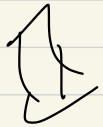
Other references

- Freed - Uhlenbeck Instantons and Four Manifolds
- Lawson

Gauge Fixing

Parallel between gauge theory

in 3+4 dimensions



symplectic geometry.

Classification Results

Classify : 3 & 4 dim'l manifold and understand their sub manifold

Classify : symplectic manifold and their lagrangia sub manifold (isotropic or coisotropic)

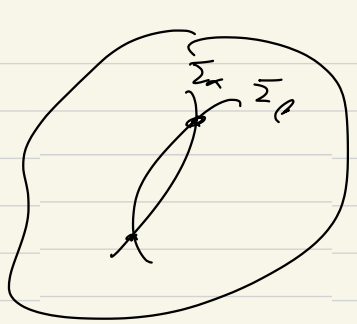
(M^{2n}, ω)

Understand $\pi_1(M^4) = \{1\}$

$\Rightarrow \pi_2(M) = H_2(M)$

$Q : H_2(M) \times H_2(M) \to \mathbb{Z}$

$$Q(\alpha, \beta) = \sum_{x \in \Sigma_\alpha \cap \Sigma_\beta} \varepsilon(x)$$



Suppose Symmetric, bilinear, unimodular $\Leftrightarrow$ $Q^\vee : H_2(M) \to H_2(M)^\vee \simeq H^2(M)$

Eg $M = \mathbb{CP}^2$, $\overline{\mathbb{CP}}^2$, $S^2 \times S^2$, ...

$[1]$ $[-1]$ $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

$M = K3 \sim 2E_8 \oplus 3\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

$M_1^4 \# M_2^4 \Rightarrow Q_{M_1 \# M_2} = Q_{M_1} \oplus Q_{M_2}$?

$$K3 \overset{?}{=} M \# S^2 \times S^2$$

$\alpha \quad \beta \quad \in H_2(K3)$

$\alpha \cdot \alpha = 0, \quad \beta \cdot \beta = 0, \quad \alpha \cdot \beta = 1$

$S^2_\alpha \longrightarrow K3$

$S^2_\beta \longrightarrow K3$

if these where 1) embedding

2) $S^2_\alpha \cap S^2_\beta = \text{pt}$

If $\Sigma \subset M^4$ embedded $\Rightarrow N(S^2_\alpha)$ or $N(S^2_\beta)$

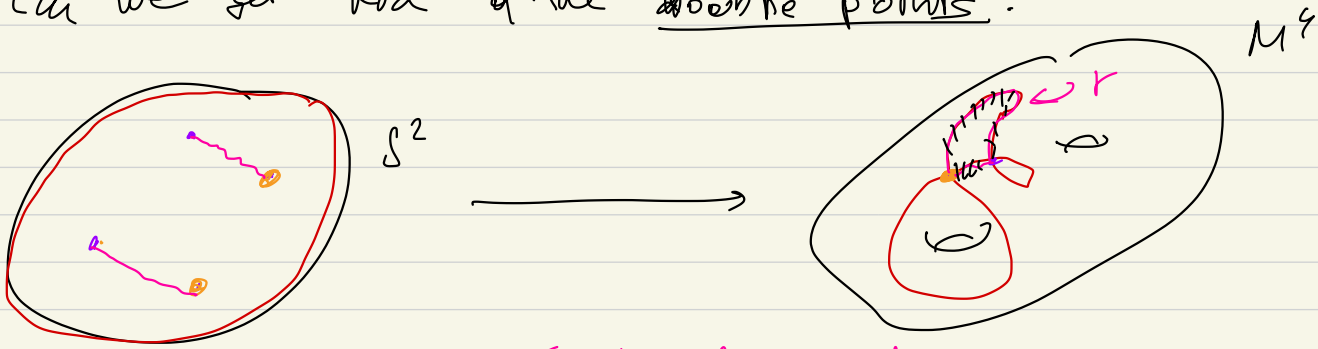
$\Sigma \cdot \Sigma = \langle e(N_\Sigma), [\Sigma] \rangle$

case trivial

$\Rightarrow \quad N(S^2_\alpha), N(S^2_\beta) = S^2 \times D^2 \subset B^4$

S^2 a generic embedding has self intersections

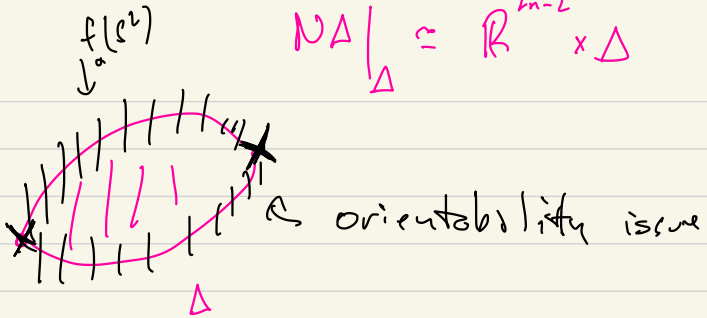
Can we get rid of the double points?



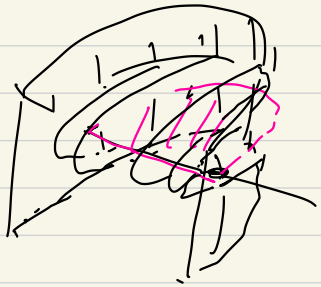
γ bounds a disk Δ

If we are in dim > 4 Δ could perturbed to be embedded but in dimension 4 it will have double points

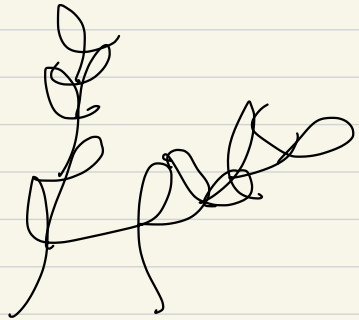
cf. Whitney Trick ~1940 (Used in the hard Whitney embedding theorem $M^n \subset \mathbb{R}^{2n}$ $\forall n$.



$S^1 \times \mathbb{R}^2 \subset \mathbb{R}^4$



$S^1 \times \mathbb{R}^2 \subset \mathbb{R}^4$



M. Freedman

à construction

found

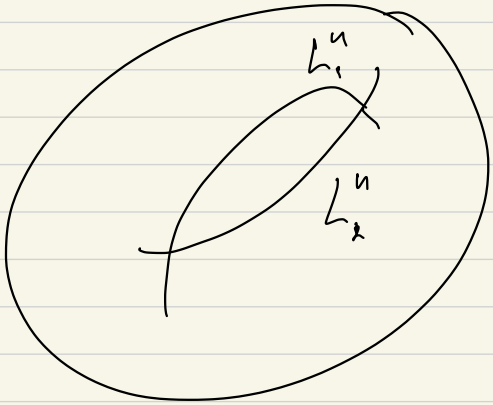
embedded

Whitney disks

⇒ classification

of TOP-4

Symplectic Arnold's Conjecture



M^{2n}

What can we say about # of intersection between L_1, L_2

h and a hamiltonian push off of L, L'

$\#(L \cap L') \geq$ # critical points of a morse function on L

$h_t \longrightarrow X_{h_t} \longrightarrow \Phi_t$, $\Phi_1(L) = L'$

Invariant of 3 and 4 dimensional manifolds.

Donaldson Invariants $\Rightarrow$

polynomial function on $H_*(M)$

$\forall k \longrightarrow$ $I_k^M : H_*(M) \to \mathbb{Q}$ homogeneous polynomial function

$\Big\{$ generating function

$\mathcal{D}^M : H_*(M) \to \mathbb{Q}$ found processes

$$D^M(\Sigma) = e^{Q/2} \sum_{i=1}^{R} a_i e^{K_i \cdot \Sigma}$$

$K_i \in H^2(X, \mathbb{Q})$.