Back of Envelope Physics

- Universe
- Galaxies, Stars and Planets
- Climate and Energy
- Earth
- Earth, Water, Air and Fire
- Nuclear Power
- Computation and Information
- Music and Art
- People and other Animals

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/sanjoy/mit/

"Speedy Neutrinos?"

Reports that neutrinos from CERN arrive 60ns early relative to light speed at Grand Sasso

If this is true,

$$\frac{\delta v}{c} \sim \frac{\delta t}{t} \sim \frac{60\,\mathrm{ns}}{1\,\mathrm{ms}} \sim 6 \times 10^{-5}$$

$c \sim 300\,\mathrm{Mm\,s^{-1}}$ $\qquad$ $1\,\mathrm{ms} \sim 300\,\mathrm{km}/c$

Delay due to finite rest mass

$$\frac{\delta v}{c} \sim \left(\frac{m_\nu}{E}\right)^2 \sim \left(\frac{1\,\mathrm{eV}}{10\,\mathrm{GeV}}\right)^2 \sim 10^{-20}\,!$$

Why?

negligible.

SN1987a in Large Magellanic Cloud at 50 kpc produced neutrinos seen at same time

$$\left|\frac{\delta v}{c}\right| \lesssim \frac{\delta t}{t} \sim \frac{\overset{1\frac{1}{2}\,\mathrm{hr}}{\overline{5 \times 10^3\,\mathrm{s}}}}{5 \times 10^{12}\,\mathrm{s}} \sim 10^{-9}\,!$$

$50\,\mathrm{kpc}/c$ $\qquad$ @ ~4MeV

Universe

- Age, Size, Density
- Matter
- Microwave Background
- Dark Energy, Planck Units

Age, Size, Density

Age $t_0 = 4\times10^{17}\,s$ 14 Gyr.

Size $R_0 = c t_0 = 10^{26}\,m$

$\sim 10^5 R_{galaxy}$

Density

when gravity is dominant,

$\rho \sim \frac{1}{G t^2}$ on dimensional grounds.

0.7×10^{-10}

eg orbiting satellite

$$\frac{GM}{R^2} \sim \omega^2 R \sim \frac{R}{t^2}$$

Lets improve this estimate by using an energy equation.

$$\frac{1}{2}\dot{R}^2 - \frac{G\frac{4}{3}\pi\rho R^3}{R} \sim \text{energy} \sim 0$$

$$\Rightarrow \rho \sim \frac{3}{8\pi G t^2} \sim \frac{1}{10\,10^{-10}\,10^{35}} \sim 10^{-26}\,kg\,m^{-3}$$

Matter

Suppose all of this were hydrogen. The number density would be

$$n_p \sim \frac{10^{-26}\,\text{kg m}^{-3}}{10^{-27}\,\text{kg}} \sim 10\,\text{m}^{-3}$$

— proton mass

In fact only 4% is hydrogen

So $n_p \sim 0.3\,\text{m}^{-3}$

About 20% is dark matter

If, as conjectured, these are ~100 GeV particles, their number density will be

proton mass

$$n_{DM} \sim 0.3 \times \frac{20}{4} \times \frac{1\,\text{GeV}}{100\,\text{GeV}} \sim 10^{-2}\,\text{m}^{-3}$$

Dark matter particles would be $(10^{-2})^{-1/3} \sim 5$ m apart.

The density within the Galaxy is $\sim 10^6$ larger, and so the spacing of DM particles here is 5 cm.

Microwave Background

The temperature is $T \sim 3K$

The mean photon energy is

$$E \sim 3kT \sim 3\times10^{-23}\times3 \sim 10^{-22}\,J$$

~6dF.

So, the wavelength becomes

$$\lambda \sim \frac{hc}{E} \sim \frac{7\times10^{-34}\times3\times10^{8}}{10^{-22}} \sim 1\,mm$$

The number density of photons is then roughly the density of states

$$n_{CMB} \sim \frac{1}{\lambda^3} \sim 10^{9}\,m^{-3}$$

Why no 2π's?

$$\sim 3\times10^{9}\times n_p\,!$$

However the energy density is

$$u_{CMB} \sim 10^{9}\times10^{-22} \sim 10^{-13}\,J\,m^{-3}$$

and the equivalent mass density is $\sim u_{CMB}/c^2 \sim 10^{-30}\,kg\,m^{-3}$ which is 10^{-4} of the total density, and negligible today (but not in the past)

Dark Energy and Planck Mass.

3/4 of the energy density is associated with an effective "cosmological constant" which behaves like material with pressure $P = -\rho c^2$, As the active gravitational mass density is $\rho + 3P/c^2$, this causes the universe to accelerate? What is this dark matter? One thought was that it was a type of vacuum energy associated with quantum gravity. However, there is a BIG problem.

A theory of relativistic quantum gravity surely involves $c, \hbar, G$. These can be combined to define a "Planck" mass using

$Gm/c^2 \sim \hbar/mc \Rightarrow m_p = (\hbar c/G)^{1/2} \sim 10^{-8}\,\mathrm{kg}$

$\hbar \sim 10^{-34}$

$\Rightarrow L_p \sim 10^{-35}\,\mathrm{m}$, $T_p \sim 10^{-43}\,\mathrm{s}$

$\rho_p \sim 10^{97}\,\mathrm{kg\,m^{-3}} \sim 10^{123}\rho_{DE}$!

This is not the explanation !

Other explanations for dark energy

$\rho_{DE} \sim 10^{-26}\ \mathrm{kg\,m^{-3}}$

c^2 $\underline{U_{DE}} \equiv \boxed{10^{17}} \times 10^{-26} \sim 10^{-9}\ \mathrm{J\,m^{-3}}$

Magnetic Field? $B \sim (\boxed{\mu_0}\, U_{DE})^{1/2} \sim 30\,\mathrm{nT}$

$\sim 4\pi \times 10^{-7} \sim 10^{-6}$

Particles?

Energy density $\propto \frac{m}{(1/m)^3} \propto m^4$

$$\Rightarrow \frac{m_{DE}}{M_{PL}} \sim \left(\frac{\rho}{\rho_{PL}}\right)^{1/4} \sim \left(10^{-123}\right)^{1/4} \sim 10^{-31}$$

Planck mass

$$\Rightarrow m_{DE} \sim 10^{-31} \times \boxed{10^{28}\,\mathrm{eV}}$$

$\sim 1\,\mathrm{meV}$

$\sim m_\nu$??

~1 TeV we guess!

$$\sim 0.1 \frac{m^2_{susy}}{M_{PL}} \sim 0.1 \frac{(10^{12})^2}{10^{28}}\ ?$$

We are just guessing!

We hope to know soon!

Galaxies, Stars, Planets

- Galaxies
  Number, stars, dark matter, nuclei, black holes.
- Stars
  Masses, radii temperature, luminosities..
- Compact objects
  White dwarfs, neutron stars, blackholes
- Planets
  masses, radii, gravity..
  asteroids

Galaxies

There are ~ 10^{11} galaxies in the universe and a typical one, like ours, contains:

~10^{12} $M_\odot$ of dark matter

solar mass

~10^{11} $M_\odot$ of stars

~10^{10} $L_\odot$ of starlight

solar luminosity

The circular speed is ~ 300 km s^{-1}

The solar radius is 10 kpc ~ 3×10^4 lt yr ~ 3×10^{20} m

Earth orbit

Also, 1 AU/pc ~ 1 sec ~ $\frac{\pi}{180 \times 60 \times 60}$ ~ 5×10^{-6}

$$\text{Hence } \frac{M_{DM}}{M_\odot} \sim \left(\frac{V_{DM}}{V_\oplus}\right)^2 \left(\frac{R_{DM}}{1\,AU}\right) \sim \left(\frac{300}{30}\right)^2 \times \frac{10^5}{5 \times 10^{-6}}$$

Earth

$\sim 10^{12}$ — A "scaling" argument

Stars

1 AU = Earth orbit radius

$$\sim 3\,\mathrm{Mlyr} \times 5\times10^{-6} \sim 1.5\times10^{11}\,\mathrm{m}$$

nb $\frac{2\pi\ 1\mathrm{AU}}{\mathrm{yr}} \sim \frac{6\times1.5\times10^{11}}{3\times10^{7}} \sim 30\,\mathrm{km\,s^{-1}}$

Solar radius $\sim 1.5\times10^{11} \times \frac{1}{4}^{\circ}$

$$\sim 1.5\times10^{11} \times \frac{1}{200} \sim 7\times10^{8}\,\mathrm{m}$$

Escape Velocity

Circular speed at solar surface $V \propto R^{-1/2}$

$$\Rightarrow V_c \sim 30\,\mathrm{km\,s^{-1}} \times (200)^{1/2}$$

$$\sim 450\,\mathrm{km\,s^{-1}}$$

$$\Rightarrow V_{esc} \sim \sqrt{2}\,V_c \sim 600\,\mathrm{km\,s^{-1}}$$

Black Hole

$$R \sim \frac{GM}{c^2} \sim 1.5\times10^{11} \times \left(\frac{30}{3\times10^{5}}\right)^2$$

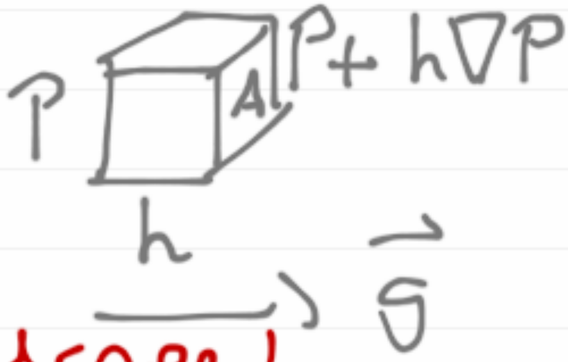
$$\sim 1.5\left(\frac{M}{M_\odot}\right)\mathrm{km}$$

Useful for scaling!

Solar Interior

Important for solar oscillations

$$h\nabla P\, A = \rho A h g$$



$$\Rightarrow \nabla P = \rho g$$

$$\Rightarrow \underbrace{\frac{P}{\rho} \sim c_s^2}_{} \overset{\text{soundspeed}}{\sim} \frac{GM}{R} \sim V_c^2 \sim (450\ \mathrm{km\,s^{-1}})^2$$

$$\sim v_p^2 \quad \text{proton speed}$$

Temperature $kT \sim m v^2$

$$T \sim T_{room} \times \left(\frac{c_{s\odot}}{c_{s\,room}}\right)^2 \times \left(\frac{\overset{m_p}{1}}{\underbrace{30}_{m_{N_2}}}\right)$$

$$\sim 300 \times \left(\frac{450\times10^3}{300}\right)^2 \times \frac{1}{30}$$

$$\sim 2\times10^7\,K \sim 2\,keV << MeV$$

cf Homework 2 Why?

Neutron star

$$N_{nucleons} \sim \frac{\overset{\text{stellar mass}}{10^{30}}}{\underbrace{10^{-27}}_{\text{nucleon mass}}} \sim 10^{57} = A_{NS}$$

$$R_{NS} \sim (10^{57})^{1/3} \times \underbrace{1\,fm}_{R_{nucleon}} \sim 10\,km$$

$$\rho_{NS} \sim \rho_{nucleon} \sim \frac{10^{-27}\,kg}{(1\,fm)^3} \sim 10^{18}\,kg\,m^{-3}$$

Solar Surface

Gravity

$$g_\odot \sim \frac{GM}{R^2} \sim \frac{V_c^2}{R} \sim \frac{(450\times10^3)^2}{7\times10^8}$$

$$\sim 300\,\mathrm{m\,s^{-2}}$$

$$\sim 30\, g_\oplus$$ - Earth gravity

Note that $g \propto \rho R$

$$\Rightarrow \rho_\odot \sim \rho_\oplus \left(\frac{g_\odot}{g_\oplus}\right)\left(\frac{R_\oplus}{R_\odot}\right) \sim 5000 \times 30 \times \frac{6\times10^3}{7\times10^5} \sim 1300\ \mathrm{kg\,m^{-3}}$$

Like H_2O!

Neutron star

$$g_{NS} \sim 300 \times \left(\frac{10^{18}}{1300}\right)\left(\frac{10}{7\times10^5}\right) \sim 3\times10^{12}\ \mathrm{m\,s^{-2}}$$

Temperature scaling

From HW 2, for black body

$T_\odot \sim 6000\,\mathrm{K}$, $L_\odot \sim 4\times10^{26}\,\mathrm{W}$, $R_\odot \sim 7\times10^5\,\mathrm{km}$

For other bodies,

$$L \propto R^2 T_{surf}^4$$

Climate and Energy

- Basic properties of the earth and its atmosphere
- Green house effect
- Global energy consumption
- Coal; carbon and water cycles
- Gas, autos and planes.

The Earth - Basics

$M_\oplus \sim 6\times10^{24}\,\mathrm{kg}$, $R_\oplus \sim 6.4\,\mathrm{Mm}$, $g_\oplus \sim 10\,\mathrm{m\,s^{-2}}$

cf Sun $P_\oplus \sim \rho g R \sim 5\times10^3 \times 10 \times 6\times10^6 \sim 3\times10^{11}\,\mathrm{N\,m^{-2}}$

Earth has solid Fe core Si, Fe + O, Mg

From seismology

Bulk modulus $K \sim (e^2/4\pi\epsilon_0 a_0^2)/a_0^2$

$dP/d\ln\rho$ $\sim 3\times10^{11}\,\mathrm{N\,m^{-2}}$

$\Rightarrow \delta\rho/\rho \sim 1$

Central density of earth is $\sim 12000\,\mathrm{kg\,m^{-3}}$ $\sim 1.5\,\rho_{Fe}$

Central temperature is $\sim 6000\,\mathrm{K}$
(~ melting temperature of hardest material)

core mass

$10^{-25}\,\mathrm{kg}$ Internal Energy $\sim \frac{M_\oplus}{m_{Fe}} \times \boxed{3k}T$ ~ specific heat Dulong-Petit

$U_\oplus \sim 10^{31}\,\mathrm{J}$

~150 Ps

$L_\oplus \sim U_\oplus / t_\oplus \sim 60\,\mathrm{TW} \sim 2\times$ actual

From ν's Radioactivity (+Tidal heating) ~20 TW

$\langle f_\oplus \rangle \sim \frac{30+20\,\mathrm{TW}}{5\times10^{14}\,\mathrm{m^2}} \sim 0.1\,\mathrm{W\,m^{-2}}$

heat flux

Atmosphere – Basics

1 atm.

$P \sim 10^5 \, \mathrm{N\,m^{-2}}$; $\Sigma \sim \frac{P}{g_E} \sim 10^4 \, \mathrm{kg\,m^{-2}}$

30 GeV

$\rho \sim \frac{m_{N_2} P}{RT} \sim 1 \, \mathrm{kg\,m^{-3}}$

$\frac{1}{40}$ eV

Scale height, $H \sim \Sigma / \rho \sim 10 \, \mathrm{km}$.

from HW 2

Solar flux $F_\odot \sim 1.4 \, \mathrm{kW\,m^{-2}}$

Solar Power $\sim F_\odot \pi R_E^2 \sim 200 \, \mathrm{PW}$ >> internal, support power

If re-emit as blackbody

$$\sigma T_E^4 4\pi R_E^2 \sim F_\odot \pi R_E^2$$

$$\Rightarrow T_E \sim 270 \, \mathrm{K}$$

Cosmic rays $\sigma \sim (1 \, \mathrm{fm})^2 \times A^{2/3}$ ~14 for N

$m_N \sim 14 \times m_p$

"Probability" of stopping a cosmic ray

$$\approx \frac{\Sigma}{m_n} \sigma \sim 10$$

Atmosphere shields us!!

Composition (Richter)

$N_2 \sim 0.8$, $O_2 \sim 0.2$, $A \sim 0.01$

Green-house gases

$CO_2 \sim 4 \times 10^{-4}$ (was 3×10^{-4})

$H_2O \sim .004$ (height integrated)

$\sim .01 - .04$ at surface.

UV $O_3 \sim 10^{-7}$

Blackbody peak wavelength.

$$\lambda_{max} \sim 600\,nm \times \left(\frac{6000K}{T}\right)$$

yellow

solar temp

So for $T \sim 270K$

$\lambda_{max} \sim 13\mu$

This is where CO_2, H_2O make the atmosphere opaque leading to heating, so that $\lambda_{max} \sim 12\mu$ and the incident radiation can escape

Greenhouse Effect

CH_4 stored in frozen clathrates on ocean floor are 25× as serious a greenhouse gas.

Energy Consumption [Global US ~ 0.3]

FOSSIL

very rough

5 TW (oil) + 4 TW (coal) + 4 TW (gas) + 1 TW (nuclear) + 1 TW (solar, wind, bio ...)

≃ 15 TW

not clearly defined

≃ 5 TW (transportation) + 5 TW (domestic) + 5 TW (manufacturing)

Population: 7 Gp
(China, India ~ 0.15, US ~ 0.05, UK ~ 0.01,

Power consumption ~ 2 kW p^{-1}
~1.5 kW p^{-1} in 1995

cf Personal power ~ 70 W p^{-1}

US power consumption ~ 10 kW p^{-1}

(UK ~ 5 kW p^{-1})

Electricity ~ 2 TW requiring 5 TW of power

Unusual Units

1 Quad ~ 1 EJ, 1 barrel ~ 160 l

Powers of 10^3: k M G T P E Z Y X

unofficial

Depends on economics
takes 10^7 yr to form

Large exchanges of H_2O, CO_2.

cf incinerating waste 2 MJ kg^{-1}

Coal

Consumption ~ 200 M$g\,s^{-1}$ $\}$ ⇒ $\tau_{use} \sim R/C$

Reserves ~ 1 Eg $\}$ ~ 200 yr

Heat Content ~ 30 MJ kg^{-1} ≡ 0.3 eV/n

cf 1 kt_{TNT} ≡ 4 GJ ≡ 0.04 eV/n ~ dynamite!

cf 3 MeV/n – nuclear, 300 MeV/n gravity

⇒ Thermal power ≃ HC × C = 6 TW $\boxed{E=mc^2}$

3 TW of this produces 1 TW electrical power with 0.3 efficiency.

Remainder → steel, heating...

Carbon cycle

C Fraction CO_2/C

CO_2 production ~ 200 M$g\,s^{-1}$ × 0.8 × $\frac{44}{12}$

~ 600 M$g\,s^{-1}$ from coal

Total CO_2 ~ 1 G$g\,s^{-1}$ (+oil, gas)

However biomass ⟷ Atmosphere ⟷ Ocean exchange CO_2 at a rate ~ 10 G$g\,s^{-1}$ each in equilibrium. Effect of coal burning is to disturb this balance.

Water cycle

M_{ocean} ~ 1000 kg m^{-3} × $4\pi R_E^2$ × 2 km (mean depth)

~ 1 Yg.

Precipitation ~ 20 T$g\,s^{-1}$ ⇒ $\tau_{ocean} \sim M/P$ ~ 2000 yr.

Gas, Autos, Planes

Crude Oil consumption $\sim 100\ \mathrm{Mg\ s^{-1}}$ } $\tau \sim 40$ y

Reserves ~ 300 Pg

~ 0.5 TW electricity

1/3 crude oil

Autos

10^9 worldwide (~1/4 US)

gasoline usage $\sim 30\ \mathrm{Mg\ s^{-1}}$

km/mi

gas mileage ~ 30 mpg $\simeq 30 \times \frac{8}{5} \times \frac{1}{4} \sim 10\ \mathrm{km\ kg^{-1}}$ ~ l/gallo

"mean speed" $\sim \frac{30 \times 10^3}{10^9} \times 10 \sim 300\ \mathrm{mm\ s^{-1}}$

$\sim 30\ \mathrm{km\ d^{-1}}$ reasonable.

CO_2 rate $\sim \frac{0.1}{2} \times \frac{44}{12} \sim 0.2\ \mathrm{kg\ km\ p^{-1}}$

#passengers

Planes

Boeing 747 Jet uses 2.4×10^5 l to fly 9000 mi carrying 400 p.

density

$$CO_2 \text{ rate} \sim \frac{2.4 \times 10^5 \times .8 \times \left(\frac{44}{12}\right)}{9000 \times 8/5 \times 400} \sim 0.1\ \mathrm{kg\ km^{-1}\ p^{-1}}$$

cf Autos

→ planes also cause damage with fuel additives, ozone at high altitude and water → clouds.

Solids, Liquids, Gases (RVW notes)

- Pressure (kinetic + degeneracy)
- Atoms, molecules
- Nuclear binding.
- Bulk modulus, flow (mountains)
- Surface tension (raindrops)
- Dimensions, Buckingham π theorem
- Sedov-Taylor explosions.
- shallow, deep, capillary waves.

Earth, Water, Air & Fire

- Earth - Geothermal
- Water - Hydro, waves.
- Air - Wind; land, sea
- Fire - soler power.

Geothermal Power

present consumption

p15 $U_\oplus \sim 10^{31}\,\mathrm{J} \sim 15\,\mathrm{TW} \times 20\,\mathrm{Gyr}$!

Can "mine" heat.

good ? $\langle F_\oplus \rangle \sim 0.1\,\mathrm{W\,m^{-2}} \sim 50\,\mathrm{TW}/A_\oplus$

More in hotspots like Iceland

Thermal conductivity of hot rock $\sim 3\,\mathrm{W\,m^{-1}\,K^{-1}}$ dictated by phonon mfp

cf. Mine Can drill to $\sim 3\,\mathrm{km} \Rightarrow \Delta T \sim \frac{0.1\,\mathrm{W\,m^{-2}}\,3\,\mathrm{km}}{3\,\mathrm{W\,m^{-1}\,K^{-1}}} \sim 100\,\mathrm{K}$

Pump water down, steam up

Efficiency

Carnot cycle $\varepsilon \sim \frac{W}{Q_0} \sim \frac{\Delta T}{T_0} \sim \frac{1}{4}$

Typical $\varepsilon \sim 0.1$

Current production $\sim 10\,\mathrm{GW} \Rightarrow 1\,\mathrm{TW}$??

Problems: Qe, CO_2, environment, earthquakes, capital cost...

Aside ε for Heat Pumps $= \frac{Q_0}{W}$

Carnot cycle $= Q_0/W = T_0/\Delta T$

A little work can raise heat in temperature.

Hydroelectricity.

Check

p19 $\langle \text{Precipitation} \rangle \sim 20\,\text{Tg}\,\text{s}^{-1} \equiv 1\,\text{m}\,\text{yr}^{-1}$

$S_\oplus$,

$$\text{Power} \sim 20\,\text{Tg}\,\text{s}^{-1} \times 10 \times 100\,\underbrace{h_2}_{h/100\text{m}} \times f_\oplus \times \varepsilon$$

$$\sim 20\,h_2\,\underbrace{f_\oplus}_{<.01}\,\underbrace{\varepsilon}_{<.1}\,\text{TW}$$

Globally minor, locally feasible especially if dam large rivers

ε low because of evaporation loss, friction.

Environmental issues!

Three Gorges Dam

Yangste river 20 GW eventually out of ~500 GW total power

Overall hydropower produces ~0.5 TW globally.

Wave Power

RVW showed that $\omega^2 = gHk^2$
for shallow water waves

Deep water waves independent of H
so $\omega^2 = gk \times \#$ (# = 1, actually tanh kH in general)

$KE \sim \frac{1}{2}\frac{\rho\,\omega^2 h^2}{k} \sim \rho h \times \frac{gh}{2}$ GE

Group velocity $V_g = \omega/2k$ GE

Energy flux: $F = \frac{H}{2}\underbrace{\frac{\rho\,\omega^2\langle h^2\rangle}{2}}\times 2 \times V_g$ — per area

per length

$Z = \frac{1}{2k}$

$$F = \frac{1}{2k}\frac{\rho\,\omega^2 h^2}{2}\frac{\omega}{2k}$$

$$= \frac{\rho g^2 h^2}{8\omega}$$

$\omega \sim 2\pi/10\,s$

$V_\phi = g/\omega \sim 16$, $\lambda \sim 160\,m$ $h \sim 1\,m$

$$F \sim \frac{10^{3+2+1}}{50} \sim 20\,kW\,m^{-1}$$

$$P \sim 20\times10^3 \times \underset{2\pi R_\oplus}{40\times10^6} \sim 1\,TW$$

$\sim 1\,TW$ Globally insufficient

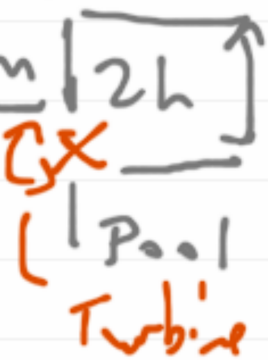
HW4

Tidal Power

$☽ : ⊙ = 7:3$

$$\frac{GM_{\odot}}{a^3} R_{\oplus} \sim \frac{GM_{\oplus}}{R_{\oplus}^{2}} h$$

$$h/R_{\oplus} \sim (1.5\,hr/310\,d)^2 \sim \underset{\sim\sim}{0.2m}$$



Must move because shallow water (cf Tsunami)

Power density $\sim \frac{2\rho g h^2}{6hr} \sim 3\varepsilon\ W m^{-2}$ for $h \sim 3m$.

10 TW would need $(1\,Mm)^2$

LaRance (St Malo, France)

$L \sim 300m$, $h \sim 8m$, $P_{max} \sim 240MW$

$\langle P \rangle \sim 70MW$

Aside need larger tides

tidal flow in a shallow channel

$F \sim \frac{1}{2}\rho V^3$

$$V \sim \frac{D}{6hr} \times \frac{h}{H}$$

$$\sim \frac{10^5}{2\times10^4} \times \frac{10}{100} \sim 1\,m\,s^{-1} \text{ say.}$$

~ wind

Energy from ⊕ rotation!

Tsunami

Wind Power

$P = \left[\frac{1}{2}\rho v^{\boxed{3}}\right]\varepsilon A \sim .5 \times 1 \times 10^3 \times \frac{1}{2} \times 30^2 \sim 300\,\mathrm{kW}$

also 3/2 PV Shadowing ⇒ $f \sim .02$ on farm ~ filling factor

~10^{-3} solar 1-2 W m^{-2} on land.

horizontal axis – several designs

largest is 200 m high with 130 m diameter → 8 MW!

nb this is peak power typical power can be much less

Largest wind farms are ~ 500 MW ~ 10 GW planned

Offshore is better – have floating turbines when in deep water.

However, serious problems with gales, corrosion, cost of servicing . . .

Capital costs

40% UK 2 kW p^{-1} requires 1000 kg p^{-1} of concrete and steel

Global. 10 TW ≡ 1 W m^{-2} × $(3\,\mathrm{Mm})^2$!

Locally impactful, globally minor

Solar Power – Generalities

$P \sim 1.5\,kW\,m^{-2} \times \pi \times (6000\,km)^2$

$\sim 200\,PW \sim 10^4 \times$ consumption

Solar power is dilute.

Effective duty cycle $\delta \sim 1/8 - 1/4 \sim .15$ / AZ

Efficiency $\varepsilon \sim 0.1 - 0.5$ n'E

Integrated area $\sim 10^4\,km^2\,(\delta\varepsilon)^{-1}$ of collectors for global consumption.

- Biomass (burning trees, plants, waste) ~ 1TW
- Direct heating ~ 100GW
- Photovoltaic ~ 10 GW

Commercial viability requires $\sim 1\,\$\,W^{-1} \equiv 1\,B\$\,/\,GW$ capital cost

Electricity price ~ \$20/40 GJ^{-1} wholesale/retail variable!

- Domestic usage is total consumption

Biomass

- Plants, trees etc → fuel, gas, food
- Efficiency of photosynthesis is very low ~ 1 $W\ m^{-2}$
  [Corn ethanol ~ 20 $mW\ m^{-2}$ according to MacKay]
- Growing plants can absorb CO_2 but burning them creates more CO_2 unless you have carbon sequestration
- Biofuel produces 30 $MJ\ kg^{-1}$

p19 Waste produces 2 $MJ\ kg^{-1}$

- Most biofuel usage is not being renewed (but could be).

Direct heating

For domestic hot water, space heating

$\sim 1/4 \times$ $\quad \sim 1/2 \times$ domestic

Circulate water from reservoir onto roof and heat exchange with domestic water supply.

eg $1\,m^3$ reservoir $\equiv 10^3\,kg$

heat by 30 K for ~1 d

$$\text{Power} \sim \frac{10^3\,kg \times 4\,kJ\,kg^{-1}\,K^{-1} \times 30\,K}{10^5\,s}$$

$\sim 1\,kW$, $\varepsilon S \sim 0.1$

Need an area $\sim 10\,m^2$

Also skylights, windows contribute to space heating.

Photovoltaic Devices (For electricity)

Flatscreen $\varepsilon \sim 0.1-0.2$

Example Fig 6.6 MacKay

$2m^2$ panel rated @ 300W

produce a peak power ~250W and

and an average power ~ 40W

(a dim light bulb)

Full domestic electricity production

requires ~ 100 panels > 1 roof!

However partial power is practical

Economics changing because

capital costs are falling.

and solar power more promising

Cylindrical Mirrors

Intensity (≡ temperature) of sunlight in $W\,m^{-2}\,Hz^{-1}\,sterad^{-1}$ is conserved through optical system. Sun subtends an angle ~ 0.01 rad so if focus in 1D can increase flux by ~100 by increasing the solid angle to ~1 rad.

Andasol power station (Spain)

Pipe containing molten salt at 1000K

cylindrical mirror

$$\frac{dM_{salt}}{dt} \sim \frac{50\,MW}{6\,kJ\,kg^{-1}K^{-1}\,1000K}$$

\$8/W!

specific heat.

$\sim 1\,kg\,s^{-1} \sim 100\,Mg\,d^{-1}$

Can hoard turbine and run generators at night.

Spherical Mirrors

2D Focusing - concentrate flux by up to 10^4 (typically $10^3 \sim$ MWm^{-2})

Prototype facility REhnu in Arizona

Detector area

$$\sim \frac{10\,m^2}{1000}$$

$\sim$ 100 × 1 cm^2 detectors with $\varepsilon \sim 0.4$

10 kW ε each! cooling?

cost ~ 1\$ W^{-1}! Load balance?

"Tower of Power"

Nuclear Power

- Nuclear fuel. Fission, fusion.
- Light Water reactors
- Global implications
- Bottomline.

Nuclear Fuel

Fission $U^{235}+n \to \ldots + 1\,\mathrm{MeV}/n$

$\sim 10^{-3}c^2$

Yield: $10^{14}\,\mathrm{J\,kg^{-1}} \times \cdot 3 \times 0\cdot 007 \sim 200\,\mathrm{GJ\,kg^{-1}}$ of U ore

1MeV/n ε f_{235}

p19 cf coal/gas/oil $\sim 30 \times \cdot 4 \sim 10\,\mathrm{MJ\,kg^{-1}}$

$U^{238}+n \to Pu^{239}+2e^-$ ε

RVW notes A-bombs: $20\,\mathrm{kt} \times 4\,\mathrm{GJ\,t^{-1}} = 80\,\mathrm{TJ}$

Pu^{239} is fissile; smallest critical mass!

Fusion $d+t \to \alpha + n + 23\,\mathrm{MeV}$, also dd, Li

absorb n on Li, regenerate t

HW Temperature $\sim 10^8$ K to overcome Coulomb barrier

- Magnetic confinement (Tokamaks, ITER)
  10 T field, $\tau \sim 1$ s for breakeven.
- Inertial confinement (Lasers, NIF)

Ablation: $\frac{\text{Momentum}}{\text{Energy}}$ $\boxed{\sim \frac{2}{v}\ (\mathrm{NR})}$ and $\frac{1}{c}$ photon

H bombs: 10 – 50 MT ~ 40–200 PJ!

Light Water Reactors

Commercial reactors are "once through" LWR

- use centrifuges to increase $f_{235} \sim .04$
- burn .01 of fuel and store waste
- PWR - 2 stage, BWR - 1 stage
- Rankine cycle (isobaric Carnot)

- breeder reactors burn most of the fuel
  - create Pu^{239}
  - concern of proliferation of nuclear weapons

dd

~100 reactors in US ~ 400 world

Power ~ .1 - 1 GW

~ 60 y lifetime

France, Sweden ~ 800 W p^{-1} electricity, US 300 W p^{-1}

high grade waste ~ $\frac{0.02 \times 2\,TW}{2\,GJ/kg} \sim 20.1\,kg\,s^{-1}$

300 power stations (100 in US)

None since 80s..

2 TW need 2000 × GW or 20,000 × 100 MW.

Global Implications.

p18 Suppose we try to supply the total global electrical power today 2TW by nuclear power we need
$2\,TW / 200\,GJ\,kg^{-1} \sim 10\,kg\,s^{-1}$ fuel for a breeder reactor and
$1\,Mg\,s^{-1}$ for a once through cycle.

The known reserves of U are 30 Tg which would be exhausted in
$30\,Tg / 10\,kg\,s^{-1} \sim 100\,yr$ and $\sim 1\,yr$ for a once through cycle?

- 5 Pg of oceanic U turns over $\sim 10^3\,yr$

p19 supply $\sim 200\,kg\,s^{-1}$ U.

Economics: Dominated by capital cost

- ~ 5-$7\,\$\,W^{-1}$ (Europe, US), $\sim 2\,\$\,W^{-1}$ (China)
- Advantages: No CO_2 (except for construction), load balances, cheap running costs
- Issues: Waste disposal (political) proliferation, accidents. TMI, Chernobyl Fukushima

Bottom Lines

- Two power crises - CO$_2$ release
  - increase in demand.
- Address with combination of power sources + conservation
- Economic and political problem
- Brutal arithmetic not romantic vision
- All sources have drawbacks; seek least problematic
- Still plenty of opportunity for new ideas and cost reduction.
- Solutions take > a decade lead time
- Power consumption increasing
- CO$_2$ increasing

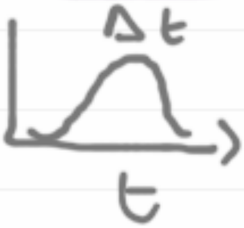
Go figure!

Computation and Information

- Fourier Transform
- Storage
- Computation
- Power
- Brain

Fourier Transforms

VP!

$V(t)$  $= \int_{-\infty}^{\infty} df\, e^{2\pi i f t}\, \tilde{V}(f)$  $\sim \Delta t^{-1}$

Shannon: data rate ~ Bandwidth $\log_2\left(\frac{S}{N}\right)$
Human audio range ~ 20 kHz
Sampling rate on audio CD ~ 40 kHz.
Power: If f is amplitude for a finite length T of data.

$$\frac{\int dt\, V^2}{T} = \int df\, \boxed{\frac{|\tilde{V}|^2}{T}} \quad P_f$$

$$C(\tau) = \int dt\, V(t)\, V(t+\tau) \sim P_f$$

(Wiener-Khintchine)

Fast Fourier Transform

Discretize $\hat{V}_k = \sum_n V_n e^{-2\pi i k n/N}$

N^2 operations.

Cooley Tukey algorithm reduces this to $O(N \ln N)$ (generally set $N = 2^p$)

- ## Storage

red!

Audio CD $R \sim 6$ cm, $\lambda_{\text{laser}} \sim 800$ nm

Diffraction grating ⇒ pitch $\sim 1\,\mu$

⇒ $L \sim 2\pi \times \cdot 05\,\text{m} \times \underbrace{4\times10^4}_{\#\text{ rings}} \sim 10$ km !

Storage in "pits" which change brightness
can store up to 700 MB ~ 6 Gb(its)
read out in 80 m ~ 5 ks ⇒ rate $\sim 1\,\text{Mb}\,\text{s}^{-1}$
⇒ bits ~ $1\,\mu$ apart. $V \sim 2\,\text{m}\,\text{s}^{-1}$!

Minimum storage for music ~ 40 kHz × 5 hrs
~ 200 Mb

Flash memory

"Non-moving" memory. Up to 250 GB
Read/Write up 5 $\text{Gb}\,\text{s}^{-1}$
limited # of read/writes

Hard Drive - Disk Storage

Spinning ferromagnetic disk R/W with moving head. Can spin up to 250 Hz and store several TB (for 100\$ TB^{-1}) magnetic domain size ~ 10 nm. (Storage ~ 10^4 × CD)

Data size ~ 10 PB now > 100 PB soon!

Arithmetic, Error and Computation

- I/O, communication, memory (cache), arithmetic, management all important. Many coding schemes in use! Most common is 64 bit / double precision / 8 bytes. floating point.

$$\pm m \times 2^{e-1023}$$

precision $\sim 2^{-52} \sim 10^{-15}$.

- Clock rate ~ 3 GHz; $c/\nu \geq 1$ cm chip

$$\lesssim 10 \text{GHz}$$

- CPU ~ 4-10 cores (1000 planned for GPUs)
- Chips contain ~ 10^{10} transistors spacing ~ 30 nm. Tunneling serious for ≤ 10 nm
- Performance improvement limited by slowest parts of operation (codified in Amdahl's law) and improved by parallelization.

Power

Although reversible computation is theoretically possible, practical computers are dissipative and this limits their expansion.

Energy loss in wires is $\sim 1\,nJ\,m^{-1}s^{-1}$ and so short communications important

Rough!

	Power W	FLOPs	Energy/f (nJ)
Cellphone	1	na	na
Brain	15	na	na
Laptop	30	TBD	TBD
Supercomputer	3×10^6	10^{15}	3
Googlesite	10^8	3×10^{16}	3

<u>Servers</u>

US total ~ 10 GW ~ \$10B y^{-1} r28

Global total ~ 30 GW ~ \$30B y^{-1}

(likely to be underestimates)

- Cost of power and cooling of computers already exceed capital cost
- 10^5 core clusters are being constructed
- 10^3 core CPU^{-1} GPUs under construction for special purpose operation.
- E-waste is major problem

Brain

- $\sim 10^{11}$ neurons, $\sim 10^4$ synapses (electrical, chemical connections) per neuron.
- $\tau \sim 10$ms; Mean rate ~1 Hz $\Rightarrow 10^{15}$ SOPS (synaptic operations per s) possible
- typical operation consumes ~1 pJ
- Power ~5 kW ~15 W measured. suggesting that duty cycle ~ .01 [2g of food @ 30 MJ $\text{kg}^{-1} \cong$ 60 MJ, good for a lecture!]
- V ~ 1 l (M ~ 1.5 kg) spacing ~ 30 μm (neuron size ~ 3 - 100 μm)
- Evolved for low power
- "memory" estimated at ~1 byte/syn $\Rightarrow$ ~1 PB. What does this mean?
- Reasons for low power include low "clock rate", 3d connectivity and massive parallelization.
- The IBM Blue Gene/Brain project needs 100 kW to simulate 10^5 neurons

Summary

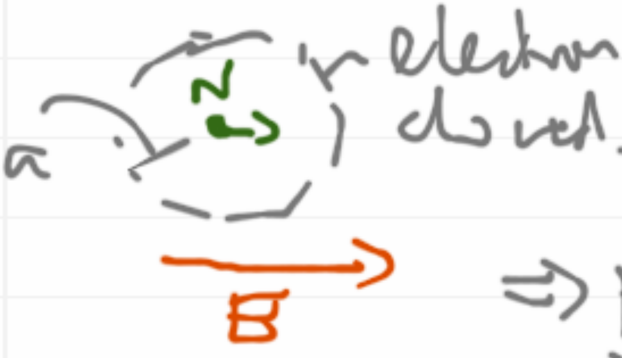
- Computation currently at Petascale (petaflops, petabytes)
- Needs to get to Exascale in ~5yr
- Total power usage of current computation is > 0.01 of electrical power usage
- Physics limits clock speed, component size.
- Future improvement in energy per flop must come from chip design parallelization and minimizing communication.
- The brain can execute ~PHZ spikes using 15W!
- We may learn from the brain!!

Optics and Acoustics

- Geometrical Optics
- Ionosphere
- Gravitational Lenses
- Fraunhofer diffraction
- Instruments
- Fresnel diffraction

Geometrical Optics – Refraction

Atomic Polarizability.



$$\frac{Ze}{4\pi\epsilon_0 x^2}\left(\frac{x}{a}\right)^3 \sim ZeE$$

$$\Rightarrow p \sim Zex \sim 4\pi\epsilon_0 a^3 E$$

$$\Rightarrow \chi = \frac{n p}{\epsilon_0 E} \sim 4\pi N a^3$$

~1 for water, glass

$$\Rightarrow v \simeq \frac{c}{n} = \frac{c}{(\chi+1)^{1/2}} \sim \frac{c}{1.5}$$

refractive index

For air $\chi - 1 \approx n - 1 \sim \frac{N_{air}}{N_{solid}} \sim 10^{-3}$ (3×10^{-4} at STP)

$k_{\parallel} = \text{const} \sim n \sin\theta$

Snell's Law

$n = \frac{ck}{\omega}$

const.

Electron density in ionosphere

AM – 0·5 – 1·6 MHz reflects

FM – 90 – 110 MHz no reflect

"cut off" for normal incidence ~10 MHz

Consider electron in EM wave $Ee^{i\omega t}$

Maxwell $\nabla\times B = \mu_0 \dot{j} + \epsilon_0 \frac{\partial E}{\partial t}$; $\nabla\times E = -\frac{\partial B}{\partial t}$

$$\dot{j} \sim Nev \sim i\frac{Ne^2}{m\omega}E$$

$$\Rightarrow \nabla\times B = \mu_0\epsilon_0\left(1-\underbrace{\boxed{\frac{Ne^2}{m\epsilon_0}}}_{}\frac{1}{\omega^2}\right)\frac{\partial E}{\partial t}$$

ω_p^2 plasma freq.

So cut off when

$$N \sim \frac{m\epsilon_0\omega^2}{e^2} \sim \frac{10^{-30}\,10^{-11}\,(6\times10^7)^2}{(1.6\times10^{-19})^2}$$

$2\pi\nu$

$$\sim 10^{12}\,\mathrm{m}^{-3}$$

refractive index

$$\boxed{n}^2 = \frac{ck^2}{\omega^2} = 1-\frac{\omega_p^2}{\omega^2}$$

Dispersion relation

Oblique

$n\sin\theta = \text{const}$

$\sim n = n_c = \sin\theta_0$

$$\Rightarrow \omega_c = \omega_p\sec\theta_0$$

Gravitational Lenses

Solar deflection

Newtonian

$$\alpha_N \sim \frac{GM}{b^2} \cdot \frac{2b}{c} / c \sim 2\frac{GM}{bc^2} \sim \frac{1}{2}\alpha_{GR}$$

$$\sim 2\frac{V_{esc}^2}{c^2} \sim 10^{-5}$$

600 km s^{-1}

Galaxy. $\frac{GM}{b} \sim (2)\sigma^2$

from hydro equm

1D vel disp

$$\Rightarrow \alpha \sim 8\frac{\sigma^2}{c^2} \sim 10^{-5} \sim 1''$$

Multiple Imaging

use Fermat P!

$$2b \sim D\theta \sim (D/2)\alpha \sim \frac{D}{2}\frac{4GM}{bc^2}$$

"Rule of Thumb"

$$\Rightarrow \Sigma > \Sigma_c \equiv \frac{M}{\pi b^2} \sim \frac{c^2}{\pi D G} \sim \frac{10^{17}}{3\,10^{26}\,7\,10^{-11}} \sim 2\,\mathrm{kg\,m^{-2}}$$

Fraunhofer Diffraction

Diffraction – extended source
Interference – few beams.

transmission

Amplitude $\psi(\theta) \sim \int dx\, t(x)\, e^{i\boxed{kx\theta}}$

FT

phase

Intensity $I \sim |\psi|^2$

If aperture size is a, $\theta \sim \frac{1}{ka}$

Young 2 narrow slits

$\psi \sim \cos ka\theta/2$

2 finite slits

slit width δ

Convolve with $\overset{\delta}{\sqcap}$: with width δ

multiply FT by function with width $\Delta\theta \sim 1/k\delta$

Instruments

Telescope

Angular resolution ~ λ/D HW

Interferometer

baseline.

Angular resolution ~ $\lambda/b << \frac{\lambda}{D}$

Grating

N slits of width δ

convolution multiplication

HW

Fresnel Diffraction

Observer/detector near aperture.

$\phi \sim k[(d^2+x^2)^{1/2}-d]$

$\sim kx^2/2d$

$$\psi \sim \int dx\, t(x)\, e^{i\frac{kx^2}{2d}}$$

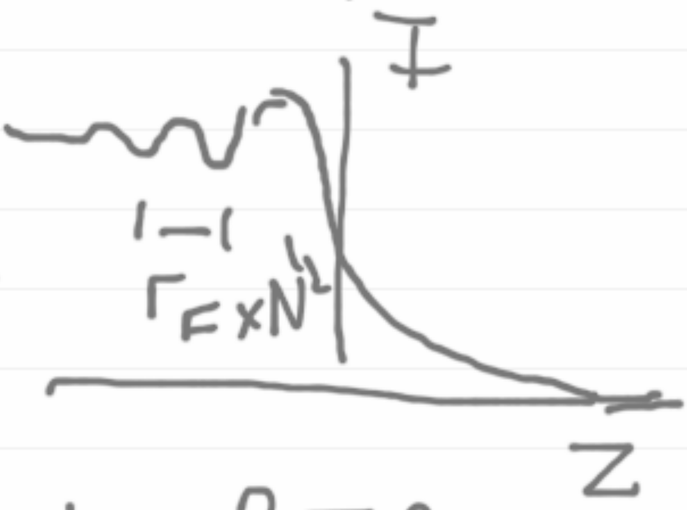
Fresnel length $r_F \sim (\lambda d)^{1/2}$

Fraunhofer if $a << r_F$

Diffraction by straight edge

$$\psi \sim \int_z^\infty dx\, e^{i\frac{kx^2}{2d}}.$$

Plot in complex plane



$kz^2/2d \sim 2\pi n$

Sound

Speed – air $\sim 300\,\mathrm{m\,s^{-1}} \sim (\gamma P/\rho)^{1/2} \propto T^{1/2}$

$\lambda \sim 30\,\nu^{-1}_{\mathrm{kHz}}$ cm – Longitudinal

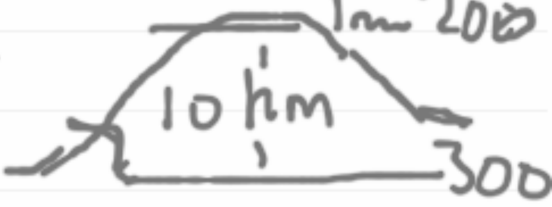
$F \sim \rho v^2 c$, 1 db $\sim 10^{-12}$, 120 db $\sim 1\,\mathrm{W\,m^{-2}}$

threshold   rock concert

$v \sim 50\,\mathrm{nm\,s^{-1}}$ $\sim 50\,\mathrm{mm\,s^{-1}}$

$\delta\rho/\rho \sim \delta P/P \sim M \sim 10^{-10}$ $\sim 10^{-4}$

Refraction – Stratosphere



Also above lake

Generation – Scalar field / wave; $\omega = ck$

Monopole radiation from radial oscillation of sphere

$V(kR)^2$ $v_r \sim V\dfrac{(kR)^2}{kr}$

Near field !   Wave field

—— k^{-1} ——

—— r ——

$$P_M \sim \rho\left[V\frac{(kR)^2}{kr}\right]^2 c\,4\pi r^2 \sim \frac{(4\pi R^2\rho a)^2}{4\pi\rho c} \qquad \frac{dV}{dt} = \omega V$$

$$P_{quad} \sim \left(\frac{V}{c}\right)^2 P_{dip} \sim \left(\frac{V}{c}\right)^4 P_M \text{ etc.}$$

Aerodynamic Sound Generation

From jet engines etc.
In a free flow, fluctuations must conserve momentum

⇒ Quadrupolar emission

⇒ Aerodynamic power per unit volume from turbulence with characteristic speed V and characteristic length l is

$$\sim \frac{(\rho l V^2)^2}{\rho c l^3} \left(\frac{V}{c}\right)^4$$

$$\sim \frac{\rho c^3}{l} \left(\frac{V}{c}\right)^8$$

very sensitive to speed

Aside on EM, Grav. Radiation

EM - vector waves

most power in electric dipole.

dipole moment

$\uparrow \mu \cos\omega t$ $E \sim \frac{\mu R^3}{4\pi\epsilon_0}$ $E \sim \frac{\dot{\mu} R^3}{4\pi\epsilon_0 R r}$

Near — R^{-1} —— ! Wave

—————————— r ——————

$$P_{dip} \sim \epsilon_0 \left(\frac{\mu R^2}{4\pi\epsilon_0 r}\right)^2 c\, 4\pi r^2 \sim \frac{(\omega^2 \mu)^2}{4\pi\epsilon_0 c^{\boxed{3}}}$$ ~dip

GR - tensor waves

most power in mass quadrupole

$Q \sim MR^2$ $\phi \sim G R^3 Q$ $\phi \sim \frac{G R^3 Q}{R r}$

Near —— R^{-1} - - ⋮ Wave

—————————— r ——————

$$P_{quad} \sim \left(\frac{c^5}{G}\right) \left(\frac{R\phi}{c^2}\right)^2 4\pi r^2 \sim \frac{4\pi G(\omega^3 Q)^2}{c^{\boxed{5}}}$$ quad

dimensional power dimensionless amplitude

Music

Equal tempered scale eg Piano

16 ~250 $\boxed{440}$ 500......5000Hz

C^4 $\boxed{A^4}$ C^5

(middle)

$$\frac{\nu_{A\#}}{\nu_A} = 2^{1/12} \simeq e^{\frac{\ln 2}{12}} \simeq 1.06 \text{ etc.}$$

(Commonly tune to whole number ratios)

An individual note contains many (mellifluous) harmonics

eg

Must tune instruments

Power spectrum of "music"

However fluctuations and phases matter! Ear imposes filters.

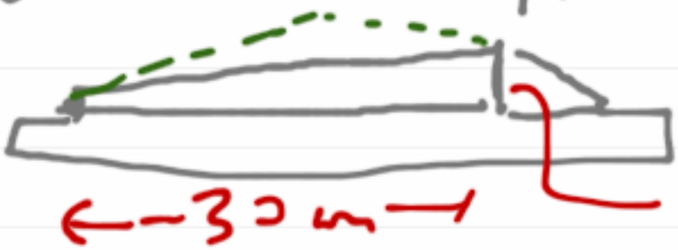
Guitar, violin - chordophone

eg plucked violin string

(Bow plucks many times per stroke)

Sound amplified and "improved"

by resonance effects in wood.



open G string.

$\lambda \sim 60\,\mathrm{cm}$; $\nu \sim 200\,\mathrm{Hz}$, $C_T \sim 120\,\mathrm{m}$

mass per length

$$\mu \sim \overset{\rho}{4 \times 10^{3}} \times \overset{A}{10^{-6}} \sim 4 \times 10^{-3}\,\mathrm{kg\,m^{-1}}$$

$$\Rightarrow T \sim \mu C_T^2 \sim 4 \times 10^{-3} \times 10^{4} \sim 5\,\mathrm{N}$$

tension

$$\text{Strain} \sim T/EA \sim \frac{50}{10^{11} \times 10^{-6}} \sim 5 \times 10^{-4}$$

indep of A

Temperature sensitivity of tuning.

To change by a semitone - C_T changes by 3% strain by 6% $\sim 3 \times 10^{-5}$.

Typical coefficients of linear expansion $\sim 10^{-5}$ So a temperature change of a few degrees will detune a violin

Trumpet, – Aerophone

We can simplify this as a bent pipe of length ~ 1·2 m (allowing for the bell) with a node at the mouthpiece end and an antinode at the bell. The standing wave can be tuned at the lips and changed with the valves. The lowest note played is the first overtone

more accurate c

$$\lambda \sim \frac{4}{3} \times 1{\cdot}2\text{ m} \sim 1{\cdot}6\text{ m}$$

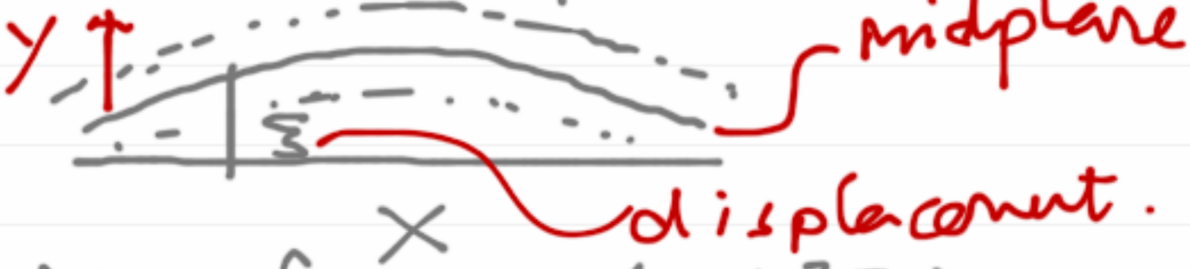
$$\nu \sim \frac{340}{1{\cdot}6} \sim 215\text{ Hz}$$

close to B^b

A semitone change c requires a 6% change in c and a 12% change in T ~ 30°. Can tune by lengthening tube.

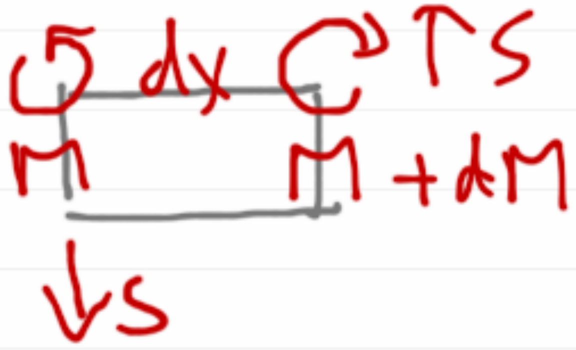
Idiophone – Xylophone

Wooden bar composed of "fibres"



Bending Moment $M \sim \underbrace{\int dy\,dz}_{\text{Area}}\ \underbrace{E}_{\text{Young's Mod.}} \underbrace{\left(y\frac{\partial^2\xi}{\partial x^2}\right)}_{\text{Strain}} \underbrace{y}_{\text{lever arm}}$

$$M \sim w t^3 E \frac{\partial^2\xi}{\partial x^2}$$



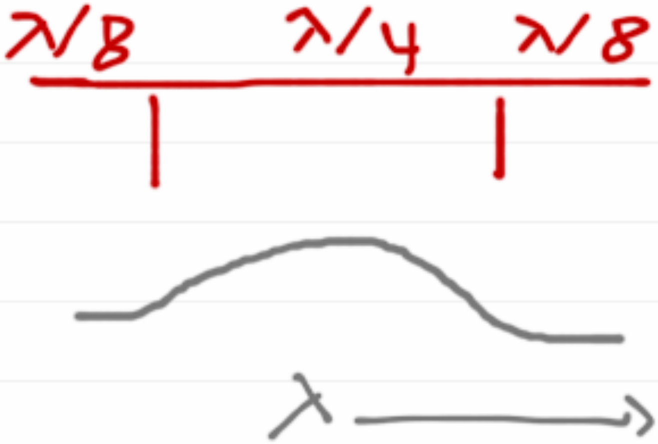
$$dM \simeq S\,dx$$

$$dS \simeq \underbrace{\mu}_{\text{mass/length}} dx \frac{\partial^2\xi}{\partial t^2}$$

$$w t^3 E \frac{\partial^4\xi}{\partial x^4} = -\mu \frac{\partial^2\xi}{\partial t^2}$$

This equation, unlike the wave equation has hyperbolic as well as harmonic solutions. In order to make music, we need to impose boundary conditions which will select harmonic solutions.

eg



$3 \times 10^3 \times 10^{\cancel{4}}$

$$k \sim 2\pi / .15 \sim 40\, m^{-1}$$

$$\omega \sim \left(\frac{W t^3}{m} E\right)^{1/2} k^2 \quad \propto k^2 !$$

$$\sim \left(\frac{E}{2\rho}\right)^{1/2} k^2 t$$

$$\Rightarrow \sim \left(\frac{10^{10}}{10^3}\right)^{1/2} \frac{40^2\, 10^{-3}}{6}$$

$$\sim 600 \text{ Hz}.$$

People and other animals

- Allometry
- Blood flow
- Turbulence
- Structural limit

Allometry (Scaling relations)

Range of animal sizes. ρ const.

$\rho \sim 1000\ \mathrm{kgm^{-3}}$

line below?

Mycoplasma	100 fg	500 nm (10^{-4} r
Bee	100 mg	5 mm
Human	100 kg	500 mm
Blue Whale (H_2O) (>dinosaurs!)	100 Mg	5 m
	$\times 10^{21}$ range	$\times 10^{7}$ range

Scaling :- Schmidt Nielsen.

Many approximate relations.

eg M_{brain}

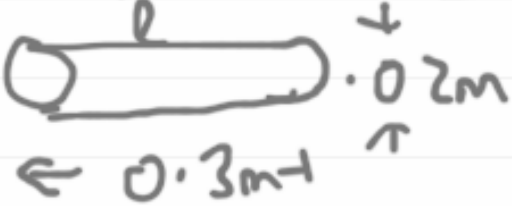
$\propto M_{body}^{1.2}$

$$\left(\frac{\text{Metabolic rate}}{100\,\mathrm{W}}\right) \sim \left(\frac{M_{body}}{100\,\mathrm{kg}}\right)^{.7}$$

$\sim$ Area ?

Blood Flow

Healthy human circulates 5l of blood min^{-1}

Aorta 2a 

$v \sim \frac{5\times10^{-3}}{60\times4\times10^{-4}} \sim .2ms^{-1}$

Navier-Stokes equation

$$\frac{d\bar{v}}{dt} = -\frac{1}{\rho}\nabla P + \bar{g} + \nu\nabla^2\bar{v}$$

Kinematic viscosity

	H_2O	blood	air
$\nu\ m^2s^{-1}$	10^{-6}	3×10^{-6}	10^{-5}

$\nu \sim v\, l_{mfp}$. for gas

Pipe flow $d\bar{v}/dt, g\sim0$ $v \sim a^2P'/\rho\nu$

$$\Rightarrow \Delta P \sim \frac{10^3 . 2\ 3\times10^{-6} . 3}{10^{-4}} \sim 20Nm^{-2}$$

Blood pressure 110-60 mmHg $\sim .05\times1.4\times10^4\times10$
$\sim 7k.Nm^{-2}$

Most of pressure pushes flow through capillaries.

Oxygen supply 250ml O_2 per minute

"Power" $\approx \frac{.25\times10^{-3}}{60}\times1\times30MJ\,kg^{-1} = 120W$

cf 70W heat

Turbulence

We assumed the flow is laminar. If we increase the pressure gradient, the flow becomes irregular. The critical parameter is:

Flow is aided by elastic waves in wall

Reynolds' No $\sim \frac{\left|\frac{\partial \vec{v}}{\partial t}\right|}{|\nu \nabla^2 \vec{v}|} \sim \frac{VL}{\nu}$

$R \lesssim 1000$ - flow laminar ignore inertia

$R \gtrsim 1000$ - flow unstable - turbulent.

Flows with same R are similar.

For aorta ~ $R \sim \frac{.2 \times .02}{3 \times 10^{-6}} \sim 1000$

cf nuclear reactor design!

A heart as small as can be and remain efficient. Most animals have evolved this performance

As devolve to N arteries / capillaries of size a, expected Flow $\propto NaR$

$\Rightarrow N \sim 1/a$, $P' \sim a^{-3}$?

Swimming $R_{\text{whale}} \sim 10^8$ - turbulent wake

$R_{\text{bug}} \sim .1$ - inertia unimportant

flow reversible. Propel with waves, screws.

Structural limits

Bone has $\rho \sim 1500\,kgm^{-3}$, $E \sim 20GNm^{-2}$
mass fraction ~ $0.06(M/M_{kg})^{1.1}$

Xylophone $M \sim \frac{Ed^4\xi}{l^2}$ $\downarrow F_g$

$> F\xi$ for restoring force to prevent

Buckling

So stress $< E(d/l)^2 \sim 200\,MNm^{-2}$ for $d \sim .1l$.

cf $\frac{\overset{\text{2 femurs}}{50kg} \times \overset{g}{10}}{(.1)^2} \sim 50\,MNm^{-2}$

So 4g places you in danger of fracture! nb cracks, very rough

Larger animals are more vulnerable
Elephants have $d/l \sim .2$
Blue whales ($M \sim L^3$, Stress $\sim L$)
cannot stand on land.

Some Helpful Suggestions

- Estimation needs to precede or obviate a proper calculation
- Sometimes we need a 20% estimate; sometimes an order of magnitude in the exponent suffices!
- Its OK to know something or look it up and use it!
- Physicists use strange, idiosyncratic units. Regrettable but true. Use the units that are useful and practice changing units.
- Practice mental arithmetic
- If you don't know, guess and learn
- Dimensions matter!
- Be bold. You won't "bat 100".
- Look out for "sanity checks"

- Scaling arguments can bypass looking up lots of constants
- Thinking about whether or not 2π s should be there - eg $\hbar$ or h - can lead to greater accuracy.
- Scaling arguments can bypass solving hard problems.
- Temper your enthusiasm for understanding a new area with an appreciation for how subtle many "real world" problems can be
- <u>Be</u> skeptical of numbers from the web or newspapers especially on politically-charged issues
- Physical descriptions can be expressed as functional relationships between independent dimensionless combinations of relevent dimensional quantities
- Learn how to skim large amounts of material for the key facts, numbers etc you need and then check!
- Follow the energy!

- Many problems are flow processes. Consider what is conserved and estimate flow rates and residence times.
- Handle acoustic, electromagnetic and gravitational radiation by evolving near field to $r \sim R^{-1}$ and wave field beyond.
- Flows typically become turbulent when the Reynolds number $R \sim LV/\nu \gtrsim 1000$
- Structures typically fail when (buckle) when the moment associated with a small displacement of an external force exceeds the elastic restoring force.

Some Important Numbers

$c \simeq 300\,\mathrm{Mm\,s^{-1}} \sim 3\times10^{5}\,\mathrm{km\,s^{-1}}$

Sound speed $c_s \sim 300\,\mathrm{m\,s^{-1}} \sim 10^{-6}\,c$!

$1\,\mathrm{pc} \sim 3\,\mathrm{ltyr} \sim 3\times10^{16}\,\mathrm{m}$

Age of Universe $t_0 \sim 14\,\mathrm{Gyr}$

$G \sim 7\times10^{-11}\,\mathrm{kg^{-1}\,m^{3}\,s^{-2}}$

$\hbar \sim 10^{-34}\,\mathrm{J\,s}$

$\sigma \sim 6\times10^{-8}\,\mathrm{W\,m^{-2}\,T^{-4}}$

$M_\odot \sim 2\times10^{30}\,\mathrm{kg}$, $L_\odot \sim 4\times10^{26}\,\mathrm{W}$

$T_\odot \sim 6000\,\mathrm{K}$, $\mathrm{AU} \sim 1.5\times10^{11}\,\mathrm{m}$

$1\,\mathrm{eV} \sim 1.6\times10^{-19}\,\mathrm{J} \simeq 10^{4}\,\mathrm{K}$
$\sim 1.2\,\mu \sim 240\,\mathrm{THz}$

$M_\oplus \sim 6\times10^{24}\,\mathrm{kg}$; $R_\oplus \sim 6.4\,\mathrm{Mm}$

$F_\oplus^{\odot} \sim 1.4\,\mathrm{kW\,m^{-2}}$ $g_\oplus \sim 10\,\mathrm{m\,s^{-1}}$

1 Calorie $\simeq$ 1000 calories $\simeq$ 4 kJ

1 ton $\simeq$ 1000 kg

1 kiloton (TNT) $\simeq$ 4 GJ

$a_0 \sim 50\,\mathrm{pm}$, $m_p \sim 1.7\times10^{-27}\,\mathrm{kg}$

$\sim 1\,\mathrm{GeV}$

$m_e \sim 10^{-30}\,\mathrm{kg}$

$\sim 0.5\,\mathrm{MeV}$

Powers of 1000

yzafpnμm|kMGTPEZY X(unofficial)

1 Gallon $H_2O \sim 4\,\mathrm{kg}$

$1'' \sim 5\times10^{-6}$ rad.

4π steradians $\sim 40{,}000\,\mathrm{deg}^2$

$\mu_0 = 4\pi\times10^{-7} \sim 10^{-6}\,\mathrm{Hm^{-1}}$

$\epsilon_0 = c^{-2}\mu_0^{-1} \sim 10^{-11}\,\mathrm{Fm^{-1}}$

$\rho_{air} \sim 1\,\mathrm{kgm^{-3}}$ $\rho_{H_2O} - 1\,\mathrm{Mg\,m^{-3}}$

$\nu_{air} \sim 10^{-5}$, $\nu_{blood} \sim 3\times10^{-5}$ $\nu_{H_2O} \sim 10^{-6}\,\mathrm{m^2 s^{-1}}$

Blood volume $\sim 5\times10^{-3}\,\mathrm{m^3}$ (5l)

circulation time ~ 1 minute.

Bone $\rho \sim 1500\,\mathrm{kgm^{-3}}$, $E \sim 2\times10^{10}\,\mathrm{Nm^{-2}}$

Sketch Solutions

1 a. $\frac{M}{M_p} \sim \underset{.04}{f} \frac{c^3 \overset{\text{age}}{T}}{G M_p} \sim .01 \frac{3\times10^{25}\times 4\times10^{17}}{7\times10^{-11}\times 1.7\times10^{-27}}$

$\sim 5\times10^{78}$

b. $P \sim \rho g R \sim 5\times10^{3}\times1\times7\times10^{6}$

$\sim 3\times10^{11}\ \mathrm{N\,m^{-2}}$

c. $F_{\oplus} \sim \frac{B^2}{\mu_0} c \sim 1.4\times10^{3}$

$\Rightarrow B \sim \left(\frac{1.4\times10^{3}}{3\times10^{14}}\right)^{1/2} \sim 2\ \mu\mathrm{T}$

d. $N_{\text{trees}} \sim \frac{(7\times10^{6})^{2}(.3)^{2}}{10}\times\frac{1}{3}$

$\sim 2\times10^{11}$

e. $\omega \sim \frac{6}{24\times3600} \sim 7\times10^{-5}$

$I \sim \frac{2}{5}\underset{\rho H}{10^{4}\times4\pi}\ \underset{R}{(7\times10^{6})^{4}} \sim 10^{32}\ \mathrm{kg\,m^2}$

" $T \sim \frac{1}{2} I \omega^2 \sim 200$ ZJ

f. $\tau \sim \frac{2\pi R}{8c} \sim \frac{6 \times 5 \times 10^5}{10^4 \times 3 \times 10^8} \sim .1$ ns

g. $\# \left(\frac{0.1}{3 \times 10^{-6}} \right)^2 \sim 3 \times 10^6$

$\Rightarrow \Sigma V_{hair} \sim \frac{3 \times 10^6 \times .01\,m}{3 \times 10^6\,s}$.

$\sim 0.01\,m$

h. $\lambda_{green} \sim \sqrt{400.700} \sim 520\,nm$.

$\nu_{green} \sim 3 \times 10^8 / 5.2 \times 10^{-7} \sim 600\,THz$

$\lambda_{FM} \sim \frac{3 \times 10^8}{10^8} \sim 3\,m$.

i. $\underbrace{\frac{v}{2^{1/2}}}_{v_\perp} \times \underbrace{2 \frac{v}{2^{1/2} g}}_{t} \sim d \sim 100\,m$

$\Rightarrow v \sim 30\,m\,s^{-1}$

(an underestimate because of air resistance. Pitcher throws @ 40 m s^{-1})

j. $\frac{50\,kg}{\underbrace{16 \times 1.7 \times 10^{-27}}_{A}} \sim 2 \times 10^{27}$

M_{H_2O}

2. $\frac{M}{m_p} \sim \left(\frac{m_e c R}{\hbar}\right)^3 \sim \frac{\boxed{\frac{GM^2}{R}}}{m_e c^2}$ ~ internal energy

N_e

$\Rightarrow R \sim \frac{GM}{c^2}\left(\frac{m_p}{m_e}\right)$

a) $\frac{M}{m_p} \sim \left(\frac{M m_p}{m_{pL}^2}\right)^3 \Rightarrow M \sim \frac{m_{pL}^3}{m_p^2}$

b) $R \sim \frac{m_{pL}^2}{m_e m_p} r_{pL}$

c) $F \sim \frac{c^5}{G}\left(\frac{\phi}{c^2}\right)^2\left(\frac{\omega}{c}\right)^2 c \sim \frac{\omega^2\phi^2}{c.G}$

power

1

dimensionless amplitude

$\sim 1/\text{Area}$

d) ⓪ c/ω, $L \sim \frac{\omega^2}{cG} \times \left(\frac{c}{\omega}\right)^2 \times \left(\frac{GMR^2\omega^3}{c^3}\right)^2$

Area $\qquad \phi$

$\sim GM^2R^4\omega^6c^{-5} \sim (c^5/G)(m_e/m_p)^5$

e) # in $\frac{GM^2}{R}\left(\frac{\omega}{L}\right) \sim (m_e/m_p)^{5/2}$

3a) $\lambda \sim 2\times 0.3\,\text{m} \sim 0.6\,\text{m}$

b) $\nu \sim 250\,\text{Hz} \Longrightarrow v \sim 150\,\text{m s}^{-1}$

$T \sim \underset{m/L}{\sigma} v^2 \sim 8\times10^3\times10^{-6}\times 2\times10^4$

$\sim 200\,\text{N}$

c)

$$\mu \frac{\partial^2\phi}{\partial x^2} = \rho \frac{\partial^2\phi}{\partial t^2}$$

$$\underbrace{a^2}_{\text{area}}\, d\underbrace{\left(\mu a \overbrace{\frac{\partial\phi}{\partial x}}^{\text{strain}}\right)}_{\text{stress}} \underbrace{a}_{\text{lever arm}} \sim \underbrace{\rho a^2 a^2 dx}_{\text{MI}} \underbrace{\frac{\partial^2\phi}{\partial t^2}}_{\text{ang. acc}^{\text{n}}.}$$

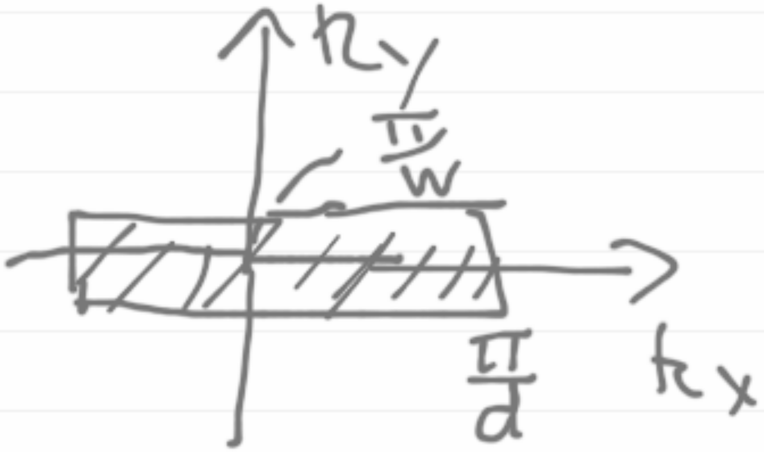
No tension because no extension

d) $v \sim \left(\dfrac{10^{11}}{5\times10^3}\right)^{1/2} \sim 4\,\text{km s}^{-1}$

$\nu \sim 4/0.6 \sim 7\,\text{kHz}$

4.

a)



b) $\omega_{max} \sim k_B T/\hbar$

c) $U_{rad} \sim \left(\frac{\omega_{max}}{c}\right)^3 k_B T\, w^2 d$

$$\sim \frac{k_B^4 T^4}{\hbar^3 c^3} w^2 d.$$

d) ZPE because mode is an oscillator and UP dictates it cannot have "x" = "p" = 0

e) $\frac{\Delta U}{w^2 d} \sim \left(\frac{1}{d}\right)^3 \left(\frac{\hbar c}{d}\right) \Rightarrow \Delta P \sim -\frac{1}{w^2}\frac{\partial \Delta U}{\partial d} \sim \frac{\hbar c}{d^4}$

f) $|\Delta P| \sim \frac{10^{-34}\, 3\times 10^{8}}{10^{-28}} \sim 300\,\mathrm{N\,m^{-2}}$

5a) Duck: $d \sim 0.15\,\mathrm{m}$ $\rho \sim 300\,\mathrm{kg\,m^{-3}}$

$\Rightarrow m \sim 2\,\mathrm{kg}$

b) $v \sim c \sim 300\,\mathrm{m\,s^{-1}}$

$$n \sim \frac{1}{2\times10^{-27}\times 30} \sim 2\times10^{25}\,\mathrm{m^{-3}}$$

$\sigma \sim 10^{-18}\,\mathrm{m^2}$

$\Rightarrow l \sim 1/n\sigma \sim 100\,\mathrm{nm}$

$\Rightarrow \nu \sim \frac{1}{3} v l \sim 10^{-5}\,\mathrm{m^2\,s^{-1}}$

c) $R \sim Vd/\nu \sim 2\times10^{4}(V/\mathrm{m\,s^{-1}})$

$\gg 1000 \Rightarrow$ turbulent flow

d) $\rho v^2 d^2 \sim mg$

$$\Rightarrow v \sim \left(\frac{2\times10}{1\times.04}\right)^{1/2} \sim 25\,\mathrm{m\,s^{-1}}$$