Investment in Equipment and Structures, 1929-1961

Robert E. Hall

A Bachelor's thesis submitted in fulfillment of the requirements of the Undergraduate Honors Program of the Department of Economics, University of California.

This paper is also listed as Working Paper No. 51 in the series by the Committee on Econometrics and Mathematical Economics at the Institute of Business and Economic Research, University of California, Berkeley.

May, 1964

I. Introduction

This study attempts to test a relatively simple hypothesis of investment behavior based on a profit-maximizing principle against yearly investment data for the United States since 1929. It is to some extent a measure of the inadequacy of a single semester of an undergraduate's time that only the preliminary stages of the testing have been carried out. The most promising leads uncovered by this study have yet to be followed up. Still, the hypothesis appears to have survived the first round, and in the positivistic world of the modern economist, this is perhaps the most that can be expected.

II. Investment and Capital Stock

In general, the level of capital services available to the firm is a function of the firm's investment in capital goods at each point in its past and perhaps of the rate of use of its capital stock. We assume that there is a function g(t) such that the relation between the rate of input of capital services k(t) and gross investment I(t) can be written

$$k(t) = \int_{-\infty}^{t} g(t-\tau)\, I(\tau)\, d\tau$$

where $g(0) = 1$, $\frac{dg}{dt} \leq 0$, and $\lim_{t\to\infty} g(t) = 0$.

This appears to be a good approximation under the hypothesis that the distribution of investment among capital goods of different lifetimes is always the same and that the rate of depreciation is independent of the intensity of use of capital.

Replacement investment R(t) is defined as the difference between gross investment I(t) and net investment $N(t) = \frac{dk}{dt}$,

$$R(t) = I(t) - \frac{d}{dt} \int_{-\infty}^{t} g(t-\tau)\, I(\tau)\, d\tau$$

$$= I(t) - g(0)\, I(t) - \int_{-\infty}^{t} g'(t-\tau)\, I(\tau)\, d\tau$$

$$= - \int_{-\infty}^{t} g'(t-\tau)\, I(\tau)\, d\tau \quad .$$

In this study, replacement is approximated by the sum

$$R(t) = \sum_{\tau=t-\tau_o}^{t} W(t-\tau)\, I(\tau)$$

It is by no means clear what distribution w(t) is appropriate for this calculation. Two approaches suggest themselves: first, assuming that the distribution has a small number of parameters and estimating these along with the other parameters of the investment model, or second, making assumptions *a priori* about the distribution. Fortunately the values derived for replacement and net investment are relatively insensitive to the form of the distribution. Figure 1 illustrates the differences brought about by two estimates of w(t) in the case of manufacturing equipment investment.

The assumption that the replacement distribution is geometric, that is, that

$$R(t) = [\delta L + \delta(1-\delta)L^2 + \delta(1-\delta)^2 L^3 + \ldots] I(t)$$

(where L is the lag operator: $L^n x(t) = x(t-n)$)

has particular computational advantages. In that case,

$$R(t) = \frac{\delta L}{1-(1-\delta)L} I(t)$$

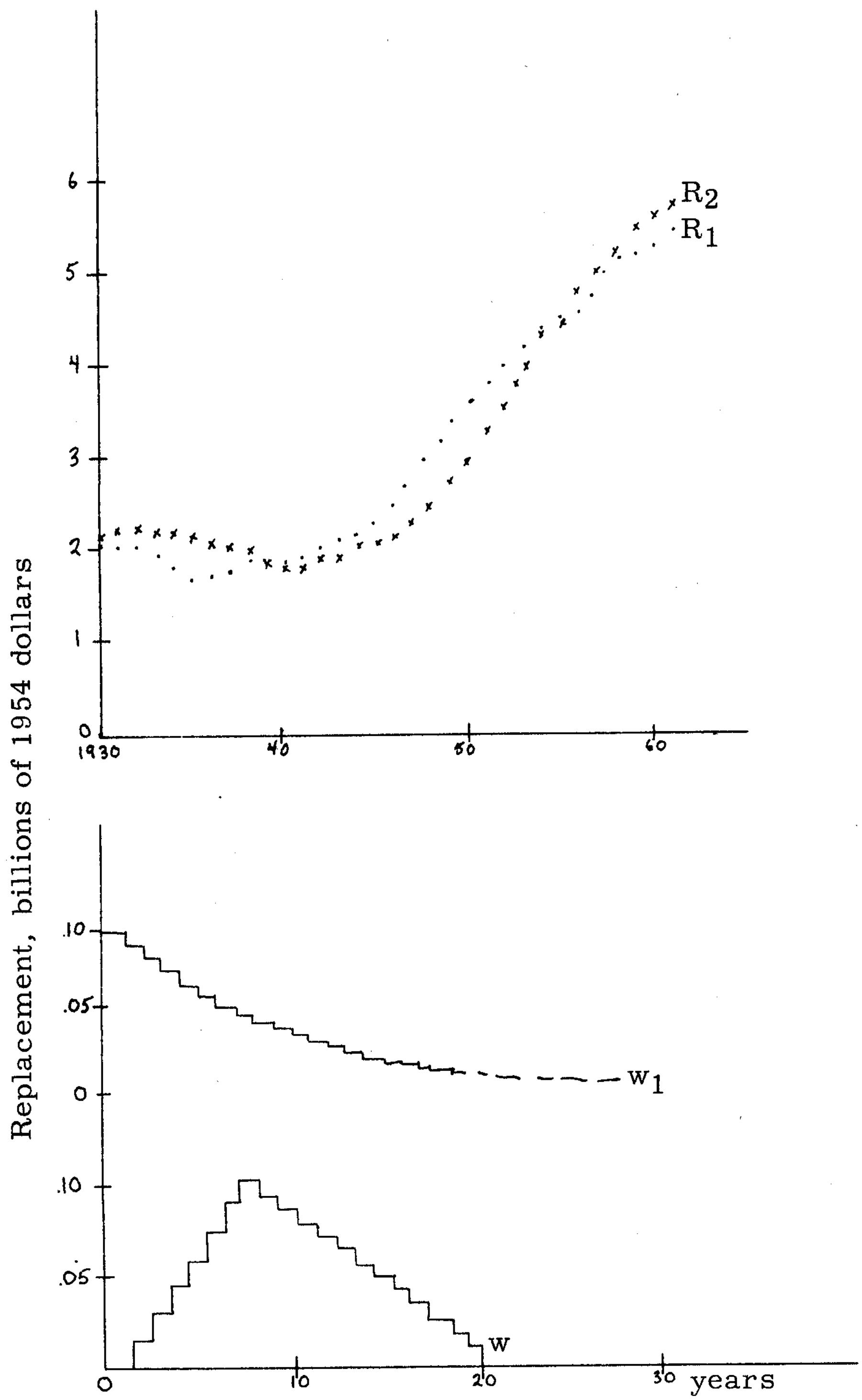
$$N(t) = I(t) - R(t)$$

$$= \frac{1-L}{1-L+\delta L} I(t)$$

$$k(t) = \frac{L}{1-L} N(t)$$

$$= \frac{L}{1-L+\delta L} I(t)$$

or, $$R(t) = \delta k(t)$$



Replacement, billions of 1954 dollars
6
5
4
3
2
1
0
1930
40
50
60
R_2
R_1
.10
.05
0
w_1
.10
.05
w
0
10
20
30
years

Figure 1

III. The Demand for Capital

In deriving the demand function for capital services we take the following view of the firm: capital stock is acquired at the beginning of a period of length τ at a price q(t) and sold at the end at a price $q(t+\tau)$. Output sells at a price p in a competitive market, the wage rate is s, and the cost of funds is r. Then the cost of holding a unit of capital stock for the period is $q(t) - q(t+\tau) + r\tau q(t)$ and the cost of labor input is $s\tau l$. A production function describes the technology of the firm:

$$Q = f(k, \ell)$$

We assume that the firm maximizes profit for the period, where profit is given by

$$\pi \tau = p \tau Q - s \tau \ell - [q(t) - q(t+\tau) + r \tau q(t)] k$$

If f(k,l) is suitably convex, profit is maximized when

$$\frac{\partial \pi \tau}{\partial \ell} = p \tau \frac{\partial Q}{\partial \ell} - s \tau = 0 \text{ , or}$$

$$\frac{\partial Q}{\partial \ell} = \frac{s}{p} \text{ ,}$$

and

$$\frac{\partial \pi \tau}{\partial k} = p \tau \frac{\partial Q}{\partial k} - [q(t) - q(t+\tau) + r \tau q(t)] = 0,$$

or

$$\frac{\partial Q}{\partial k} = \frac{1}{p \tau} [q(t) - q(t+\tau) + r \tau q(t)] \text{ .}$$

Now taking the limit as $\tau \to 0$,

$$\frac{\partial Q}{\partial k} = \frac{-\frac{dq}{dt} + r\,q(t)}{p} = \frac{c}{p}$$

where

$$c = -\frac{dq}{dt} + r\,q(t)$$

is the user cost of capital.

Thus far we have glossed over the important question of how the level of capital services is related to the price q(t) of the capital goods delivering this level. Suppose that a unit of capital costing q_o dollars when new delivers k(t) units of capital services during a lifetime of λ years, and suppose that prices for capital goods of each age are constant over time. The user cost of this unit of stock is

$$c = \frac{1}{k(t)} \left[-\frac{dq}{dt} + r\,q(t) \right]$$

Under the assumption of a well-developed market in capital goods with many buyers, market indifference conditions require that c be constant over the age of the capital. Now in most ordinary accounting practice, q(t) is taken to be

$$q(t) = q_o \left(1 - \frac{t - t_o}{\lambda} \right) , \quad t_o \leq t \leq t_o + \lambda ,$$

the case of straight-line depreciation.

$$-\frac{dq}{dt} = \frac{q_o}{\lambda}$$

$$c = \frac{1}{k(t)} \left[\frac{q_o}{\lambda} + r\,q_o \left(1 - \frac{t - t_o}{\lambda} \right) \right]$$

Choosing units so that $k(t_o) = 1$

$$c = q_o \left[\frac{1}{\lambda} + r \right]$$

and

$$k(t) = \frac{1}{1+\lambda r} \left[1 + r(\lambda - t + t_o) \right]$$

Explicit account may be taken of the income tax structure by supposing that a proportion w of the cost of funds and a proportion v of depreciation expense are deductible for tax purposes. Then if the rate of taxation of taxable income is u, profit is given by

$$\pi = p\,Q - s\,\ell - (\delta + r)k - u[p\,Q - s\,\ell - (v\,\delta + w\,r)k]$$

where $\delta = \frac{1}{\lambda}$

which achieves a maximum under the conditions described above when

$$\frac{\partial \pi}{\partial k} = p\frac{\partial Q}{\partial k} - (\delta + r) - u\,p\,\frac{\partial Q}{\partial k} + u\,w\,r + u\,v\,\delta = 0$$

or

$$\frac{\partial Q}{\partial k} = \frac{c}{p} \qquad \text{where } c = \frac{1 - uv}{1 - u}\,\delta + \frac{1 - uw}{1 - u}\,r$$

and

$$\frac{\partial Q}{\partial \ell} = \frac{s}{p}$$

The demand function for capital stock is the solution of this system.

If the production function is Cobb-Doublas,

$$Q = a\,k^{\alpha}\,\ell^{\beta} \quad , \quad \alpha + \beta < 1$$

$$\frac{\partial Q}{\partial k} = \alpha\, a\, k^{\alpha}\, \ell^{\beta}\, k^{-1}$$

$$= \alpha\, Q\, k^{-1}$$

$$\alpha\, Q\, k^{-1} = \frac{c}{p}$$

$$k^{*} = \alpha\,\frac{p\,Q}{c} \quad , \text{ desired capital stock.}$$

Now if the market in capital goods is well-organized and reacts instantaneously, the market will be cleared:
$k = k^{*}$, in which case

$$c = q_o(\delta + r) = \alpha\,\frac{p\,Q}{k}$$

or

$$q_o = \frac{\alpha\, p\, Q}{k(\delta + r)}$$

Investment is then given by the supply function for the capital goods industry. However, this view of the supply side of investment is not empirically fruitful at the moment because of the difficulties inherent in specifying a supply function which is suitable for long periods of time.

The theory does, however, yield a testable hypothesis of more limited scope:

$$\frac{d}{dt}\, k(t) = \frac{d}{dt}\, k^{*}(t)$$

or

$$N(t) = \frac{d}{dt}\, k^{*}(t)$$

which is obtained simply by differentiating the equilibrium condition. Jorgenson [3] has formulated this model in a more sophisticated manner so as to recognize the existence of a lag in the response of investment as measured to changes in k^*. His model, which is formulated in discrete terms for the purpose of empirical application, proposes that new investment put in place, N(t), and new investment projects begun, $N_B(t)$, are related in this way:

$$N(t) = \mu(L)\, N_B(t)$$

where $\mu(L)$ is a power series in the lag operator L. He further assumes that new projects are begun in period t until the sum of all uncompleted projects equals the difference between $k^*(t)$ and k(t):

$$\frac{1 - L\,\mu(L)}{1 - L}\, N_B(t) = k^*(t) - k(t)$$

or

$$[1 - L\,\mu(L)]\, N_B(t) = (1 - L)\,[k^*(t) - k(t)]$$

but

$$(1 - L)\, k(t) = N(t-1) = L\ \mu\,(L)\, N_B(t)$$

so

$$N_B(t) = (1 - L)\, k^*(t)$$

and

$$N(t) = \mu(L)\,(1-L)\, k^*(t)$$

$$= \mu(L)\, \Delta k^*(t)$$

Combining this with the value for k^* derived earlier,

$$N(t) = \mu(L)\alpha\left[\frac{p(t)\,Q(t)}{c(t)} - \frac{p(t-1)\,Q(t-1)}{c(t-1)}\right]$$

For the purposes of estimating the parameters of this equation, we assume $\mu(L)$ has the form

$$\mu(L) = \frac{\gamma_o + \gamma_1 L}{1 - \omega_1 L}$$

In this case,

$$(1 - \omega_1 L)\,N(t) = (\gamma_o + \gamma_1 L)\ \alpha\left[\frac{p(t)\,Q(t)}{c(t)} - \frac{p(t-1)\,Q(t-1)}{c(t-1)}\right]$$

or, writing it out in full,

$$N(t) = \gamma_o\,\alpha\left[\frac{p(t)\,Q(t)}{c(t)} - \frac{p(t-1)\,Q(t-1)}{c(t-1)}\right]$$

$$+\ \gamma_1\alpha\left[\frac{p(t-1)\,Q(t-1)}{c(t-1)} - \frac{p(t-2)\,Q(t-2)}{c(t-2)}\right]$$

$$+\ \omega_1\,N(t-1)$$

From the assumption that all investment which is begun is completed eventually, we have

$$\sum_{t=0}^{\infty} \mu_t = 1$$

which in this case reduces to the condition

$$\gamma_o + \gamma_1 + \omega_1 = 1$$

The mean lag, $\bar{\mu}$, is

$$\bar{\mu} = \sum_{t=1}^{\infty} t\,\mu_t = \sum_{t=1}^{\infty} t\,(\gamma_o\,\omega_1 + \gamma_1)\,\omega_1^{t-1}$$

$$= \frac{\gamma_o\omega_1+\gamma_1}{(1-\omega_1)^2}$$

IV. Estimation of the Parameters of the Model

Serious problems arise in trying to specify the stochastic element which must be added to this model before it can be fitted to the real worl. Perhaps the most plausible specification provides that k^* is observed without error and that there is a stochastic element, either an error of observation or a random disturbance, in the net investment terms. In this case,

$$\begin{aligned} N(t) &= \mu(L)\,\Delta k^*(t) + \nu(t) \\ &= \gamma_o\,\Delta k^*(t) + \gamma_1\,\Delta k^*(t-1) \\ &\qquad + \omega\quad N(t-1) + \nu(t) - \omega_1\,\nu(t-1) \end{aligned}$$

Koyck [5], Klein 4], and Nerlove [6] have devoted attention to the problem of estimating the parameters of a model of this type. In general, ordinary least squares estimates are asymptotically biased since $\nu(t-1)$ is not distributed independently of N(t-1). However, in the special case where $\nu(t)$ is generated by the stationary stochastic process,

$$\nu(t) = \omega_1\,\nu(t-1) + \epsilon(t)$$

and $\epsilon(t)$ is independently and identically distributed for each t, the composite error term $\nu(t) - \omega_1\,\nu(t-1)$ reduces to $\epsilon(t)$ and the least squares estimator has all its usual desirable properties. It is not clear whether there is an appropriate statistical test of this hypothesis based on the results of fitting the equation by least squares. In any case, the estimates obtained by least squares for $\alpha\gamma_o$ and $\alpha\gamma_1$

are unbiased under the hypothesis ω_1 = est. ω_1 and efficient if the composite errors are serially uncorrelated.

Nerlove [6] and Klein [4] suggest deriving an estimator for this type of equation by the principle of maximum likelihood. No analytical solution appears to exist to the problem of finding values of the parameters which maximize the appropriate likelihood function. Since under the hypothesis that ω_1 has some particular value $\bar{\omega}_1$, consistent estimates of γ_o and γ_1 can be obtained directly, the problem reduces to one of finding the maximum of the likelihood function over only one variable, $\bar{\omega}_1$, and a graphical solution is feasible.

Koyck [5] suggests a consistent estimator based on iterated least squares which Klein [4] shows is equivalent to a limited information maximum likelihood estimator which ignores the constraint that the same error appears in two successive equations.

A second stochastic specification of some interest provides that $\Delta k^*(t)$ is observed with an error $\eta(t)$ while the true value of N(t) is observed. In this case we may rewrite $\mu(L)$ as

$$\mu(L) = \frac{1 + \frac{\gamma_1}{\gamma_o} L}{\frac{1}{\gamma_o} - \frac{\omega_1}{\gamma_o} L}$$

in which case,

$$\Delta k^*(t) = -\frac{\gamma_1}{\gamma_o} \Delta k^*(t-1) + \frac{1}{\gamma_o} N(t) - \frac{\omega_1}{\gamma_o} N(t-1) + \eta(t) + \frac{\gamma_1}{\gamma_o} \eta(t-1)$$

Exactly the same problems of estimation are involved with this equation as with the first equation.

V. Application to U.S. Investment, 1929-1961

The model was fitted to investment data for equipment and structures separately for manufacturing, nonfarm-nonmanufacturing, and farm sectors. The results will be presented in detail for manufacturing equipment and will be summarized for other assets and sectors.

Figure 2 presents estimates of actual capital stock and a multiple of desired stock for the period. The replacement distribution is assumed to be geometric with $\delta = .1034$. Desired capital stock fluctuates around the actual value to a greater extent than predicted by the theory, which allows only short-run fluctuations resulting from the lagged response of investment. No doubt this is in great part a reflection of the disparity between our assumptions about the nature of the market in capital goods and the true state of the real world. For example, although we have assumed the contrary, it is not often possible to sell off redundant capital stock in slack times. However, another important cause of the discrepancy between these estimates of desired and actual stock is probably the inadequacy of the price index for capital goods. This is particularly true for the war years, for which ordinary price indices are entirely artificial and are useful only for deflating investment data. For this reason the years 1943 to 1948 were excluded in making all further estimates. In addition, however, there is a serious question as to how adequately available indices measure either short or long run changes in capital goods prices even in relatively stable times. It appears that weaknesses in price data are one of the major impediments to the empirical application of investment theory.

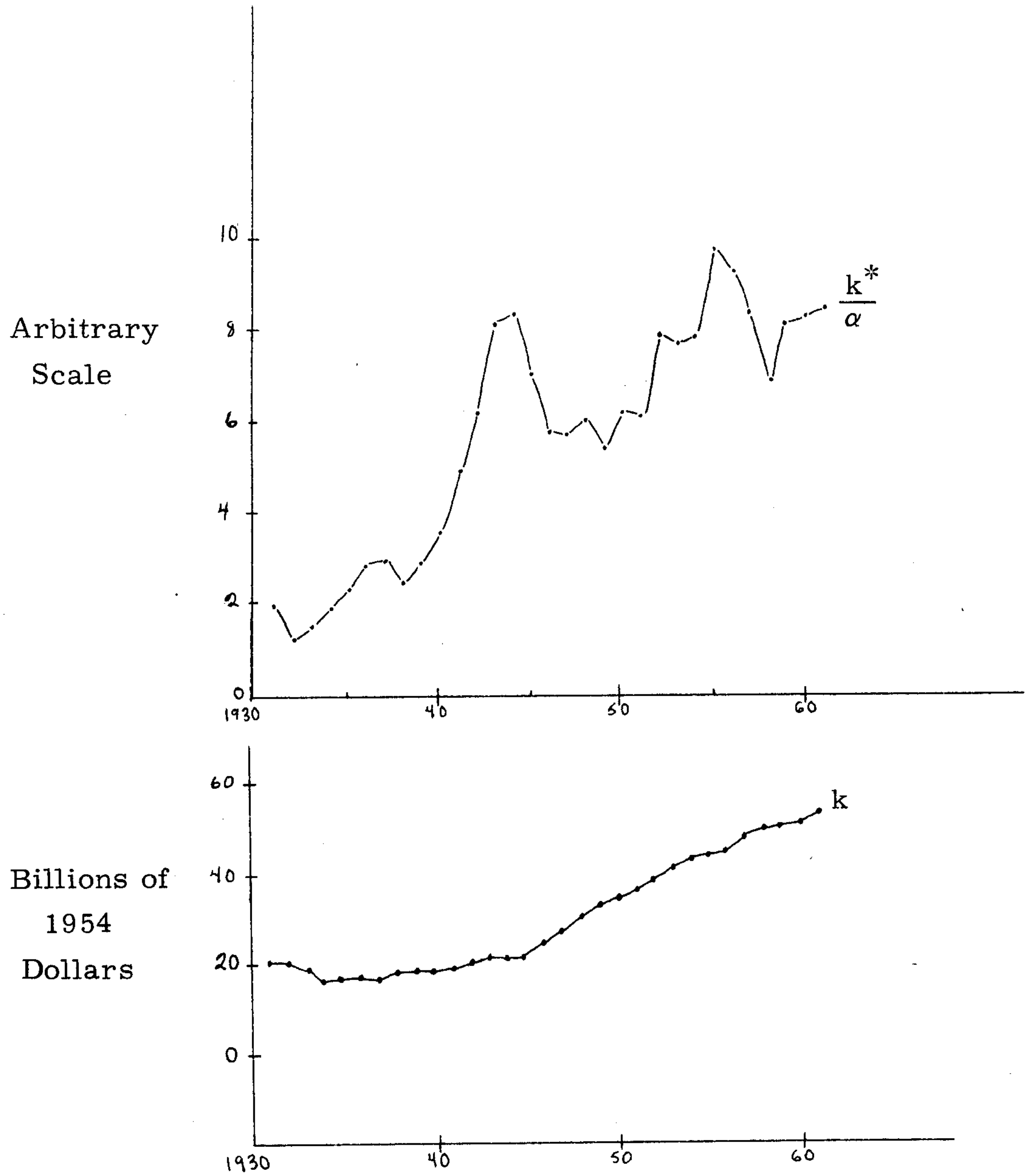

Arbitrary
Scale
10
8
6
4
2
0
1930
40
50
60
$\frac{k^*}{\alpha}$
Billions of
1954
Dollars
60
40
20
0
1930
40
50
60
k

Figure 2

Net investment and change in desired stock are shown in Figure 3. The following estimates of the parameters of the model were made from these data using the ordinary least squares estimator with net investment as the dependent variable:

1931-42: $$N(t) = .00394 \left[\frac{\Delta k^*(t)}{\alpha}\right] + .00117\left[\frac{\Delta k^*(t-1)}{\alpha}\right] + .5258\ N(t-1)$$

est. α = .01708

est. γ_o = .3655, est. γ_1 = .1085, est. ω_1 = .5258

est. $\bar{\mu}$ = 1.337 years

1949-61: $$N(t) = .00369 \left[\frac{\Delta k^*(t)}{\alpha}\right] + .00571 \left[\frac{\Delta k^*(t-1)}{\alpha}\right] + .7485\ N(t-1)$$

est. α = .03738

est. γ_o = .0987, est. γ_1 = .1528, est. ω_1 = .7485

est. $\bar{\mu}$ = 3.584 years

1931-42 and 1949-61: $$N(t) = .00280\left[\frac{\Delta k^*(t)}{\alpha}\right] + .00438\left[\frac{\Delta k^*(t-1)}{\alpha}\right] + .7166\ N(t-1)$$

est. α = .02534

est. γ_o = .11049, est. γ_1 = .1728, est. ω_1 = .7166

est. $\bar{\mu}$ = 3.137 years

In general, the standard errors of the estimates of the first two coefficients vary between .001 and .002 and those of the third from .1 to .25. These are not of course meaningful except under the

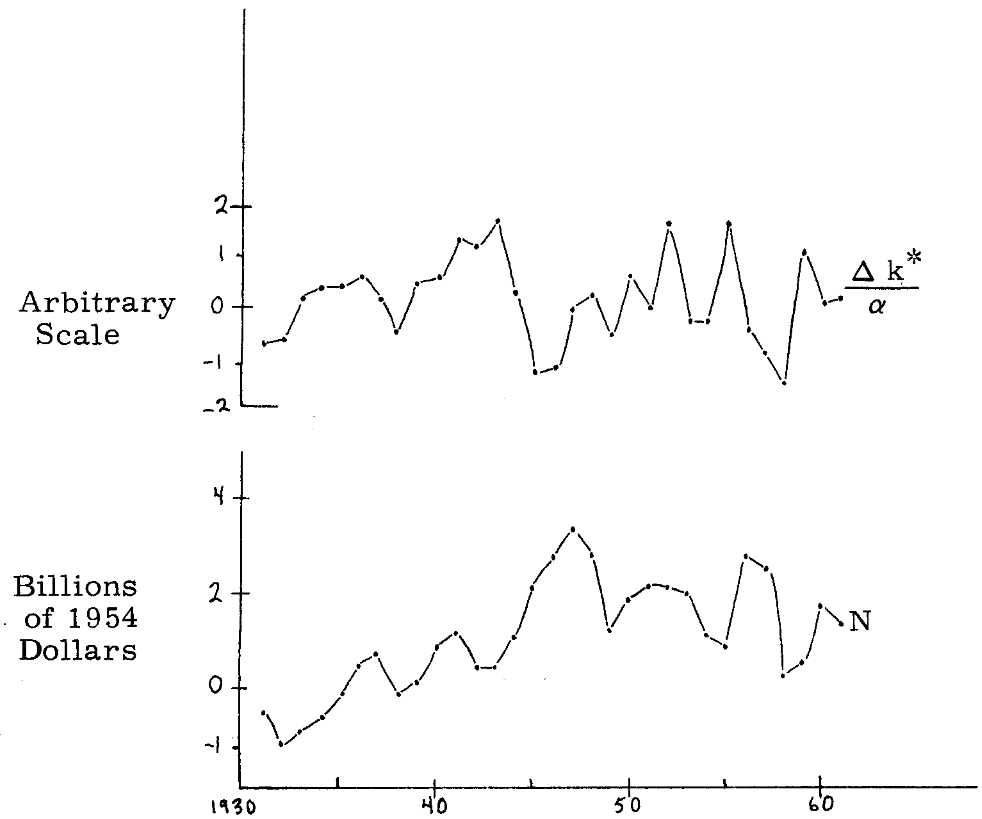

Arbitrary
Scale
2
1
0
-1
-2
Δ k*
α
Billions
of 1954
Dollars
4
2
0
-1
N
1930
40
50
60

Figure 3

strong hypothesis discussed above. Neither of the usual goodness-of-fit statistics is relevant to this model: little interest attaches to the hypothesis associated with R^2 that all the coefficients are 0, while the Durbin-Watson statistic is not applicable directly to this form [1]. Figure 4 presents the actual and fitted values for net investment derived from these estimates.

Two alternative sets of estimates were prepared for the case of manufacturing equipment investment. First, under the stochastic specification of errors in observations of $\frac{\Delta k^*}{\alpha}$ the following estimates were made by least squares:

	est. α	est. γ_o	est. γ_1	est. ω_1	est. $\bar{\mu}$
1931-42	.0350	.5400	-.3800	.840	2.875
1949-61	.10159	.1683	.0727	.755	3.328
1931-42 and 1949-61	.07659	.2690	.0281	.703	2.830

Analogous least squares estimates for all sectors appear in Table 1.

Second, a set of estimates was prepared for the whole period by the Koyck-Klein limited information method for both stochastic specifications:

	est. α	est. γ_o	est. γ_1	est. ω_1	est. $\bar{\mu}$
Disturbances in N(t)	.0354	.0784	.1130	.8141	5.10
Errors in $\frac{\Delta k^*(t)}{\alpha}$	.322	.122	.037	.8399	5.42

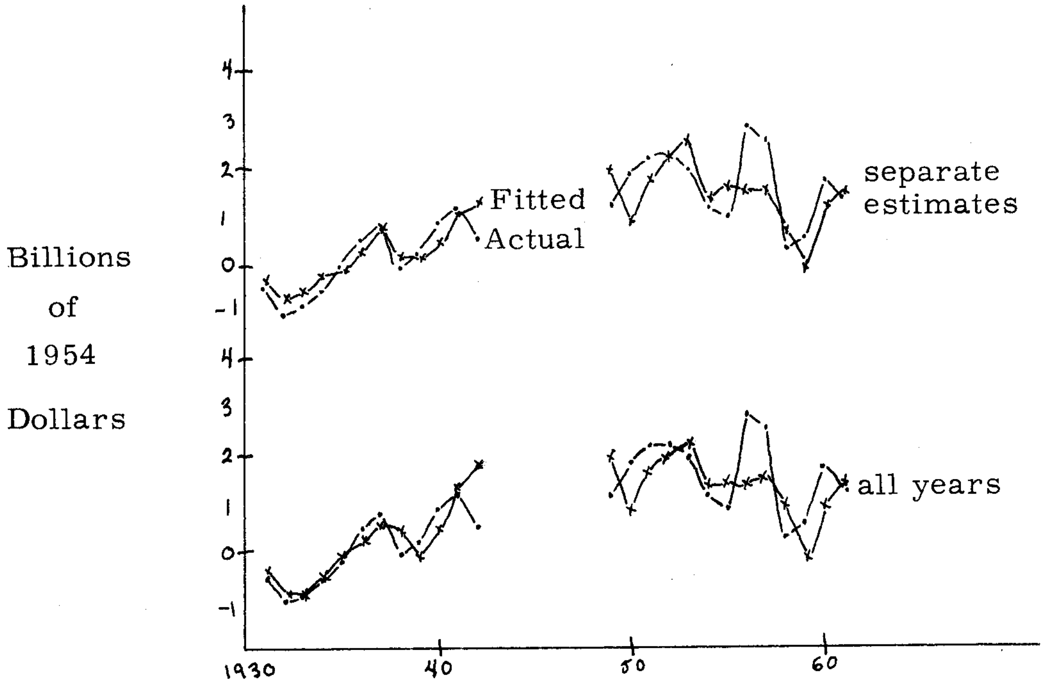

Billions
of
1954
Dollars
4
3
2
1
0
-1
4
3
2
1
0
-1
Fitted
Actual
separate
estimates
all years
1930
40
50
60

Figure 4

Table 1

	est. α	est. γ_0	est. γ_1	est. ω_1	est. δ	est. $\bar{\mu}$
Manufacturing Equipment						
1931-1942						
a)	.01708	.3655	.1085	.5258		1.337
b)	.01779	.3953	.0811	.5233	.09757	1.268
c)	.0350	.5400	-.3800	.8400		2.875
1949-1961						
a)	.03738	.0987	.1528	.7485		3.584
b)	.01040	.1740	.4942	.3318	.1238	1.236
c)	.10159	.1683	.0727	.7550		3.328
1931-1942 and 1949-1961						
a)	.02534	.11049	.1728	.7166		3.137
b)	.01303	.1505	.3461	.5033	.1149	1.710
c)	.07659	.2690	.0281	.7030		2.83
Manufacturing Structures						
1931-1942						
a)	.004289	.5409	-.2518	.7109		1.583
b)	.002922	.6776	.1424	.1801	.03050	.3933
c)*						
1949-1961						
a)	.01679	.0655	.1304	.8040		4.768
b)	.005838	.1067	.3340	.5593	.05556	2.027
c)	.08179	.1137	.0313	.855		6.119
1931-1942 and 1949-1961						
a)	.007790	.1437	.1211	.7352		3.229
b)	.008684	.1381	.1111	.7507	.04662	3.456
c)*						

---

* est. $\alpha < 0$.

Table 1 (continued)

	est. α	est. γ_o	est. γ_1	est. ω_1	est. δ	est. $\bar{\mu}$
Nonfarm, Nonmanufacturing Equipment						
1931-1942						
a)	.02388	.1235	.5544	.3221		1.293
b)	.02670	.2161	.4872	.2967	.1303	1.115
c)*						
1949-1961						
a)	.02661	.3221	.0245	.6534		1.952
b)	.01357	.4642	-.0492	.5850	.1510	1.292
c)	.09457	1.2690	-.5139	.2440		-.357
1931-1942 and 1949-1961						
a)	.02969	.2479	.1593	.5928		1.850
b)	.03329	.2463	.1505	.6032	.1443	1.899
c)	.12000	1.0850	-.5250	.4400		-.152
Nonfarm, Nonmanufacturing Structures						
1931-1942						
a)*						
b)	.01006	.1143	.2773	.6083	.04138	2.261
c)*						
1949-1961						
a)	.2430	.00856	.01514	.9763		40.78
b)	.01965	.0814	.1486	.7700	.05945	3.994
c)	.1410	.1305	.0645	.8050		4.463
1931-1942 and 1949-1961						
a)	.4305	.003716	.008083	.9882		81.66
b)*						
c)*						

Table 1 (concluded)

	est. α	est. γ_o	est. γ_1	est. ω_1	est. δ	est. $\bar{\mu}$
Farm Equipment						
1912-1942						
a)	.06141	.1789	.0896	.7315		3.058
b)	.04949	.2155	.1198	.6646	.1253	2.344
c)	.1078	.1735	.0765	.7500		3.305
1949-1961						
a)	.01253	-.0287	.2219	.8069		5.327
b)*						
c)*						
1912-1942 and 1949-1961						
a)	.0919	.0970	.0513	.8518		6.090
b)	.0906	.0987	.0524	.8490		6.020
c)*						
Farm Structures						
1912-1942						
a)	.003281	.0585	.0259	.9156		11.26
b)	.002542	.0854	.0338	.8808	.02062	7.697
c)*						
1949-1961						
a)	.001872	.01378	.01752	.9687		32.12
b)	.001328	.01935	.02545	.9552	.02172	21.69
c)*						
1912-1942 and 1949-1961						
a)	.004717	.04133	.01887	.9398		16.03
b)	.005202	.04017	.01420	.9456	.0211	17.89
c)*						

---

a) Least squares estimate, N(t) dependent: $N(t) = \gamma_o \Delta k^*(t) + \gamma_1 \Delta k^*(t-1) + \omega_1 N(t-1) + \epsilon(t)$

b) Least squares estimate, I(t) dependent: $I(t) = \gamma_0 \Delta k^*(t) + \gamma_1 \Delta k^*(t-1) + \omega_1 N(t-1) + \delta k(t) + \epsilon(t)$

c) Least squares estimate, $\Delta k^*(t)$ dependent:

$$\Delta k^*(t) = -\frac{\gamma_1}{\gamma_o} \Delta k^*(t-1) + \frac{1}{\gamma_o} N(t) - \frac{\omega_1}{\gamma_o} N(t-1) + \epsilon(t)$$

It is by no means a simple task to evaluate the success of this hypothesis on the basis of these estimates. In most cases the estimate of α, the share of capital in the production function $Q = a\, k^{\alpha} \ell^{\beta}$, is much smaller than appears to be reasonable. This may be a result of the failure of the capital goods price index to record short-run changes in the price, in which case $\frac{\Delta k^*}{\alpha}$ would be overestimated and hence α underestimated. It also appears that the mean of the lag distribution is generally overestimated, particularly in those cases where the fit is poor, as for example in the case of postwar manufacturing equipment. This may be a result of an incorrect specification of the lag distribution; perhaps the assumption

$$\mu(L) = \frac{\gamma_0 + \gamma_1 L}{\gamma - \omega_1 L - \omega_2 L^2}$$

would yield better results. For yearly data, however, it is unlikely that the addition of higher order terms would contribute much to the explanatory value of the hypothesis.

A third source of difficulty may perhaps lie in the postwar estimates of v, the proportion of depreciation expense deductible for tax purposes:

year	user cost, man. eq.	v, manufacturing
49	.1250	.8670
50	.1288	.8950
51	.1544	.8910
52	.1234	1.1740
53	.1379	1.0740
54	.1255	1.1800
55	.1169	1.2940
56	.1293	1.2910
57	.1477	1.2630
58	.1671	1.1140
59	.1622	1.2310
60	.1611	1.2490
61	.1564	1.2800

v accounts for a substantial part of the variance of $\frac{\Delta k^*}{\alpha}$ in the postwar years. Because of this, it would probably be worthwhile to attempt to find a better measure of v than the one used, which was simply

$$V = \frac{D}{q \delta k}$$

where D is accounting depreciation. Some measure of the marginal proportion of depreciation deductible for tax purposes would be more appropriate and would not tend to overestimate the effect of changes in the tax laws relative to the basis of depreciation of assets in an inflationary period.

Finally, considerable uncertainty surrounds the choice of the appropriate replacement distribution. While all estimates were based on a geometric distribution with an assumed mean δ, this is not necessarily the most plausible form, although its use is partly justified by the fact that for a constant rate of growth of capital, in the limit, all replacement distributions behave as if they were geometric. It proved useful to attempt to estimate the parameter δ from the data. Under the hypothesis of a geometric replacement distribution, replacement investment, R(t), is a constant fraction of capital stock; hence

$$N(t) = I(t) - \delta k(t)$$

It is thus possible to estimate the rate of replacement by fitting the following equation:

$$I(t) = \gamma_o \Delta k^*(t) + \gamma_1 \Delta k^*(t-1) + \omega_1 N(t-1) + \delta k(t)$$

Since N(t-1) and k(t) are obtained by assuming a value for δ in the first place, the estimate of δ made by this procedure is not closely related to its true value. Nonetheless, interesting results have been obtained in carrying out these estimates: δ assumed = .1034

	est. α	est. γ_o	est. γ_1	est. ω_1	est. δ	est. $\bar{\mu}$
1931-42	.01779	.3953	.0811	.5233	.09757	1.268
1949-61	.01040	.1740	.4942	.3318	.1238	1.236
1931-42 and 1949-61	.01303	.1505	.3461	.5033	.1149	1.710

These estimates of γ_o, γ_1, and ω_1 are much more closely in accord with what we might expect <u>a priori</u> than were those obtained previously. Two tentative conclusions may be drawn from this set of estimates: first, the rate of replacement assumed for manufacturing equipment was probably too small; second, the rate of replacement may have increased substantially since the war, relative to the depression rate. Results from other sectors tend to reinforce this second conclusion:

	δ assumed	est. δ 1931-42	est. δ 1949-61	est. δ 1931-42 and 1949-61
Manu. eq.	.1034	.09757	.1238	.1149
Manu. st.	.04778	.03050	.05556	.04662
NFNM eq.	.1465	.1303	.1510	.1443
NFNM st.	.05197	.04138	.05945	.04778
Farm eq.	.1190	.1253	.1146	.1193
Farm st.	.02149	.02062	.02172	.02110

VI. The Data

A. Gross Investment

Estimates of gross investment in equipment and structures for each sector were obtained from the OBE Capital Goods Study [2]. The OBE has prepared estimates of gross investment in current and constant (1954) dollars; the capital goods price used throughout this study was the deflator implicit in these estimates. While these deflators are notoriously poor since they measure in most cases prices of inputs to the capital goods industries, there is no reasonable alternative to using them in most cases.

B. Gross Income

Estimates of net income by sector were obtained from U.S. Income and Output. (IO) Estimates of manufacturing depreciation allowances before 1946 were made by inflating corporate manufacturing depreciation allowances by estimates of the ratio of all assets to corporate assets in manufacturing derived by interpolating Census of Manufactures data for 1919, 1929, 1947, and 1954. The postwar estimates were obtained from IO, as were estimates of farm depreciation for years from 1929. Nonfarm, nonmanufacturing depreciation allowances were estimated by subtracting the above estimates from estimates of total depreciation allowances obtained from the National Income Supplement to the Survey of Current Business. Estimates of total capital outlays charged to current expense and allowances for accidental damage from IO were allocated in an arbitrary manner to manufacturing and nonfarm, nonmanufacturing sectors after subtracting estimates of of farm capital outlay charged to current expense. All consumer bad debts were allocated to the nonfarm, nonmanufacturing sector. Other business transfer payments were distributed arbitrarily among the sectors. Gross income in each sector was estimated by adding the above estimates of depreciation allowances, capital outlays charged to current expense, and allowances for accidental damage to the estimates of net income. For the farm sector, the estimates actually used were obtained by subtracting indirect business taxes from farm gross income as estimated in IO. These estimates agreed quite closely with those obtained by the procedure indicated above, providing a check for those calculations.

C. User Cost, Non-farm

Capital goods prices were obtained directly from the OBE study. The tax rate was taken to be the statutory corporate income tax rate for all years except those in which a significant excess profits tax was levied, in which case the average rate paid by all corporations was calculated. The proportion of depreciation expense deductible for tax purposes, v, was estimated by calculating the "economic" depreciation expense $q \delta k$ for equipment and structures separately and applying the formula

$$V = \frac{\text{actual depreciation charge}}{(q\delta k) \text{ equip.} + (q\delta k) \text{ struc.}}$$

The marginal proportion of interest costs deductible for tax purposes, w, was estimated by taking the ratio of new bonds to total new corporate issues, using figures tabulated by Moody's from the SEC and the Commercial and Financial Chronicle.

The marginal cost of funds, r, was estimated by taking a weighted average of Moody's figures for average yield on 125 industrial stocks and average yield on 40 industrial bonds, the weights being 1-w and w respectively.

D. User Cost, Farm

For the farm sector, it was not possible to take account of the influence of the income tax structure, but data were available for the rate of taxation of real assets. Thus user cost was estimated as

$$c = q(\delta + t + r)$$

where q is the price of capital goods, estimated as before, t is the farm property tax rate based on "full value" prepared by the USDA Economic Research Service (Agricultural Finance Review, Supplement to Vol. 24, December 1963, p. 51, Table 28), and r is the interest rate on farm mortgages recorded. The last series is available annually to 1935 and biennally from 1941. The values used for 1936 to 1961 were obtained by fitting a regression to the biennial data, using the bond rate from above as the independent variable. The mortgage rate was obtained from Major Statistical Series of the USDA, Vol. 6.

VII. Computational Aspects

For the special case of a geometric replacement distribution estimates of replacement and capital stock were prepared with the aid of a computer program written by Hodson Thornber of the University of Chicago and described by Jorgenson in the statistical supplement to "Capital Theory and Investment Behavior."

For the general case involving an arbitrary replacement distribution, estimates of replacement, capital stock, and depreciation in constant and current dollars were made with the program described in the appendix.

The calculations of user cost were carried out by the second program described in the appendix.

All regressions were calculated by the MBRV stepwise multiple regression program written by Don Wyman of IBM and Gordon Rowe of the University of California.

Computer time on the IBM 7090 of the University of California Computer Center was supplied under a grant to Professor Jorgenson from the National Science Foundation.

References

[1] J. Durbin and G. S. Watson, "Testing for Serial Correlation in Least Squares Regression, II." Biometrika, 38, June, 1951, pp. 159-178.

[2] G. Jaszi, R. Wasson, and L. Grose, "Expansion of Fixed Business Capital in the United States." Survey of Current Business, 42, November 1962, pp. 9-18.

[3] D. W. Jorgenson, "Anticipations and Investment Behavior." Working Paper No. 40, Committee on Econometrics and Mathematical Economics, Institute of Business and Economic Research, University of California. (A Monograph)

[4] L. R. Klein, "The Estimation of Distributed Lags." Econometrica, 26, October 1958, pp. 553-565.

[5] L. M. Koyck, Distributed Lags and Investment Analysis, Amsterdam, North Holland: 1954.

[6] M. Nerlove, Distributed Lags and Demand Analysis for Agricultural and Other Commodities, USDA Agricultural Handbook No. 141, 1958.

Appendix A

Program for calculation of replacement and depreciation.

This program calculates replacement investment, net investment, capital stock depreciation in constant dollars and depreciation in current dollars. It requires as input the replacement distribution, the prices of capital goods, gross investment in constant dollars, and a value for the rate of depreciation. A typical deck set-up is the following:

Monitor control cards
Binary or source deck
Data Control card
Program control card

col.	1-3	555
	4-6	Number of terms in replacement distribution
	7-9	Number of observations
	10-12	Number of investment data, equals number of replacement terms plus number of observations
	13-18	Rate of depreciation, F6.6
	19-22	Problem number, I4

Replacement distribution cards--starting with the value for the longest-lived assets. The last value is assumed to apply to the year before the year for which replacement and capital stock are being calculated. Format 12F5.4
Price cards--one value for each observation, format 12F5.4
Investment cards--one card per figure, format F5.3
Second program control card
etc.

```
C       A PROGRAM TO CALCULATE REPLACEMENT AND DEPRECIATION
C
      DIMENSION WGHT(500), PRICE(500), DINV(500)
    5 READ 60,ITEST,NOW,NOB,NOI,DELTA,NCASE
      IF (ITEST-555) 10,11,10
   10 PRINT 80
      CALL EXIT
   11 READ 61, (WGHT(I),I=1,NOW)
      READ 62, (PRICE(I),I=1,NOB)
      READ 63,(DINV(I),I=1,NOI)
      CUMW=0.
      CAP=0.
      DO 21 I=1,NOW
      CUMW=WGHT(I)+CUMW
      CAP=CAP+CUMW*DINV(I)
   21 CONTINUE
      PRINT 82, NCASE,DELTA,CAP
      DO 22 I=1,NOB
      REPL=0
      DO 23 J=1,NOW
      L=I+J-1
      REPL=REPL+WGHT(J)*DINV(L)
   23 CONTINUE
      K=NOW+I
      DNINV=DINV(K)-REPL
      DEP=CAP*DELTA
      DEPM=DEP*PRICE(I)
      PRINT 83,I,DINV(K),REPL,DNINV,CAP,DEP,DEPM
      CAP=CAP+DNINV
   22 CONTINUE
      GO TO 5
   60 FORMAT (4I3,F6.6,I4)
   80 FORMAT (28H1PROGRAM TERMINATED BY ITEST)
   61 FORMAT (12F5.5)
   62 FORMAT (12F5.4)
   63 FORMAT (F5.3)
   82 FORMAT (44H1CALCULATION OF REPLACEMENT AND DEPRECIATION///
     124H THIS IS PROBLEM NUMBER ,I3,///7H DELTA=,F6.6,///19H CAPITAL BE
     2NCHMARK=,F7.3,///2X,2HNO,10X,3HINV,9X,4HREPL,6X,7HNET INV,6X,
     37HCAPITAL,10X,3HDEP,6X,7HMON DEP//)
   83 FORMAT (2X,I2,3X,F10.4,3X,F10.4,3X,F10.4,3X,F10.4,3X,F10.4,3X,F10.
     14)
      END
```

Appendix B

Program for calculation of user cost

This program calculates user cost according to the formula

$$c = q \left[\frac{1 - uv}{1 - u} \delta + \frac{1 - uw}{1 - u} r \right]$$

where c is user cost, q is the price of capital goods, u is the income tax rate, v is the proportion of depreciation charges deductible for tax purposes, δ is the rate of depreciation, w is the proportion of the marginal cost of capital deductible for tax purposes, and r is the marginal cost of capital.

A typical deck setup consists of the following cards:

Monitor control cards
Binary or source deck
Data Control card
Program control card

col.	1-3	999
	4-6	Number of observations
	7-12	Rate of depreciation, F6. 5
	13	Punch 1 to use price data from previous problem, blank for new data
	14	Punch 1 to use income tax rate data from previous problem, blank for new data
	15	Punch 1 to use v data from previous problem, blank for new data
	16	Punch 1 to use w data from previous problem, blank for new data
	17	Punch 1 to use r data from previous problem, blank for new data.

Price cards, format 12F5. 4, if needed
Tax rate cards, format 12F5. 4, if needed
v cards, format 12F5. 4, if needed
w cards, format 12F5. 4, if needed
r cards, format 12F5. 5, if needed
Second program control card
etc.

```
C      A PROGRAM TO CALCULATE USER COST
C
       DIMENSION Q(100),U(100),V(100),W(100),R(100)
     4 READ 40,ITEST,NOB,DELTA,IFQ,IFU,IFV,IFW,IFR
       IF(ITEST-999) 5,9,5
     5 PRINT 41
       CALL EXIT
     9 PRINT 410, NOB,DELTA
    10 IF(IFQ-1) 11,12,11
    11 READ 42,(Q(I),I=1,NOB)
    12 IF (IFU-1) 13,14,13
    13 READ 43, (U(I),I=1,NOB)
    14 IF (IFV-1) 15,16,15
    15 READ 44, (V(I),I=1,NOB)
    16 IF (IFW-1) 17,18,17
    17 READ 45, (W(I),I=1,NOB)
    18 IF (IFR-1) 19,20,19
    19 READ 46, (R(I),I=1,NOB)
    20 DO 21 I=1,NOB
       C=Q(I)*(((1.-U(I)*V(I))/(1.-U(I)))*DELTA+((1.-U(I)*W(I))/(1.-U(I))
      1)*R(I))
       PRINT 47,I,C,Q(I),U(I),V(I),W(I),R(I)
    21 CONTINUE
       GO TO 4
    40 FORMAT (I3,I3,F6.5,5I1)
    41 FORMAT (28H1PROGRAM TERMINATED BY ITEST)
    42 FORMAT (12F5.4)
    43 FORMAT (12F5.4)
    44 FORMAT (12F5.4)
    45 FORMAT (12F5.4)
    46 FORMAT (12F5.5)
    47 FORMAT (6X,I3,4X,F7.4,4X,F7.4,4X,F7.4,4X,F7.4,4X,F7.4,4X,F7.4)
   410 FORMAT (25H1CALCULATION OF USER COST//5H NOB=I3//7H DELTA=F6.5///
      16X,1HI,9X,1HC,10X,1HQ,10X,1HU,10X,1HV,10X,1HW,10X,1HR)
       END
```