High and Low Unemployment Equilibria, Self-Selection, and Screening in the Labor Market

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Introduction

Are unexploited arbitrage profits available in the labor market when unemployment is high? The answer to this question is central to understanding recessions and regional depressions. The arbitrage opportunity that might be available in high unemployment is the following: Firms can advertise jobs that are immediately available at wages below prevailing levels. Job-seekers rationally trade quicker job-finding for lower wages. The labor market quickly finds its equilibrium at a lower wage and lower unemployment. There is no question that workers flock to possible job openings in slack markets; in every recession and in many depressed areas, newspapers carry stories about thousands of workers lining up to apply for advertised job openings.

The persistence of unemployment in the national economy, and the even stronger persistence of unemployment in some regions, is completely inconsistent with the availability of an arbitrage opportunity. What feature of the high-unemployment labor market prevents the arbitrage of lower wages for easier job-finding? The answer considered here is that employers face much higher screening costs in a high-unemployment market. I explore this issue within a model with multiple equilibrium unemployment rates. The model is related to Peter
Diamond's [1982] idea of thick and thin markets, though the assumptions about the technology are different. The key similarity is in the complementarities across economic actors that exist when incomplete markets give rise to search.

The basic idea in the model is that selecting employees is a costly activity to firms. The efficiency of recruitment is highest if job applicants self-select perfectly. If a firm knew that every applicant was a good match, it could dispense with screening and probationary employment and simply take workers on as they applied. On the other hand, if large numbers of poorly matched workers show up whenever there is a job opening, the firm has to incur important costs to figure out which of the many applicants to hire. From the worker's point of view, the decision about where to seek work depends on the likelihood of employment. If the worker knows that a particular job is a good match, and that the employer automatically hires all applicants, then the worker will go straight to that employer and not apply anywhere else. Workers will self-select the appropriate jobs. In the resulting efficient or good self-selection equilibrium, both job search and job filling are quick and cheap. On the other hand, there can be an inefficient or bad equilibrium. In the bad equilibrium, workers know there is only a small chance of employment at any given employer. They apply at a great
many places, including those where they do not have a comparative advantage. Because every job opening attracts a flood of applicants, including many who are not good matches, employers use costly, imperfect screening methods to decide whom to hire. The efficiency of job filling is much lower than in the good equilibrium. Unemployment is much higher. No single firm sees any arbitrage possibilities because they cannot induce self-selection of applicants.

Obviously the coexistence of the two possible equilibria depends on assumptions and parameter values. If firms had perfect screening methods, they could induce self-selection by denying the possibility of rewards to workers who are poor matches. Workers need to have superior knowledge of the quality of the match, else they could not self-select.

1. A simple model

Consider a group of workers who are homogeneous as far as employers can tell without costly screening. There are two employers and $2M$ workers. For each employer, there are $M$ workers with comparative advantages at that employer. Workers know where they have their comparative advantages. Before employment occurs, there
are opportunities in sequence for workers to apply for jobs. In each of these periods, each worker draws one or the other employer as a possible place to apply for work. The draw is random with equal probabilities. The worker then chooses whether or not to apply for a job at the assigned potential employer. Each employer can screen any number of applicants. The screening process has a probability $p_Q$ of a true positive and a probability $p_N$ of a true negative. Employers do not remember the result of screening a worker from one search period to the next.

Workers have a personal cost $\gamma_N$ of working in a job where they do not have a comparative advantage (absent this cost, workers would not have an incentive to self-select in the good equilibrium). Workers face a cost $\gamma_A$ of each job application and a cost $\gamma_T$ of spending an extra period looking for work (this cost is not foregone work, which occurs only in the final period, but is some other foregone opportunity).

For suitable values of these parameters, there are two equilibria—a good one with self-selection and a bad one with screening.
Search equilibrium

When workers visit employers one after another to find jobs, there is room for opportunistic behavior on the part of employers, who automatically have a certain amount of monopsony power. If all other employers pay the same prevailing wage, one employer ought to be able to engage a worker for slightly less than the prevailing wage. The worker's reservation wage should be the prevailing wage less the incremental search cost if the worker declines the current offer and continues searching for a job at the prevailing wage. In addition, an employer may benefit from a lower wage offer because it deters unqualified workers more than qualified ones. Thus an equilibrium with a single prevailing wage at all employers is impossible. The equilibrium must involve a distribution of wages across employers. I believe this point is extraneous to the issues of this paper and, further, has little practical significance. Here, I will assume that employers forego the monopsony power available from undercutting the prevailing wage. Firms attract job applicants by developing reputations for paying the prevailing wage.

I will assume that firms act as wage-takers in the labor market. Firms are price takers in the output market as well. Their technology is
as follows: A worker with a comparative advantage with an employer produces one unit of output. A worker without a comparative advantage produces \( \theta < 1 \) units of output.

*The good equilibrium with self-selection*

In the self-selection equilibrium, firms automatically hire all applicants without screening. This behavior is optimal only if the applicants self-select—only those with a comparative advantage in the job apply for that job. From the worker's point of view, self-selection is optimal. The real wage is one unit of output. This wage is available from all employers, including those where the worker does not have a comparative advantage. However, the personal cost, \( \gamma_N \), of working in an unsuitable job guarantees that workers will actually take only a job with comparative advantage. The application cost, \( \gamma_A \), ensures that workers will not make frivolous applications where they do not have comparative advantages.
The bad equilibrium with screening

In the bad equilibrium, employers find it necessary to screen applicants. Self-selection does not occur. Because jobs are hard to find, workers apply for and accept jobs where they do not have comparative advantages. Given the likelihood of receiving applications from unsuitable workers, employers use screening.

Consider a job seeker in a screening equilibrium, if one exists. All firms pay the same wage, $w$. The worker will obviously take up the option of applying at the employer where the worker has a comparative advantage. One strategy would be to apply only at that employer and not apply if assigned to the other employer. The probability of the worker being assigned to the well matched employer and getting a job at that employer in one search period is $h_5 = p_Q/2$. The worker expects to make half a job application per period, so the search cost per period is $\gamma_A/2 + \gamma_T$. The expected number of periods of search is the reciprocal of the probability of success in each period, so the value of the strategy of searching only at the employer where the worker has a comparative advantage is

$$w - \frac{\frac{1}{2} \gamma_A + \gamma_T}{h_5}$$

(1.1)
The second strategy is to apply no matter which employer is drawn. Then the probability of getting a job in a given search period includes a term for the probability of passing the screen by mistake at the other employer:

\[ h_A = \frac{1}{2} p_Q + \frac{1}{2} (1 - p_N) \]  \hspace{1cm} (1.2)

In each search period, the probability that a job, if found, will not be one where the worker has a comparative advantage is:

\[ p_B = 1 - \frac{h_S}{h_A} \] \hspace{1cm} (1.3)

This is also the probability that the job eventually found will be unsuitable. Under the strategy of applying everywhere, the worker incurs one application cost each period, and the total search cost per period is

\[ \gamma_A + \gamma_T \] \hspace{1cm} (1.4)

The value of the apply-everywhere strategy is the wage earned on the job
less expected search costs and less the expected personal cost of an unsuitable job:

\[ w - \frac{\gamma_A + \gamma_T}{h_A} - p_B \gamma_N \]  

(1.5)

The screening equilibrium will exist if the apply-everywhere strategy has the higher value. Some algebra shows that this condition is

\[ p_B \gamma_N + \frac{\gamma_A}{2h_A} \leq \left( \frac{1}{h_s} - \frac{1}{h_A} \right)(\gamma_A + \gamma_T) \]  

(1.6)

The quantity on the left side is expected mismatch costs plus the cost of the extra application. The sum must not exceed the reduction in expected search cost on the right side to merit the choice of the apply-everywhere strategy.

It remains to show that employers will choose to screen when workers apply everywhere. With free entry, the wage will be bid up to the point where the profit from employing workers is zero. If firms do not screen, the zero-profit wage is

\[ w = \frac{1+\theta}{2} \]  

(1.7)
If screening offers a chance to shift the mix of hires to something better than equal probabilities of good and bad matches, positive profit will be available. The condition for firms to screen is simply that the screen have some discriminatory power:

\[ p_Q \geq 1 - p_N \]  \hspace{1cm} (1.8)

That is, the screen has a higher chance of resulting in hiring a qualified worker than of hiring an unqualified worker.

I summarize in

**Proposition 1.** An inefficient screening equilibrium can exist in the model labor market if the discriminatory power of the screen is low \( \left( \frac{1}{h_S} - \frac{1}{h_A} \text{ is high} \right) \), if the application cost, \( \gamma_A \), is low, if the personal cost of a job mismatch, \( \gamma_N \), is low, if the probability of a false positive screen is high, and if the per-period cost of job search, \( \gamma_T \), is high, sufficient to make the inequality 1.8 hold.
2. *A model with variable screening effort*

The nature of the missing-market complementarity is further clarified in a model where employers make a continuous choice of screening effort. The results of screening are expressed by the probabilities $p_Q$ of a correct positive result and $p_N$ of a true negative result. The employer chooses the two probabilities given a cost, $\mathcal{C}(p_Q, p_N)$, of screening one worker sufficiently accurately to achieve the probabilities. Cost is increasing and convex in the two probabilities.

*The employer's recruiting strategy*

As before, the firm is a wage-taker at the market wage $w$ and hires every worker with a non-negative expected profit. The firm gains $1 - w$ from a qualified worker and loses $w - \theta$ from a badly matched worker. Expected profit for the representative job applicant is

$$p_Q q(1 - w) - (1 - p_N)(1 - q)(w - \theta) - \mathcal{C}(p_Q, p_N).$$

(2.1)

First-order necessary conditions for the maximization are
\[ q(1-w) = \frac{\partial C}{\partial p_Q} \]  

(2.2)

\[ (1-q)(w-\theta) = \frac{\partial C}{\partial p_N}. \]  

(2.3)

The two first-order conditions can be solved for the optimal screening probabilities given the fraction of qualified applicants, \( q \), and the wage, \( w \). Call them \( p_Q^*(q,w) \) and \( p_N^*(q,w) \). The resulting demand for labor is perfectly elastic at the wage that just extinguishes the net profit from hiring a worker. Labor supply is inelastic at the level of \( 2M \) workers. Equilibrium in the labor market occurs at full employment with the wage that sets profit per worker at zero. The equilibrium wage is the root of

\[ p_Q^*(q,w)q(1-w) - [1-p_N^*(q,w)](1-q)(w-\theta) = C(p_Q^*(q,w),p_N^*(q,w)). \]  

(2.4)

Let \( p_Q^E(q) \) and \( p_N^E(q) \) be the equilibrium screening probabilities when a fraction \( q \) of job applicants are qualified.
The worker’s search strategy

Consider a worker deciding on a search strategy in a labor market where a fraction \( q \) of job applicants are qualified and all employers have adopted screening methods with success probabilities \( p^S_Q(q) \) and \( p^S_N(q) \). A worker chooses between the self-selection strategy of applying only at the employer where the worker is well matched and the apply-everywhere strategy. Under the second strategy, there is some chance that the worker will inflict a cost on both the employer and the worker. If it is sufficiently costly to find a job at the well matched employer, the worker will be willing to settle for the bad match at another employer.

Each employer screens every applicant in each search period. Under the self-selection strategy, the probability \( h_S \) that a worker will be hired at the one place the worker applies is the probability, one-half, of drawing that employer times the probability, \( p_Q \), of passing the screen, given that the worker is qualified. Under the apply-everywhere strategy, the worker has a probability \( p_Q/2 \) of being hired at the one employer where the worker is qualified and a probability \( (1-p_N)/2 \) of being hired at the other employer. The probability of being hired somewhere is

\[
h_A = \frac{1}{2} p_Q + \frac{1}{2} (1-p_N) \quad . 
\]  

(2.5)
The probability that the job will be a bad match, given that some job offer is received, is

$$p_B = 1 - \frac{h_S}{h_A}. \quad (2.6)$$

I assume that there is a distribution across workers of the personal cost of a job mis-match, $\gamma_N$: the fraction of workers with $\gamma_N$ below a specified value, $\tilde{\gamma}_N$ is $F(\tilde{\gamma}_N)$. If a worker's personal cost is the same for all search periods, the composition of the population of searchers will change as search proceeds—those with the less aversion to mismatches will find jobs first, because they apply at both employers. To avoid dealing with this technical issue, I will assume that each worker draws $\gamma_N$ independently each search period. Then the composition of the remaining searchers is the same in every search period.

To characterize the choice of search strategy by a worker, I will proceed in the following way: The worker assumes that he will use the same strategy as everybody else in all future periods. From a candidate value of the fraction of applicants who are qualified, $q$, and corresponding screening probabilities, $p^S_0(q)$ and $p^S_N(q)$, I calculate the expected search cost and expected personal cost of a possible mismatch. Then I examine
the worker's choice of strategy in one particular period. When that choice generates a population of searchers such that the fraction of applicants who apply for each job is \( q \), then that value of \( q \) is a labor market equilibrium.

Let \( a \) be the fraction of searchers who adopt the apply-everywhere strategy. It is easy to show that \( a \) is related to \( q \) by

\[
q = \frac{1}{1+a}.
\]  
(2.7)

The probability that a worker drawn at random will find a job in a given search period is

\[
ah_A + (1-a)h_S.
\]  
(2.8)

and the expected number of searches is the reciprocal of this quantity. The expected search cost per period for a worker drawn at random is

\[
\gamma_T + \frac{1}{2}(1+a)\gamma_A.
\]  
(2.9)

Expected search cost is the product of the expected number of searches and expected search cost per period because the choice of search method
this period is statistically independent of the success of search in past periods. Hence expected search cost is

$$S = \frac{\gamma_T + \frac{1}{4}(1+a)\gamma_A}{ah_A + (1-a)h_S} . \quad (2.10)$$

The searcher has a probability $ap_B$ of taking a poorly matched job and an expected personal cost of $E[\gamma_N | \text{apply-everywhere strategy}]$. Expected bad-match cost, $B$, is the product.

Under the self-selection strategy for the current period, search cost is $\gamma_A/2 + \gamma_T$. The strategy has a probability $h_s$ of success. The value of the strategy is the wage ultimately earned less expected current and future search costs:

$$w - \frac{1}{2}\gamma_A - \gamma_T - (1-h_A)(S+B) . \quad (2.11)$$

The value of the apply-anywhere strategy involves higher current application costs, a current cost of a possible bad match, and a higher likelihood of finding a job:

$$w - \gamma_A - \gamma_T - (h_A - h_s)\gamma_N - (1-h_A)(S+B) . \quad (2.12)$$
The condition for superiority of the apply-everywhere strategy is

\[(h_A - h_S)(S+B) \geq \frac{1}{2} \gamma_A + (h_A - h_S) \gamma_N\] \hspace{1cm} (2.13)

That is, expected future search costs avoided by choosing the apply-everywhere strategy with its higher job-finding probability should be at least as large as the extra application costs plus the expected mismatch cost associated with applying for poorly matched jobs. A worker will choose the apply-everywhere strategy if his mismatch cost is less than a certain critical value:

\[\gamma_N \leq S+B - \frac{\frac{1}{2} \gamma_A}{h_A - h_S}\] \hspace{1cm} (2.14)

Recall that the procedure for finding an equilibrium was to suppose that the worker under consideration sees himself as behaving like everybody else in the future. I will use this assumption to evaluate the expected mismatch cost, \(B\). The cost is

\[B = a p_B E(\gamma_N | \gamma_N < \gamma_N^*)\] \hspace{1cm} (2.15)
In the candidate equilibrium, a fraction $q$ of applicants are qualified. The corresponding fraction of job-seekers who are using the apply-everywhere strategy is

$$a = \frac{1-q}{q}.$$  \hspace{1cm} (2.16)

Then the value of $\gamma_N^*$ can be calculated from

$$a = F(\gamma_N^*).$$  \hspace{1cm} (2.17)

From $\gamma_N^*$ the value of the conditional expectation of the lower tail of the distribution can be calculated and then the value of $B$ follows from equation 2.15.

Finally, equation 2.14 gives the cutoff value for $\gamma_N$ for the job-seeker's choice of strategy in the current period. Call the value $\gamma_N^W$. Then

$$q^W = \frac{1}{1 + F(\gamma_N^W)}.$$  \hspace{1cm} (2.18)

This is the fraction of job applicants this period who are poorly matched to the job, given this period's optimal choice on the assumption that in
future periods a fraction \( q \) of applicants will be qualified. In equilibrium, the two fractions must be equal.

3. Example

In the example, the screening technology is quadratic:

\[
C(p_Q, p_N) = \frac{b}{2}(p_Q + p_N - 1)^2 + \frac{c}{2}p_Q^2 + \frac{d}{2}p_N^2 .
\]

The distribution of the personal cost of a mismatch is uniform:

\[
F(\gamma_N) = \min(1, \max(0, \frac{\gamma_N - \gamma_N}{\gamma_N - 2N})) .
\]

Figure 1 shows the resulting value of the fraction of applicants who are qualified, \( q^w \), for different values, \( q \), assumed by employers for the current period and by workers for future periods. Any point where \( q^w \) crosses the 45\(^\circ\) line is an equilibrium.
Figure 1. Equilibrium fraction of applicants qualified

Fraction

Candidate
Searcher choice

Fraction