Clashing Theories of Unemployment*

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Abstract

General-equilibrium models for studying monetary influences in general and the zero lower bound on the nominal interest rate in particular contain implicit theories of unemployment. In some cases, the theory is explicit. When the nominal rate is above the level that clears the current market for output, the excess supply shows up as diminished output, lower employment, and higher unemployment. Quite separately, the Diamond-Mortensen-Pissarides model is a widely accepted and well developed account of turnover, wage determination, and unemployment. The DMP model is a clashing theory of unemployment, in the sense that its determinants of unemployment do not include any variables that signal an excess supply of current output. In consequence, a general-equilibrium monetary model with a DMP labor market generally has no equilibrium. After demonstrating the clash in a minimal but adequate setting, I consider modifications of the DMP model that permit the complete model to have an equilibrium. No fully satisfactory modification has yet appeared.

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> A landing on the non-Walrasian continent has been made. Whatever further exploration may reveal, it has been a mind-expanding trip: We need never go back to $\dot{p} = \alpha(D - S)$ and $q = \min(D, S)$.
>
> Phelps and Winter (1970)

With the short-term nominal interest rate near its minimum feasible value of zero in the U.S. and some other advanced economies for the past few years, macroeconomics has renewed and advanced the study of the implications of the zero lower bound for economic activity in general and unemployment in particular. According to the models, when the interest rate is held above its market-clearing level, the supply of current output exceeds demand. Actual current output falls short of its market-clearing level and unemployment is above its normal level. The models provide a widely accepted account of the low levels of output and high levels of unemployment in recent years. Notwithstanding Phelps and Winter's declaration four decades ago, the $q = \min(D, S)$ model has a firm grip on macroeconomic thinking.

At the same time, the Diamond-Mortensen-Pissarides (DMP) model of unemployment is widely accepted as the most realistic account of unemployment based on a careful and full statement of the underlying economic principles governing labor turnover and wage determination. The model is relentlessly non-Walrasian—it describes transactions, not market-clearing.

Although the zero lower bound stimulated this paper and much other recent work, I frame the issue somewhat more generally, as the consequences of a short-term risk-free nominal interest held above the level that would clear the intertemporal output market. The zero lower bound is one reason for an elevated nominal rate, but central-bank policy is another. Whenever a central bank raises its policy rate to a high level to head off inflation, the issues in this paper are in play.

The starting point for this paper is the incompatibility of the DMP account of the labor market and the view of unemployment embedded in a simple general-equilibrium model that generates high unemployment as a consequence of an elevated nominal rate. A compact explanation is that, in the general-equilibrium model with the interest rate held above its market-clearing level, the model lets unemployment be a free variable that adjusts to clear the current output market. On the other hand, the DMP model prescribes the unemployment rate as a function of a limited set of variables, none of which responds to excess supply in the current output market. The two theories of unemployment clash. Another way to express the conclusion is that a simple GE model extended to include the DMP labor market has no solution with an elevated nominal rate.

The clash just described is only the starting point for the paper, because extensions of the DMP model do let unemployment respond to forces from the product market. Much of the paper considers those extensions in the hope of finding a general-equilibrium model with a DMP labor market that retains the ability of a general-equilibrium model to account for the large increase in unemployment that generally occurs when the zero interest bound is binding, while retaining DMP's account of that high level of unemployment as an equilibrium of the labor market.

A reconciliation of general-equilibrium models with DMP would deal with a significant deficiency in a number of existing applications of general-equilibrium modeling to the zero-lower-bound economy in particular and an elevated nominal rate in general. Those models arbitrarily (but totally realistically) pick out the labor-market-clearing condition to fail, while requiring the other equilibrium conditions of the model to hold. In particular, consumers continue to satisfy their Euler equations. Adding a new equation to a general-equilibrium model—the equation that pins the nominal interest rate at zero or some other elevated value—seems to call for dropping one existing equation. The obvious choice is labor-market-clearing. But no rationale for that choice, based on economic fundamentals, has appeared in the literature on monetary policy in general equilibrium.

The DMP model of the labor market replaces the simple labor-market-clearing condition of a general-equilibrium model. But it is not a candidate to be dropped out when the zero lower bound is binding. Some other free variable must appear in the model to allow the model to have a solution in which employment in the product market is consistent with unemployment that satisfies the DMP equations.

I consider several potential reconciliations of clashing unemployment theories. One makes the inflation rate the needed free variable. If expected inflation is high enough, an elevated nominal rate does not imply an elevated real rate, and the economy can have a standard equilibrium where the DMP equations provide the equivalent of a simple labor-market clearing condition. That is obvious and uninteresting. The Federal Reserve has been hoping for that much inflation since 2008 but has been unable to achieve it. There is a second possible equilibrium, the one that might describe economies with high unemployment at the zero bound, notably the United States. Here is the scenario: Adverse developments in the product market—the collapse of durables demand in general and homebulding in particular—bring a negative equilibrium real interest rate. The rate of inflation falls a bit. With the zero lower bound on the nominal rate elevating it above the market-clearing level, the unemployment rate consistent with the outcome in the product market is high. In the DMP part of the model, employers face low incentives to recruit new workers. The reason is that the starting real wage is a fixed amount—a social norm. But the wage remains constant in nominal terms for a couple of years, so with diminished inflation, the present value of the real wage is higher when inflation is lower. The DMP model makes unemployment highly sensitive to this present value, because the employer's incentive to recruit depends on the difference between present value of the worker's marginal product and the present value of the wage. The higher real wage causes the DMP unemployment rate to be higher. There may be an inflation rate—possibly negative—where the two models agree and the clash resolved. I emphasize the "may"—in the model of this paper, it is hard to generate such an equilibrium. In many cases there is no such inflation rate and the combined model has no solution.

A second reconciliation restores unemployment as a free variable, but with an equilibrium interpretation. In the DMP model of Mortensen and Pissarides (1994), the unemployment rate is one of the determinants of the net value to an employer from a new hire. When unemployment is high, a bargaining job candidate's outside option is less valuable, because finding another employment opportunity is harder. In the Nash bargain, the employer gains more of the surplus. Employers recruit more aggressively because the payoff to a new hire is greater. High unemployment disappears. Absent some other change, such as a decline in productivity, unemployment has a unique equilibrium. All this follows from the Nash-bargained wage. Now consider an alternative bargaining protocol that makes the employer's share of the surplus decline slightly with unemployment. With just the right amount of decline, the DMP equilibrium becomes indeterminate. This setup violates no principle of bargaining. I call this the *flexible unemployment hypothesis*. This version of the DMP model provides a full rationale for the otherwise arbitrary assumption of many existing GE models that the unemployment rate is a free variable.

Neither of these reconciliations is truly satisfactory. Both rest on arbitrary properties of the wage-determination part of the DMP model. In the first, under deflation, employers suffer a disadvantage arising from the unexplained stickiness of nominal wages. In the second, employers suffer a similar disadvantage from paying their workers a bit more when unemployment is high.

A third approach—as yet undeveloped—interprets the product market as remaining out of equilibrium when the zero bound is binding. Employers perceive low gains to adding a worker because they cannot sell the extra output the worker produces. Most informal discussions of the macroeconomics of the zero lower bound seem to adopt this view. The time may be ripe for a formalization.

Lastly, an active body of recent research has documented a reduction in matching efficiency in the labor market during the period of the zero lower bound. The DMP model implies a rise in unemployment when the matching process generates fewer hires given unemployment and vacancies. That part of the bulge in unemployment is outside the reconciliation problem identified here. It is only a fraction of the total increase in recent years.

The paper starts with a discussion of the labor market and the properties of the DMP model. Then I develop a general-equilibrium model embodying the DMP model of the labor market and a model of the product market based on standard principles. I explore the conditions under which the model has equilibria with high unemployment and an elevated nominal interest rate. These conditions are quite restrictive.

1 The Labor Market

I adopt a simple version of the DMP theory of unemployment. Like much of the recent literature on the DMP model, I consider modes of wage determination different from the Nash bargain of the canonical model of Mortensen and Pissarides (1994). Also, I simplify the treatment of labor-market dynamics by considering only the stochastic equilibrium of labor turnover, which means that the unemployment rate u measures the tightness of the labor market. The vacancy rate enters the picture only in fast transitional dynamics of the matching process, which can be ignored in a quarterly model without losing much. Thus the recruiting success rate is an increasing function $h(u)$ of the unemployment rate. Success is higher when unemployment is higher and employers find qualified job-seekers more easily. Hall (2009) discusses this approach more fully.

Without loss of generality, I decompose the wage paid to the worker into two parts, corresponding to a two-part pricing contract (the decomposition is conceptual, not a suggestion that actual compensation practices take this form). The worker pays a present value J, the *job value*, to the employer for the privilege of holding the job and then receives a flow of compensation equal to the worker's marginal product.

The essence of the DMP model of unemployment is a pair of equations involving the job value. The first holds that, in equilibrium, firms expect zero profit from recruiting workers. The cost of recruiting (holding a vacancy open) is γ per period, taken to be constant in output terms. The zero-profit condition for recruiting equates the expected benefit of recruiting to its cost:

$$h(u)J = \gamma. \tag{1}$$

Thus unemployment rises if the job value J falls. In slack markets with lower J, a worker pays less for a job. Because $h(u)$ is a stable function of unemployment alone and γ is a constant, the DMP model implies a stable relationship, $J_Z(u)$, between unemployment and the job value.

The second equation—which I call *wage determination*—states the job value $J = \tilde{J}(u, \eta)$ as a function of u and certain other determinants. In the canonical model of Mortensen and Pissarides (1994), a worker and an employer make a Nash bargain that sets a wage to divide their joint surplus in fixed proportion. Unemployment is one of the determinants of the Nash job-value function—when unemployment is high, the match surplus arising from labor-market frictions is greater. The job value, a fixed share of that surplus, is also higher. The worker has to pay more for the job because jobs are harder to find. Two other variables—the marginal product of labor, p, and the flow value of time spent not working (as an improvement over working), z, also enter the Nash job-value function. These are the two elements of the vector η in $\tilde{J}(u, \eta)$. The DMP literature has concentrated on explaining movements in unemployment as responses to changes in total factor productivity, which is the fundamental underlying determinant of the marginal product of labor. Movements in the flow value of not working, z, rarely figure in explanations of unemployment.

Another potential source of shifts of the wage-determination equation is inflation. The job-value function will deliver a higher value of J when inflation is expected to be higher, to the extent that inflation erodes the real value of the bargained initial wage during the course of a job. Gertler, Sala and Trigari (2008) pursues this approach.

Figure 1 shows the DMP account of the increase in a recession as explained in Mortensen and Pissarides (1994). In consequence of a drop in productivity, the Nash wage determination curve shifts downward. The new equilibrium occurs down and to the right along the stable zero-profit curve.

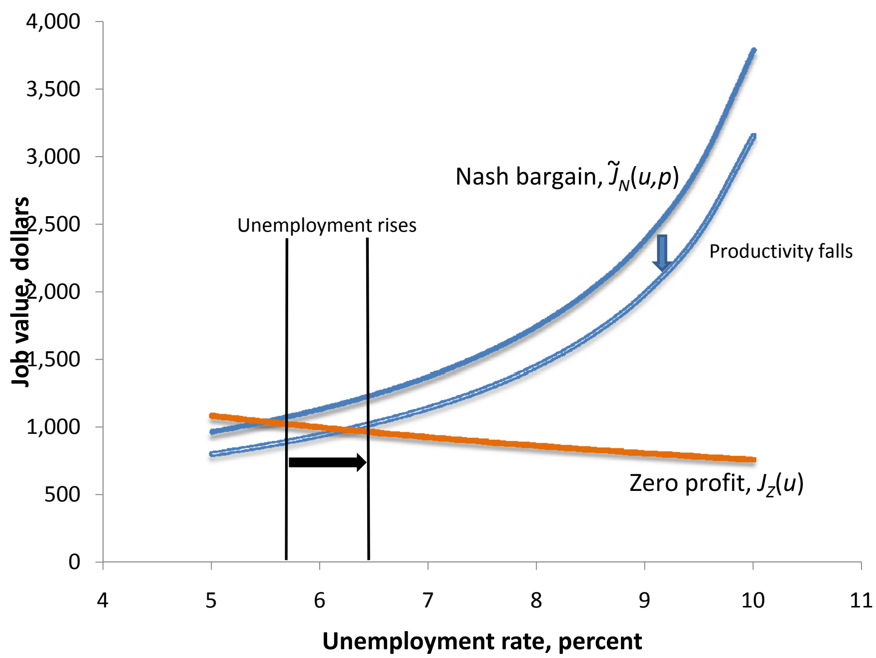

Figure 1: DMP Account of an Increase in Unemployment Caused by a Decline in Productivity

Two developments have cast doubt on the relevance of the recession mechanism of Figure 1. First, Shimer's (2005) influential paper showed than it would take a gigantic drop in productivity to cause the rise in unemployment in a typical recession, based on realistic values of the parameters of the DMP model. Second, productivity has *increased* in recent recessions. Figure 2 shows the Bureau of Labor Statistics' measure of total factor productivity in U.S. business since 1999, along with a projection of its levels in 2008 and 2009 had it continued to grow in those years at the same rate as from 1999 through 2007. Productivity grew almost at normal rates during the huge contraction that started in 2008. To generate an increase in unemployment driven by productivity, an actual decline in productivity would be needed.

Shimer's paper has stimulated an interesting literature—surveyed in Rogerson and Shimer (2010)—that alters the canonical DMP model to boost the response of unemployment to productivity. But with rising productivity in a recession, the stronger response is an embarrassment, making it even harder to square the behavior of the U.S. economy with the DMP model.

1.1 Shifts of the zero-profit curve

The DMP account of recessions generally took the zero-profit curve to be stable and viewed the rise in unemployment in a recession as a movement along that stable curve, resulting

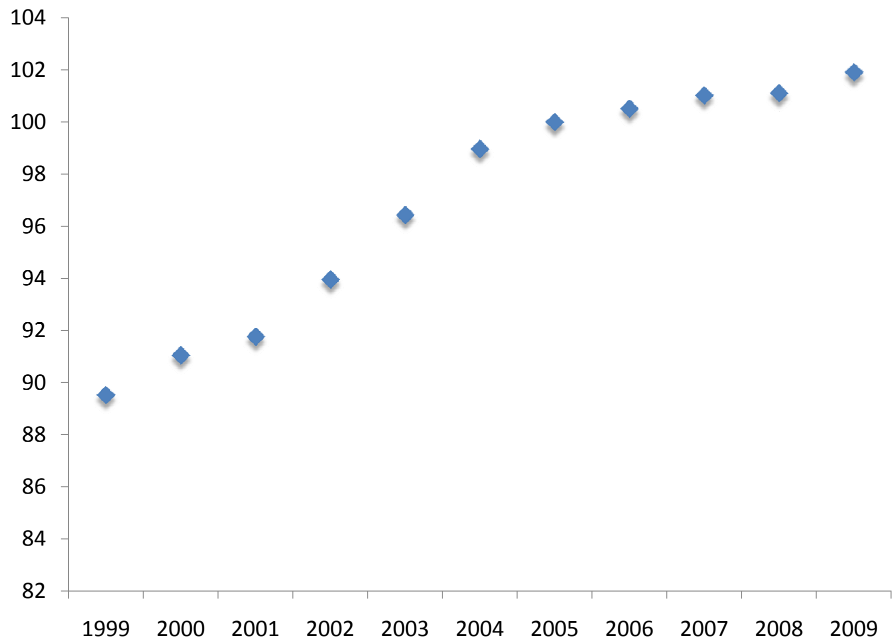

Figure 2: Actual Total Factor Productivity, 1999-2009, and Levels in 2008 and 2009 Projected at Earlier Growth Rate

from a shift of the wage-determination curve. Recently that view has come into question, as the matching process has become less effective and unemployment has remained high even though vacancies have risen. I explore this topic briefly at the end of the paper. I am skeptical that shifts of the zero-profit curve played a dominant role in the rise in unemployment since 2008, but recognize that something happened in the labor market that shifted the curve adversely.

1.2 The wage-determination curve

In this discussion, I assume that the separation rate is a constant s. Data from JOLTS and the CPS support this assumption as a useful rough approximation. The separation rate has declined along a smooth trend in recent decades.

Figure 3 shows two extremes among the many models of wage determination under recent discussion. The figure also includes the zero profit curve of equation (37). The upward-sloping wage-determination curve labeled *Nash bargain* plots the employer's half of the surplus under a symmetric Nash bargain in the DMP model, according to equation (4) in Shimer (2005):

$$\tilde{J}_N(u) = 0.5\frac{p - z}{r + 0.5\phi(u) + s}. \tag{2}$$

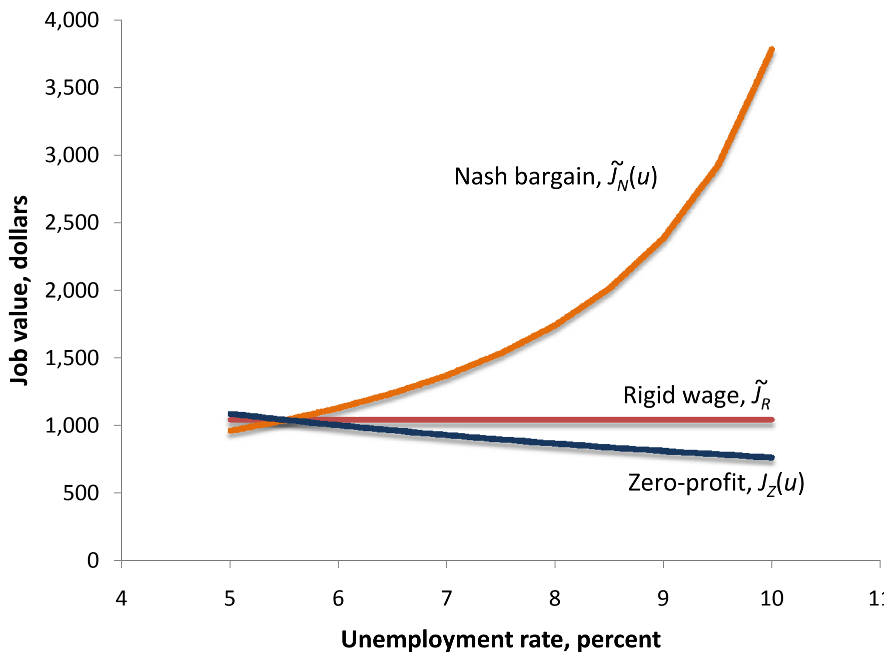

Figure 3: Alternative Views of Equilibrium Unemployment

Here $\tilde{J}_N(u)$ is the job value, the part of the surplus the employer captures, z is the flow benefit of not working, which I take to be 70 percent of p (see Hall and Milgrom (2008)), r is the real interest rate, which I take to be 5 percent per year, $\phi(u)$ is the job-finding rate function, which I take from the lower panel of Table 1, and s is the separation rate, which I take to be 5 percent per month. I take p to have the value that generates the job value shown in Figure 12 at 5.5 percent unemployment, $J = \$1{,}040$.

The only plausible source of a major shift of the Nash wage-determination function is a decline in $p - z$. In turn, given the implausibility of an increase in the value of non-employment activities z as the causal force of a recession, the only possible source of a jump in unemployment is a decline in the marginal product of labor, p. Shimer (2005) showed that only a large decrease could bring about the 5-percentage-point increase in unemployment that occurred in 2008 and 2009. But productivity actually rose during that period. Shifts of the Nash wage-determination function are unlikely candidates for explaining the recent rise in unemployment or any other important movements in unemployment as well. Here I am excluding Hagedorn and Manovskii's (2008) Nash-bargaining model, which makes the contrary claim, on the grounds that its parameters imply much too high a Frisch elasticity of labor supply.

The flat line in Figure 3 labeled *Rigid wage* illustrates Shimer's (2004) proposition that a rigid wage, unresponsive to conditions in the labor market, would lead to a highly unstable wage-determination curve and thus support explanations of large movements of unemployment in the DMP framework in response to shifts in productivity. With a rigid wage w, the job value is

$$J_R = \frac{p - w}{r + s}. \tag{3}$$

A small decline in p or a small increase in w would lower the rigid-wage line in Figure 3 and generate a large increase in unemployment. Note that the wage-rigidity model is quite specific about the driving force—fluctuations in p or w have huge effects, but other potential driving forces do not shift the wage-determination curve at all and cannot explain a large increase in unemployment in the DMP framework.

Other contributions to the recent DMP wage-determination literature have not extended the sources of shifts of the wage-determination curve outside those just discussed, except for **?**, which invokes nominal stickiness. Hagedorn and Manovskii (2008) present alternative parameter values for the Nash wage-determination case, but the source of shifts of the wage-determination curve remains productivity. Hall and Milgrom (2008) adopt an alternating-offer bargaining setup, which brings delay costs into the model as additional determinants of unemployment, but these costs are not portrayed as sources of fluctuations—they are taken as stable parameters.

1.3 Specifying the labor-market model

I adopt a specification for the DMP wage-determination equation that nests an equation with properties similar to those used in the DMP literature and an equation that captures the basic feature of New Keynesian models where workers have higher bargaining power when the price level is declining, because the real value of the wage norm rises. The norm of wage determination is that the initial wage bargain is a constant fraction $\bar{w}$ of the stationary value of the marginal product of labor. Wages are indexed to prices with an elasticity of $1 - \omega$. In the standard DMP specification, $\omega = 0$. If the wage is fixed in nominal terms for the duration of the job, $\omega = 1$. Thus ω is a measure of nominal wage stickiness. Jobs last an average of just over two years in the model, so this specification is plausible. The wage-determination function with positive ω resembles the New Keynesian formulation in, for example, Christiano, Eichenbaum and Evans (2005), where workers set wages in nominal terms and can only adjust them at random intervals. However, the function is simpler because it does not hypothesize that wage setting anticipates the wage erosion from inflation and offsets it by raising the initial wage.

The job value is the present value of the difference between the marginal product of labor and the wage. Although there is no obstacle to evaluating the job value exactly, I introduce two approximations for other reasons. The first is to use the stationary value of the marginal product of labor rather than the realized stochastic value. The result is to stabilize the job value in much the same way that the Nash bargain in the DMP model stabilizes it. In that model, a higher marginal product enhances the worker's outside option almost in proportion, so the surplus is not much affected by higher productivity. The job value is the employer's given share of the surplus, so the job value does not respond much to changes in the marginal product of labor. Shimer (2010) describes the settings where, as in the specification adopted here, shifts in productivity have no effect on unemployment.

The second simplification is to use the utility discount ratio β rather than the economy's stochastic discounter in evaluating the job value. This simplification has no effect on any conclusion of the paper. I make the assumption to rule out unemployment fluctuations from changing discount rates.

The economy has two state variables, the capital stock k and exogenous purchases g. I treat all other variables as functions of the state variables, but, where clear, I omit writing the arguments explicitly. I denote next period's value of a variable with a prime ($'$).

The components of the job value, $J = L - W$, are

$$L = \frac{M^*}{1 - \beta(1 - s)}, \tag{4}$$

the present value of the stationary value of the marginal product of labor M^*, and W, the present value of the wage. When $g = 0$, W obeys the recursion,

$$W(k, 0) = \bar{w}M^* + \beta \cdot (1 - s)\pi(k, 0)^{-\omega}[(1 - \rho)W(k', 0) + \rho W(k', \bar{g})]. \tag{5}$$

The worker hired last quarter has the same wage prospects as the worker hired this quarter, except that the earlier hire enjoys the benefit of an increase in wages from wage stickiness in the face of deflation that applies in the same proportion to all future wages on this job. Hence the present value of wages in the next quarter is the same proportion of the present value to be received by new hires in that quarter, $W(k', 0)$ or $W(k', \bar{g})$. Because no deflation occurs once g pops up,

$$W(k, \bar{g}) = \bar{w}L. \tag{6}$$

Unemployment u solves the zero-profit condition,

$$h(u)J = \gamma. \tag{7}$$

With a linear hiring-rate function $h(u)$,

$$h(u) = h_0 + h_1 u, \tag{8}$$

so

$$u(k, 0) = \frac{\frac{\gamma}{J(k,g)} - h_0}{h_1}. \tag{9}$$

Table 1 in section 7 presents estimates of the parameters h_0 and h_1.

Once g rises, π is constant at one, so unemployment is also constant:

$$u(\bar{g}) = u^*. \tag{10}$$

Employment is

$$n(k, g) = 1 - u(k, g). \tag{11}$$

2 The Product Market with an Elevated Nominal Interest Rate

Now I will turn to the product market, with a focus on what happens when monetary policy creates high unemployment by holding the nominal risk-free interest rate above the level that clears the intertemporal output market, possibly because of the zero lower bound. The point of the discussion is that a macro model of the type considered in the literature on monetary policy and the zero lower bound is a self-standing theory of unemployment, unconnected with the principles of the DMP theory of unemployment. Macroeconomics has a pair of clashing views about unemployment.

2.1 Model

Much of the model I develop in this section follows in the footsteps of Krugman (1998) and the more recent work on the zero lower bound, stimulated first by Japan's experience with the bound and most recently and abundantly by experience in the United States and other countries during the worldwide slump that began in 2008. See, in particular, Eggertsson and Woodford (2003), Christiano, Eichenbaum and Rebelo (2011), Guerrieri and Lorenzoni (2011), and Eggertsson and Krugman (2011).

The literature on the zero lower bound mostly stays close to the New Keynesian principles exemplified in Christiano et al. (2005). These principles dominate current discussions of how monetary policy in general influences output and employment. Although I adopt a streamlined model drawn from the zero lower bound literature, my remarks apply as well to New Keynesian and related models.

To simplify many of the equations in the model, I use ratios rather than rates. Thus R_n is the nominal interest ratio, 1 plus the nominal interest rate, and π is the inflation ratio, next quarter's price level divided by this quarter's.

2.2 Elevated risk-free nominal interest rate

The zero lower bound on the nominal interest rate is currently the salient cause of an elevated nominal rate. The lower bound arises because investors have the option of holding currency. If the safe nominal rate were negative, currency would be a safe alternative yielding a higher return, so financial markets could not be in equilibrium. A more general statement of the same proposition is that if the government offers a security at a fixed nominal rate—such as zero on currency—the safe nominal interest rate cannot be less than that fixed rate. Modern monetary policy, as adopted by many central banks in advanced economies, including those of Canada and Sweden, controls the safe nominal rate by paying one rate on bank reserves and lending to banks at a slightly higher rate. Equilibrium in financial markets must occur at a nominal rate in the corridor between the two rates. The analysis of this paper applies equally to the analysis of the effect of variations in the central bank's policy rate. When the central bank raises the policy rate to cool the economy off, the issue that occupies this paper arises directly: Does unemployment rise because the higher rate causes an excess supply of current output, or because it induces a shift in the equilibrium of the labor market according to the principles of the DMP model? The goal of the paper is to find a combined model where the answer is "both."

The zero lower bound arises from the impracticality of collecting negative interest from holders of currency—see Buiter (2009). If the risk-free interest rate fell below zero, currency would become an asset that paid above its appropriate return—it would be a government giveaway because it would pay a risk-free return of zero. The model cannot have such a giveaway, as discussed in Hall (2011) and the prior literature cited there. Thus $R_n \geq 1$.

When the central bank sets a reserve rate at R_r and a lending rate R_ℓ, the nominal rate is bounded on both sides:

$$R_r \leq R_n \leq R_\ell. \tag{12}$$

Central banks using this strategy generally set R_r and R_ℓ quite close to each other, so for simplicity I will take the policy to be

$$R_n = R_p, \tag{13}$$

where R_p is the policy rate: $R_p = .5(R_r + R_\ell)$. The policy rate sets the nominal risk-free rate throughout the economy. That rate cannot drop below the policy rate for the same reason that it cannot drop below zero in the presence of currency—reserves would become a security paying a giveaway rate. The policy rate is also the upper bound on the nominal rate—no bank would hold reserves if other risk-free investments paid more. Central banks using the corridor strategy do not have binding reserve requirements.

The policy rate also controls the risk-free nominal rate under the old style of central banking that the Fed practiced until late 2008, where reserves earned no interest and banks avoided holding reserves above the level of reserve requirements. The shadow return to reserves, as revealed in the interbank lending rate, plays the same role as explicit interest on reserves in modern central banking. The analysis in this paper describes the consequences of a policy rate set above the level that clears the current market for output. In particular, it applies to the period from 1980 to 1982, when the Fed pushed the interbank rate to extraordinary levels and unemployment soared.

2.3 Inflation

The standard approach in modern macroeconomics is to use the Taylor rule to describe the actions of the monetary authority. The rule dictates the nominal interest ratio given inflation and the unemployment rate:

$$R_n = \max(\phi_0 + \phi_\pi \pi - \phi_u u, 1). \tag{14}$$

A binding lower bound disables the Taylor rule. In the economy subject to the bound, one can think of the inflation rate as a free variable, provided

$$\pi < \frac{1 - \phi_0 + \phi_u u}{\phi_\pi}. \tag{15}$$

The corresponding risk-free real interest ratio is

$$R_f = \frac{R_n}{\pi}, \tag{16}$$

the Fisher equation. For convenience, I treat current inflation, π, as known in the current period, even though it is in principle the ratio of the stochastic p' to the current p. Near-term inflation is highly predictable in the U.S., so little is lost by this simplification.

In this paper, I mostly treat inflation as a free variable, even when a Taylor rule might actually influence its value. For some purposes, I take the inflation ratio as given. I also investigate the inflation ratio that would reconcile the DMP model and the product-market model. As a general matter, this paper sidesteps the truly difficult question of what determines the rate of inflation.

2.4 The driving force: rising exogenous component of product demand

For simplicity, I take the driving force of the model to be a temporary shortfall in an exogenous component of product demand. I call this variable *exogenous purchases*. In Hall (2011) I measured the temporary burden of repaying the bulge of debt that household took on during the 2000s and I inferred that this burden resulted in a shortfall of consumption among households that are at the corner of their intertemporal allocation problem and thus borrow as much as they can. When lenders cut back in consumer credit, these households are forced to cut consumption during the period when their outstanding debt is falling. Households not at the corner, whose Euler equations govern their consumption growth, anticipate that their consumption will fall once the constrained households resume normal levels of consumption. During the period of expected consumption shrinkage, the real interest rate is low. In fact, in the calibration I use, the real rate is negative. Here, I take the simplest version of this idea. An exogenous variable g, a component of product demand, is temporarily low but it is public knowledge that it will return to its normal level fairly soon.

The expectation that an exogenous component of product demand will rise in the future implies that resources are expected to become scarcer in the future and thus the interest rate may be negative to induce consumers to consume more in the present to take advantage of the lower scarcity. Thus, the anticipation of later higher exogenous purchases can cause the zero lower bound on the interest rate to bind.

The condition that generates the negative rate is the expected increase in exogenous purchases g. To overcome consumers' tendency to equalize current and future consumption and induce them to consume more in early years when output is more plentiful because exogenous demand is lower, the interest rate is negative. Krugman (1998) makes the related assumption in an endowment economy that the endowment declines from the first to subsequent periods. Guerrieri and Lorenzoni (2011) generate negative rates from tightening of borrowing limits which cause households to raise precautionary saving and to plan later increases in consumption, which, in equilibrium, are partially discouraged by the negative rates. My setup mimics theirs in a broad sense.

Exogenous purchases g, take on only two values, 0 and $\bar{g}$. The economy begins with $g = 0$ and switches permanently to $g = \bar{g}$ sometime thereafter. Its law of motion is

$$\text{Prob}[g' = \bar{g} | g = 0] = \rho. \tag{17}$$

and

$$\text{Prob}[g' = \bar{g} | g = \bar{g}] = 1. \tag{18}$$

Until purchases pop up to $\bar{g}$, the economy expects resources to become scarcer in the future and consumers correspondingly expect their consumption to decline. With sufficient expected decline, the equilibrium interest rate is negative and the zero bound may bind.

2.5 Technology, capital, and consumption

Let n be employment, $x = (k' - k)/k$ be the investment/capital ratio, v be resources expended in recruiting workers:

$$v = \frac{\gamma s n}{h(u)}, \tag{19}$$

and c be consumption. At the beginning of the period, labor and capital form gross output according to

$$n^{\alpha} k^{1-\alpha}. \tag{20}$$

At the end of the period, consumption and exogenous purchases occur. Remaining output is invested. Adjustment costs are:

$$\frac{\kappa}{2} k x^2. \tag{21}$$

The parameter κ controls adjustment cost. Capital deteriorates at rate δ. Material balance requires

$$n^{\alpha} k^{1-\alpha} + (1 - \delta) k = c + kx + \frac{\kappa}{2} k x^2 + v + g. \tag{22}$$

I let q be the market or shadow price of installed capital. Firms solve the atemporal capital-installation problem:

$$\max_x \; q \cdot (x + 1) k - \frac{\kappa}{2} k x^2 - (x + 1) k. \tag{23}$$

The first-order condition is:

$$\kappa x = q - 1. \tag{24}$$

Tobin's investment equation. The coefficient κ controls capital adjustment. If κ is very large, capital does not adjust at all; the economy is the endowment economy of Lucas (1978). If $\kappa = 0$, capital adjusts without impediment and q is always one.

Households can buy and sell a claim to a unit of installed capital with price q. Its realized return ratio is

$$R(k, g, g') = \frac{(1-\delta)q' + (1-\alpha)n'^{\alpha}k'^{-\alpha}}{q}. \tag{25}$$

Households have an intertemporal elasticity of substitution of σ. Their realized intertemporal marginal rate of substitution is

$$m(k, g, g') = \beta \left(\frac{c'}{c}\right)^{-\frac{1}{\sigma}}. \tag{26}$$

Households plan consumption to satisfy the Euler equation,

$$\mathbb{E}_{g=0}\,(mR) = (1-\rho)m(k, g, 0)R(k, g, 0) + \rho m(k, g, \bar{g})R(k, g, \bar{g}) = 1 \tag{27}$$

and

$$\mathbb{E}_{g=\bar{g}}\,(mR) = m(k, g, \bar{g})R(k, g, \bar{g}) = 1. \tag{28}$$

The risk-free return ratio is

$$R_f(k, 0) = \frac{1}{\mathbb{E}_{g=0}\,(m)} = \frac{1}{(1-\rho)m(k, 0, 0) + \rho m(k, 0, \bar{g})} \tag{29}$$

and

$$R_f(k, \bar{g}) = \frac{1}{\mathbb{E}_{g=\bar{g}}\,(m)} = \frac{1}{m(k, \bar{g}, \bar{g})}. \tag{30}$$

3 Solutions

3.1 The model for $g = \bar{g}$

Because g remains fixed at $\bar{g}$ once it reaches that level, the model for $g = \bar{g}$ is independent of the model for $g = 0$. As I noted above, the DMP labor-market model implies a constant unemployment rate of 5.5 percent once $g = \bar{g}$. There is no inflation when $g = \bar{g}$: $\pi(k, \bar{g}) = 1$.

The model has only one unknown function, $x(k, \bar{g})$. I approximate $x(k, \bar{g})$ with an orthogonal polynomial in k. A low order polynomial is a completely adequate approximation because the range of variation of k that I consider is quite limited. Given a candidate for that function, the following equations yield functions for the other variables: L, constant: (4), $W(\bar{g})$, constant: (6), $u(\bar{g})$, constant: (10), $n(\bar{g})$, constant: (11), $v(k, \bar{g})$: (19), $c(k, \bar{g})$: (22), $q(k, \bar{g})$: (24), $R(k, \bar{g})$: (25), $m(k, \bar{g}) = \mathbb{E}\, m$: (26), $R_f(k, \bar{g})$: (30), and $R_n(k, \bar{g})$: (16). Then the solution process finds the values of the coefficients of the polynomial $x(k, \bar{g})$ such that the Euler equation (28), holds for a grid of values of k that span the domain of k.

3.2 The model for $g = 0$

After solving the model for $g = \bar{g}$, I solve the model for $g = 0$ in the same way. The model for $g = 0$ uses the previously derived function $x(k, \bar{g})$ and other variables derived from it where transitions to the state $g = \bar{g}$ occur. I consider three versions of the model:

- *Full model*: The risk-free nominal return ratio, $R_n(k, 0)$, takes on a specified value $\bar{R}$ (1 in the case of the zero lower bound), and the inflation ratio function $\pi(k, 0)$ and unemployment $u(k, 0)$ are equilibrium objects.

- *Product-market model*: The nominal return ratio $R_n(k, 0) = \bar{R}$ and the inflation ratio function $\pi(k, 0)$ are given, and unemployment $u(k, 0)$ is an equilibrium object not controlled by the DMP labor-market model.

- *Labor-market model*: The inflation ratio function $\pi(k, 0)$ is given, and unemployment $u(k, 0)$ is an equilibrium object controlled by the DMP labor-market model by itself.

In the full model with $R_n(k, 0) = \bar{R}$ given, there are three unknown functions: $x(k, 0)$, $W(k, 0)$, and $\pi(k, 0)$, each represented as a polynomial in k. Given candidates for those functions, the following equations yield values of the other variables: L, constant: (4), $u(k, 0)$: (9), $n(k, 0)$: (11), $v(k, 0)$: (19), $c(k, 0)$: (22), $q(k, 0)$: (24), $R(k, 0, g')$: (25), $m(k, 0, g')$: (26), and $R_f(k, 0)$: (29). The solution process finds polynomials $x(k, 0)$, $W(k, 0)$, and $\pi(k, 0)$ that solve the Euler equation (27), the Fisher equation (16), and equation (5) for the present value of starting wages, on a grid of values of k. The solved $\pi(k, 0)$ in these solutions is very close to constant over k.

In the product-market model with $R_n(k, 0) = \bar{R}$ and $\pi(k, 0) = \pi_0$ given, the equations of the DMP labor-market model do not appear. Unemployment is an unknown function $u(k, 0)$ along with $x(k, 0)$. Given candidates for the unknown functions, the following equations yield values of the other variables: $n(k, 0)$: (11), $v(k, 0)$: (19), $c(k, 0)$: (22), $q(k, 0)$: (24),

$R(k,0,g')$: (25), $m(k,0,g')$: (26), and $R_f(k,0)$: (29). The solution process finds $x(k,0)$ and $u(k,0)$ that solve the Euler equation (27) and the Fisher equation (16). The product-market model resembles the models used in earlier work on the zero lower bound, where unemployment is a free variable that replaces the interest rate, so to speak, in clearing the current-period output market.

In the labor-market model with $\pi(k,0) = \pi_0$ given, $u(k,0) = u(0)$ is the unknown variable, calculated directly from: L, constant: (4), $W(0)$, constant over k: (5), $u(0)$, constant over k: (9).

4 Parameters

I use generally accepted parameter values: The elasticity of output with respect to labor input is $\alpha = 0.646$, the utility discount is $\beta = 0.9997$ at a quarterly rate, capital deterioration is $\delta = 0.0188$ per quarter, capital adjustment cost is $\kappa = 8$, the intertemporal elasticity of substitution is $\sigma = 0.5$, and the labor turnover rate is $s = 3 \times 0.04 = 0.12$ per quarter. I specify the process for growth of exogenous purchases as $\bar{g} = 0.234$ (5 percent of stationary output) and probability of remaining at zero of $\rho = 0.9$, so the expected growth of g is 0.5 percent of stationary output per quarter.

5 Equilibria

5.1 Equilibria of the full model without a sticky starting wage

If $\omega = 0$, so that the equilibrium unemployment rate is the constant u^* in both g states, equilibrium in the product market when $g = 0$ requires that the real interest ratio have the value R_f^* such that unemployment from the product-market model is equal to u^*. I calibrate the DMP equations so that $u^* = 5.5$ percent. Then, for a give inflation ratio π_0, the unique nominal interest ratio satisfies the Fisher equation:

$$R_n = \pi_0 R_f^*. \tag{31}$$

Two conclusions follow: First, if the inflation rate is flexible and responds immediately to monetary policy, the choice of the nominal interest ratio implies an equilibrium inflation ratio:

$$\pi_0 = \frac{R_n}{R_f^*}. \tag{32}$$

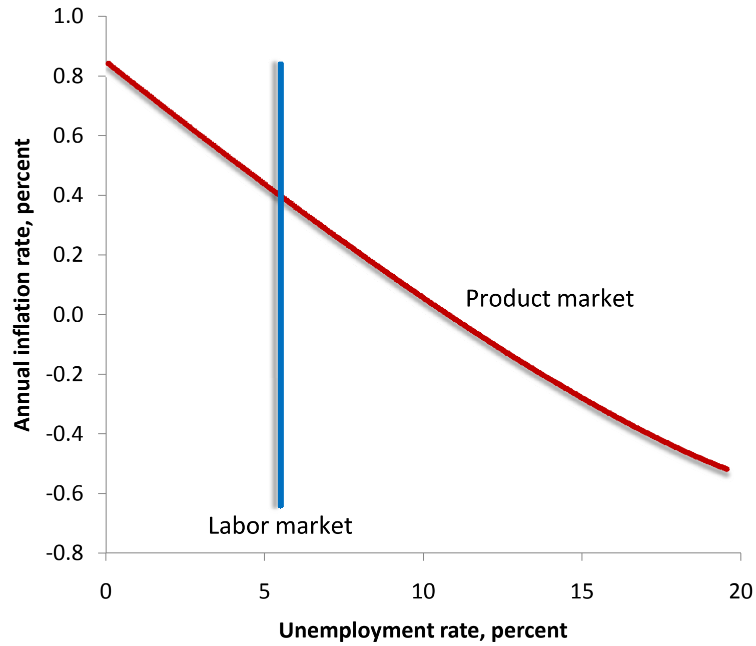

Figure 4: Equilibrium without sticky nominal wages

Second, if the inflation rate is given, the only value of the nominal interest ratio compatible with equilibrium is $R_n = \pi_0 R_f^*$. At any other rate, the two models of unemployment would clash.

Existing models of the zero lower bound ($R_n = 1$) have not considered this issue because they take the inflation rate as somewhat flexible and because they do not typically have any model of the labor market. They take unemployment as a free variable.

Figure 4 shows the unemployment rates from the labor- and product-market models with $R_n = 1$ for a range of rates of inflation and a prospective increase in exogenous purchases of three percent of stationary output. Inflation has no role in the labor market, so that line is vertical at 5.5 percent. Unemployment from the product-market model is higher for lower inflation rates because the real interest rate is higher and the excess supply of current output is correspondingly higher.

5.2 Equilibria of the full model with a sticky starting wage

Figure 5 repeats Figure 4 for the case $\omega = 1$, so the initial nominal wage remains fixed for the duration of the job. The prospective increase in exogenous purchases is one percent of stationary output. Now the unemployment rate from the DMP labor-market model also slopes downward. At positive inflation rates, real wages erode in the course of a job because

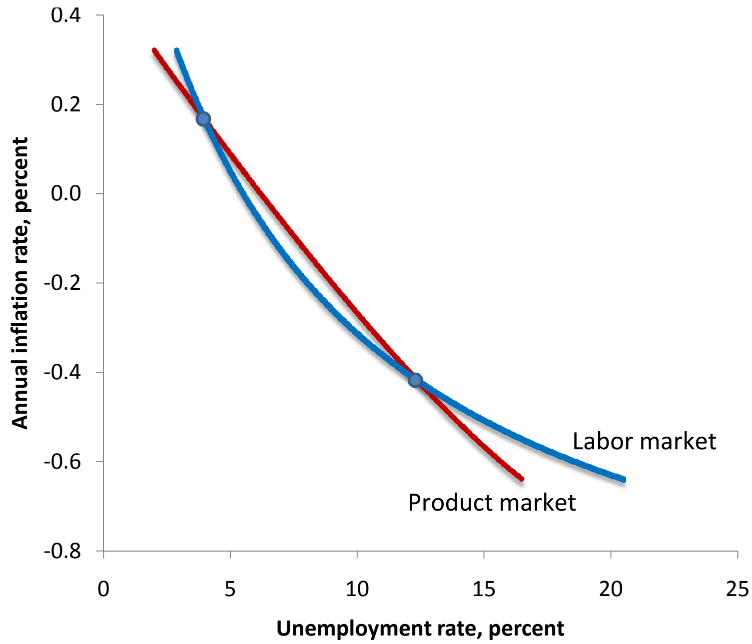

Figure 5: Equilibrium with sticky nominal wages

they remain constant in nominal terms. The job value—the present value of the employer's benefit from the employment relationship—is higher. Employers recruit more aggressively and the unemployment rate is lower than its value at the zero-inflation calibration point, 5.5 percent. With deflation, the reverse happens and unemployment is high.

Figure 5 shows two intersections of the labor- and product-market curves. One occurs at a positive inflation rate and a tight labor market. The zero nominal rate set by the central bank results in a negative real interest rate, which implies a high level of output and labor demand from the product-market model. It is hard to see why this case would arise under modern central banking, because it places interest-rate policy far off any plausible Taylor rule.

The interesting feature of Figure 5 is the second intersection with deflation and high unemployment. This equilibrium seems to resemble what happened in the United States starting in late 2008, when the Fed set the nominal rate close to zero. It shows that the simple model of this paper, with sticky nominal wages during the course of employment, can overcome the apparent clash between unemployment theories.

For inflation rates between the two points marked with discs, the full model has no equilibrium. Again, the clash between the two theories of unemployment is unresolved. Depending on one's view of the determination of the rate of price change, this finding implies either that the model is incapable of explaining what happens at those intermediate rates of change or that the equilibrium rate of price change does not lie in that interval.

To generate an equilibrium satisfying the zero lower bound with deflation, two things happen. First, the high unemployment while $g = 0$ results in a level of consumption during that time that is *below* its level when g pops up. By contrast, in the case of fixed unemployment ($\omega = 0$), consumption contracts when g pops up. In the model of Figure 5, at the lowest price change ratio shown ($\pi_0 = 0.9984$ at a quarterly rate), at the stationary level of capital, consumption is 2.763. It *rises* to 2.970 when g pops up. By contrast, in the low unemployment equilibrium with shown at the top, consumption falls from 3.048 to 2.992 when g pops up. The expected growth of consumption lowers the discounter m and raises $R_n = \pi_0 / \mathbb{E}\ (m)$ enough to offset the low value of π_0.

The second feature of the equilibrium is that the behavior of Tobin's q provides enough boost to the return to capital to offset the reduction in the marginal product of capital that results from the low level of employment. q rises from 0.979 in the depressed economy when g remains at zero to 1.002 after g pops up. The return to capital is essentially the sum of the marginal product of capital net of depreciation and the rate of change of q, so the potential jump in q offsets the low marginal product.

Figure 6 shows that the high unemployment deflationary equilibrium is not universal. It shows the same economy as the previous figure except that the jump in exogenous purchases will be three percent of stationary output rather than one percent. The line showing the solutions to the product-market model ends before the labor-market line crosses it. The product-market model has no solutions for higher rates of deflation.

Figure 7 and Figure 8 describe all of the equilibria of the full model—the intersections shown in Figure 5. Each marker shows a calculated equilibrium corresponding to the nominal riskless rate on the horizontal axis, for a large set of closely spaced rates. As in the Figure 5, the nominal wage is sticky in the course of employment and the pending increase in exogenous spending is small, only one percent of stationary output. In a range of the nominal interest rate from just below zero to 0.4 percent, two equilibria occur, corresponding to the bifurcation in Figure 5. In this range, the low-unemployment equilibria have positive rates of inflation and lower real interest rates and the high-unemployment ones involve deflation. For all more negative interest rates, unemployment is high and becomes higher with more negative rates. Figure 7 shows the reason. Because the product-market model cannot easily generate low

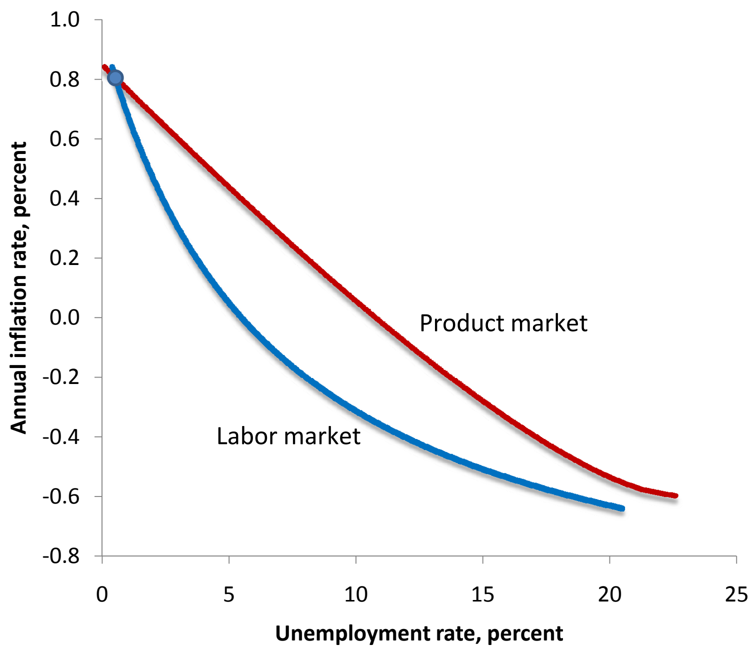

Figure 6: Equilibrium with Sticky Nominal Wages and Larger Increase in Exogenous Purchases

rates of return, the equilibrium rate of deflation must be greater if the nominal interest rate is lower. But the labor-market model calls for higher unemployment with more deflation, as the reward to employers declines when workers benefit from higher real wages as their jobs progress.

5.3 Conclusions about sticky nominal wages as a reconciliation of the clash between unemployment theories

Sticky nominal wages can account for high unemployment during episodes of deflation. When a fixed norm determines the starting real wage, but the real wage rises in the course of a job because the wage is sticky in nominal terms, the value of a new worker to an employer declines, employers put less effort into recruiting, and the labor market softens. Of course, the result is a naive Phillips curve, subject to all the objections to the implicit behavioral assumptions of almost any theory of nominal rigidity.

As an account of the behavior of the economy in a deflationary slump, the hypothesis of sticky nominal wages has three defects. First, under the assumption that the central bank has set the nominal rate at zero, the rate of inflation has to take on the unique value that reconciles the two models of unemployment. As yet, neither actual experience with the

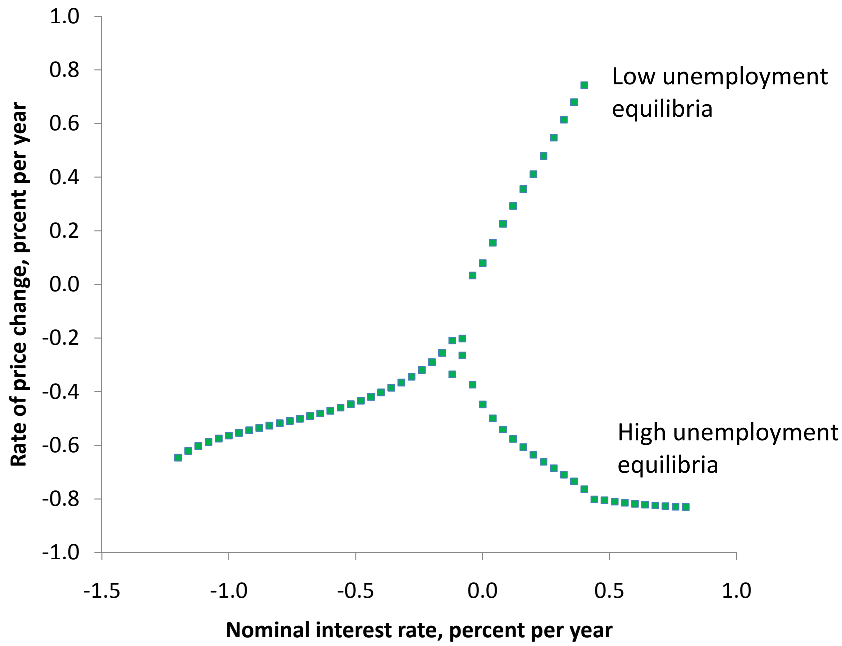

Figure 7: Equilibrium rates of price change given the nominal interest rate

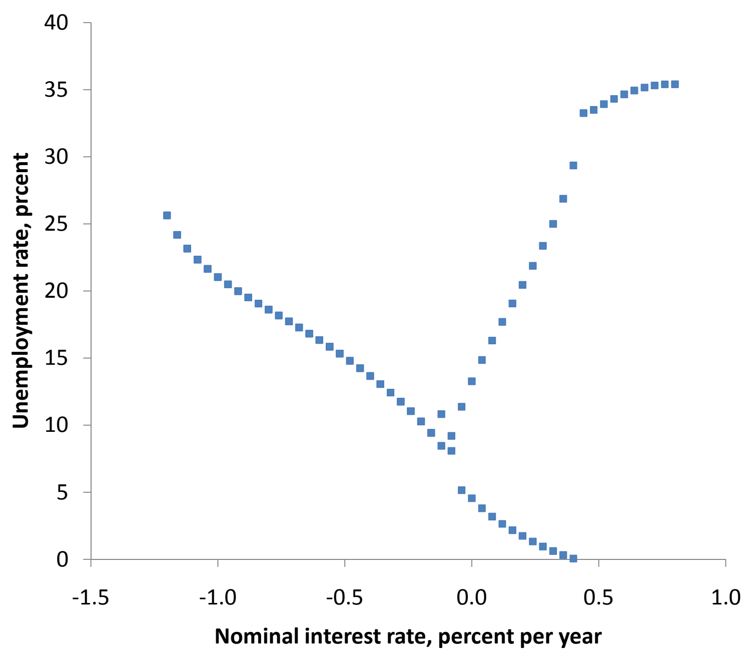

Figure 8: Equilibrium unemployment rates given the nominal interest rate

zero lower bound nor models have reached any reliable conclusion about the determination of price-level change at the lower bound. The issue of the clash of unemployment theories would arise if the amount of inflation or deflation was not at its equilibrium value.

Second, when the lower bound is binding, or if the nominal rate is fixed at some value other than zero, unemployment cannot fall in the range in Figure 8 between the high- and low-unemployment equilibria. Unemployment would jump from a low level associated with positive inflation to a high level associated with deflation.

Third, as Figure 6 shows, a deflationary high-unemployment equilibrium may not exist in the model with sticky nominal wages. Non-existence of equilibrium is a defect of a model, not a statement about the economy.

6 Other Approaches to Reconciliation

6.1 Variations in market power

Early New Keynesian models featured variations in the markup of product prices over cost that arose from price stickiness. Most of these models neglected unemployment and generated employment volatility through variations in the marginal revenue product of capital. The first part of Christiano, Trabandt and Walentin (2010) extends the labor market in an early New Keynesian model to include unemployment. The extension does not follow the principles of the DMP model. Instead, employers face a conventional labor supply schedule and the external labor market clears. Families have an internal labor market where members have a probability less than one of finding a job each period. The probability rises with search effort. The model of the first part of the paper generates cyclical fluctuations in unemployment as total employment moves along the labor-supply curve. A similar result would follow in a DMP model, where the marginal revenue product of labor would fall in a monetary contraction that enlarged markups as firms kept older, higher prices when the contraction lowered their costs.

More recent New Keynesian models tend to follow Christiano et al. (2005) in placing the most important nominal stickiness in the labor market. The second part of Christiano et al. (2010) adds unemployment as described above to that type of setup. No clash of unemployment theories occurs in the paper, because nominal wage stickiness results in rising market power for workers under a monetary contraction, so unemployment rises. One can think of the simple nominal stickiness that I have added to the DMP model as pursuing the same idea with a different specification of the search and matching process. Gertler et al. (2008) also combine a sticky nominal wage with a DMP labor market.

Papers in the New Keynesian framework have not reported non-existence of equilibrium in their models or the discontinuities in unemployment demonstrated in Section 5 of this paper. Given the complexity of New Keynesian models, it is a considerable challenge to track down the differences that allow the models to overcome the defects of the simple model considered here.

6.2 Excess supply in the product market

The second approach drops the strong assumption that no other endogenous variable enters the DMP model. Instead, the solution to the product-market model is viewed as one of excess supply in the product market and a corresponding constraint on the sales of each firm. So far, to my knowledge, no author in this area has worked out the mechanism that maps excess supply in the market to this constraint on individual firms. The proposition seems dangerously close to Aristotle's fallacy of division, attributing to each component of an entity a property of that entity.

The logic of excess supply with a binding lower bound is straightforward. Given a zero interest rate when only a negative one would clear the product market, producers are unable to sell as much current output as much as they would find it remunerative to produce. The analysis of the consequences of excess supply is close to the simple Keynesian expenditure model—when full-employment output would result in an excess of saving over investment, output falls to a lower level where saving and investment are equal.

In the conditions created by a binding lower bound on the interest rate, firms face constraints on the amount they can sell. To incorporate the DMP analysis of the labor market in that setting, one must take a stand on the benefit that accrues to a constrained firm by hiring another worker. I'm not aware that the issues involved in characterizing the benefit have yet been thought through.

The marginal benefit of adding a worker is the key connection in the DMP model between the product and labor markets. To generate high unemployment in a regular DMP model, the marginal benefit needs to drop below its normal level. Although it is tempting to conclude that a firm with constrained output has no benefit from an added worker, factor substitution stands in the way of that simple conclusion. If the firm cannot sell more output, the firm can still substitute away from other inputs when it hires a worker. Material inputs seem the most likely substitution opportunity. No framework yet exists to measure the marginal benefit of labor.

I believe that the hypothesis of excess product supply is ripe for further development.

6.3 The flexible unemployment hypothesis

Now I turn to a third and quite different approach to reconciling the excess-supply theory of unemployment and the DMP theory. This approach—based on what I call the *flexible unemployment hypothesis*—drops the assumption that the wage-determination function cannot slope downward and lets it do so just enough to lie on top of the zero-profit function. Recall from Figure 3 that only a small clockwise twist of the wage-determination function is enough to accomplish this goal. The wage determination function becomes

$$\tilde{J}_F(u) = \frac{\gamma}{h(u)}. \tag{33}$$

Under this hypothesis, workers pay less for their jobs when unemployment is higher. By coincidence, the relation is just negative enough to call forth lower recruiting effort by employer that ratifies the higher unemployment.

Inserting this job value into the zero-profit condition that determines the unemployment rate in the DMP model yields

$$h(u)\frac{\gamma}{h(u)} = \gamma, \tag{34}$$

which is satisfied for all levels of unemployment. Unemployment is no longer constrained by the DMP model.

When the interest rate is pinned at zero, unemployment takes its value from the product-market model discussed earlier. Figure 9 shows unemployment as a function of the rate of price change, given a zero nominal interest rate (as in Figure 6, $\omega = 1$ and the pending increase in exogenous product demand is three percent of stationary output). In an economy with a determinate rate of price change, this model would provide a reasonable account of high unemployment in deflationary slumps. If some exogenous force caused inflation to be close to zero or negative at the same time that some force caused low interest rates, high unemployment would occur and would last as long as inflation and product demand remained below normal.

Obviously the main shortcoming of the flexible unemployment hypothesis is its foundation upon a coincidence in the positions of the zero-profit and wage determination curves.

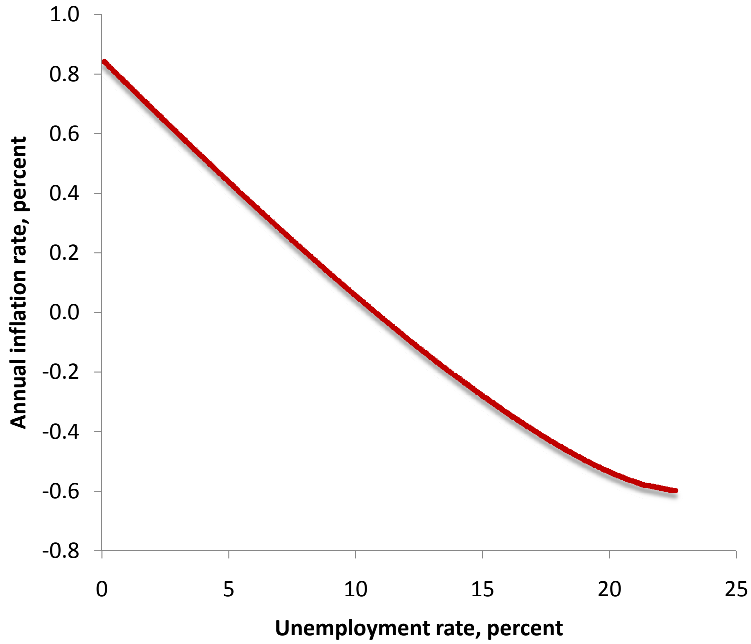

Figure 9: Unemployment as a function of inflation under the flexible unemployment hypothesis

A second issue is the fundamental indeterminacy of unemployment and other variables. One can think of Figure 9 as describing the relationship between the real interest rate and unemployment—variations in the rate of inflation given the nominal rate correspond to variations in the real rate. The economy can have a low real rate and a low unemployment rate (at the upper left, with higher inflation) or a high real rate and high unemployment rate (lower right). These conditions last until the transition to normal g. Given a rate of inflation, the central bank could choose the nominal rate to determine the unemployment rate. To the extent that the inflation rate is free to change, the central bank is not assured of control over unemployment.

The flexible-unemployment hypothesis implies a modest negative slope of the wage- determination function—when unemployment is high, workers pay less for their jobs. This property strikes me as reasonably intuitive. A worker pays the job value to the employer for the privilege of holding the job. If a higher investment of search time is also required to gain the job, shouldn't the worker pay less for the job?

Figure 10 shows the implications of the wage-determination function

$$\tilde{J}_F(u) = \frac{\gamma}{h(u)} \tag{35}$$

for hourly compensation, with the marginal product of labor held constant. I assume that

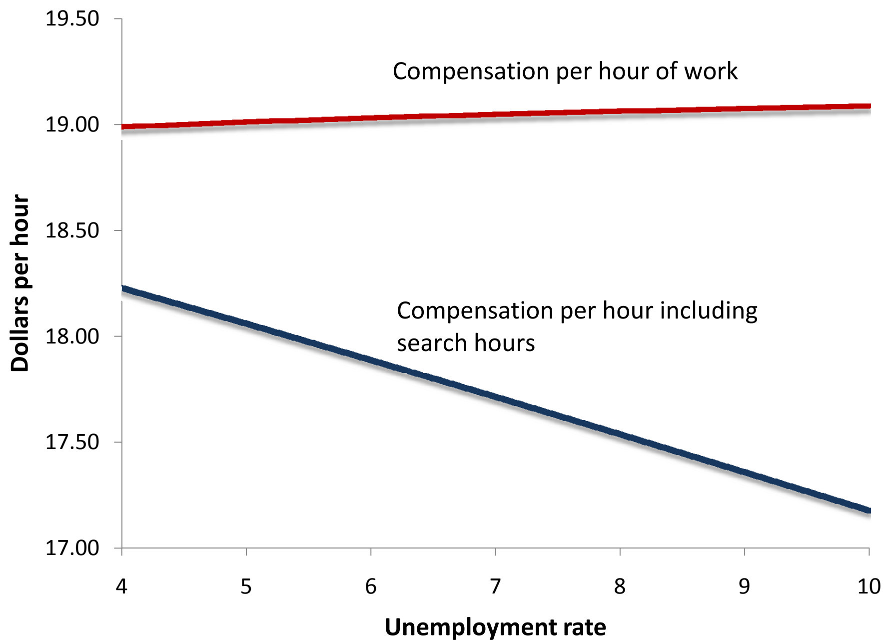

Figure 10: Relation between Hourly Compensation and Unemployment, with Marginal Product of Labor Held Constant

the job value is deducted from compensation in equal amounts spread over the duration of the job. To estimate that duration, I take the reciprocal of the total separation rate reported in JOLTS, averaged over the period December 2000 through December 2010. The average rate is 4.2 percent per month, implying an average duration of 24 months per job. Average pay in the U.S. economy in January 2011 was $19.07. I add $0.23 as the amortized hourly job value, to get an estimate of $19.30 as the average marginal product of labor. I use the fitted hiring rate function $h(u)$ described earlier. The line labeled *Compensation per hour of work* is

$$\$19.30 - \frac{\gamma}{h(u)}. \tag{36}$$

The line labeled *Compensation per hour including search hours* is compensation per hour multiplied by one minus the unemployment rate.

Compensation per hour rises slightly with unemployment, while compensation per hour including search hours falls substantially. One reason that $J_Z(u)$ rises with u may be that job-seekers get a slight reward to the extra time spent waiting for work to begin when unemployment is higher. That is, job-seekers are willing to pay less for jobs when jobs take longer to find.

The value lost from higher unemployment is vastly greater than the value gained from the decline in the job value. Thus higher unemployment unambiguously lowers the earnings of members of the labor force even though it gives workers a better deal.

Figure 10 understates the rise in pay per hour of work as unemployment rises in the model that follows, because the marginal product of labor rises with declining employment.

The flexible-employment hypothesis specifies a modest negative relation between unemployment and the job value negotiated for new jobs by workers and employers. The job value describes the point along the contract curve resulting from the parties' bargain. Any non-negative job value not exceeding the job-seeker's productivity is in the bargaining set and is therefore consistent with the fundamental principles of bilateral bargaining. The necessary negative relation between unemployment and the job value could arise as an equilibrium selection rule. The other way is that the bargaining protocol—such as alternating-offer bargaining—has a unique equilibrium that happens to match the needed relationship.

Hall (2005) discusses the fundamentals of the wage bargain. The most general view of a bilateral bargain limits the outcome in only one way—if the parties have a joint surplus, they will successfully bargain to a point where each receives a non-negative share of that surplus. The division of the surplus is indeterminate. For example, if the employer and worker engage in the two-sided demand auction of Chatterjee and Samuelson (1983), any wage in the bargaining set is a Nash equilibrium (not to be confused with a Nash bargain). In that auction, the parties submit wage bids simultaneously. If the employer's bid does not fall short of the job-seeker's bid, employment occurs at the average of the two bids. Otherwise, the parties do not contract. The employer's best response to a wage bid from the job-seeker that is no higher than the job-seeker's productivity is to match the wage. The job-seeker's best response to a wage bid that is no lower than the job-seeker's reservation wage is to match the bid. The bargaining set runs from the reservation wage to productivity. Hence any wage in the bargaining set is a Nash equilibrium.

My earlier paper demonstrated that a wide class of state-contingent wages satisfies the condition that the wage is in the bargaining set in every state. All of these state-contingent wage functions are candidate equilibria of the wage bargaining problem. For example, they are all Nash equilibria of the demand auction.

In the setup considered in this paper, the bargaining is over the job value J rather than the wage itself, but the point is the same. The employer's reservation value of J is zero—it is remunerative to hire a worker for any positive job value that the worker pays. The worker's reservation value is the worker's opportunity cost of taking the job. The bargaining set runs from zero to the opportunity cost.

7 Employment Fluctuations Resulting from Shifts in the Zero-Profit Curve

Part of the increase in unemployment since late 2008 in the United States appears to have arisen from an adverse shift of the zero-profit curve. Such a shift does not belong in a list of ways that models could resolve the clash between unemployment theories. Rather, in a model that had succeeded in incorporating the DMP model in general equilibrium, the adverse shift would raise the unemployment rate through a mechanism the DMP model explains.

Recent work has studied the adverse shift from the perspective of the Beveridge curve (the joint behavior of unemployment and vacancies) and the matching function, I will approach the topic via the zero-profit curve to relate to the analysis in Figure 1 as closely as possible. I document a limited amount of instability of the zero-profit function. The adverse shift is not large enough to be a candidate to explain more than a fraction of the increase in unemployment in 2008 and 2009.

From equation (1), the job value is

$$J_t) = \frac{\gamma}{h_t}. \tag{37}$$

To evaluate J_t, I need a value for the vacancy-posting cost γ and an estimate of the recruiting-success rate h_t.

Hall and Milgrom (2008) calculate that the daily cost of maintaining a vacancy is 0.43 days of pay, based on data from Silva and Toledo (2008), or $\gamma = \$66$ per day for the average U.S. employee in January 2011.

The daily hiring success rate h is

$$h_t = \frac{H_t}{21 V_t}, \tag{38}$$

where H_t is the number of hires during a month and V_t is the average number of vacancies open during the month, approximated as openings at the beginning of the month. Both series are from the BLS's Job Openings and Labor Turnover Survey (JOLTS). I divide by 21 as the number of working days in a month.

		Intercept	Slope	Trend	Standard error of the regression
Daily recruiting sucess rate	$h(u)$	0.0371 (0.0020)	0.545 (0.037)	-0.000082 (0.000013)	0.0037
Daily job-finding rate	$\varphi(u)$	0.064219 (0.001173)	-0.593 (0.022)	-0.000019 (0.000008)	0.0022

Table 1: Estimates of Parameters of the Hiring and Job-Finding Functions

To estimate the hiring success rate function $h(u)$, I regress h_t on the unemployment rate u_t from the Current Population Survey for the period from December 2000 (the onset of JOLTS) to June 2009 (omitting data from the anomalous period in the second half of 2009 and 2010). I also include a linear trend. The identifying assumption—lack of correlation between the unemployment rate and the disturbances in the hiring rate—is consistent with the theme of the paper that the product market determines the unemployment rate, not factors relating to the labor market. The regression appears in the top panel of Table 1. It shows a robust positive relationship between the recruiting success rate and the unemployment rate.

The lower panel of Table 1 shows estimates of the daily probability $\phi(u)$ that a job-seeker will find a job. The left-hand variable is the ratio of the number of hires reported in JOLTS to the number of unemployed workers in the CPS, divided by 21 to place it on a daily basis. There is a robust negative relation between the job-finding rate and the unemployment rate.

Figure 11 shows the calculated job value J_t since the inception of JOLTS. The job value is strongly pro-cyclical. New workers pay their employers—in the form of a wage below their marginal products—more in good times, such as the middle of the decade of the 2000s, and less during slumps, such as 2001 to 2003 and 2008 to 2011. The small light squares in the figure show the job values calculated from the fitted values of h_t from the regression.

Figure 12 is a scatter diagram with unemployment on the horizontal axis and the job value J_t on the vertical axis. It distinguishes the cycle—contraction and expansion—running from 2000 to 2007 and shown with a solid red line from the second cycle that started in 2008, shown as a double blue line. The earlier cycle follows a reasonably well-defined negatively sloped line, both during the contraction and during the subsequent expansion. The contraction starting in 2008 (not yet retraced by the modest expansion that has occurred to date) is somewhat flatter than the earlier one. Unemployment rose dramatically, but the hiring rate did not rise as fast as it would have if the labor market had retraced the contraction that

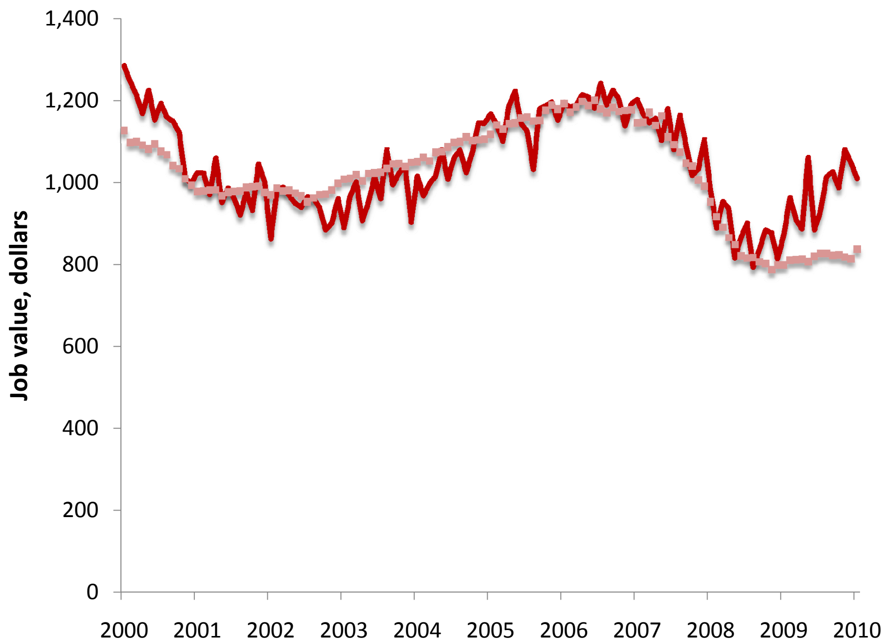

Figure 11: Job Value Calculated from JOLTS

started in 2001.

A second anomaly appears at the trough of the contraction in 2009 and 2010, when unemployment lingered just below 10 percent but the hiring rate fell by as much as it would have in a substantial expansion. This behavior has been widely discussed. It corresponds to an inward shift of the matching function. See Hall (2010).

Figure 13 is a scatter diagram with the unemployment/vacancy ratio on the horizontal axis and job value J_t on the vertical axis. That ratio or its reciprocal is the exact measure of labor-market tightness in the standard DMP model. Comparison of the two scatter diagrams suggests that not much of the anomalous behavior in Figure 12 is the result of the neglect of the dynamics of job matching, which in principle are handled exactly in Figure 13, but without much improvement in fit.

The broad joint movements over the period from 2000 to 2011 of unemployment and the calculated job value J_t suggest a roughly stable relationship —unemployment was high early in the period when the job value was low, fell to reasonable levels in the middle of the decade when J was high, and rose to a very high peak when J fell starting in 2008. Notice that unemployment is not an input to the calculation of J. Of course, tracking down the sources of the departures from stability is still an interesting undertaking. Nothing in what follows would be altered in substance if a more refined version of the zero-profit condition contained

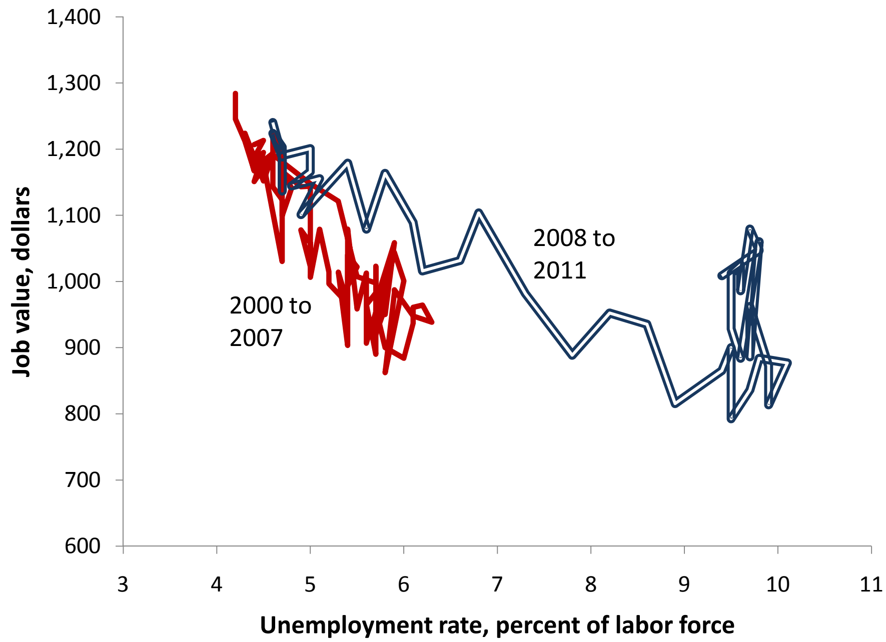

Figure 12: Job Value J Plotted against Unemployment

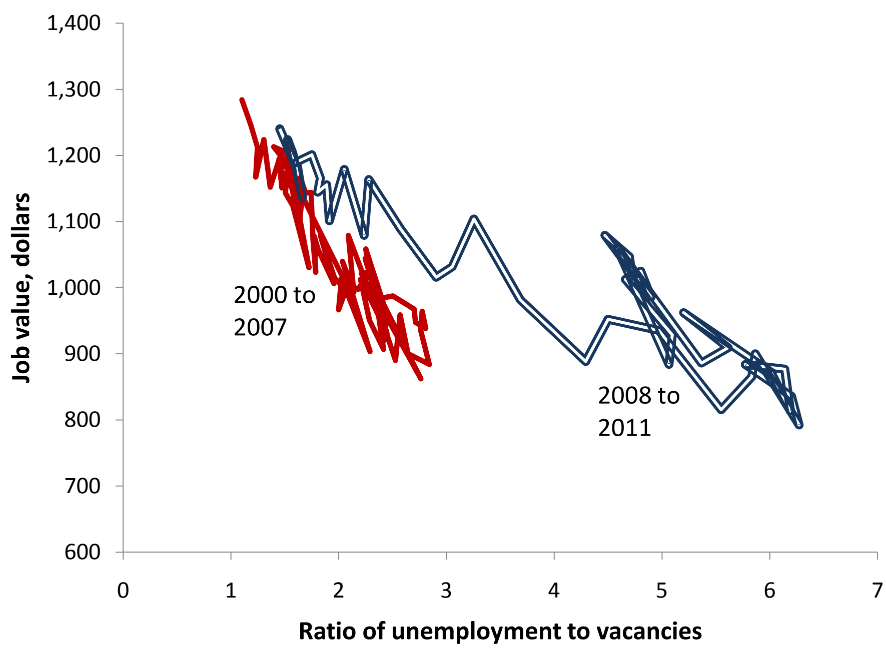

Figure 13: Job Value J Plotted against the Unemployment/Vacancy Ratio

additional variables to account for what appear to be shifts in the joint behavior of u and J.

8 Concluding Remarks

Macroeconomics badly needs to resolve the conflict between a theory of unemployment based on excess supply in the current product market and the DMP theory based on equilibrium in the labor market. A regular DMP model—one where a slack labor market improves the bargaining outcome for the employer—cannot coexist with a standard model of excess supply resulting from a binding lower bound on the interest rate. Either (1) the excess-supply model needs to include an element that reduces the benefit to the employer from a new hire or (2) the DMP model needs to include the flexible-employment property that a slack labor market improves the bargaining outcome for the worker by just the right amount to make the DMP unemployment rate indeterminate.

An extended period of high unemployment in the U.S. economy with the risk-free interest rate pinned at zero has left believers in the DMP model puzzled about the forces that caused such a large change in the labor market. Although some of the rise in unemployment appears to be the associated with an adverse shift of the matching function, most seems to be the result of forces that operate in the product market, much amplified by the inability of the interest rate to fall enough to restore current product demand to normal levels.

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