Friday, July 9, 2021 9:12 AM

Should be this. $\exists$ Typo in the lecture.

Claim $DP_{[g]}(m)(\alpha) = \Pi(m(\alpha))$ $\forall m \in T(\mathrm{Hom}(\Lambda^-,\Lambda^+))$, $\alpha \in \mathcal{H}^-$

where $\Pi : \Omega^2 \to \mathcal{H}^2$ is the L^2-proj.

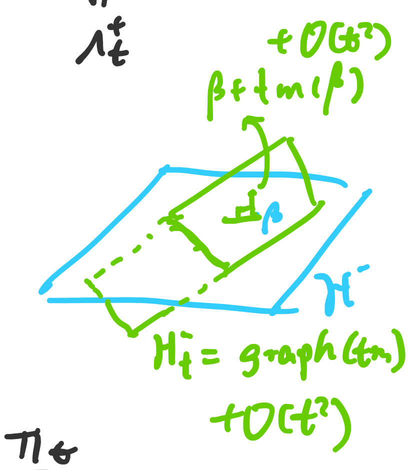
Pf of Claim $P : \mathcal{C} \to U \cong \mathrm{Hom}(\mathcal{H}^-,\mathcal{H}^+)$ — as subspaces of $H^2(X;\mathbb{R})$ via $\mathcal{H}^2 \cong H^2$

$\mathrm{graph}(\varphi) \mapsfrom \varphi$

Let $[g_t] \in \mathcal{C}$ w/ $[g_0] = [g]$, $\frac{d}{dt}\big|_{t=0}[g_t] = m$.

Let $*_t$, $(\ )^{+_t}$, $\mathcal{H}^2_t$, ... be those wrt. $[g_t]$.

Then $\forall \beta \in \Lambda^-$ we have $\beta = \underbrace{(\beta + tm(\beta) + O(t^2))}_{\in \Lambda_t^-} + \underbrace{(-tm(\beta) + O(t^2))}_{\in \Lambda_t^+}$



$\Rightarrow *_t\beta = -\beta - 2tm(\beta) + O(t^2)$

$\Rightarrow \frac{d}{dt}\big|_{t=0} *_t = -2m$ on Λ^-.

Denote $P([g_t])(\alpha) = x \in \mathcal{H}^+$.

Should have $\alpha + x \in \mathcal{H}_t^-$. $(*)$

Careful: $\mathcal{H}$, $\mathcal{H}_t$ are identified via $\mathcal{H} \underset{\Pi}{\overset{\Pi_t}{\rightleftarrows}} H^2 \cong \mathcal{H}_t$. Same Hodge theory

Therefore $(*)$ is really $\Pi_t(\alpha + x) \in \mathcal{H}_t^-$.

$\Rightarrow \Pi_t x = -(\Pi_t\alpha)^{+_t} \Rightarrow x = -\Pi(\Pi_t\alpha)^{+_t}$.

$\Rightarrow DP_{[g]}(m)(\alpha) = \frac{d}{dt}\big|_{t=0}\left(-\Pi(\Pi_t\alpha)^{+_t}\right)$

$= -\frac{1}{2}\frac{d}{dt}\big|_{t=0}\Pi(1 + *_t)(\Pi_t\alpha)$

$= -\frac{1}{2}\frac{d}{dt}\big|_{t=0}(\alpha + \Pi *_t \Pi_t\alpha)$

$= -\frac{1}{2}\Pi\left(\frac{d}{dt}\big|_{t=0} *_t\right)\alpha - \frac{1}{2}\Pi * \frac{d}{dt}\big|_{t=0}\Pi_t\alpha$

$= \Pi(m(\alpha)) - \frac{1}{2}\Pi * d\beta$ $\quad \Pi_t\alpha \in \alpha + d\Omega^1$

$= \Pi(m(\alpha))$. $\quad *d\beta = \pm d^*(*\beta) \in \ker \Pi$ $\square$