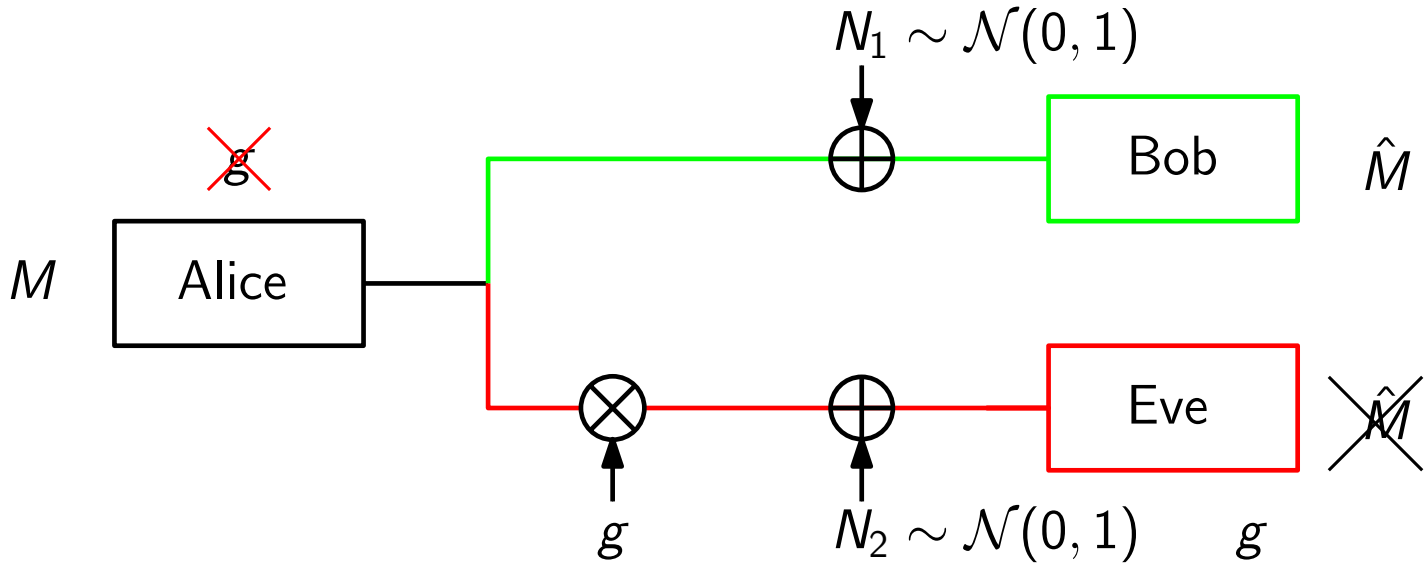


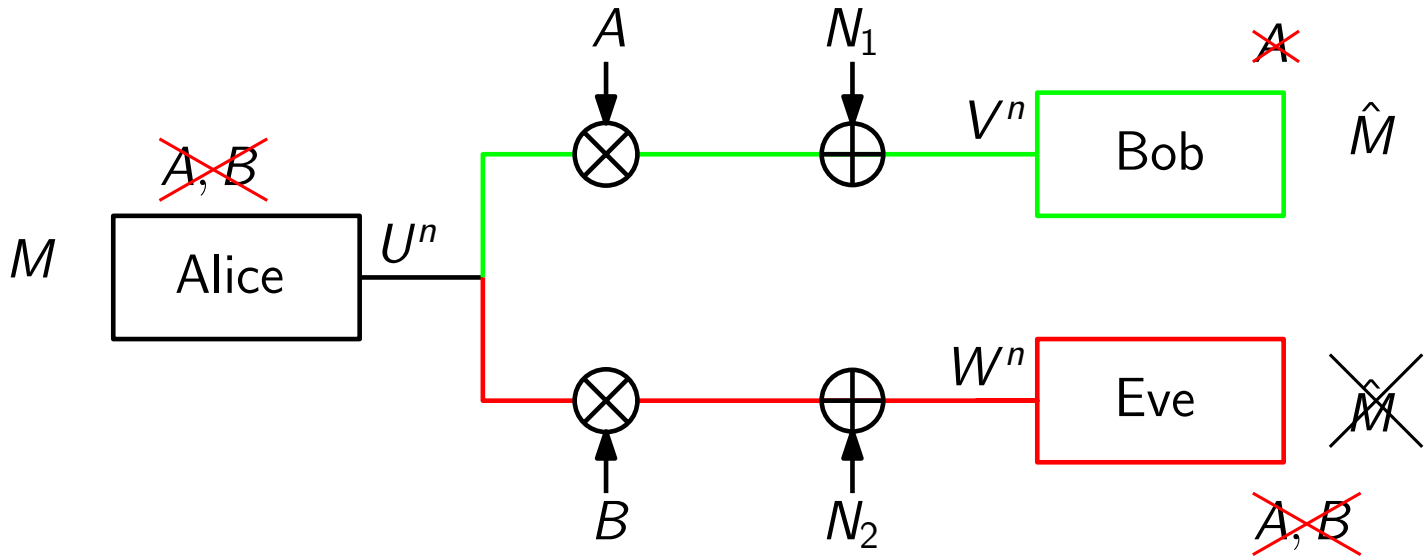
- **Fixed** main channel, **fading** eavesdropper channel.
- Codelength $\gg$ Coherence time $= 1$
- When main channel is better, Gaussian signaling is almost optimal.
- When main channel is worse, M-QAM outperforms Gaussian signaling with artificial noise and bursting.

Fading wiretap channel with no CSI anywhere

- *Very* fast fading scenario: small coherence time.

Fading wiretap channel with no CSI anywhere

- *Very* fast fading scenario: small coherence time.
- Training requires too much overhead.



$$N_1 \sim \mathcal{CN}(0, \frac{1}{2}\sigma_1^2 I) \qquad N_2 \sim \mathcal{CN}(0, \frac{1}{2}\sigma_2^2 I)$$

$$V = AU + N_1$$
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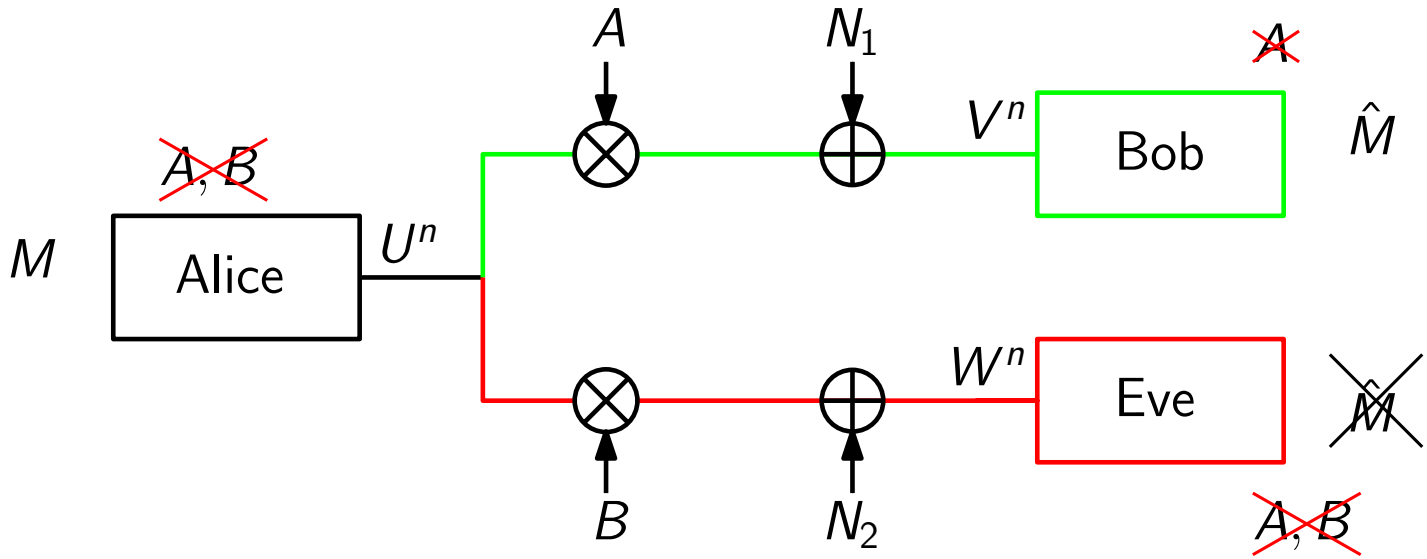

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- Wiretap extension of Abou-Faycal, Trott, Shamai (2001).

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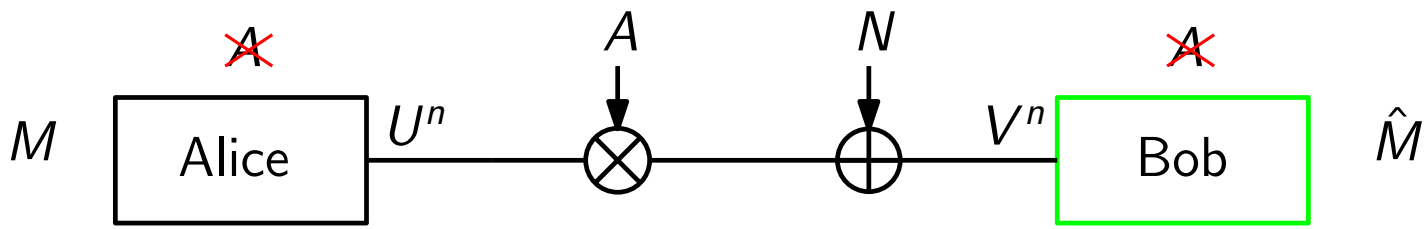
- Studied by Abou-Faycal, Trott, Shamai (2001).



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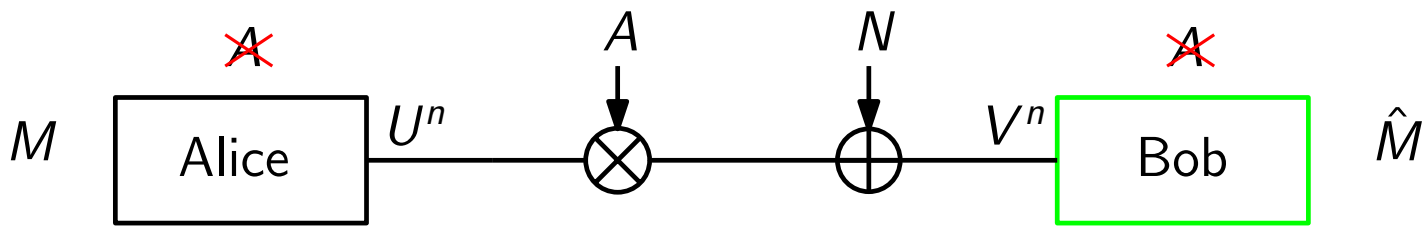

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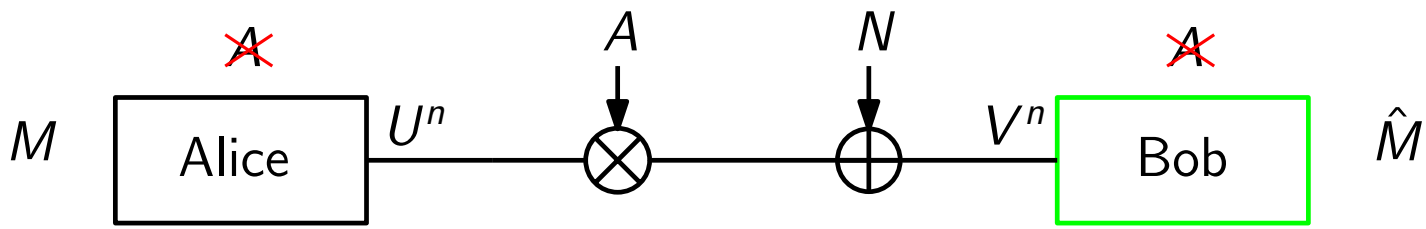

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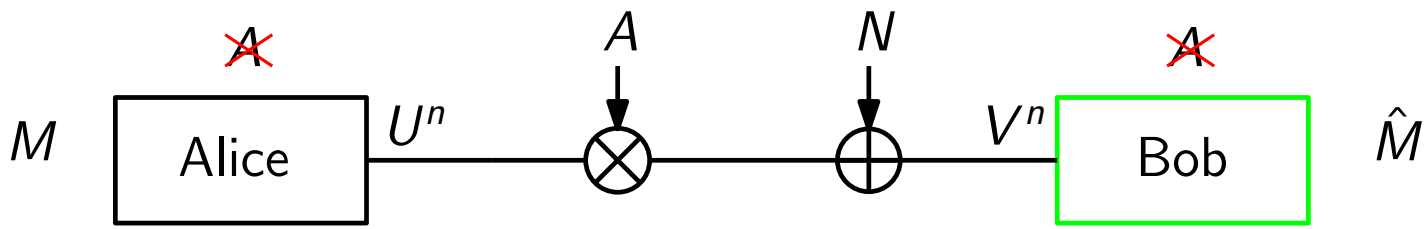

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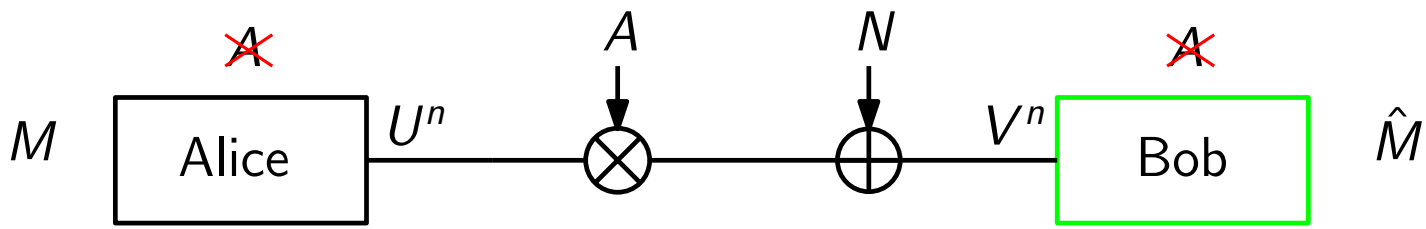

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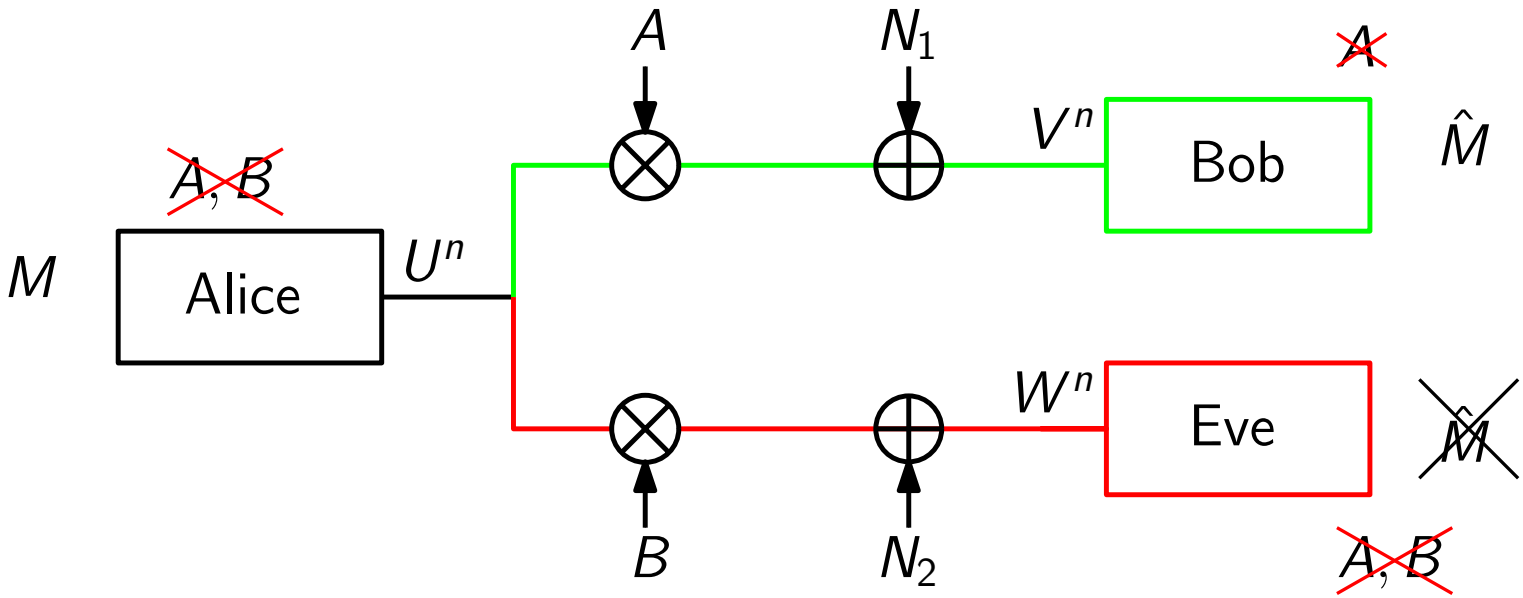
- The phase does not carry any information.
- The optimal input distribution is discrete with a finite number of mass points.

Equivalent degraded wiretap channel

- The secrecy capacity depends only on the marginal distributions $p(u, v)$ and $p(u, w)$, **not** $p(u, v, w)$.



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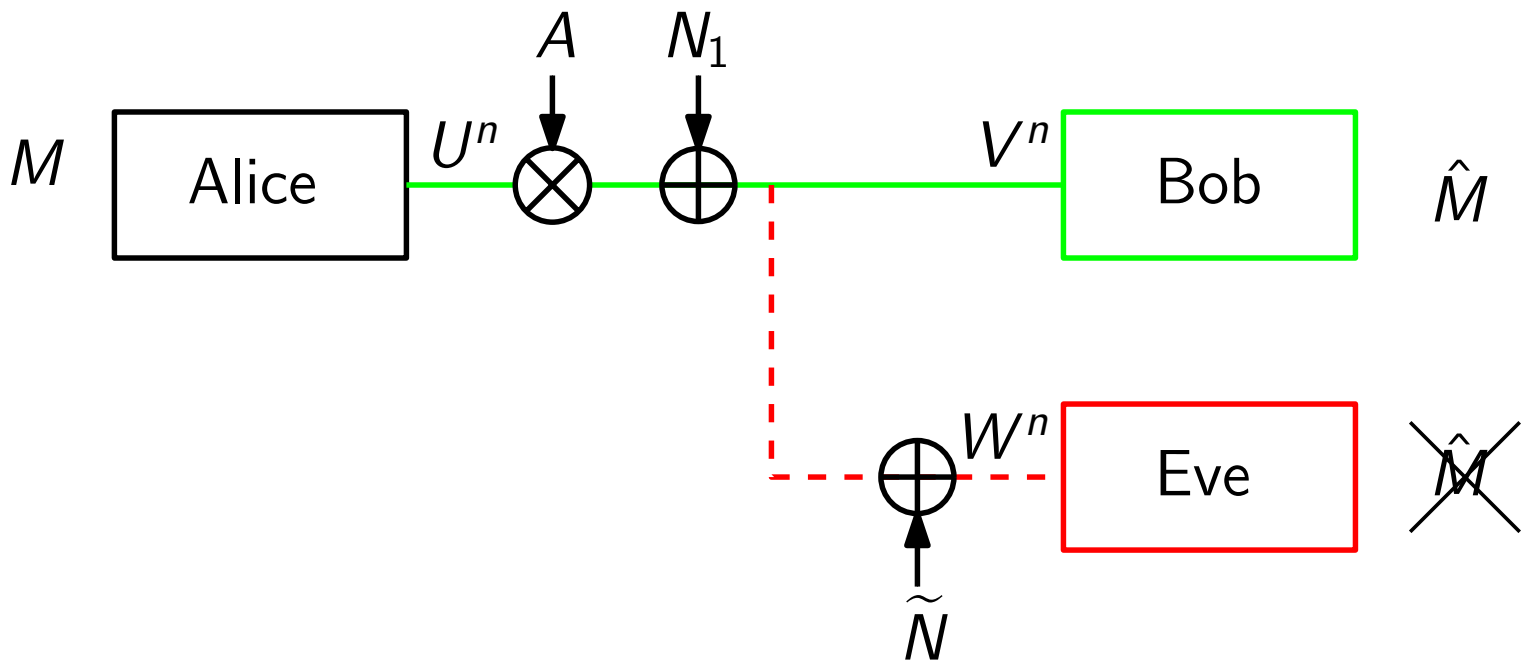

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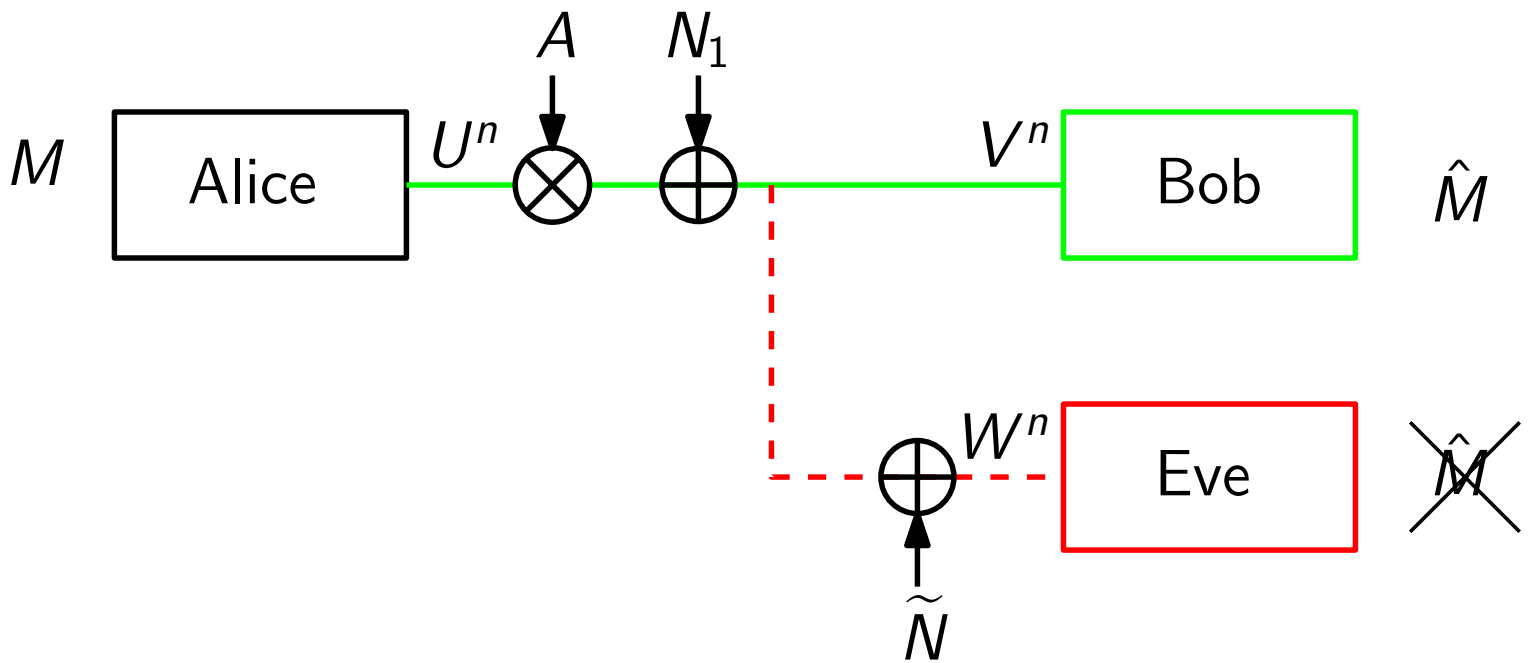

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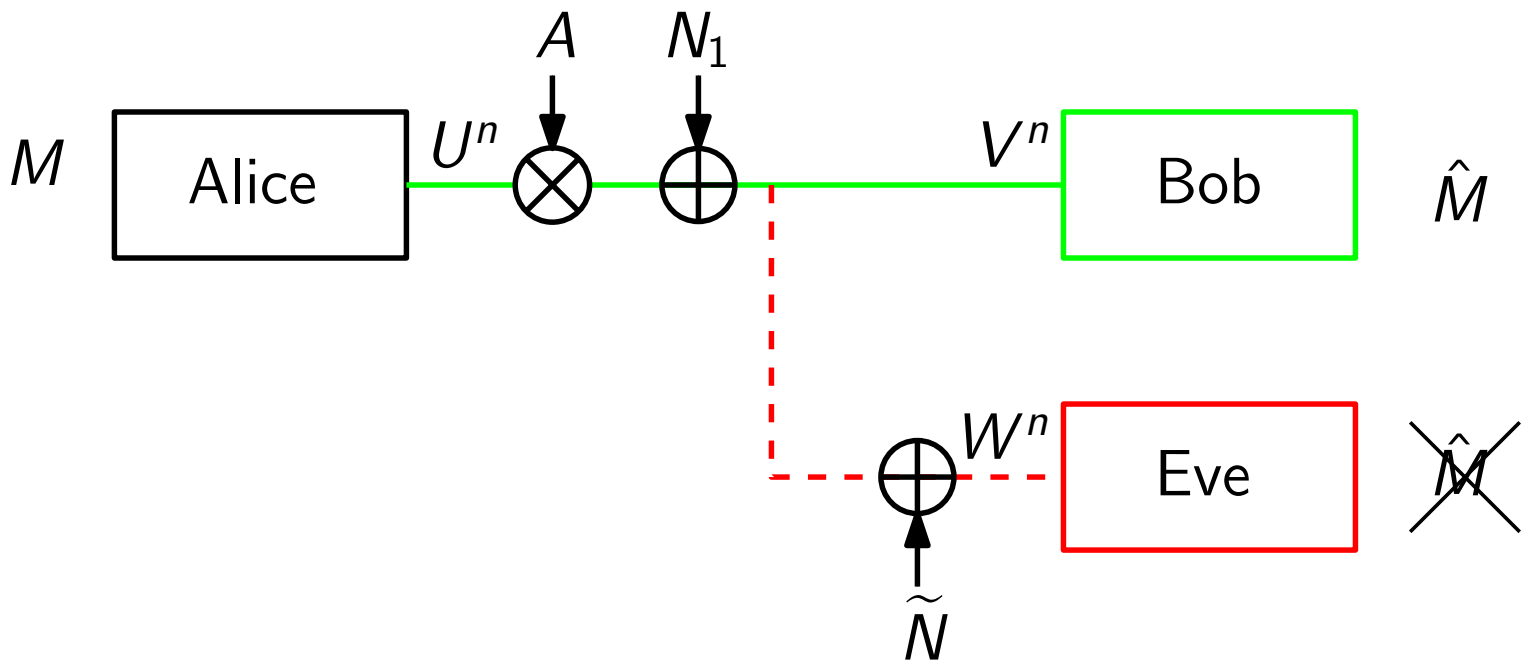

  - Equivalent with respect to secrecy capacity.
  - Apply Wyner's result.

The optimization problem

- Wyner's result yields

$$C_s = \sup_{F \in \mathcal{F}} I(U;V) - I(U;W)$$

where F : distribution of U and $\mathcal{F} = \left\{ F : \mathbb{E}_F \left[|U|^2 \right] \leq P \right\}$

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- Phase carries **no** information, so, if $X = |U|$, $Y = |V|^2$, $Z = |W|^2$,

$$I(U;V) = I(X;Y) \text{ and } I(U;W) = I(X;Z)$$

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- Prove $X \to Y \to Z$.

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- Prove $X \to Y \to Z$.
- Generalize van Dijk: convex optimization problem.

The optimization problem (contd.)

- Functional optimization problem: Smith (1971).

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- Abou-Faycal, Trott, Shamai: the supremum is achieved.

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with equality iff $x \in \operatorname{supp} X^*$, and,

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- Analyse KKT conditions to get the results.

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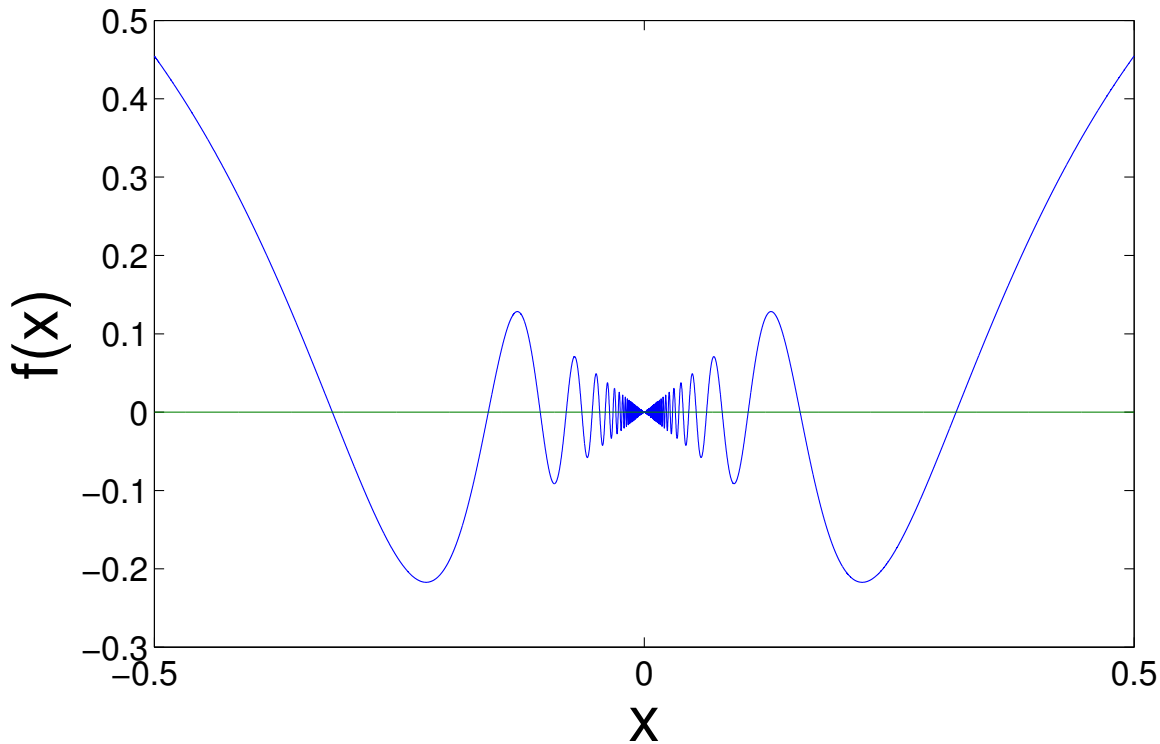

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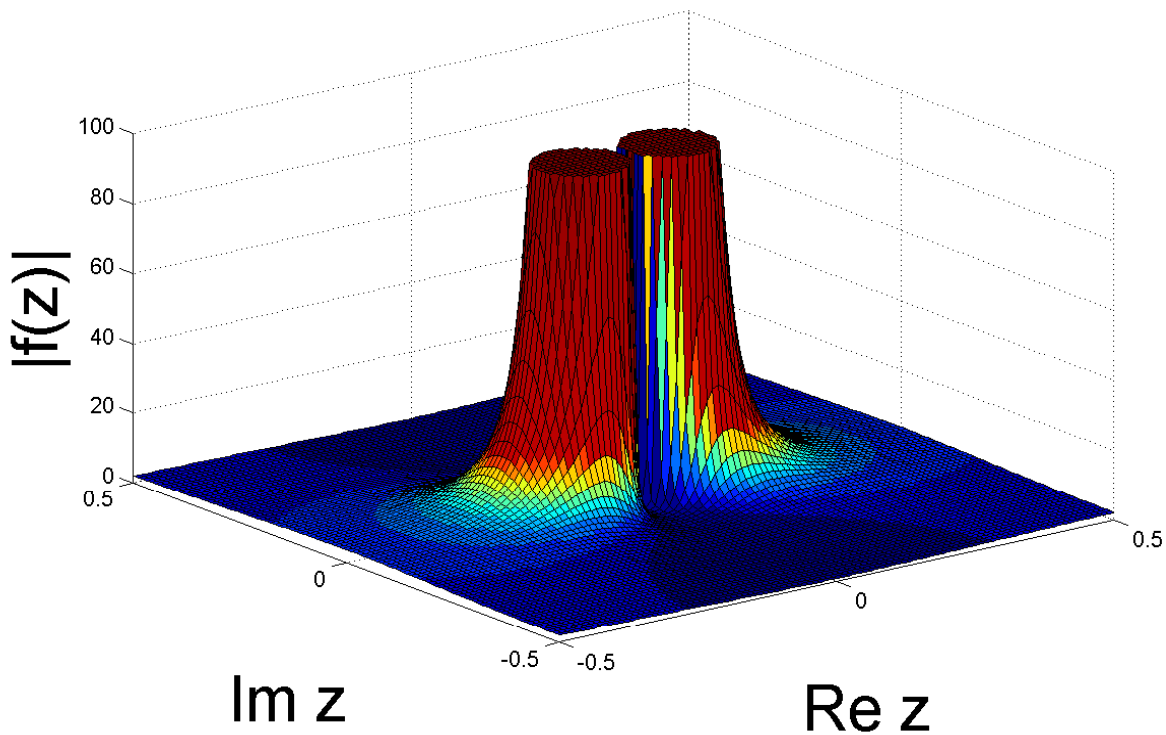

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- Simplify and use Laplace transform along with other bounds on $p(y)$ to yield the contradiction.

More results

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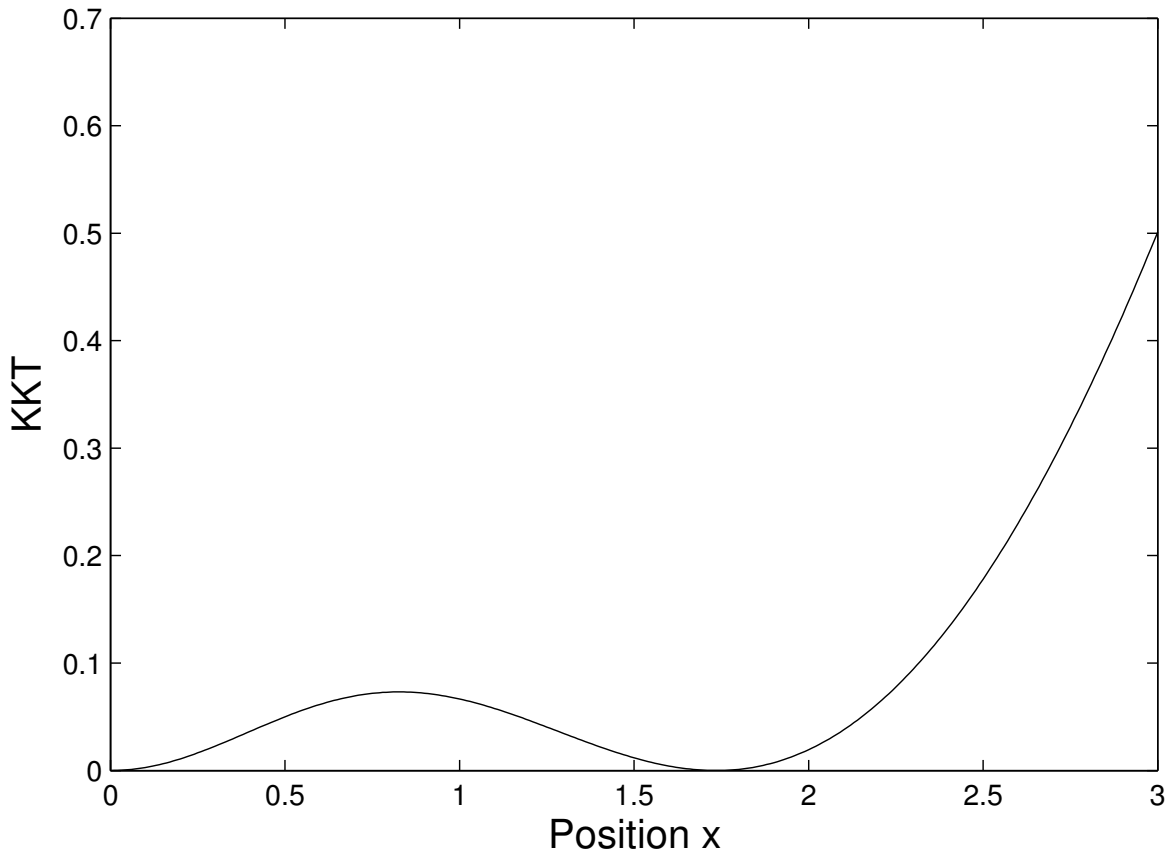
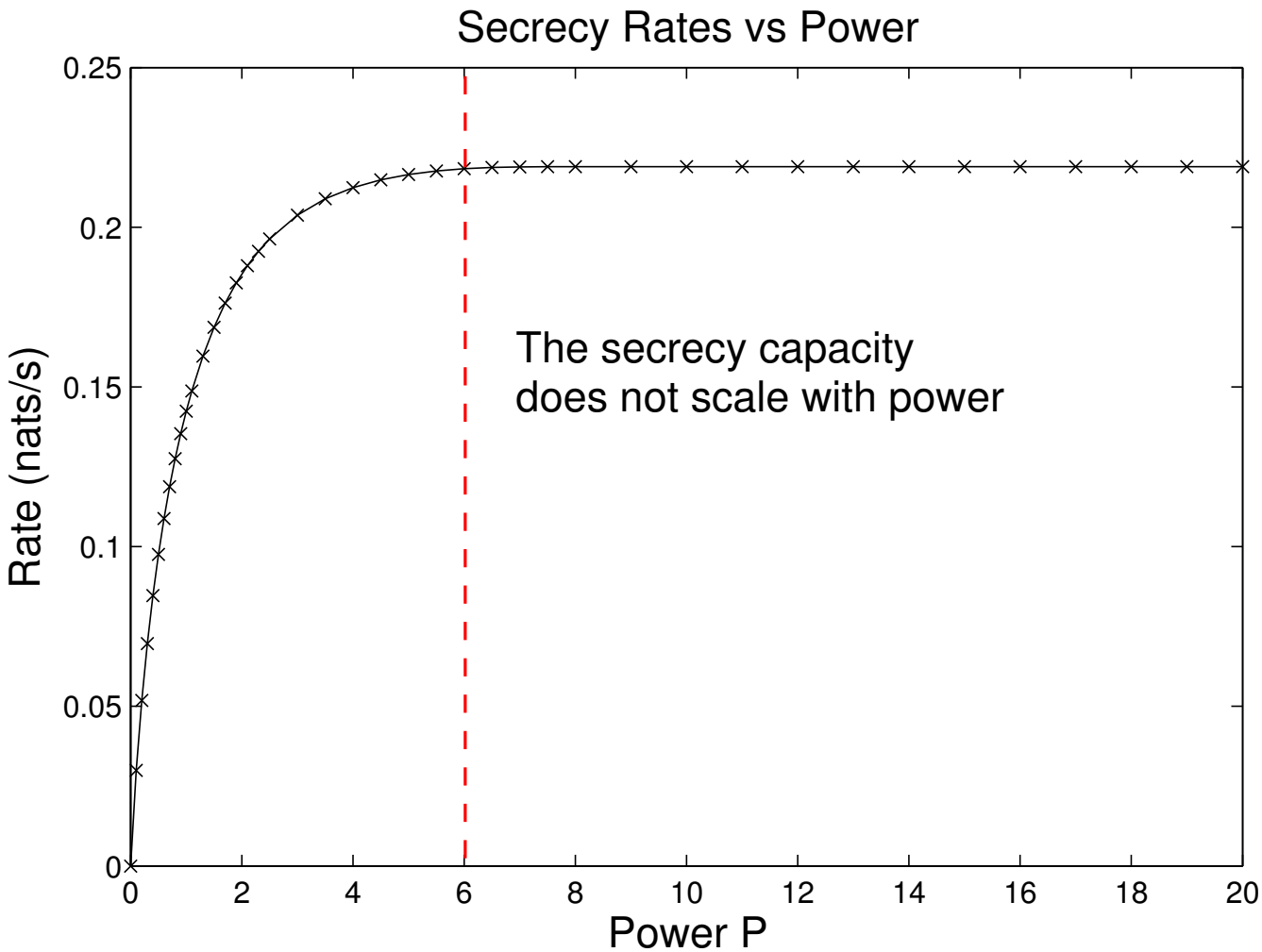
- Proof by contradiction.
- Observe continuous differentiability of $f(x)$.
- Analyse $f'(x)$ and use other bounds on $p(y)$ to reach a contradiction.

Numerical results



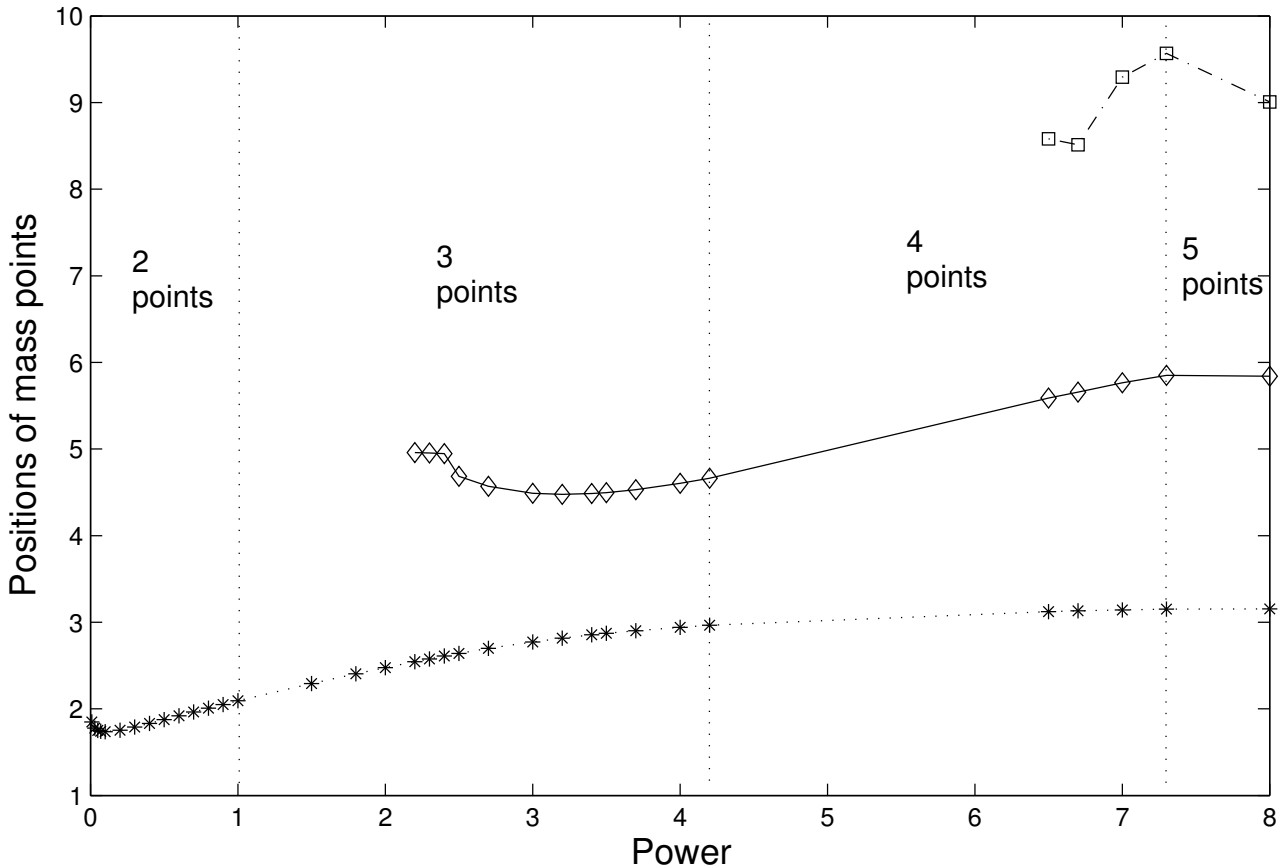
Satisfying the KKT conditions with $P = 0.1$, $\sigma_h = \sigma_1 = 1$, $\sigma_2 = 2$, $\gamma = 0.2461$, $C_s = 0.03$ and $F(x) = 0.9668\delta(x) + 0.0332\delta(x - 1.7348)$.

Numerical results



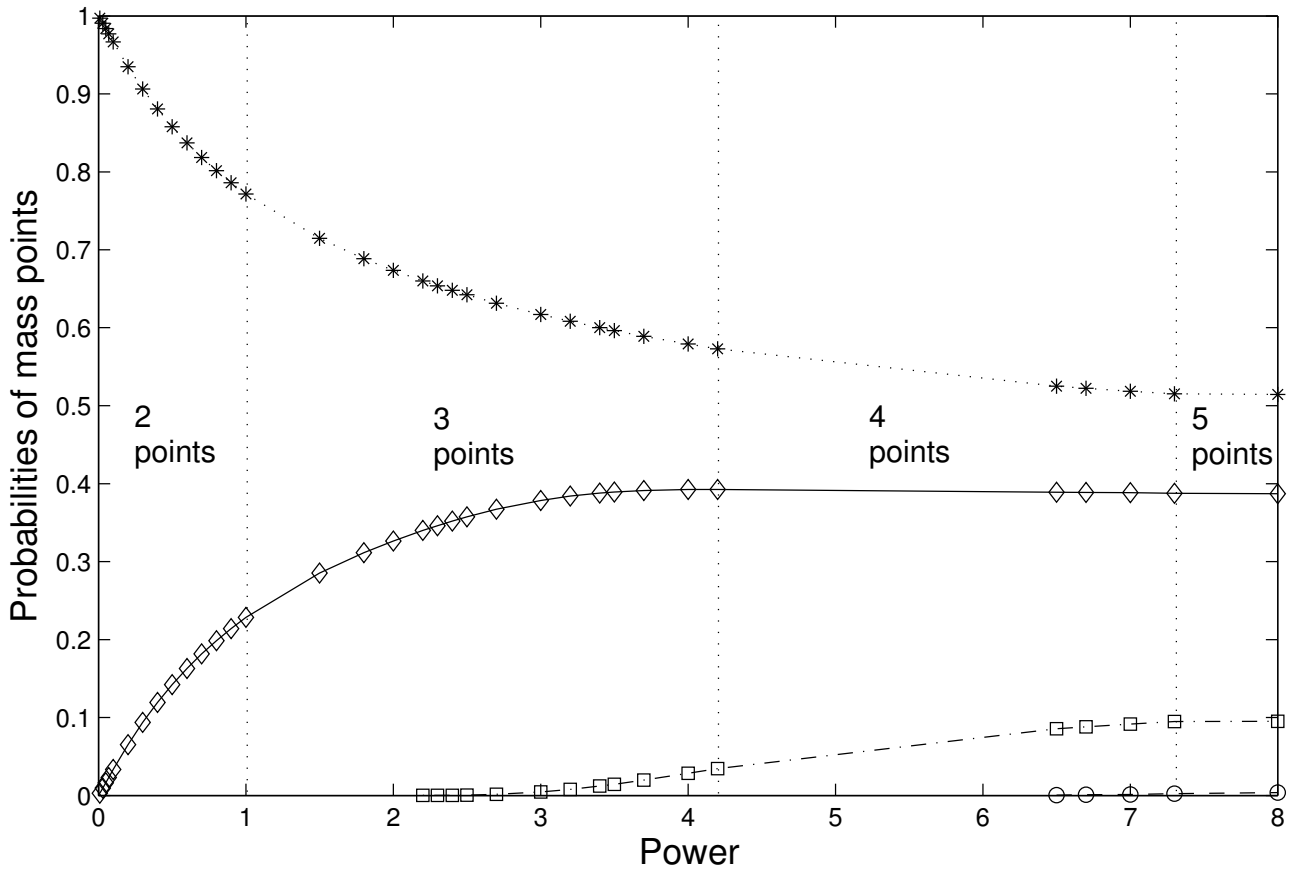
Secrecy capacity versus power.

Numerical results



The position of the mass points versus power.

Numerical results



The probabilities of the mass points versus power.

Conclusions

- Studied the Rayleigh fast fading wiretap channel with coherence time 1 and no CSI anywhere.

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- Proved that the optimal input distribution is discrete with finite number of mass points.