The Role of the Growth of Risk-Averse Wealth in the Decline of the Safe Real Interest Rate *

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Abstract

Over the past few decades, worldwide real interest rates have trended downward. The real interest rate describes the terms of trade between risk-tolerant and risk-averse investors. Debt pays off equally across contingencies at a given future date, so debt is valuable to risk-averse investors to smooth consumption across those contingencies. In an equilibrium with trade between investors who differ in attitudes toward risk, the risk-tolerant investors borrow from the risk-averse ones, shifting the risk to those whose preferences favor taking on risk. Heterogeneity in risk aversion takes two forms in the model of the paper: variation in coefficients of relative risk aversion and variation in beliefs about the probabilities of seriously adverse outcomes. If the composition of wealth shifts into the hands of investors with higher coefficients of relative risk aversion and investors who believe in higher probabilities of bad events, the real interest rate falls. The paper calculates likely magnitudes of the decline and presents evidence in favor of a shift in the composition of wealth toward the holdings of the more risk-averse. In particular, the United States absorbs large amounts of risk by borrowing from more risk-averse countries, notably China, which thereby shed corresponding amounts of risk.

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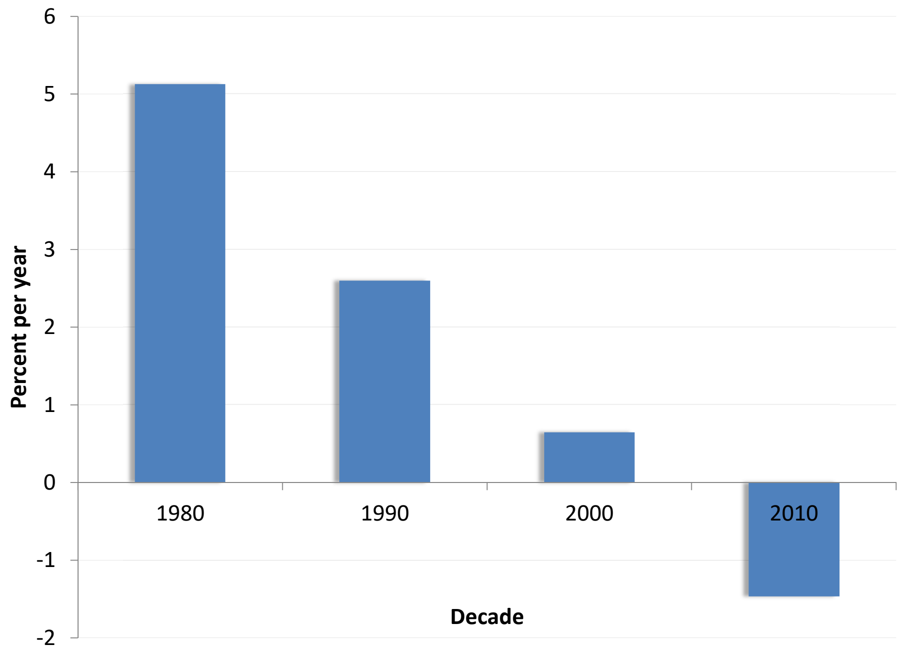

Figure 1: Decade Averages of the Real Interest Rate on 5-Year Treasury Notes

1 The Issues

Figure 1 shows the realized real interest rate on one-year obligations of the United States Treasury, averaged over recent decades. Since the decade of the 1980s, the rate has declined monotonically, reaching a negative level in the current decade. Real rates on other safe obligations around the world have declined in parallel. The decline in real interest rates has been welcome in some respects, notably in reducing the burden of the national debt. But the decline has created a challenge for monetary policy, because it has lowered the headroom of monetary policy to offset an incipient recession. Central banks fear that, in future recessions, even if mild, monetary policy's expansionary effect will be limited because the interest rate will become pinned at its effective lower limit.

Bernanke, Bertaut, DeMarco and Kamin (2011) laid out an explanation of the decline in the real rate based on the "global saving glut", an idea that Bernanke had advanced in speeches before and during his service at the Federal Reserve. Their paper cites some of the evidence considered here, but does not rest on an explicit model of global financial equilibrium. Caballero, Farhi and Gourinchas (2008) offer a portfolio-based explanation based on an heterogeneity across economies in the ability to create safe assets to fund risky real investments. A later section surveys some of the many earlier papers dealing with heterogeneous investors and declining real interest rate.

This paper uses established principles of portfolio economics to study the saving glut, formulated in terms of the buildup of holdings of safe debt in China and among similar investors, taken as more risk-averse than the investors in debt-issuing countries. Building on Barro and Mollerus (2014) and a large literature in finance, the paper pursues the implications of heterogeneity in risk-aversion among investors who trade with each other in a risky environment. The investors trade claims in a capital market to take advantage of the gains to trade that exist among heterogeneous agents. In the post-trade equilibrium, risk-tolerant investors absorb more of the downward shocks and gain more from the upward shocks. The risk-averse investors, on the other hand, have more equal returns, losing less in bad times and gaining less in good times. In effect, the risk-tolerant investors sell insurance to the risk-averse ones. Safe debt-type claims play a role in the equilibrium by paying off the same amount in both good and bad times. Throughout, the paper embodies the principle of modern financial economics that securities are packages of the underlying fundamental risk factors of the economy.

This research explores the implications of observed patterns of trade in financial claims for the implied safe real interest rate, the terms under which investors swap pre-specified non-contingent claims across time periods. The interest rate is a useful summary of the terms under which the risk-tolerant help the risk-averse stabilize consumption, because providing a non-contingent return is a powerful tool for such stabilization. A growing fraction of wealth around the world in the hands of risk-averse investors could be an important part of the explanation of the declining safe real interest rate.

An upward trend in the proportion of wealth held by investors with higher coefficients of relative risk aversion is one potential explanation of the decline in the safe real interest rate. A second form of heterogeneity may also play a role. A rising share of wealth held by investors who believe in a higher probability of bad future outcomes raises the volume of trade in safe claims. Interest rates based on trade between risk-tolerant optimists and risk-averse pessimists can be substantially lower than occur with trade between investors with common beliefs about the distribution of returns. This paper joins many others in recent financial economics that emphasize heterogeneous beliefs.

An important dimension of heterogeneity is the relative size of the resources commanded by different types of investors. In the economy considered here, the interest rate is particularly low if the risk-tolerant investors own fewer resources than do the risk-averse. It follows that the real safe interest rate will trend downward if the resources in the hands of the risk-averse grow faster than those in the hands of the risk-tolerant.

By the short safe real rate, I mean the rate shown in Figure 1, with a maturity of a year, long enough to avoid the stronger effects of liquidity services from shorter maturities but still eligible to be called short.

The paper focuses on the decline in the safe real rate associated with the growth of risk-averse wealth relative to risk-tolerant wealth, but makes no claim that the growth is the only force at work in the interest-rate decline. In particular, a decline in expected consumption growth—excluded by assumption in the model of this paper—would result in a decline in the safe real rate in a model that considered that force. Further, while the paper exploits ideas in modern financial economics, it does not introduce any new ideas to that field. The ideas that trade in a risky environment takes the form of the sale of insurance by the risk-tolerant to the risk averse, and that the safe real interest rate describes the terms of that trade, are hardly new. The novelty here is to study the implications of the changing distribution of resources between risk-tolerant and risk-averse investors, in a simple theoretical model and in the global economy of recent decades.

2 Model

The model describes an endowment economy with two types of investor-consumers. They consume a variety of products, each involving the same physical good, but delivered in different time periods and different states of the world. The economy has complete Arrow-Debreu markets. Investors own endowments of the products. There are two types of investors. One type is risk-tolerant in two respects and the other is risk-averse. At the beginning of time, the investors make contingent trades whose general character is to transfer risk from the risk-averse to the risk-tolerant. The two types are equally impatient, so intertemporal trade is not an important feature of the economy. Both types believe in the same expected rate of growth of the endowment, so heterogeneity in beliefs about growth is not a factor in determining the real interest rate. The paper concentrates on the role of financial markets in the distribution of risk in the determination of the safe real interest rate.

The products are numbered by the index i. Product $i = 1$ is immediate delivery of the good with certainty. This product serves as numeraire, with price $p_1 = 1$. The risk-tolerant investors consume c_i of product i and the risk-averse c_i^*. The preferences of the risk-tolerant are expressed in a utility function $U(c_1, \ldots, c_N)$. They solve the problem

$$\max U(c_1, \ldots, c_N) \text{ subject to } \sum_i p_i(c_i - \omega y_i) = 0. \tag{1}$$

Here y_i is the economy's endowment of product i and ω is the fraction of the endowment owned by the risk-tolerant investors. The outcome of the choice is a set of excess supply functions $d_i(p, y)$ describing the amount of product i that the risk-tolerant offer to trade. Variables without subscripts are vectors of the corresponding variables with subscripts. The risk-averse solve a similar problem and have excess supply functions $d_i^*(p, y)$. An equilibrium of the economy is a vector of prices p such that $d_i(p, y) + d_i^*(p, y) = 0$, for all products i. Such a vector exists, according to standard continuity principles—this economy satisfies all of the standard properties of general-equilibrium models in the Arrow-Debreu tradition.

This setup has no explicit role for the probability that the delivery of a given product occurs. Probabilities of states of the world are bundled into the utility functions. Nothing requires that risk-tolerant investors agree with the risk-averse on the probabilities. Heterogeneity in beliefs about probabilities plays a large and growing role in the finance literature because it appears to help understand many features of the operation of financial markets.

Under the assumption of state- and time-separable preferences, the utility function has the special form

$$U(c_1, \ldots, c_N) = \sum_{i=1}^{N} \beta_{\tau(i)} \phi_i u(c_i). \tag{2}$$

Here $\tau(i)$ is the time period when product i is delivered, β_t is a time-weight describing impatience (if β_t declines with t), and ϕ_i is the belief of the investor about the probability. The probabilities sum to one within each time period:

$$\sum_{i \text{ such that } \tau(i)=t} \phi_i = 1. \tag{3}$$

The notation's asymmetric treatment of i and t saves a lot of double subscripts in what follows. The utility function of a risk-averse investor is

$$U^*(c_1^*, \ldots, c_N^*) = \sum_{i=1}^{N} \beta_{\tau(i)}^* \phi_i^* u^*(c_i^*). \tag{4}$$

I make the assumption

$$\beta_t = \beta_t^* = \beta^t \tag{5}$$

to impose equal impatience on both investor types and to adopt the standard geometric pattern of discounting future utilities.

2.1 Variation of the endowment

Uncertainty arises in the economy entirely through stochastic properties of the endowment. I assume that, for each product i, a ternary event occurs that changes the endowment associated with i, y_i, to one of three values in the next period:

$$\{y_i(1-\Delta_-), y_i, y_i(1+\Delta_+)\}. \tag{6}$$

Thus each product except those in the terminal period has three successor products in the following period. The two types of investors agree on the set of values that the endowment can take—agreement is essential for meaningful trading.

There is one possible value of the endowment in the first period, normalized at one. There are three possible values in the second period, $\{1-\Delta_-, 1, 1+\Delta_+\}$. Among the nine products in the third period, there are six possible values,

$$\{1-\Delta_-, 1, 1+\Delta_+, (1-\Delta_-)^2, (1-\Delta_-)(1+\Delta_+), (1+\Delta_+)^2\}. \tag{7}$$

The set of values becomes correspondingly richer with each passing period. The assumptions I make turn out to imply that the two-period economy closely resembles the first two periods of an economy with more than two periods.

2.2 Beliefs about probabilities

For each product in a given period, the risk-tolerant investors believe that the probabilities are π_- that the endowment will fall in the next period, π_+ that it will rise, and $\pi_0 = 1-\pi_- - \pi_+$ that it will remain the same. The corresponding beliefs of the risk-averse investors are π_-^* that the endowment will fall, π_+^* that it will rise, and $\pi_0^* = 1 - \pi_-^* - \pi_+^*$. Both types of investors believe that the expected change in the endowment is zero, so

$$\pi_-\Delta_- = \pi_+\Delta_+ \tag{8}$$

and

$$\pi_-^*\Delta_- = \pi_+^*\Delta_+. \tag{9}$$

Thus both types believe that the endowment is an untrended random walk, though they disagree about the probabilities of the changes in endowment. The risk-averse investors believe that the dispersion of the endowment fans out over time at a higher rate than the risk-tolerant investors believe.

The ternary setup in the process is necessary to accommodate heterogeneity in beliefs under the random-walk restriction. With a binary increment to the endowment, only a single pair of probabilities would be consistent with the random walk.

The other restriction is that none of the probabilities can be negative. There is a one-dimensional subspace of probabilities that satisfies the random walk and non-negativity. The set of probability beliefs satisfying all the constraints is

$$0 \leq \pi_- \leq \frac{\Delta_+}{\Delta_+ + \Delta_-} \tag{10}$$

and

$$\pi_+ = \frac{\Delta_-}{\Delta_+}\pi_-, \tag{11}$$

and similarly for π^*.

The probabilities π and π^* induce probability beliefs ϕ_i and ϕ_i^* on the products indexed by i. For example, the probabilities on the endowment values in period 3 listed earlier,

$$\{1-\Delta_-, 1, 1+\Delta_+, (1-\Delta_-)^2, (1-\Delta_-)(1+\Delta_+), (1+\Delta_+)^2\} \tag{12}$$

are

$$\{\pi_0\pi_-, \pi_0^2, \pi_0\pi_+, \pi_-^2, \pi_-\pi_+, \pi_+^2\}. \tag{13}$$

They sum to one, keeping in mind that three of them occur twice.

2.3 Utility kernel

I take the utility kernel for risk-tolerant investors to be

$$u(c) = \frac{c^{1-\gamma}}{1-\gamma}, \tag{14}$$

so marginal utility is $c^{-\gamma}$. Similarly, for risk-averse investors, marginal utility is $(c^*)^{-\gamma^*}$.

2.4 Solving the model

The problem is to find trades x_i and Arrow-Debreu prices p_i that satisfy

$$\beta^{\tau(i)}\phi_i c_i^{-\gamma} = p_i c_1^{-\gamma}, \tag{15}$$

$$c_i = \omega y_i - x_i, \tag{16}$$

$$\beta^{\tau(i)} \phi_i^* (c_i^*)^{-\gamma^*} = p_i (c_1^*)^{-\gamma^*}, \tag{17}$$

$$c_i^* = (1-\omega) y_i + x_i, \tag{18}$$

and

$$\sum_i p_i x_i = 0. \tag{19}$$

Existence of an equilibrium follows from standard continuity properties. I have found no evidence of multiple equilibria for the parameter values considered here.

3 The Safe Interest Rate

A safe (discount) bond is a package of Arrow-Debreu claims containing one unit of each of the products that are delivered in a particular time period. The package has probability one of delivering one unit. The sum of the prices of those products is the cost of a claim that yields one unit of output in a future period. It is the market price at origination of a pure discount bond that pays no coupon and makes a single unit payment in that future period. For a bond with maturity of m periods, the price is

$$P_m = \sum_{i \text{ such that } \tau(i)=m} p_i. \tag{20}$$

The interest rate in the sense of a per-period yield is

$$r_m = P_m^{-1/m} - 1. \tag{21}$$

Thus r_m traces out the yield curve for risk-free bonds in the economy.

Factors that raise P_m lower the safe interest rate. The value is

$$P_m = \sum_{i \text{ such that } \tau(i)=m} \pi_i \beta^m \left(\frac{c_i}{c_1}\right)^{-\gamma} = \mathbb{E}\ \beta^m \left(\frac{c_i}{c_1}\right)^{-\gamma} = \mathbb{E}\ \beta^m \left(\frac{c_i^*}{c_1^*}\right)^{-\gamma^*}. \tag{22}$$

Because the utility kernel with constant relative risk aversion belongs to the family of precautionary utility functions with convex marginal utility, $u'''(c) > 0$, Jensen's inequality implies that higher dispersion of consumption c_i raises the price of certain future output and lowers the risk-free interest rate. Forces that raise the dispersion of c_i and c_i^* will lower the interest rate.

3.1 The yield curve

The slope of the yield curve depends on how the dispersion of future consumption rises with futurity, m. The yield curve will be flat at level r if

$$P_m = \left(\frac{1}{1+r}\right)^m . \tag{23}$$

Thus, to generate a flat yield curve, the dispersion of consumption will be higher for maturity m relative to $m-1$ sufficiently to make P_m lower than P_{m-1} by the ratio $\frac{1}{1+r}$. To a reasonable approximation, this condition implies that the dispersion rises by a constant factor with each period of added maturity. A constant proportional increase in dispersion over time is a characteristic of a (geometric) random walk, so a flat yield curve goes with consumption that evolves as something like a random walk. The assumption that both types of investors believe that the endowment is a random walk turns out to imply that they believe that their consumption levels are close to random walks.

3.2 Alternatives to the contingent-product setup

In place of the Arrow-Debreu setup, where investors purchase contingent products at the beginning of time, one could study an equivalent setup where investors make contingent one-period contracts that are more like the financial contracts seen in the real world. Each contract would specify contingent payments. One is in units of product i, interpreted as the purchase of a security. There are three others specified in units of the successor products, interpreted as the (possibly) random payoffs of the security in the succeeding period.

One of the reasons to consider the setup with one-period contracts is that the Arrow-Debreu setup appears to involve breathtaking issues of commitment, with all contracts made once-and-for-all at some mythical starting time. Putting aside the issues of finite lifetimes and yet-unborn investors and assuming that the one-period contracts are sufficiently detailed, an economy where investors wait to make contracts until the time when they become operative will have the same rational-expectations equilibrium as the Arrow-Debreu equilibrium.

		Base value	Alternative value
Δ_+	Increment to endowment	0.04	
Δ_-	Decrement to endowment	0.60	
ω	Share of endowment owned by risk-tolerant consumers	0.75	0.25

Table 1: Parameter Values of the Physical Environment

3.3 Characterizing the volume of insurance implicit in the equilibrium allocation

In the model, the only motive for trade between the two types of investors is the shifting of risk from the risk-averse to the risk-tolerant. Pure diversification is absent because both types are exposed to the same risks. As a measure of the amount of risk shifting, I follow the literature on international trade in goods and services by calculating the ratio of trade to the endowment. I weight the amount of output changing hands, in absolute value, by the homogeneous-investor Arrow-Debreu prices of the output and divide by the endowment share of the risk-tolerant investors. I interpret the ratio as a measure of the amount of insurance that the risk-tolerant investors provide to the risk-averse investors.

4 Parameters

Table 1 shows the parameter values of the physical environment for the calculations that follow. In all calculations, the upward jump in the joint endowment y is $\Delta_+ = 0.04$ and the downward jump is $\Delta_- = 0.6$. The first set of calculations sets the share of the endowment owned by the risk-tolerant investors to $\omega = 0.75$. Later, to illustrate the role of the growing fraction of world wealth held by the risk-averse investors, notably China, I show calculations for lower values down to $\omega = 0.25$.

Table 2 describes the preferences of the investors, including their beliefs about the probability distribution of the endowments. The left column for each type of investor shows the base values, which are the same for both types—the coefficient of relative risk aversion is $\gamma = 2$ and the probability of the extreme bad value of the endowment is $\pi_- = 0.01$ (once in a hundred years), implying that the probability of the outcome with positive endowment growth is $\pi_+ = 0.15$. The right column shows alternative values for the parameters. The alternative values have the property of preserving the equally weighted geometric means of

	Parameter	Risk tolerant		Risk averse	
		Base value	Alternative value	Base value	Alternative value
γ	Coefficient of relative risk aversion	2.00	1.74	2.00	2.30
π_-	Belief about probability of bad decrement to endowment	0.0100	0.0071	0.0100	0.0140
π_+	Belief about probability of good increment to endowment	0.150	0.107	0.150	0.210

Table 2: Parameter Values of Preferences Including Beliefs about Probabilities

the parameters. The alternative values of the coefficient of relative risk aversion are $\gamma = 1.74$ and $\gamma^* = 2.30$, and the alternative values of the disaster probability are $\pi_- = 0.0071$ and $\pi_-^* = 0.0140$. The beliefs of both types about the probability of an improvement in the endowment are the values described earlier needed to ensure that both types of investors believe that the expected change in the endowment is zero.

The values of the utility discount parameter β vary across the parameter sets and are given in the tables showing the outcomes by parameter set. The values are chosen to give approximately the same interest rate in the set describing homogeneous investors.

5 Equilibrium in the Model

Trade in the model takes the form of non-zero values of contracted transfers, x_i, from the risk-tolerant investors to the risk-averse investors. Positive values of x_2 arise if the risk-averse find their endowments $(1-\omega)y_2$ to be more painful than do the risk-tolerant. This motivation for trade exists even when the two types of investors agree on the probabilities of the endowment process. Positive values also arise for x_2 when the risk-averse perceive higher probabilities of the bad outcome 2 than do the risk-tolerant. This motivation for trade would exist even if the two types of investors were equally risk-averse. Thus trade arises from a mixture of two kinds of heterogeneity among investors.

The numerical results in this section are from the model with $T = 2$, that is, one date when trades in contingent products are made and second date when the contingencies occur.

	Endowment, y	Trade, x	Consumption/endowment		A-D price, p	Probability, π	SDF, p/π
			Risk-tolerant	Risk-averse			
Initial	1	0.0000	1.000	1.000	1	1	1
After one year	0.4	0.000	1.000	1.000	0.058	0.010	5.81
	1	0.000	1.000	1.000	0.781	0.840	0.93
	1.04	0.000	1.000	1.000	0.129	0.150	0.86
Utility discount factor, β			0.930				
Expected consumption growth			0.0000	0.0000			
Price with certainty			0.968				
Annual interest			3.27				
Gross trade			0.0000				

Table 3: Latent Prices with No Heterogeneity among Investors

5.1 Equilibrium without heterogeneity among investors

Absent both sources of heterogeneity, the equilibrium involves no trade. Lucas (1978) describes the equilibrium. Latent Arrow-Debreu product prices are the marginal rates of substitution between product 1 and the other products. Table 3 shows the prices, along with the values of the endowment, and the corresponding probabilities perceived by both types of investors. Parameter values are the base values shown in Table 1 and Table 2. The upper panel describes the situation at the time that prices are set. The middle panel shows the three possible realizations a year later. The top line describes the worst realization, with an endowment of 0.4. The price of that product conditional on its realization is 0.058/0.01=5.81 (the value of the economy's stochastic discount factor). Output is quite valuable when the endowment is so low. For the other two realizations, with endowments of 1 and 1.04, output is not particularly valuable, with prices conditional on realization of 0.93 and 0.86. In all outcomes, both types of consumer-investors consume exactly their endowments—the third and fourth columns of the table show the consumption/endowment ratios as 1.

The bottom panel of Table 3 shows the price of the package of products that delivers one unit of output with certainty—the sum of the numbers in the corresponding column above: $P = 0.968$. The corresponding annual interest rate or yield is 3.27 percent. It is

	Endow-ment, y	Trade, x	Consumption/ endowment: Risk-tolerant	Consumption/ endowment: Risk-averse	A-D price, p	Proba-bility, π	SDF, p/π
Initial	1	-0.0004	1.000	0.999	1	1	1
After one year	0.4	0.019	0.937	1.188	0.052	0.010	5.15
	1	0.000	1.000	0.999	0.785	0.840	0.94
	1.04	-0.002	1.003	0.991	0.130	0.150	0.87
Utility discount factor, β			0.935				
Expected consumption growth			0.0001	-0.0004			
Price with certainty			0.967				
Annual interest			3.37				
Gross trade			0.0042				

Table 4: Equilibrium with Heterogeneity in Risk-Aversion Coefficients

representative of the historical real yield on Treasury debt in recent decades, though not in the current decade, as shown in Figure 1. The volume of debt implicit in the zero-trade allocation is, of course, zero.

5.2 Equilibrium with trade resulting from heterogeneous risk aversion

Table 4 shows the equilibrium with heterogeneity in preferences, with the same parameter values as in Table 3, except that the risk-tolerant investors have coefficients of relative risk aversion of 1.74 and the risk-averse have coefficients of 2.3 (the two values are equally distant from 2 in proportional form—$1.74 \times 2.3 = 2$). Trade occurs in this economy. In the first period, the risk-averse investors pay the risk-tolerant ones 0.0004 units of output. In the most common outcome a year later—with probability 0.84, as before—essentially no transfer occurs. In the bad outcome, with a joint endowment of only 0.4, the risk-tolerant provide 0.019 units of output to the risk-averse to cushion them against the low value of the endowment. In that outcome, the risk-tolerant consume 0.937 of their endowments while the risk-averse consume 1.188 units. The Arrow-Debreu price of the product is 0.052, and the conditional price is $0.052/0.01 = 5.15$, somewhat lower than in the case of homogeneous investors in Table 3.

In the best outcome, with an endowment of 1.04 units, the risk-averse provide the risk-tolerant 0.002 units. Output is cheap in that contingency—the conditional price is only 0.87. The effect of trade is to compress the range of consumption levels for the risk-averse, relative to the no-trade case and relative to the consumption of the risk-tolerant.

Because the calculation in Table 4 uses a slightly higher discount, $\beta = 0.935$, than the value in Table 3, $\beta = 0.930$, the interest rate is essentially the same in both equilibria. This paper makes no claim that changes in risk aversion are responsible for the decline in the real interest rate. Rather, as later calculations will show, heterogeneity in risk aversion makes the interest rate sensitive to changes in the ratio of wealth held by risk-averse investors to wealth held by risk-tolerant investors.

The bottom panel of Table 4 shows that neither type of investor has expected consumption growth more than slightly different from zero and that gross trade is quite small. These two results are connected—because trade is small, consumption is close to the level of the endowment, and by construction, expected endowment growth is zero. Trade is small because the local effect of the mean-preserving spread in the coefficients of relative risk aversion is zero and the higher-order effects are small given the relatively small actual spread.

5.3 Equilibrium with trade resulting from heterogeneity in beliefs about probabilities

Table 5 illustrates the risk-shifting trade that occurs in an economy where some investors are risk-tolerant in the sense that they believe that low realizations of the endowments are unlikely and others are risk-averse in the sense that they believe that those endowment realizations are more likely. The parameter values are the same here as in Table 3 except that $\pi_- = 0.0071$ and $\pi^*_- = 0.014$, so the risk-averse investors believe that adverse shocks to the endowment are twice as likely as risk-tolerant investors believe. The corresponding values for the probabilities of the good outcome are $\pi_+ = 0.107$ and $\pi^*_+ = 0.021$, as dictated by the assumption of no expected change in the endowment.

Table 5 shows much more trade than occurs in Table 4. As in the earlier case, trade cushions the risk-averse against the bad outcome—the risk-tolerant provide 0.027 units to the risk-averse in that contingency. The driving force of trade is powerful because the risk-averse believe that the bad outcome is twice as likely as the risk-tolerant believe. But the risk-averse also believe that the best outcome, on the bottom line of the middle panel, is

	Endow-ment, y	*Trade, x*	*Consumption/ endowment*		*A-D price, p*	*Proba-bility, π*	*SDF, p/π*
			Risk-tolerant	*Risk-averse*			
Initial	1	0.0002	1.000	1.001	1	1	1
After one year	0.4	0.027	0.909	1.274	0.051	0.007	7.12
	1	-0.012	1.016	0.952	0.807	0.886	0.91
	1.04	0.071	0.909	1.274	0.113	0.107	1.05
Utility discount factor, β			0.941				
Expected consumption growth			0.0040	0.0232			
Price with certainty			0.971				
Annual interest			3.03				
Gross trade			0.0260				

Table 5: Equilibrium with Heterogeneity in Beliefs about Probabilities

twice as big as the risk-tolerant believe. As a result, the risk-tolerant also provide a large amount of output, 0.071 units, in that contingency, the opposite of what happens with heterogeneity in the coefficient of relative risk aversion. The risk-averse pay for these two contingent transfers by paying the risk-tolerant 0.012 units in the most likely contingency, an endowment of 1.

Consumption growth expected by the risk-tolerant is close to zero, but the risk-averse, who assign higher probabilities to changes in the endowment, expect consumption to grow by over two percent even though their endowments have zero expected growth.

Table 6 combines the two forms of heterogeneity. Not surprisingly, the results are similar to those in Table 5 without heterogeneity in the coefficient of relative risk aversion. The next subsection shows that curvature in utility matters a lot for the basic point of the paper, the response of the equilibrium to changes in the parameter ω, the fraction of the endowment held by risk-tolerant investors.

	Endow-ment, y	*Trade, x*	*Consumption/ endowment*		*A-D price, p*	*Proba-bility, π*	*SDF, p/π*
			Risk-tolerant	*Risk-averse*			
Initial	1	0.0001	1.000	1.000	1	1	1
After one year	0.4	0.047	0.843	1.472	0.045	0.007	6.28
	1	-0.011	1.015	0.955	0.817	0.886	0.92
	1.04	0.064	0.918	1.245	0.110	0.107	1.03
Utility discount factor, β			0.947				
Expected consumption growth			0.0038	0.0210			
Price with certainty			0.972				
Annual interest			2.88				
Gross trade			0.0256				

Table 6: Heterogeneity in Both Probability Beliefs and Risk Aversion Coefficients

5.4 Implications of a decline in the endowment share of risk-tolerant investors

Table 7 compares the equilibria in the previous three tables, with the risk-tolerant endowment share at 0.75 with the equilibrium in an otherwise similar economy with a share of 0.25. The two panels on the left show the parameter values. The third panel shows the difference in the safe rale interest rate between the equilibria with the high and low endowment shares and the fourth panel shows the volume of trade. Even though the case described in the first row of the table generated little trade with $\omega = 0.75$ and not a lot more with $\omega = 0.25$—only 0.0095 relative to that level of the endowment—the economy with a low endowment share has a substantially lower interest rate, 1.45 percentage points below the rate in the economy with $\omega = 0.75$. The second line shows that the difference in the interest rate in the economy with heterogeneous beliefs about probabilities is similar, at 1.39 percentage points below the rate in the economy with a high risk-tolerant endowment. And the third line considers the effect when both forms of risk aversion are present. It is a decline of 2.87 percentage points, enough to drive the rate to essentially zero in the low-risk-tolerant endowment economy.

CRRAs		*Bad outcome beliefs*		*Interest rate*			*Volume of trade as a ratio to risk-tolerant endowment share*		
Risk tolerant	*Risk averse*	*Risk tolerant*	*Risk averse*	*α = 75 percent*	*α = 25 percent*	*Difference*	*α = 75 percent*	*α = 25 percent*	*Difference*
1.74	2.30	0.0100	0.0100	3.37	1.93	-1.45	0.0027	0.0095	0.0067
2.00	2.00	0.0071	0.0140	3.03	1.64	-1.39	0.0271	0.0811	0.0540
1.74	2.30	0.0071	0.0140	2.88	0.01	-2.87	0.0264	0.0918	0.0654

Table 7: Comparison of the Interest Rate and the Volume of Trade with High and Low Fractions of the Endowment Held by Risk-Tolerant Investors

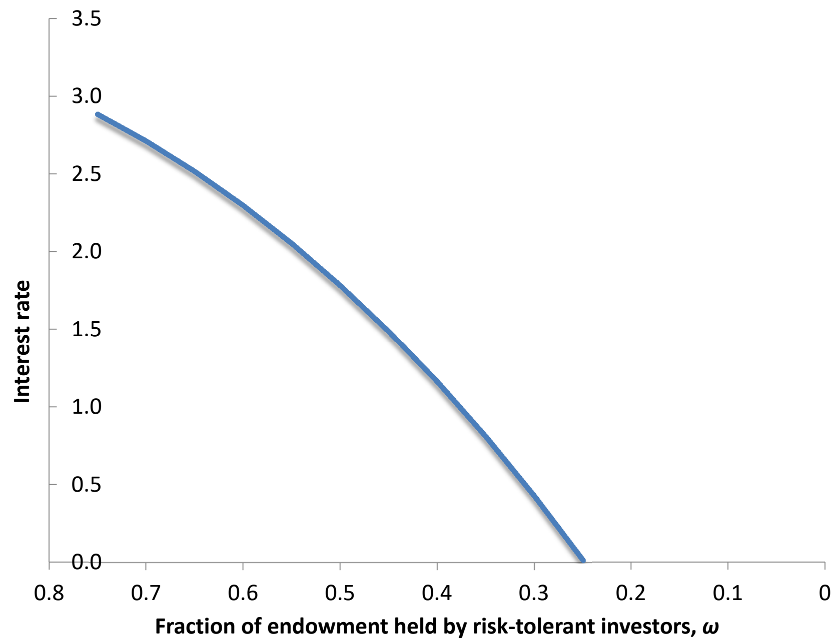

Figure 2: The Interest Rate as a Function of the Endowment Share of Risk-Tolerant Investors

Figure 2 shows how the interest rate in the economy with both types of heterogeneity varies with the share of the endowment held by risk-tolerant investors. The rate falls smoothly as the share declines, moving to the right in the figure. Figure 3 shows the gross volume of trade as a ratio to the contingent endowment held by the risk-tolerant investors. That measure of trade rises as the endowment share declines. The effect occurs almost entirely from the heterogeneity in beliefs about probabilities and not from heterogeneity in coefficients of relative risk aversion.

The table and the two figures demonstrate the basic point of the paper: With heterogeneity in risk aversion, trade in the asset market enables the risk-tolerant investors to insure

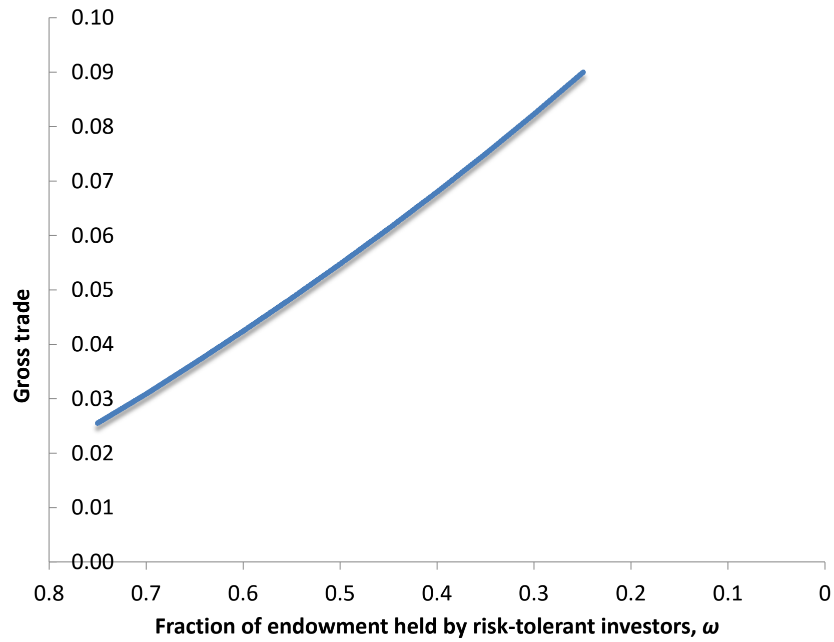

Figure 3: Gross Trade as a Function of the Endowment Share of Risk-Tolerant Investors

the risk-averse investors, to the mutual benefit of both types of investors. The interest rate describes the terms of trade between the two types of investors. As the endowment share of the risk-tolerant investors declines, the terms of trade shift in favor of the risk-tolerant investors, as the demand for insurance rises and the supply falls. Thus risk-tolerant investors can borrow more cheaply. That change occurs in economies with either type of heterogeneity. With heterogeneous beliefs about probabilities, the volume of trade is higher and more responsive to the endowment share.

5.5 Importance of improbable highly adverse outcomes

All the results in this section are based on $\Delta_- = 0.6$, which implies that investors believe that every 100 or 50 years, the endowment drops by 60 percent. Volatility of this character is essential to the large effects on interest rates found here. An extensive literature emphasizes the importance of large unfavorable events with low probabilities in explaining the equity premium. Barro and Mollerus (2014) survey that literature and discuss the issue in connection with the demand of risk-averse investors for safe debt-type investments.

6 Evidence

This paper is about risk-splitting. In an economy with heterogeneous risk aversion among investors, institutions will arise that split the basic risk in the economy so that risk-tolerant investors hold disproportionate shares of that risk. The main points are (1) that risk-splitting is socially efficient and will arise naturally in a market economy where risk-averse investors can hold low-risk claims on risk-tolerant investors, and (2) as the fraction of wealth in the hands of the risk-averse investors rises, the terms of trade shift in favor of the risk-tolerant investors, and the safe real interest rate declines. This section identifies financial institutions that facilitate risk-splitting. It provides data on the current volume of debt issuance of the institutions and on the trend, generally but not always upward, in the volume.

6.1 Debt

A reasonably safe debt instrument has the effect of splitting risk to impose more on the borrower and less on the lender. The amount of debt issued by the agents in an economy is a good metric of the general importance of the risk-splitting within the economy, or, in an open economy, of the importance of providing debt to domestic and foreign holders. Table 8 shows U.S. debt in trillions of dollars and as a ratio to GDP in 2015. To avoid double-counting debt held on both sides of the balance sheets of financial institutions, the table omits the financial sector. The items in the table are in rough order of safety, inferred from the relative yields of the debt. Federal debt stands at the top, in both safety and quantity. Just below are the obligations that are as safe as federal debt, the debt of the government-sponsored enterprises (Fannie Mae and Freddie Mac) and guarantees on mortgages securitized by the GSEs. Debt claims on households are at the bottom of the table.

The table shows that government has a large role in creating safe debt. It's obvious that U.S. government debt provides safe investments to investors around the world who want to shed risk. It's somewhat less obvious that they do so by moving that risk to risk-tolerant investors. Taxpayers are the effective equity holders in the government. A substantial fraction of incremental government revenue, apart from the federal payroll tax earmarked for retirement benefits, arises from high-income taxpayers with high marginal tax rates, who are presumably more risk-tolerant. Thus, by issuing debt, the government facilitates value-enhancing trade between risk-tolerant taxpayers and risk-averse investors around the world.

	$ trillions	*Ratio to GDP*
Federal government debt	15.2	0.85
Federally guaranteed GSE debt and guaranteed mortgages	8.1	0.45
State and local government debt	3.0	0.17
Non-financial business, bonds and loans	12.8	0.71
Non-guaranteed household mortgages	1.4	0.08
Other debt of households	4.7	0.26
Total	45.1	2.52

Table 8: Debt of U.S. Investors in 2015, in Trillions of Dollars and as Ratios to GDP

Government guarantees of mortgage and other types of debt have the same effect—the safe guaranteed bonds suit risk-averse investors and the risk falls on risk-tolerant taxpayers.

Non-financial business issues bonds and borrows in the loan market in large volumes. Some businesses participate actively in risk-splitting by taking on debt and thus concentrating risks on their shareholders. Others, including some of the most valuable corporations, do the opposite, by accumulating debt on the asset side of their balance sheets. It is beyond the scope of this paper to explain the heterogeneity in corporate leverage. The important fact for this paper is that non-financial businesses as a group are major participants in splitting risks between debt held by risk-averse investors and equity held by risk-tolerant ones.

6.2 Growth of institutions that facilitate trade between risk-tolerant and risk-averse investors

The canonical institution whose existence is rationalized by heterogeneity in risk aversion is the bank. A traditional bank holds risky assets, funded by risk-averse depositors and by equity supplied by risk-tolerant investors. The latter generally consider deposits as risk-free debt, because the depositors have a primary claim over all of the assets, and the value of the assets is well above the promised value of the debt. The debt is over-collateralized. The equity holders face not just the risk of the assets, but the risk magnified by the prior claim of the depositors. It is difficult to analyze the growth of banks in the framework of this paper, for two reasons. One is that banks hold many debt-type claims as assets. The other is that big banks take large positions in derivatives that are not reported to the public in normal accounting disclosures in a way that helps understand banks' role in splitting risk.

	Government			Private				
Decade	*Consoli-dated govern-ment debt*	*GSE debt*	*GSE guaran-teed debt*	*Private equity funds*	*Securitiz-ations*	*Non-financial corporate debt*	*Repos*	*Non-mortgage household debt*
1980s	0.469	0.061	0.091		0.012	0.163	0.103	0.186
1990s	0.611	0.101	0.204		0.086	0.211	0.166	0.204
2000s	0.574	0.203	0.293	0.058	0.233	0.238	0.237	0.239
2010s	0.936	0.126	0.347	0.140	0.109	0.275	0.221	0.251

Table 9: Examples of the Scale of Risk-Splitting Institutions

Table 9 gives some major examples of risk-splitting institutions and the level and growth of their volumes, stated as in the previous table as ratios to U.S. GDP. The left panel describes government institutions and the right panel private ones. The most important way the consolidated government meets the needs of risk-averse investors around the world is the issuance of debt. The volume of debt was a bit below half a year's GDP averaged over the 1980s. In the 1990s the ratio reached over 0.6, then fell slightly in the 2000s (as deficits fell), then jumped upward in period 2010 through 2015, thanks to the financial crisis, recession, and poor growth thereafter. Changes in income tax rates starting in the 1990s shifted the burden of paying for government upward in the wealth distribution and presumably onto more risk-tolerant investors. A substantial widening of the wealth distribution compounded the tendency for taxpayers weighted by wealth to be increasingly risk-tolerant. The other two columns of the table relating to the government quantify programs that create safe debt instruments through government guarantees, that is, to the debt of the GSEs and to the debt guaranteed by the GSEs. Guarantees transfer the risk to the taxpayers in the same way as direct issuance of debt. GSE debt grew as a fraction of GDP until the crisis, then fell in the 2010s. Guarantees have grown monotonically.

On the private side of Table 9, the first column refers to private-equity funds (data are available only starting in 2000; these funds were small in prior decades). These funds sell limited partnership interests to risk-tolerant investors, including high-wealth families and endowments. The funds sell debt or obtain loans from financial institutions, which directly or indirectly serve the needs of risk-averse investors. Private-equity funds are a good example of the principle of creating safe debt through over-collateralization. By contract, the debt-holders have rights to the value of the risky assets of the fund, up to the face value of the

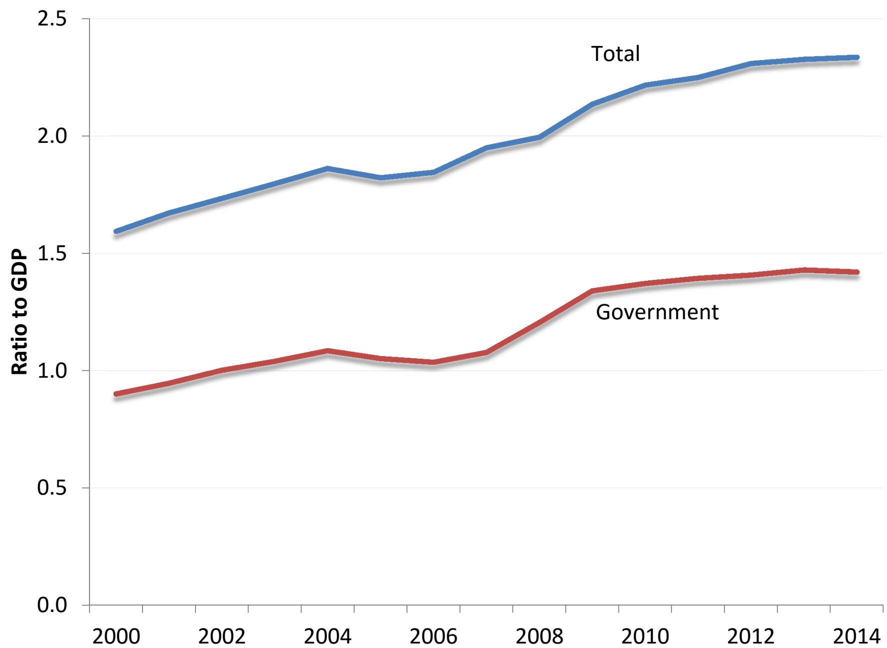

Figure 4: Scale of Risk-Splitting Institutions Relative to GDP

debt. The debt is over-collateralized as long as the value of the assets exceeds the face value of the debt. At issuance, that margin is usually substantial, so default on the debt is unlikely. The value of the assets in private-equity funds grew rapidly after 2000, averaging 0.14 years of GDP in the years 2010-2014.

Corporate bonds are often heavily over-collateralized, so bonds are a safe assets serving the interests of the risk-averse while making corporate equity correspondingly riskier, serving the interests of the risk-tolerant.

Modern institutions that split risks of underlying assets and offer low-risk instruments to risk-averse investors include (1) the repo, which is over-collateralized by the repo haircut, (2) the securitization, which creates a safe tranche by over-collateralizing the promised cash flows to the holders of the bonds in that tranche and creates a complementary equity tranche with magnified risk, (3) the levered hedge fund, and (4) the private-equity debt-funded buyout, which issues overcollaterilzed debt and concentrates the risk on risk-tolerant limited-partner investors.

6.3 International trade in risk-splitting assets

A good deal of risk splitting occurs across international borders. Lane and Milesi-Ferretti (2007) provide estimates of international asset positions for 188 countries for the years 1970

Country	Net equity owned	Net debt owed	Average	Average/ GDP
United States	2,521	6,651	4,586	0.304
United Kingdom	383	806	594	0.271
France	128	1,035	581	0.239
Spain	-83	1,192	554	0.199
Netherlands	598	354	476	0.322
Canada	206	453	329	0.393
Australia	-198	651	227	0.127
Sweden	175	220	197	0.132
Greece	22	262	142	0.264
Norway	348	-75	137	0.471
Denmark	159	80	120	0.244
Austria	79	98	88	0.265
Finland	99	65	82	0.197
United Arab Emirates	239	-101	69	0.262
Portugal	-59	193	67	0.196
Mauritius	95	7	51	0.215
Iceland	3	79	41	3.645
Germany	488	-414	37	2.640
New Zealand	-31	81	25	0.007
Other countries, sum	-229	322	47	

Table 10: Countries that Absorb Risk by Holding Positive Amounts of Net Foreign Equity or by Borrowing from Foreign Lenders

through 2011. From their database, I have calculated net equity owned as the difference between the value of equity claims on foreign countries less foreign equity claims, and calculated net debt owed as the difference between debt owed and debt claims on other countries owned. In terms of the ideas in this paper, both of these measures indicate a country's financial position as a risk absorber, with high equity claims on other countries or high debt obligations to other countries, or as a risk shedder, with the opposite position. Because a pure leveraging transaction—issuing debt to foreigners to purchase foreign equity—raises equity owned and debt owed by the same amount, it is logical to use the average of equity owned and debt owed as a single measure, though the separate measures are also informative.

Table 10 lists all of the countries with more than $25 billion in foreign risk exposure in 2011, ordered by the amount of their exposure. The U.S. with $4.6 trillion in exposure, completely dominates other countries in this dimension. With the exceptions of Mauritius and the United Arab Emirates, all of the significantly exposed countries are advanced and have high incomes. A reasonable conclusion is that these countries find it advantageous to take advantage of their higher risk tolerance by trading risk with risk-averse countries.

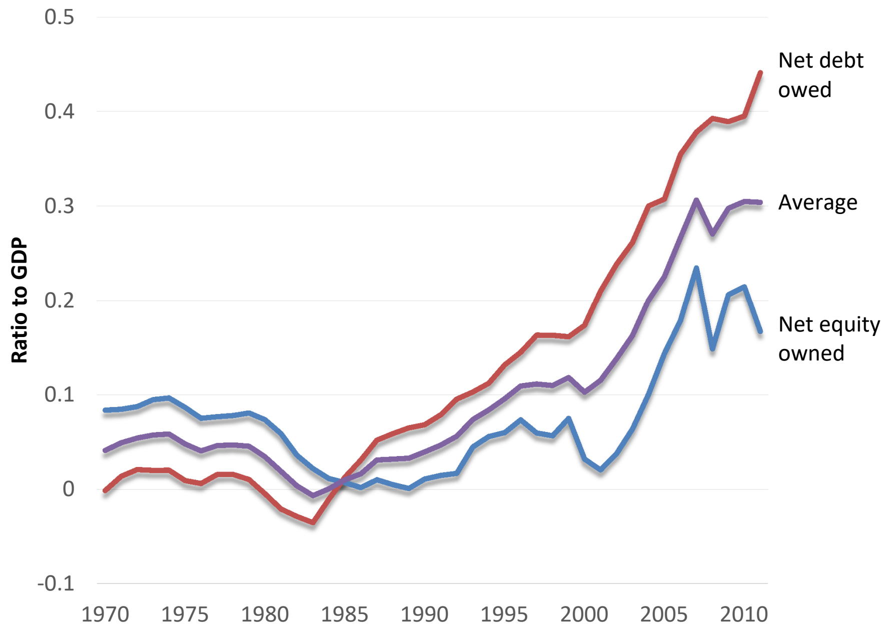

Figure 5: Risk Absorption by the United States, 1970-2011

Table 11 lists all of the countries with more than $25 billion in average risk shedding, either with negative net foreign equity positions or positive net holdings of foreign debt. In this table, China completely dominates other countries. Other east Asian economies are included, along with many lower-income countries and oil-rich countries.

Figure 5 shows the data for the U.S. as time series since 1970. The growth of U.S. borrowing from the rest of the world began in 1982. Growth in the accumulation of equity claims on the rest of the world began a decade later. From 2000 through 2007, the U.S. grew remarkably rapidly as a levered hedge fund.

Figure 6 shows the data for mainland China since 1981. Until the early 1990s, China was financially isolated from the rest of the world. Thereafter, until 2007, China loaned huge and growing volumes of funds to other countries, notably the United States. At the same time, the rest of the world purchased huge and growing volumes of equity claims on Chinese businesses.

Figure 7 shows the worldwide evolution of trade in risky and safe financial claims since 1980. Initially, about 70 percent of world GDP was generated in countries with net financial positions near zero (specifically, in a band between -5 and +5 percent of GDP). As international trade in financial claims proliferated, almost all countries specialized as risk absorbers, like the United States, or risk shedders, like China. GDP in risk absorbers rose in parallel

Country	*Net equity owned*	*Net debt owed*	*Average*	*Average/ GDP*
China,P.R.: Mainland	-1,706	-3,243	-2,474	-0.338
Luxembourg	-1,452	-1,511	-1,482	-24.987
Japan	530	-2,850	-1,160	-0.197
Ireland	-841	-642	-741	-3.351
Brazil	-837	-111	-474	-0.190
Saudi Arabia	6	-731	-362	-0.541
Russia	-263	-361	-312	-0.164
China: Hong Kong S.A.R.	101	-625	-262	-1.052
Singapore	1	-519	-259	-0.976
Taiwan	160	-623	-232	-0.499
Switzerland	217	-604	-194	-0.293
Mexico	-363	23	-170	-0.147
Thailand	-177	-137	-157	-0.454
Korea	-177	-90	-134	-0.120
India	-291	36	-127	-0.069
Algeria	-21	-183	-102	-0.512
Kuwait	147	-349	-101	-0.628
Indonesia	-251	55	-98	-0.116
Venezuela, Rep. Bol.	-24	-157	-91	-0.286
Iran, Islamic Republic of	-26	-134	-80	-0.161
Argentina	-66	-80	-73	-0.165
Libya	38	-165	-63	-1.830
Czech Republic	-106	-14	-60	-0.276
Kazakhstan	-73	-45	-59	-0.323
Nigeria	-64	-52	-58	-0.237
Malaysia	-48	-59	-53	-0.186
Egypt	-67	-13	-40	-0.171
Ukraine	-61	-12	-36	-0.222
Israel	-14	-53	-33	-0.137
Peru	-56	-9	-33	-0.182
China,P.R.:Macao	-6	-55	-31	-0.886
Colombia	-72	11	-30	-0.093
Azerbaijan	-3	-55	-29	-0.450
Philippines	-39	-18	-29	-0.127
Oman	-14	-43	-28	-0.401
Other countries (sum)	-1,078	-137	-608	

Table 11: Countries that Shed Risk by Holding Negative Amounts of Net Foreign Equity or by Lending Positive Amounts to Foreign Borrowers

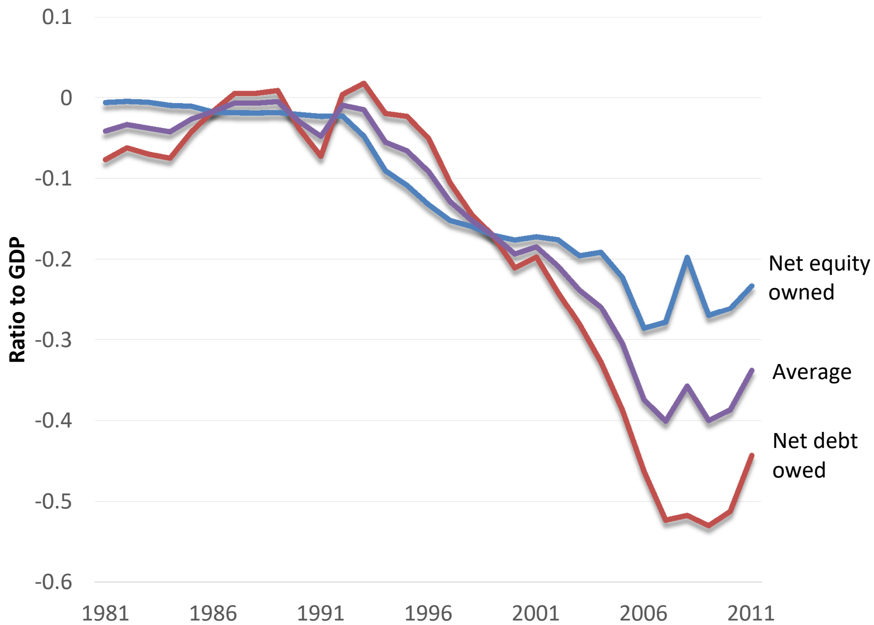

Figure 6: Risk Shedding by China, 1981-2011

with GDP in risk shedders, so the ratio of GDP among the shedders to GDP among the absorbers rose dramatically.

Figure 8 restates the data in Figure 7 as the share of the risk absorbers of the GDP arising from the globally integrated countries (risk absorbers plus risk shedders), as an indicator of the parameter ω in the model. The share declined dramatically from a peak in 1992, over a period that saw a large decline in the world safe real rate. The figure captures only the part of the risk splitting that occurs over national boundaries, so the overall decline in ω could be rather larger.

7 The Equity Premium

Though the equity premium is not the subject of this paper, it is interesting to calculate the premium for values of the parameters that shed light on the central question of the decline in the safe real interest rate. Research on the equity premium has considered many of the same issues that arise here.

The equity premium is the difference between the expected return on a risky investment and the safe return on a debt investment of the same maturity. Here I will consider two concepts of the equity premium. One is for an unlevered investment in a security that pays

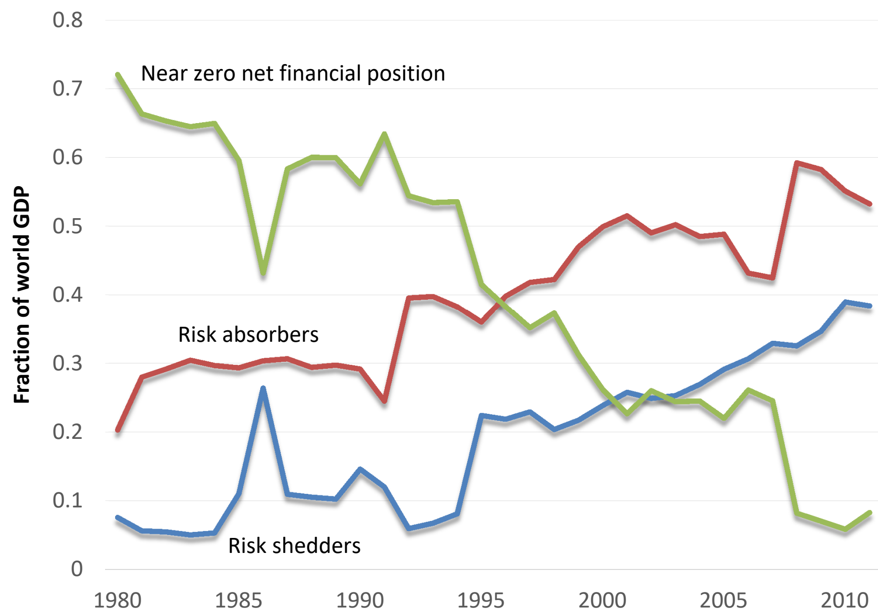

Figure 7: Fractions of World GDP, 1970-2011, for Countries with Positive, Near Zero, or Negative Net Foreign Equity and Debt as Fractions of Their GDP

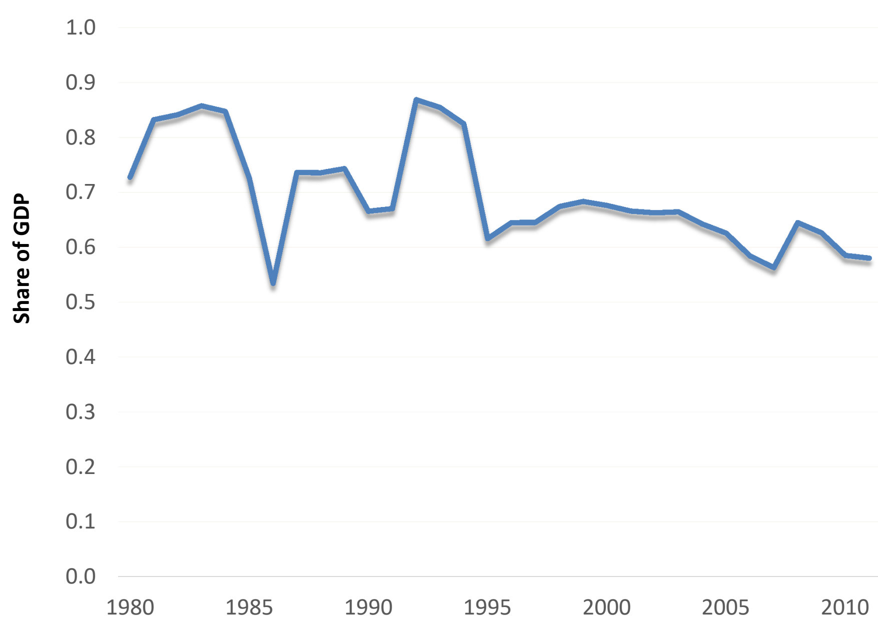

Figure 8: Share of Globally Integrated GDP Arising from Risk-Absorbing Countries

off y' one period later and the other is for a levered investment that pays off $y' - d$ one period later. Here d is the amount of borrowed to help finance the purchase. The unlevered investment is analogous to a dividend strip rather than a claim on the entire stream of future dividends, which has been the subject of most past research. In the setup of this paper, with the assumption of stationarity enforced by requiring both types of investors to believe that the expected future endowment is the same as the current endowment, the premium over a one-year span is close to the premium over a long span.

The unlevered investment costs $P_y = \sum p_i y_i$ and pays off y_i with probability π_i, so its expected payoff is 1, by assumption. Its expected return ratio is $1/P_y$. The return ratio for debt is $1/P$, so the difference, the unlevered equity premium, is

$$\frac{1}{P_y} - \frac{1}{P}. \tag{24}$$

For a levered portfolio that borrows d units at the same time as investing P_y of the endowment, the expected payoff is $1 - d$, so the return ratio is

$$\frac{1-d}{P_y - dP}, \tag{25}$$

and the levered equity premium is the difference between this ratio and $1/P$.

Table 12 shows the equity returns and premiums implied by four combinations of the parameters of the model, with $\omega = 0.5$. The top row shows the results for the case of homogeneous investors with the same coefficient of relative risk aversion of 2 and equal beliefs about the probability of a bad outcome. The expected return on equity—a claim on next year's endowment—is 6.54 percent, corresponding to an equity premium of 3.27 percent, somewhat below most estimates of the normal equity premium. These investors hold no debt. The rightmost two columns of the table show leveraged returns and premiums, with debt at $d = 0.5$. In the first row, the levered return is 10.03 percent and the levered premium is 6.75 percent, comparable to premiums in the U.S. stock market. The second row introduces mild heterogeneity in risk aversion, with a coefficient of 2.3 for the risk-averse investors. The expected return to equity is a bit lower, but the asset equity premium is essentially the same, at 3.24 percent. The levered return is a bit lower and the levered premium almost the same as in the first row.

The third row of Table 12 shows the effect of heterogeneous beliefs by themselves. The asset return is 5.85 percent and the asset equity premium is 3.35 percent. The levered return is 9.43 percent and the levered premium is 6.94 percent. The fourth row combines the two

Parameters		*Calculated values*					
γ, CRRA of risk averse investors*	*π-*, risk-averse belief about probability of bad outcome*	*Gross trade*	*Asset expected return*	*Risk-free rate*	*Asset equity premium*	*Levered equity return*	*Levered equity premium*
2.0	0.010	0.000	6.54	3.27	3.27	10.03	6.75
2.3	0.010	0.009	5.97	2.72	3.24	9.42	6.70
2.0	0.014	0.051	5.85	2.49	3.35	9.43	6.94
2.3	0.014	0.055	5.12	1.78	3.34	8.68	6.90

Table 12: The Equity Premium Implied by Selected Combinations of the Parameters

forms of heterogeneity. The asset premium is 3.35 percent and the levered premium is 6.94 percent. Heterogeneity in preferences is not an important factor in the determination of the equity premium. Rather, what is needed to explain observed premiums is substantial volatility of the Arrow-Debreu prices, a property shared by all four specifications in the table.

I conclude that, notwithstanding the model's focus on explaining the decline in the safe interest rate, it gives a reasonable account of the equity premium. It does so by harnessing the volatility of marginal utility—a key ingredient in most explanations of the magnitude of the equity premium—to the related purpose of understanding the level of the interest rate.

It is beyond the scope of the model to study the pricing of other securities that are more focused on rare events, such as BAA corporate bonds and out-of-the-money options, though these prices are informative and are the subject of a growing literature.

8 Related Literature

Furceri and Pescatori (2014) is a comprehensive review of the level of safe real interest rates in recent decades and in many countries, with results similar to those in Figure 1.

For discussions of the burgeoning literature on low real interest rates, with many recent cites, see Rachel and Smith (2015) and Juselius, Borio, Disyatat and Drehmann (2016). Sources considered in that literature include declining rates of growth of consumption, forces depressing the marginal product of capita or the spread between the marginal product and the cost of capital, rising inequality, and central-bank policy. Kocherlakota (2013) is an informal discussion of the forces depressing the real rate. See also Eichengreen (2015). None of these papers considers changing heterogeneity in risk aversion.

Caballero et al. (2008) develop a global financial model focusing on the decline in world real interest rates, an equilibrium outcome when different regions of the world differ in their capacity to generate financial assets from real investments. High growth in lower-income countries—with low capacity—results in a shift in the terms of trade in the market for safe assets against those countries and thus lower interest rates.

Gourinchas, Rey and Govillot (2010) focuses on the spread between U.S. rates of return on its foreign holdings and the interest rate paid on borrowings from foreign countries. It observes that the US is long risky claims and the rest of the world is long safe claims. This insurance regime has changed over time. The model features inter-country heterogeneity in risk aversion and incorporates disaster risk. Though it treats the size of the risk-tolerant country relative to the risk averse one as a parameter, it does not pursue the role of that dimension of heterogeneity with respect to the global safe real rate.

Barro and Mollerus (2014) study heterogeneity in risk aversion to explain trade in risk-free assets. The paper focuses on explaining the observed volume of outstanding risk-free debt based on heterogeneity in the coefficients of relative risk aversion of two populations, one risk-averse and the other risk-tolerant. The paper notes the relation between heterogeneity and the safe interest rate, but focuses on the effects of disasters on debt and interest rates rather than trends in heterogeneity.

Caballero and Farhi (2016) develop a model with heterogenous agents having extreme heterogeneity in risk aversion. Risk-neutral agents issue safe assets to risk averse agents in a process of securitization, which may be impeded by financial friction. Higher friction reduces the supply of safe assets and lowers the interest rate. Iachan, Nenov and Simsek (2015) consider the role of rising securitization and the decline in the real interest rate.

Abel (1989) is an early contribution to the finance theory with heterogeneous preferences. For recent surveys of that literature, see Basak (2005) and Xiong (2013). Wang (1996) derives closed-form solutions for safe yields in an economy with heterogeneous risk aversion and homogeneous beliefs. The paper surveys earlier contributions to the study of heterogeneous risk aversion. Duffie (2010), chapter 1, section E, discusses heterogeneous risk aversion and the existence of a representative-agent economy that mimics the complete-markets equilibrium of the economy with heterogeneity. Gârleanu and Panageas (2015) consider effects of heterogeneity in risk aversion and intertemporal substitution in a general-equilibrium model. Ilut, Krivenko and Schneider (2016) is a recent contribution to the literature on heterogeneity in beliefs using the ambiguity-aversion framework.

As I noted earlier, this paper and many others using financial models rest crucially on the hypothesis that investors are concerned about seriously adverse events that may lurk in the future. Barro (2006) and Barro (2009) make the case that events that substantially raised marginal utility have occurred and that their potential future occurrence influences asset prices significantly. Farhi and Gabaix (2016) and Seo and Wachter (2016) discuss the evidence from contingent securities prices about the probability of seriously adverse events. Goetzmann, Kim and Shiller (2016) present survey evidence that investors are concerned about disasters.

Fernndez, Schmitt-Groh and Uribe (2016) present evidence that the case of a single global shock is sufficiently close to reality to merit its role as a simplifying assumption in this paper.

9 Concluding Remarks

The modeling and data in this paper support the hypothesis that growth in the wealth in the portfolios of risk-averse investors relative to the wealth of risk-tolerant investors is a source of the downward trend in the worldwide real interest rate. Continuation of the higher growth rates in China and other countries with high propensities to hold debt rather than equity suggests that real rates may continue to decline, or at least not rise back to earlier levels. The extraordinarily low rates for real long-term U.S. Treasury bonds confirm this hypothesis—the 30-year TIPS yield at this writing is 0.92 percent.

The federal government in recent years has contributed immensely to the risk-splitting that is the efficient market response to growing wealth among risk-averse investors. Current predictions show the national debt rising moderately in relation to GDP in the next decade, followed by a return to rapid growth in later decades. The feasibility of this path depends on real rates on that debt at current rates or below. The ideas in this paper point in the direction of feasibility, as does the world's continuing willingness to treat federal debt as safe.

Most discussions of monetary policy, on the other hand, foresee normalization of short-term real rates substantially higher than current rates. For example, at the FOMC's meeting on September 21, 2016, the median in the Economic Projections document (containing the "dot plot") for the federal funds rate in the longer run, was 2.9 percent, 2.4 percentage points above the then-current actual rate. Under the FOMC's forecast, monetary policy would have that many points of headroom to stimulate the economy in the face of a mildly recessionary shock. In a world economy with continuing declines in the fundamental determinants of the safe interest rate, nowhere near that much headroom would exist. The equilibrium real rate could be minus one percent, corresponding to a nominal rate of plus one percent. The lower bound on the nominal rate would become a serious obstacle to effective monetary policy even in the face of small adverse shocks.

References

Abel, Andrew B., "Asset Prices under Heterogeneous Beliefs: Implications for the Equity Premium," February 1989. Whatron School.

Barro, Robert J., "Rare Disasters and Asset Markets in the Twentieth Century," *Quarterly Journal of Economics*, 2006, *121* (3), 823–866.

——, "Rare Disasters, Asset Prices, and Welfare Costs," *American Economic Review*, March 2009, *99* (1), 243–264.

—— and Andrew Mollerus, "Safe Assets," Working Paper 20652, National Bureau of Economic Research, October 2014.

Basak, Suleyman, "Asset Pricing with Heterogeneous Beliefs," *Journal of Banking and Finance*, 2005, *29* (11), 2849 – 2881.

Bernanke, Ben S., Carol Bertaut, Laurie Pounder DeMarco, and Steven Kamin, "International Capital Flows and the Returns to Safe Assets in the United States, 2003-2007," Technical Report 1014, Federal Reserve Board, February 2011. International Finance Discussion Papers.

Caballero, Ricardo J. and Emmanuel Farhi, "The Safety Trap," March 2016. Harvard University, Department of Economics.

——, ——, and Pierre-Olivier Gourinchas, "An Equilibrium Model of "Global Imbalances" and Low Interest Rates," *American Economic Review*, 2008, *98* (1), 358–393.

Duffie, Darrell, *Dynamic Asset Pricing Theory*, Princeton University Press, 2010.

Eichengreen, Barry, "Secular Stagnation: The Long View," *American Economic Review*, May 2015, *105* (5), 66–70.

Farhi, Emmanuel and Xavier Gabaix, "Rare Disasters and Exchange Rates," *Quarterlly Journal of Economics*, February 2016, *131* (1), 1–52.

Ferndez, Andrs, Stephanie Schmitt-Groh, and Martn Uribe, "World Shocks, World Prices, and Business Cycles: An Empirical Investigation," Working Paper 22833, National Bureau of Economic Research, November 2016.

Furceri, Davide and Andrea Pescatori, "Perspectives on Global Real Interest Rates," in "World Economic and Financial Surveys: Recovery Strengthens, Remains Uneven," http://www.imf.org/external/pubs/ft/weo/2014/01/pdf/text.pdf: International Monetary Fund, April 2014.

Gârleanu, Nicolae and Stavros Panageas, "Young, Old, Conservative, and Bold: The Implications of Heterogeneity and Finite Lives for Asset Pricing," *Journal of Political Economy*, June 2015, *123* (3), 670–685.

Goetzmann, William N., Dasol Kim, and Robert J. Shiller, "Crash Beliefs From Investor Surveys," Working Paper 22143, National Bureau of Economic Research, April 2016.

Gourinchas, Pierre-Olivier, Hélène Rey, and Nicolas Govillot, "Exorbitant Privilege and Exorbitant Duty," 2010.

Iachan, Felipe S., Plamen T. Nenov, and Alp Simsek, "The Choice Channel of Financial Innovation," Working Paper 21686, National Bureau of Economic Research, October 2015.

Ilut, Cosmin, Pavel Krivenko, and Martin Schneider, "Uncertainty aversion and heterogeneous beliefs in linear models," March 2016. Stanford University.

Juselius, Mikael, Claudio Borio, Piti Disyatat, and Mathias Drehmann, "Monetary Policy, the Financial Cycle and Ultralow Interest Rates," July 2016. Bank for International Settlements, Working Paper 569.

Kocherlakota, Narayana, "Low Real Interest Rates," April 2013. 22nd Annual Hyman P. Minsky Conference; Levy Economics Institute of Bard College, New York.

Lane, Philip R and Gian Maria Milesi-Ferretti, "The external wealth of nations mark II: Revised and extended estimates of foreign assets and liabilities, 1970–2004," *Journal of international Economics*, 2007, *73* (2), 223–250.

Lucas, Robert E., "Asset Prices in an Exchange Economy," *Econometrica*, 1978, *46* (6), 1429–1445.

Rachel, Lukasz and Thomas Smith, "Secular Drivers of the Global Real Interest Rate," Working Paper 571, Bank of England December 2015.

Seo, Sang Byung and Jessica A. Wachter, "Do Rare Events Explain CDX Tranche Spreads?," Working Paper 22723, National Bureau of Economic Research, October 2016.

Wang, Jiang, "The Term Structure of Interest Rates in a Pure Exchange Economy with Heterogeneous Investors," *Journal of Financial Economics*, 1996, *41* (1), 75–110.

Xiong, Wei, "Bubbles, Crises, and Heterogeneous Beliefs," in Jean-Pierre Fouque and Joe Langsam, eds., *Handbook on Systemic Risk*, Cambridge University Press, 2013, pp. 663–713.