Large Employment Fluctuations with Product- and Labor-Market Equilibrium

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Abstract

The current slump has repeated on an unusual scale the large movements of quantities and tiny movements of wages and prices that often characterize the business cycle. The only price that moved significantly was the interest rate, which plunged to zero in nominal terms and into negative territory in real terms. I build a model that describes these outcomes as a full equilibrium without limitations on the immediate movements of prices or wages. Even when the nominal interest rate is pinned at zero and unemployment is high, the model is in full equilibrium, a property absent from earlier models of the zero lower bound. A key feature of the model is that unemployment is a fast-moving variable that takes over the role of clearing product markets that the interest rate normally plays, once the rate hits zero. I describe the conditions that allow the unemployment rate in the Diamond-Mortensen-Pissarides framework to match the rate as elevated by the binding zero lower bound. I argue that the conditions are plausible. Monetary policy plays a key role in the model—because of the zero lower bound, real activity is not invariant to monetary policy despite the full equilibrium character of the model.

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1 Introduction

Many accounts of an economy with high unemployment fail to describe an equilibrium in product or labor markets. Sticky-price models are one example—sellers could improve profit by lowering price and raising quantity, but fail to take that step. Recent modeling of deep recession resulting from the zero lower bound on the nominal interest rate has neglected the labor-market entirely, leaving the question of equilibrium in the labor market unanswered. Here I develop an equilibrium view of both the product and the labor market, focusing mainly on an economy at the lower bound. The characterization of the product market is utterly conventional: the market is in competitive equilibrium at all times. The treatment of the labor market with high unemployment draws heavily on the equilibrium theory of unemployment pioneered by Peter Diamond, Dale Mortensen, and Christopher Pissarides (the DMP model). I derive the unique version of the DMP model that is compatible with overall equilibrium in a standard model subject to a binding lower bound of zero on the nominal interest rate.

The paper provides a resolution of the clash between two rival theories of high unemployment when the zero bound is binding. When the interest rate cannot fall far enough to clear the output market, the equations of fairly standard models that would otherwise have full employment can be solved only if output and employment are below the full-employment level. These models appear to dictate the unemployment rate—see, for example, Krugman (1998), Eggertsson and Woodford (2003), Christiano, Eichenbaum and Rebelo (2010), and Eggertsson and Krugman (2011). The second and third of these papers adopts the Calvo model of sticky-price and sticky-wage disequilibrium. The others simply find the level of output that solves a set of equations that excludes the labor market, without considering the issue of equilibrium in the labor market. Quite separately, the DMP model describes an equilibrium level of unemployment. The driving force of unemployment in most versions of the model is productivity. Nothing about the zero bound economy causes productivity to fall—quite the contrary, productivity rose in the United States during the period of rapidly rising unemployment in 2008 and 2009. I resolve the apparent conflict between the two theories with the *flexible unemployment hypothesis*, which implies that product-market equilibrium determines the unemployment rate, while retaining the DMP principle that unemployment is an equilibrium of a well-specified labor-market model.

I use the word *flexible* in connection with this version of the DMP model of unemployment because the model portrays unemployment as a fast-moving variable that clears the output market when the zero lower bound disables the interest rate from clearing that market. In earlier work on the lower bound, unemployment was generally implicitly a residual—a variable that takes on a high value when there would be an excess supply of current output at the zero interest rate and full employment. Under the flexible unemployment hypothesis, unemployment and low output occur in a full equilibrium of the economy, in the usual sense that no agent or pair of agents is failing to take advantage of an opportunity for unilateral or bilateral improvement. Employers are in equilibrium with respect to their choices of input levels, including employment—adding another worker would leave profit unchanged, considering the revenue that the worker would produce, the cost of recruiting the worker, and the pay that the worker would receive. Producers are in equilibrium with their customers—no change in the quantity traded would result in bilateral improvement. Job-seekers are in equilibrium as well—within the actions available to a job-seeker in the DMP setup, none would enable the job-seeker to gain a higher level of satisfaction. No worker-employer pair in contact with each other has any opportunity for bilateral improvement.

In the model of this paper, the role of businesses is completely standard. Product prices are fully flexible. Firms are competitive, choosing output to maximize profit. They encounter no obstacles to immediate adjustment of their control variables. They recruit workers according to the principles of the search-and-matching model—they hire up to the point that the cost of recruiting another worker equals the contribution to the firm's value from that worker. The latter is the present value of the difference between the worker's marginal product and the worker's compensation. I call it the *job value*. Times of low output correspond to times when this condition results in slack markets with high unemployment. In these times the job value is low, so recruiting effort is low and unemployment is high. An essential idea of the paper and of DMP models in general is that the job value is lower when unemployment is high. I show that this proposition plainly holds in recent U.S. data.

2 The Labor Market

I adopt a simple version of the DMP theory of unemployment. Like much of the recent literature on the DMP model, I consider modes of wage determination different from the Nash bargain of the canonical model of Mortensen and Pissarides (1994). Also, I simplify the treatment of labor-market dynamics by considering only the stochastic equilibrium of labor turnover, which means that the unemployment rate u measures the tightness of the labor market. The vacancy rate enters the picture only in fast transitional dynamics of the matching process, which can be ignored in a quarterly model without losing much. Thus the recruiting success rate is an increasing function $h(u)$ of the unemployment rate. Success is higher when unemployment is higher and employers find qualified job-seekers more easily. Hall (2009) discusses this approach more fully.

Without loss of generality, I decompose the wage paid to the worker into two parts, corresponding to a two-part pricing contract (the decomposition is conceptual, not a suggestion that actual compensation practices take this form). The worker pays a present value J, the *job value*, to the employer for the privilege of holding the job and then receives a flow of compensation equal to the worker's marginal product.

The essence of the DMP model of unemployment is a pair of equations involving the job value. The first holds that, in equilibrium, firms expect zero profit from recruiting workers. The cost of recruiting (holding a vacancy open) is γ per period, taken to be constant in output terms. The zero-profit condition for recruiting equates the expected benefit of recruiting to its cost:

$$h(u)J = \gamma. \tag{1}$$

Thus unemployment rises if the job value J falls. In slack markets with lower J, a worker pays less for a job. Because $h(u)$ is a stable function of unemployment alone and γ is a constant, the DMP model implies a stable relationship between unemployment and the job value.

The second equation—which I call *wage determination*—states the job value J as a function of u and other determinants. In the canonical model of Mortensen and Pissarides (1994), a worker and an employer make a Nash bargain that sets a wage to divide their joint surplus into fixed shares. Unemployment is one of the determinants of the Nash job-value function—when unemployment is high, the match surplus arising from labor-market frictions is greater. The job value, a fixed share of that surplus, is also higher. The worker has to pay more for the job because jobs are harder to find. Two other variables—the marginal product of labor, p, and the flow value of time spent not working (as an improvement over working), z, also enter the Nash job-value function. The DMP literature has concentrated on explaining movements in unemployment as responses to changes in total factor productivity, which is the fundamental underlying determinant of the revenue product of labor. Movements in the

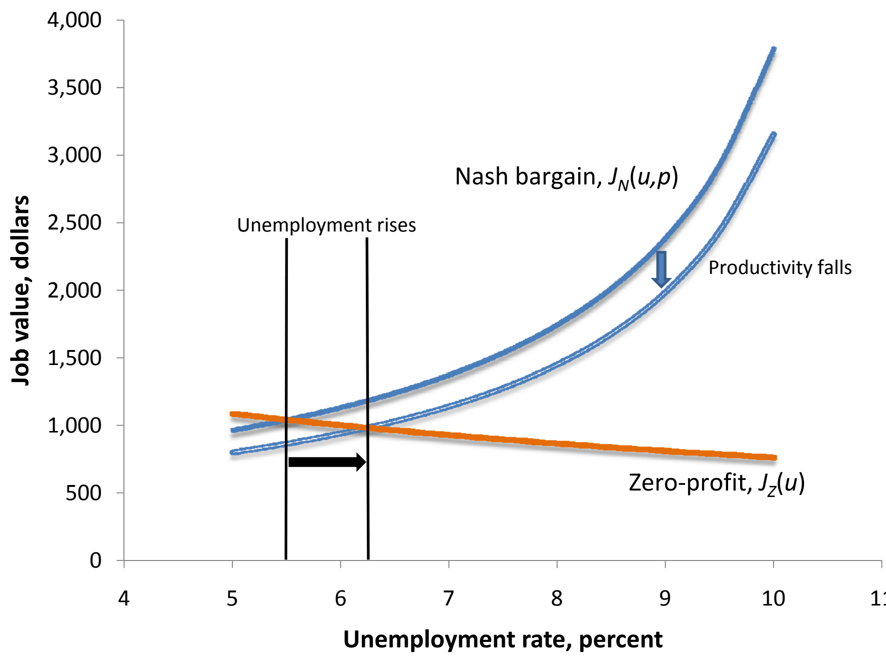

Figure 1: DMP Account of an Increase in Unemployment Caused by a Decline in Productivity

opportunity cost z rarely figure in explanations of unemployment.

Figure 1 shows the DMP account of the increase in a recession as explained in Mortensen and Pissarides (1994). In consequence of a drop in productivity, the Nash wage determination curve shifts downward. The new equilibrium occurs down and to the right along the stable zero-profit curve.

Two developments have cast doubt on the relevance of the recession mechanism of Figure 1. First, Shimer's (2005) influential paper showed than it would take a gigantic drop in productivity to cause the rise in unemployment in a typical recession, based on realistic values of the parameters of the DMP model. Second, productivity has *increased* in recent recessions. Figure 2 shows the Bureau of Labor Statistics' measure of total factor productivity in U.S. business since 1999, along with a projection of its levels in 2008 and 2009 had it continued to grow in those years at the same rate as from 1999 through 2007. Productivity grew almost at normal rates during the huge contraction that started in 2008. To generate an increase in unemployment driven by productivity, an actual decline in productivity would be needed.

Shimer's paper has stimulated an interesting literature—surveyed in Rogerson and Shimer (2010)—that alters the canonical DMP model to boost the response of unemployment to productivity. But with rising productivity in a recession, the stronger response is an embarrassment, making it even harder to square the behavior of the U.S. economy with the DMP

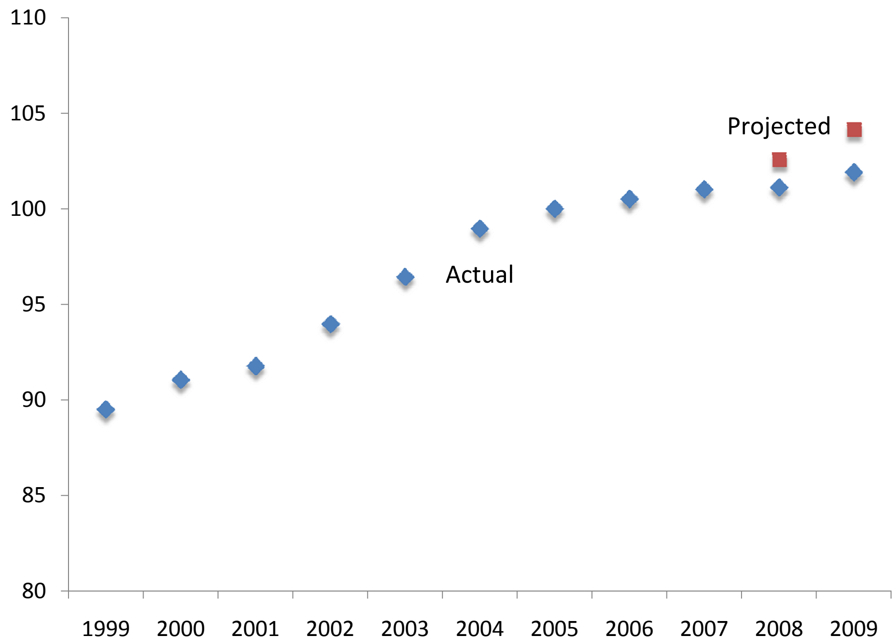

Figure 2: Actual Total Factor Productivity, 1999-2009, and Levels in 2008 and 2009 Projected at Earlier Growth Rate

model.

2.1 Stability of the zero-profit curve

The DMP account of recessions generally took the zero-profit curve to be stable and viewed the rise in unemployment in a recession as a movement along that stable curve, resulting from a shift of the wage-determination curve. Recently that view has come into question, as the matching process has become less effective and unemployment has remained high even though vacancies have risen. Recent work has studied these changes from the perspective of the Beveridge curve (the joint behavior of unemployment and vacancies) and the matching function, I will approach the topic via the zero-profit curve to relate to the analysis in Figure 1 as closely as possible. I document a limited amount of instability of the zero-profit function but not enough to make shifts of the zero-profit function a candidate to explain the gross magnitude of the increase in unemployment in 2008 and 2009.

From equation (1), the zero-profit function is

$$J(u) = \frac{\gamma}{h(u)}. \tag{2}$$

To evaluate J, I need a value for the vacancy-posting cost γ and an estimate of the recruiting-success function $h(u)$.

		Intercept	*Slope*	*Trend*	*Standard error of the regression*
Daily recruiting sucess rate	$h(u)$	0.0371 (0.0020)	0.545 (0.037)	-0.000082 (0.000013)	0.0037
Daily job-finding rate	$\varphi(u)$	0.064219 (0.001173)	-0.593 (0.022)	-0.000019 (0.000008)	0.0022

Table 1: Estimates of Parameters of the Hiring and Job-Finding Functions

Hall and Milgrom (2008) calculate that the daily cost of maintaining a vacancy is 0.43 days of pay, based on data from Silva and Toledo (2008), or $\gamma = \$66$ per day for the average U.S. employee in January 2011.

The daily hiring success rate h is

$$h_t = \frac{H_t}{21 V_t}, \tag{3}$$

where H_t is the number of hires during a month and V_t is the average number of vacancies open during the month, approximated as openings at the beginning of the month. Both series are from the BLS's Job Openings and Labor Turnover Survey (JOLTS). I divide by 21 as the number of working days in a month.

To estimate the hiring success rate function $h(u)$, I regress h_t on the unemployment rate u_t from the Current Population Survey for the period from December 2000 (the onset of JOLTS) to June 2009 (omitting data from the anomalous period in the second half of 2009 and 2010). I also include a linear trend. The identifying assumption—lack of correlation between disturbances in the hiring rate and the unemployment rate—is consistent with the theme of the paper that the product market determines the unemployment rate, not factors relating to the labor market. The regression appears in the top panel of Table 1. It shows a robust positive relationship between the recruiting success rate and the unemployment rate.

The lower panel of Table 1 shows estimates of the daily probability $\phi(u)$ that a job-seeker will find a job. The left-hand variable is the ratio of the number of hires reported in JOLTS to the number of unemployed workers in the CPS, divided by 21 to place it on a daily basis. There is a robust negative relation between the job-finding rate and the unemployment rate.

Figure 3 shows the job value since the inception of JOLTS, calculated as

$$J_t = \frac{\gamma}{h_t}. \tag{4}$$

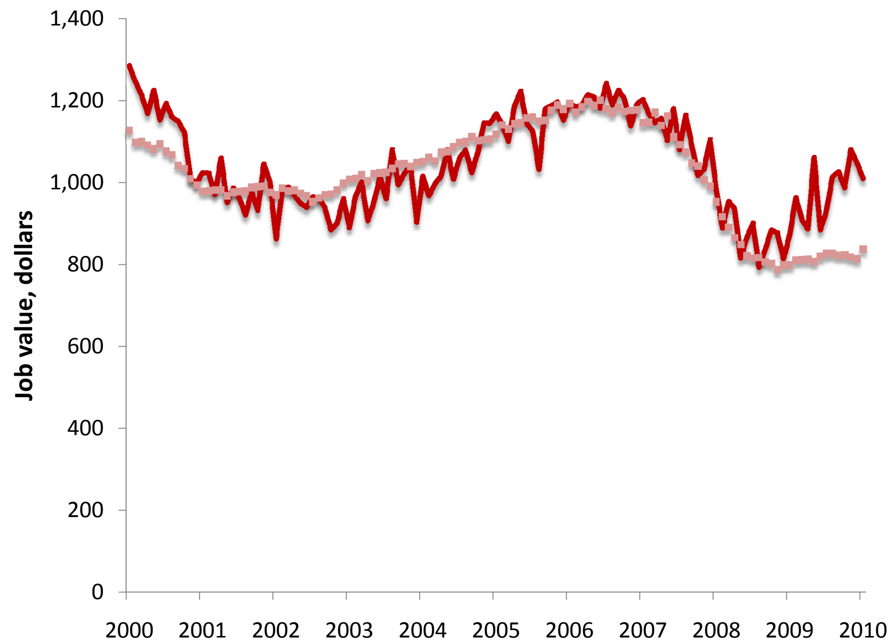

Figure 3: Job Value Calculated from JOLTS

Job value is strongly pro-cyclical. New workers pay their employers—in the form of a wage below their marginal products—more in good times, such as the middle of the decade of the 2000s, and less during slumps, such as 2001 to 2003 and 2008 to 2011. The small light squares in the figure show the job values calculated from the fitted values of h_t from the regression.

Figure 4 is a scatter diagram with unemployment on the horizontal axis and the job value J_t on the vertical axis. It distinguishes the cycle—contraction and expansion—running from 2000 to 2007 and shown with a solid red line from the second cycle that started in 2008, shown as a double blue line. The earlier cycle follows a reasonably well-defined negatively sloped line, both during the contraction and during the subsequent expansion. The contraction starting in 2008 (not yet retraced by the modest expansion that has occurred to date) is somewhat flatter than the earlier one. Unemployment rose dramatically, but the hiring rate did not rise as fast as it would have if the labor market had retraced the contraction that started in 2001.

A second anomaly appears at the trough of the contraction in 2009 and 2010, when unemployment lingered just below 10 percent but the hiring rate fell by as much as it would have in a substantial expansion. This behavior has been widely discussed. It corresponds to an inward shift of the matching function. See Hall (2010).

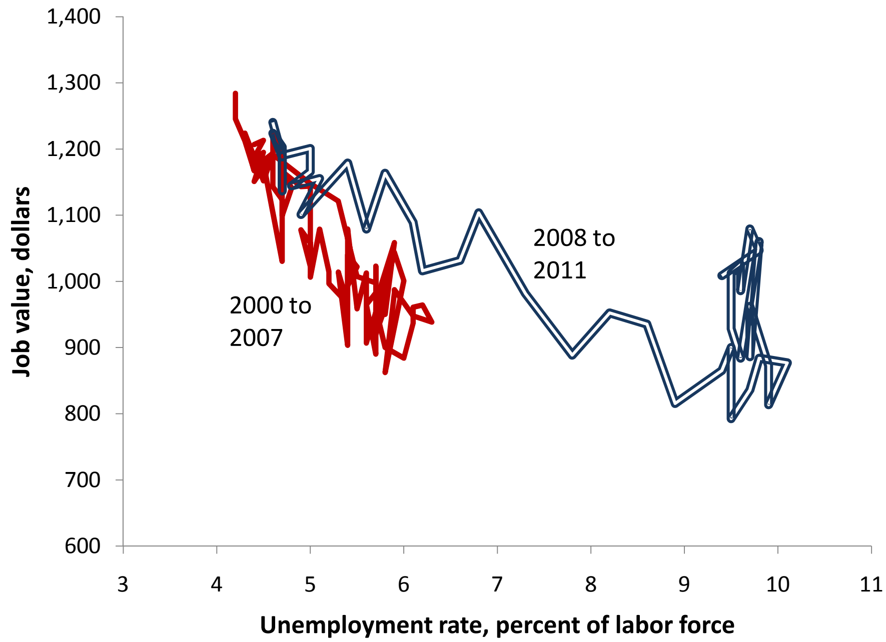

Figure 4: Job Value J Plotted against Unemployment

Figure 5 is a scatter diagram with the unemployment/vacancy ratio on the horizontal axis and job value J_t on the vertical axis. That ratio or its reciprocal is the exact measure of labor-market tightness in the standard DMP model. Comparison of the two scatter diagrams suggests that not much of the anomalous behavior in Figure 4 is the result of the neglect of the dynamics of job matching, which in principle are handled exactly in Figure 5, but without much improvement in fit.

The broad joint movements over the period from 2000 to 2011 of unemployment and the calculated job value J_t suggest a roughly stable relationship —unemployment was high early in the period when the job value was low, fell to reasonable levels in the middle of the decade when J was high, and rose to a very high peak when J fell starting in 2008. Notice that unemployment is not an input to the calculation of J. Of course, tracking down the sources of the departures from stability is still an interesting undertaking. Nothing in what follows would be altered in substance if a more refined version of the zero-profit condition contained additional variables to account for what appear to be shifts in the joint behavior of u and J.

2.2 Shifts of the wage-determination curve

In this discussion, I assume that the separation rate is a constant s. Data from JOLTS and the CPS support this assumption as a useful rough approximation. The separation rate has

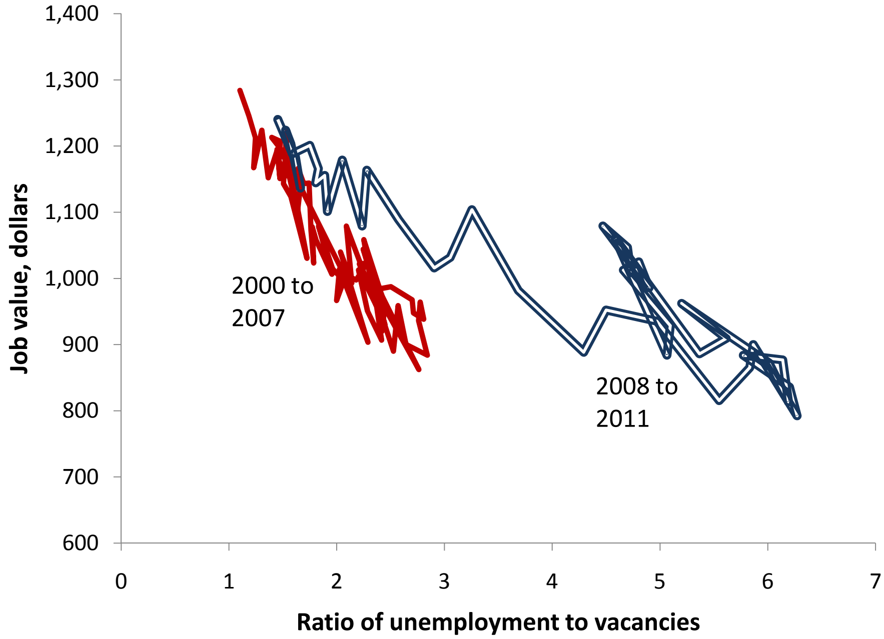

Figure 5: Job Value J Plotted against the Unemployment/Vacancy Ratio

declined along a smooth trend in recent decades.

Figure 6 shows two extremes among the many models of wage determination under recent discussion. The figure also includes the zero profit curve of equation (2). The upward-sloping wage-determination curve labeled *Nash bargain* plots the employer's half of the surplus under a symmetric Nash bargain in the DMP model, according to equation (4) in Shimer (2005):

$$J_N(u) = 0.5\frac{p - z}{r + 0.5\phi(u) + s}. \tag{5}$$

Here J_N is the job value, the part of the surplus the employer captures, z is the flow-value benefit of not working, which I take to be 70 percent of p (see Hall and Milgrom (2008)), r is the real interest rate, which I take to be 5 percent per year, $\phi(u)$ is the job-finding rate function, which I take from the lower panel of Table 1, and s is the separation rate, which I take to be 5 percent per month. I take p to have the value that generates the job value shown in Figure 4 at 5.5 percent unemployment, $J = \$1{,}040$.

The only plausible source of a major shift of the Nash wage-determination function is a decline in $p - z$. In turn, given the implausibility of an increase in the value of non-employment activities z as the causal force of a recession, the only possible source of a jump in unemployment is a decline in the marginal product of labor, p. Shimer (2005) showed that only a large decrease could bring about the 5-percentage-point increase in unemployment

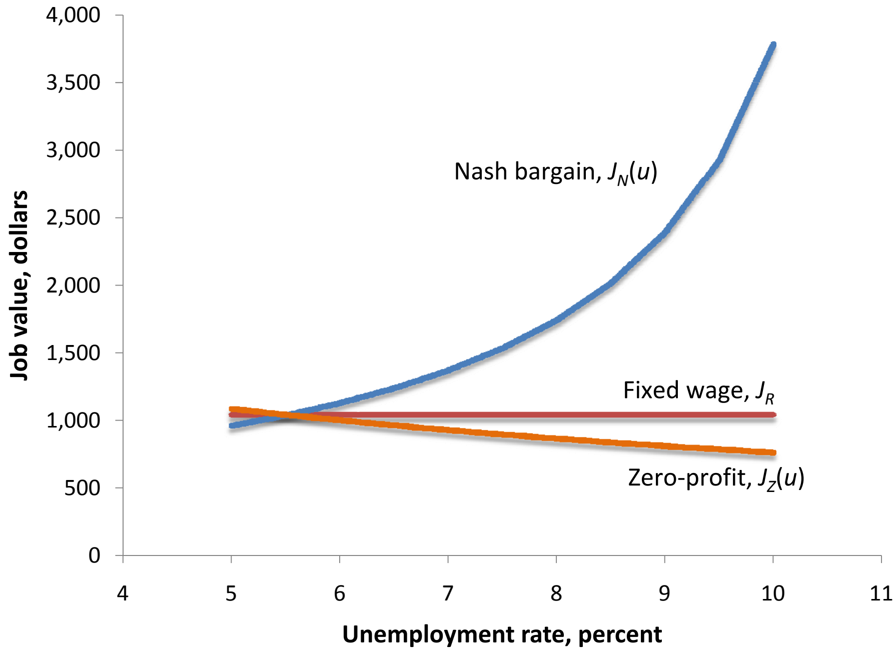

Figure 6: Alternative Views of Equilibrium Unemployment

that occurred in 2008 and 2009. But productivity actually rose during that period. Shifts of the Nash wage-determination function are unlikely candidates for explaining the recent rise in unemployment or any other important movements in unemployment as well. Here I am excluding Hagedorn and Manovskii's (2008) Nash-bargaining model, which makes the contrary claim, on the grounds that its parameters imply much too high a Frisch elasticity of labor supply.

The flat line in Figure 6 labeled *Fixed wage* illustrates Shimer's (2004) proposition that a fixed wage, unresponsive to conditions in the labor market, would lead to a highly unstable wage-determination curve and thus support explanations of large movements of unemployment in the DMP framework in response to shifts in productivity. With a rigid wage w, the job value is

$$J_R = \frac{p - w}{r + s}. \tag{6}$$

A small decline in p or a small increase in w would lower the fixed-wage line in Figure 6 and generate a large increase in unemployment. Note that the wage-rigidity model is quite specific about the driving force—fluctuations in p or w have huge effects, but other potential driving forces do not shift the wage-determination curve at all and cannot explain a large increase in unemployment in the DMP framework.

Figure 7 shows data for the past decade on the best available comprehensive measure of

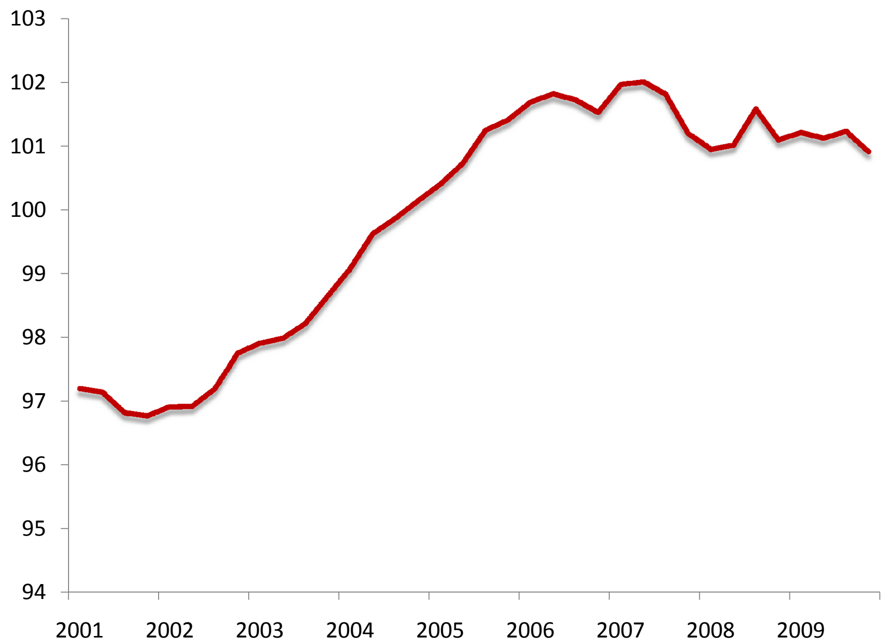

Figure 7: Ratio of Productivity to the Real Employment Cost Index, 2001-2009

the variable that drives unemployment in a DMP model that takes the wage as an exogenous driving force. It shows the ratio of total factor productivity to the real cost of labor input, measured as the BLS's Employment Cost Index deflated by the price index for GDP from the National Income and Product Accounts. Notice that the wage measure is a product wage, deflated by a comprehensive index of the price of U.S. output, not a real wage that measures the purchasing power of earnings from the worker's perspective.

The figure does not reveal any significant decline in productivity relative to the real cost of labor that would cause a large increase in unemployment in a sticky-wage DMP model. According to that model, unemployment should have fallen dramatically from 2003 to 2007, as productivity rose relative to the cost of labor, and then leveled out. Unemployment did fall in the first years when productivity gained relative

Figure 7 raises important questions about DMP models in general. Recall that the job value shown in Figure 3 is the present value of the difference between the marginal revenue product of labor and the wage for newly hired workers. On its face, Figure 7 does not appear to satisfy that relationship. Many factors would go into a reconciliation of the measures. One of the most important is that wage contracts with established workers, with substantial job-specific capital accumulated with growing tenure, provide a good deal of insurance. The bulk of total labor compensation is paid to workers with reasonable tenure—see Hall (1982) and the subsequent literature. Thus the evidence in Figure 7 may be irrelevant to the terms of employment of the newly hired. Haefke, Sonntag and van Rens (2008) study the wages of the newly hired in the CPS and find substantial flexibility, but the sampling uncertainty in the estimate of their flexibility parameter is large.

Other contributions to the recent DMP wage-determination literature have not extended the sources of shifts of the wage-determination curve outside those just discussed. Hagedorn and Manovskii (2008) present alternative parameter values for the Nash wage-determination case, but the source of shifts of the wage-determination curve remains productivity. Hall and Milgrom (2008) adopt an alternating-offer bargaining setup, which brings delay costs into the model as additional determinants of unemployment, but these costs are not portrayed as sources of fluctuations—they are taken as stable parameters.

2.3 Conclusions about the labor market in the DMP framework

The DMP model views the unemployment rate as determined by the intersection of a stable zero-profit curve and a shifting wage-determination curve. With enough wage-rigidity, the model can explain a dramatic rise in unemployment, resulting from either a decline in productivity or a spontaneous increase in the sticky wage. As an explanation of the large increase in unemployment starting in 2008, productivity is not in the running because it increased and would have lowered unemployment in a sticky-wage DMP economy. A spontaneous increase in wages to the newly hired remains in the running.

A point central to the message of this paper is that, to date, no author has described a wage-determination setup in which forces in the product market, apart from productivity, shift the wage-determination curve and cause changes in unemployment.

3 The Product Market and the Zero Lower Bound

Now I will turn to the product market, with a focus on the role of the lower bound of zero on the nominal risk-free interest rate. The point of the discussion is that a macro model of the type considered in the zero-lower-bound literature is a self-standing theory of unemployment, unconnected with the principles of the DMP theory of unemployment. Macroeconomics faces a pair of clashing views about unemployment.

3.1 Model

Much of the model I develop in this section follows in the footsteps of Krugman (1998) and the more recent work on the zero lower bound, stimulated first by Japan's experience with the bound and most recently and abundantly by experience in the United States and other countries during the worldwide slump that began in 2008. See, in particular, Eggertsson and Woodford (2003), Christiano et al. (2010), and Eggertsson and Krugman (2011). However, I have found it appropriate to be somewhat more formal in certain areas, notably the existence of solutions to the model. I avoid linearization and give near-exact results for a model with uncertainty.

The model developed here limits monetary non-neutrality to the important case of the zero lower bound on the nominal interest rate. I assume that the central bank maintains inflation at zero under all conditions. The central bank's price stabilization policy is not impeded by gradual adaptation of prices and wages to changing conditions. But, despite the full flexibility of prices and wages, allocations in the economy are not invariant to monetary policy. Strict control of inflation at zero results in excess unemployment whenever the full-employment value of the interest rate would be negative. Alternative policy rules—either higher chronic inflation or inflation that rises when the economy becomes slack—could deliver lower unemployment, a point extensively developed in the papers cited above.

Although the price level and the wage level are completely free to vary in the model, it is feasible for the central bank to achieve perfect price stability. In that case, large movements of output and employment occur with only small changes in the wage. The potential flexibility of the price and wage levels makes this outcome possible.

3.2 Technology, capital, and consumption

The economy has two state variables, the capital stock k and government purchases g. I treat all other variables as functions of the state variables, but, where clear, I omit writing the arguments explicitly. I denote next period's value of a variable with a prime ($'$). I let $\mathcal{D}$ be the relevant domain of (k, g).

Government purchases g, take on only two values, 0 and $\bar{g}$. The economy begins with $g = 0$ and switches permanently to $g = \bar{g}$ sometime thereafter. Its law of motion is

$$\text{Prob}[g' = \bar{g} | g = 0] = \rho. \tag{7}$$

and

$$\text{Prob}[g' = \bar{g} | g = \bar{g}] = 1. \tag{8}$$

Until purchases pop up to $\bar{g}$, resources are expected to become scarcer in the future and thus the interest rate may be negative to induce consumers to consume more in the present to take advantage of the lower scarcity. Thus, the anticipation of later higher government purchases can cause the zero lower bound on the interest rate to bind.

Let n be employment, $x = (k' - k)/k$ be the investment/capital ratio, v be resources expended in recruiting workers:

$$v = \frac{\gamma s n}{h(u)}, \tag{9}$$

and c be consumption. At the beginning of the period, labor and capital form gross output according to

$$n^\alpha k^{1-\alpha}. \tag{10}$$

At the end of the period, consumption occurs and the government may purchase output. Remaining output is invested. Adjustment costs are:

$$\frac{\kappa}{2} k x^2. \tag{11}$$

The parameter κ controls adjustment cost. Capital deteriorates at rate δ. Material balance requires

$$n^\alpha k^{1-\alpha} + (1-\delta)k = c + kx + \frac{\kappa}{2} k x^2 + v + g. \tag{12}$$

I let q be the market or shadow price of installed capital. Firms solve the atemporal capital-installation problem:

$$\max_x \; q \cdot (x+1)k - \frac{\kappa}{2} k x^2 - (x+1)k. \tag{13}$$

The first-order condition is:

$$\kappa x = q - 1. \tag{14}$$

Tobin's investment equation. The coefficient κ controls capital adjustment. If κ is very large, capital does not adjust at all; the economy is the endowment economy of Lucas (1978). If $\kappa = 0$, capital adjusts without impediment and q is always one.

Households can buy and sell a claim to a unit of installed capital with price q. Its realized return ratio is

$$R(k, g, g') = \frac{(1-\delta)q' + (1-\alpha)n'^\alpha k'^{-\alpha}}{q}. \tag{15}$$

Households have an intertemporal elasticity of substitution of σ. Their realized intertemporal marginal rate of substitution is

$$m(k, g, g') = \beta \left(\frac{c'}{c} \right)^{-\frac{1}{\sigma}}. \tag{16}$$

Households plan consumption to satisfy the Euler equation,

$$\mathbb{E}_{g=0} \ (mR) = (1 - \rho) m(k, g, 0) R(k, g, 0) + \rho m(k, g, \bar{g}) R(k, g, \bar{g}) = 1 \tag{17}$$

and

$$\mathbb{E}_{g=\bar{g}} \ (mR) = m(k, g, \bar{g}) R(k, g, \bar{g}) = 1. \tag{18}$$

The risk-free return ratio is

$$R_f(k, 0) = \frac{1}{\mathbb{E}_{g=0} \ (m)} = \frac{1}{(1 - \rho) m(k, 0, 0) + \rho m(k, 0, \bar{g})} \tag{19}$$

and

$$R_f(k, \bar{g}) = \frac{1}{\mathbb{E}_{g=\bar{g}} \ (m)} = \frac{1}{m(k, \bar{g}, \bar{g})}. \tag{20}$$

3.3 Negative real interest rate

My discussion will focus on cases where the risk-free interest rate would be negative absent a lower bound, that is, $R_f < 1$. The condition that generates the negative rate is the expected increase in government purchases g. To overcome consumers' tendency to equalize current and future consumption and induce them to consume more in early years when output is more plentiful because government demand is lower, the interest rate is negative (Krugman (1998) makes the related assumption in an endowment economy that the endowment declines from the first to subsequent periods).

3.4 Monetary policy

The government issues currency in amount M. The demand for currency is

$$\frac{M}{p} = D(R_n), \tag{21}$$

where the nominal return ratio R_n satisfies

$$R_f = R_n \frac{p}{p'}. \tag{22}$$

I am agnostic about the source of the demand for currency, as I am mindful that only a tiny fraction of U.S. currency is held by ordinary consumers for transaction purposes. The government pays out the revenue from issuing currency to households as lump sums. It pursues a constant-price-level policy, $p = 1$, so it sets

$$M = D(R_f). \tag{23}$$

The model omits consideration of reserves and bank deposits because they earn interest in normal times and can be subject to charges that amount to negative interest if appropriate. The issues of concern here all derive from the impracticality of collecting negative interest from holders of currency—see Buiter (2009).

For the rest of the paper, I omit the monetary sector and take the price level to be constant at one. I drop the distinction between the nominal and real interest rates.

3.5 The zero lower bound on the interest rate

If the risk-free interest rate fell below zero, currency would become an asset that paid above its appropriate return—it would be a government giveaway because it would pay a risk-free return of zero. The model cannot have such a giveaway, as discussed in Hall (2011) and the prior literature cited there. Thus $R_n(k, g) = R_f(k, g) \geq 1$ for all $(k, g) \in \mathcal{D}$.

3.6 The labor market

Labor is supplied inelastically in amount $\bar{n}$. If the interest rate bound is not binding, employment equals labor supply: $n = \bar{n}$. If the bound does bind, employment may be less than labor supply.

This characterization of the labor market has no choice-theoretic foundation. Any single firm appears to have the opportunity to employ a worker at a positive wage, sell the output, and generate a bilateral benefit to worker and firm. This view of the the economy appears implicitly in every model of the zero lower bound that I have seen. Notwithstanding its lack of economic foundation, the characterization appears to describe reality accurately—high unemployment accompanied U.S. encounters with the lower bound in the 1930s and in 2009 to the present, as well as in other times in other economies.

A solution to the equations of a model with the lower bound is often possible because unemployment becomes a variable that replaces the interest rate when it is forced to be constant at zero. It would not be appropriate to call the solution an equilibrium in the sense that I use that term here. Thus I will use the term solution in discussing the model with this characterization of the labor market. I will use the term equilibrium when I replace this characterization with the DMP labor-market equilibrium.

For convenience when I later add the DMP labor-market elements, I make two assumptions now:

1. When the economy is at full employment, there is a fixed amount u^* of unemployment; the unemployment resulting from the zero lower bound is added to this amount:

$$u = u^* + \bar{n} - n. \tag{24}$$

2. Employers incur the same recruiting cost here as they do in the DMP setup:

$$v = \frac{\gamma s n}{h(u)}. \tag{25}$$

4 The Model without Capital

The case of infinite adjustment cost and thus constant capital is instructive and easy to solve. I normalize $k = 1$ in this section. I absorb recruiting cost into output, so n is net of the cost of recruiting. One unit of labor produces one unit of output. Material balance then requires

$$n = c + g. \tag{26}$$

The other relevant equations are

$$m(g, g') = \beta \left(\frac{c'}{c} \right)^{-1/\sigma} \tag{27}$$

and

$$R_f(0) = \frac{1}{(1-\rho)m(0,0) + \rho m(0,1)}. \tag{28}$$

Absent a binding lower bound on R_f, I take employment, output, and consumption plus government purchases all to have the normalized values of one. Thus consumption is $c_0 = 1$ in the $g = 0$ state and $\bar{c} = 1 - \bar{g}$ in the $g = \bar{g}$ state. The risk-free return in the $g = 0$ state is

$$R_f(0) = \frac{1}{(1-\rho)\beta + \rho\beta(1-\bar{g})^{1/\sigma}}. \tag{29}$$

With $\beta = 0.99$, $\rho = 0.9$, $\sigma = 0.5$, and $\bar{g} = 0.05$, $R_f(0) = 0.92$. An economy with a 10-percent per period chance of moving to a permanent level of government purchases of 5 percent of output, with one percent per period impatience and an intertemporal elasticity of substitution of 0.5, will have a risk-free rate of minus 8 percent per period until the time when the purchases start. At that time it will rise to the economy's natural interest rate, its rate of impatience of one percent.

Now I impose the zero bound on the risk-free rate and allow the level of employment during the $g = 0$ period to be below one. Solving the risk-free formula for the ratio of consumption in state $g = 0$ to consumption in state $g = \bar{g}$ yields:

$$\frac{c_0}{1-\bar{g}} = \left(\frac{\beta^{-1} - 1 + \rho}{\rho}\right)^{\sigma}. \tag{30}$$

This ratio exceeds one—with the parameter values above, it is 1.006. Consumption in state $g = 0$ is $c_0 = 0.955$. Employment is 0.955 as well because government purchases are zero. Unemployment is 4.5 percent.

5 Full Model

The following equations define what I call the *derived variables*:

$$v = \frac{\gamma s n}{h(\bar{n} - n)} \tag{31}$$

$$c = n^{\alpha} k^{1-\alpha} + (1-\delta)k - kx - \kappa k x^2 - v - g. \tag{32}$$

$$q = \frac{x}{\kappa} + 1 \tag{33}$$

$$m(k, g, g') = \beta \left(\frac{c'}{c}\right)^{-\frac{1}{\sigma}}. \tag{34}$$

$$R(k, g, g') = \frac{(1-\delta)q' + (1-\alpha)n'^{\alpha} k'^{-\alpha}}{q}. \tag{35}$$

$$R_f(k, g) = \frac{1}{\mathbb{E}\,(m)} \tag{36}$$

All of the derived variables can be calculated from given k, g, n, and x. In the cases of $m(k, 0, g')$ and $R(k, 0, g')$, the formulas deliver values for $g' = 0$ and $g' = \bar{g}$.

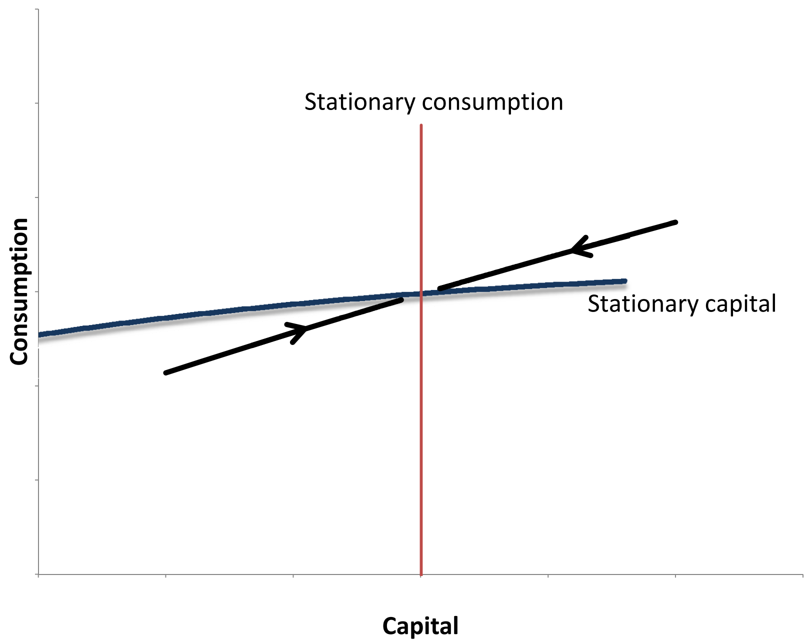

Figure 8: Solution with Constant Government Purchases, Full Employment, and No Adjustment Cost

Definition: A *no-bound solution* is an investment function $x(k, g)$, employment function $n = \bar{n}$, and values of the associated derived variables such that the consumption Euler equation,

$$\beta \, \mathbb{E} \, (m(k, g, g')R(k, g, g')) = 1, \tag{37}$$

holds for all $(k, g) \in \mathcal{D}$.

Figure 8 shows the standard phase-diagram analysis for the full-employment economy starting in and remaining in the $g = \bar{g}$ state, with no adjustment cost. Consumption and the capital stock evolve along the converging arms to the economy's stationary point. The arms form the consumption function $c(k, \bar{g})$.

5.1 Zero-lower-bound solution

The next step is to consider a solution with a binding lower bound in some conditions.

Definition: A *zero-lower-bound solution* is a non-empty set $\mathcal{Z}$, an investment function $x(k, g)$, an employment function $n(k, g)$, and associated derived variables, satisfying (1) the consumption Euler equation $\mathbb{E} \, (m(k, g, g')R(k, g, g')) = 1$, for all $(k, g) \in \mathcal{D}$, (2) the zero lower bound $R_f(k, g) = 1$ and excess unemployment $n(k, g) < \bar{n}$ for all $(k, g) \in \mathcal{Z}$, and (3) $R_f(k, g) \geq 1$ and $n(k, g) = \bar{n}$ for all $(k, g) \in \mathcal{D} - \mathcal{Z}$.

The solution within Z generally has low employment. The marginal product of capital is correspondingly depressed and it may not be possible to satisfy the condition that the capital return ratio is as high as one. Thus the existence of a solution is by no means guaranteed.

A solution is more likely to occur with convex capital adjustment costs. With adjustment costs, the rate of return to capital is the marginal product of capital plus the rate of increase of the value of installed capital. The value is Tobin's q. A negative shock causes an immediate decline in q, followed by a gradual rise in q back to normal. In principle, the decline in capital resulting from the shock is smaller with adjustment costs than without, so the decline in the marginal product of capital is larger with adjustment costs, but under reasonable parameter values, the offsetting influence is small and the q effect dominates.

5.2 Parameters

I use generally accepted parameter values: The elasticity of output with respect to labor input is $\alpha = 0.646$, the utility discount is $\beta = 0.9997$ at a quarterly rate, capital deterioration is $\delta = 0.0188$ per quarter, capital adjustment cost is $\kappa = 8$, the intertemporal elasticity of substitution is $\sigma = 0.5$, and the labor turnover rate is $s = 3 \times 0.04 = 0.12$ per quarter.

To generate a negative interest rate in the no-bound model and a binding lower bound in the model with a bound, I specify the process for growth of government purchases as $\bar{g} = 0.234$ (5 percent of stationary output) and probability of remaining at zero of $\rho = 0.9$, so the expected growth of g is 0.5 percent of stationary output per quarter.

5.3 Approximating and solving the model

Because g remains fixed at $\bar{g}$ once it reaches that level, the model for $g = \bar{g}$ is independent of the model for $g = 0$. In the no-bound case, it is defined by $x(k, \bar{g})$ alone. I approximate this function with an orthogonal quadratic polynomial in k. A low order polynomial is a completely adequate approximation because the range of variation of k that I consider is quite limited. I find the three coefficients by solving a system of three instances of the Euler equation on a grid of three values of k that span the relevant domain of k.

After solving the model for $g = \bar{g}$, I solve two models for $g = 0$. The first is the no-bound model, where unemployment and employment are constant. Again, I approximate $x(k, 0)$

with an orthogonal quadratic polynomial in k and again I find its coefficients by solving three instances of the Euler equation. The model for $g = 0$ uses the previously solved $x(k, \bar{g})$ where transitions to the state $g = \bar{g}$ occur.

The second model for $g = \bar{g}$ imposes the lower bound on the risk-free interest rate. I approximate $\mathcal{Z}$ by considering only the six points on the grid of values of (k, g) used above. There are $2^6 = 64$ possible configurations of $\mathcal{Z}$. I specify the model so that the zero bound never binds after g reaches $\bar{g}$, by avoiding considering such high values of k that the risk-free rate becomes negative because of low values of the marginal product of capital. Thus I examine only the $2^3 = 8$ possible configurations in the zero-g state. A second implication is that I can use the $x(k, \bar{g})$ from the no-bound solution in calculating the lower-bound solution.

For the lower-bound solution with $g = 0$, I approximate $x(k, 0)$ and $n(k, 0)$ as quadratics. I find their six coefficients by solving three instances of the Euler equation, n_Z instances of the equation setting the risk-free rate to zero at the locations specified in the configuration $\mathcal{Z}$ and $3 - n_Z$ instances of the DMP zero-profit condition at the locations not in the configuration (in $\mathcal{D} - \mathcal{Z}$). Barring failure caused by approximation error, either (1) one of these calculations will yield a solution, which is the zero-bound solution, or, (2), none will yield a solution, meaning that there is no zero-bound solution.

5.4 No-bound solution

For the model without a zero bound on the risk-free rate, Figure 9 shows the interest rate (vertical axis) as a function of the capital stock k (horizontal axis) and government purchases g (the two lines). The $*$ marks the stationary point of the economy, where the interest rate is 0.12 percent per year. If the economy were to start at the stationary level of capital and zero government purchases (the lower line), the initial interest rate would be -0.75 percent per year. The figure demonstrates that the expectation of a modest rise in government purchases (5 percent of output) pushes the economy into a negative interest rate regime.

5.5 Solution with binding zero lower bound

Now I look for a zero-lower-bound solution as defined above. I approximate the employment function $n(k, g)$ inside $\mathcal{Z}$ in same way as discussed earlier for $x(k, g)$

The search over all 64 of the grid points in $\mathcal{Z}$ shows that the zero bound on the risk-free rate binds for all values of k when $g = 0$ and never binds when $g = \bar{g}$. I achieve this simple

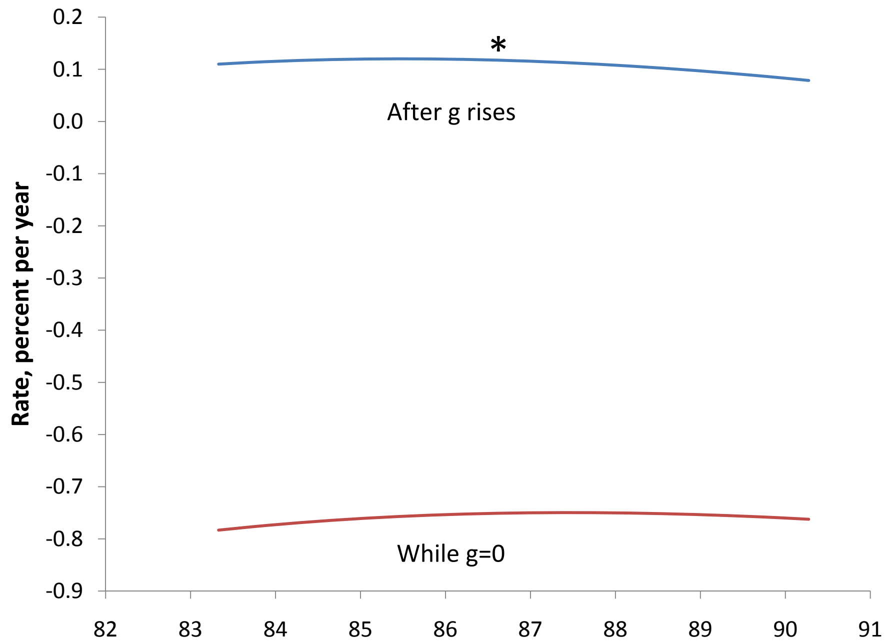

Figure 9: The Risk-Free Rate without a Bound

result by considering only a narrow band of value of k. With low enough k, the no-bound value of the risk-free rate would be positive even with $g = 0$ and with high enough k, the bound would bind even without an expectation of rising government purchases, at $g = \bar{g}$.

The risk-free rate in the ZLB economy is zero when $g = 0$ and is the same function of k as in Figure 9 when $g = \bar{g}$. As I explain shortly, the risk-free rate itself is higher at the time that g pops up to $\bar{g}$ because the capital stock is lower.

Figure 10 shows the unemployment rate in the model in the same format as the previous figure. The rate is a bit over 15 percent as long as the interest bound binds ($g = 0$). Once the government starts purchasing, the unemployment rate pops down to 5.5 percent, its normal level. I denote the unemployment rate from the model with the binding lower bound $u_Z(k, g)$. In the calculations of this section, $u_Z(k, 0) > u^*$ (the bound is binding while $g = 0$), and $u_Z(k, 0) > u^*$ (the bound never binds after g pops up to $\bar{g}$).

Equation (15) shows that the return to capital includes the capital gain term,

$$\frac{(1-\delta)q'}{q}, \tag{38}$$

which has an important role in the outcome. Table 2 shows how Tobin's q behaves. All of the numbers refer to the case when the capital stock is at its stationary level. The left column describes the economy where the risk-free interest rate is pinned at zero when $g = 0$

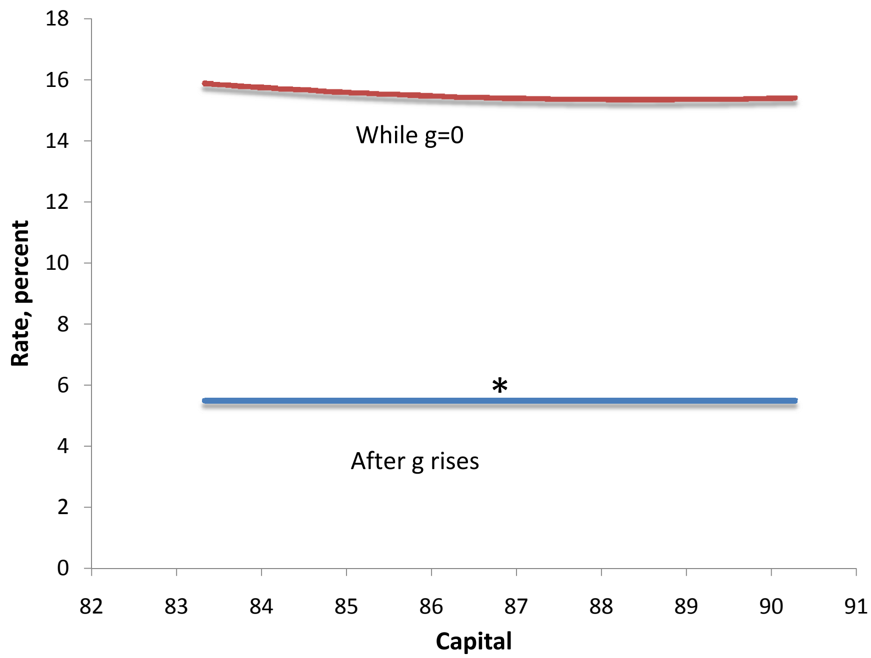

Figure 10: The Unemployment Rate as a Function of the Capital Stock and Government Purchases

		ZLB	*No bound*
While g is low	If g remains low	0.9952	1.0111
	If g becomes high	1.0003	0.9992
When g is high		1.0000	1.0000

Table 2: Tobin's q with and without the Zero Bound on the Risk-Free Interest Rate

and the right column the economy without a bound on that rate. In the top row, if the ZLB economy is in the zero-g state and remains there, q is about a half-percent below one. Investment is correspondingly low—net investment is negative. By contrast, the no-bound economy has q more than one percent above one, so investment is high. These numbers reflect the fundamental difference between the economy with the interest bound and the one without the bound: Without the bound, a negative interest rate stimulates investment, so consumption can be smoothed when government purchases begin, by drawing down the enlarged capital stock. Under the ZLB, production falls substantially, as shown by the unemployment rate in Figure 10, and investment is low instead of high.

The second row in Table 2 shows that, in the economy subject to the ZLB, q jumps up to about 30 basis points above one in the first quarter with $g = \bar{g}$. The reason is that capital is somewhat depleted by the high-unemployment, low investment conditions during the ZLB period. Along an actual trajectory, the effect would be larger because the capital stock would be below its stationary level after a few quarters of the ZLB. Recall that Table 2 holds the capital stock at its stationary level. In the no-bound economy, q drops when g pops up to $\bar{g}$. This economy has accumulated extra capital during the period of $g = 0$, when resources are plentiful, in anticipation of drawing down capital when that period ends.

The bottom line of Table 2 shows that both economies are the same once g reaches $\bar{g}$. They both have $q = 1$ when capital is at its stationary level.

6 Combining DMP and Product-Market Models

The product-market model excludes the two-equation DMP model. The combined model includes all of the product-market equations plus the two excluded equations, which are:

$$\text{Zero profit: } J = \frac{\gamma}{h(u)} \tag{39}$$

and

$$\text{Wage determination: } J = J_X(u). \tag{40}$$

Here $J_X(u)$ is some member of the post-Shimer DMP class of wage determination functions.

Definition: A *regular* DMP model combined with a product-market model has a zero-profit function that is strictly decreasing in unemployment and a wage-determination function that is weakly increasing in unemployment; both functions exclude all endogenous variables from the product-market model except unemployment.

Under the hypotheses that $h(u)$ is strictly increasing and $J_X(u)$ is weakly increasing, a regular DMP model determines unemployment u uniquely, at $u = u_X$. I assume that the two models agree on the unemployment rate in the no-bound economy: $u^* = u_X$. Then

Theorem: Consider the combination of (1) a product-market model whose solution includes points where the zero bound does and does not bind and (2) a regular DMP labor-market model. The combined model has no solution.

Proof: From the definition of a zero-lower-bound solution, the product market model has at least two values of unemployment, $u = u^*$ when the bound does not bind and $u > u^*$ when it does bind. But a regular DMP labor-market model has a single equilibrium unemployment rate u^* for both the no-bound economy and the economy subject to the bound.□

7 Building a Model that Works when the Interest Rate is Pinned at Zero

Non-existence of a solution is a problem in a model, not a statement about the economy. The economy always does something. In this section I discuss ways to overcome the existence problems exposed in the previous section.

I will consider two departures from the assumptions of the theorem that permit the combined model to have a solution. In both cases, the product market predominates—the DMP model adapts its unemployment equilibrium to the level dictated by the product-market model. In one case, the solution is an equilibrium, while in the second, the solution retains the excess-supply property emphasized in earlier work on the zero lower bound, but the product-market model is connected in a coherent way to the DMP labor-market model, which is in equilibrium.

7.1 The flexible unemployment hypothesis

This first approach—based on what I call the *flexible unemployment hypothesis*—drops the assumption that the wage-determination function cannot slope downward and lets it do so just enough to lie on top of the zero-profit function. Recall from Figure 6 that only a small clockwise twist of the wage-determination function is enough to accomplish this goal. The wage determination function becomes

$$J_F(u) = \frac{\gamma}{h(u)}. \tag{41}$$

Under this hypothesis, workers pay less for their jobs when unemployment is higher. By coincidence, the relation is just negative enough to call forth lower recruiting effort by employer that ratifies the higher unemployment.

Inserting this job value into the zero-profit condition that determines the unemployment rate in the DMP model yields

$$h(u)\frac{\gamma}{h(u)} = \gamma, \tag{42}$$

which is satisfied for all levels of unemployment. Unemployment is no longer constrained by the DMP zero-profit condition.

When the interest rate is pinned at zero, unemployment can take the value $u_Z(k, g)$ from the solution of the product-market model discussed earlier. The product-market solution resolves the indeterminacy of this special version of the DMP model.

The combined product-market model and flexible-unemployment DMP model has a full equilibrium corresponding to any solution of the product-market model. The role of the DMP model is to provide an equilibrium understanding of the unemployment rate that emerges mechanically from the product-market model. I discuss this point further in a later section.

7.2 Excess supply in the product market

The second approach drops the strong assumption that no other endogenous variable enters the DMP model. Instead, the solution to the product-market model is viewed as one of excess supply in the product market and a corresponding constraint on the sales of each firm. So far, to my knowledge, no author in this area has worked out the mechanism that maps excess supply in the market to this constraint on individual firms. The proposition seems dangerously close to Aristotle's fallacy of division, attributing to each component of an entity a property of that entity.

The logic of excess supply with a binding lower bound is straightforward. Given a zero interest rate when only a negative one would clear the product market, producers are unable to sell as much current output as much as they would find it remunerative to produce. The analysis of the consequences of excess supply is close to the simple Keynesian expenditure model—when full-employment output would result in an excess of saving over investment, output falls to a lower level where saving and investment are equal.

In the conditions created by a binding lower bound on the interest rate, firms face constraints on the amount they can sell. To incorporate the DMP analysis of the labor market in that setting, one must take a stand on the benefit that accrues to a constrained firm by hiring another worker. I'm not aware that the issues involved in characterizing the benefit have yet been thought through—the literature on the labor-market aspects of zero-lower-bound macroeconomics is nonexistent at this writing.

The marginal benefit of labor is the key connection in the DMP model between the product and labor markets. To generate high unemployment in a regular DMP model, the marginal benefit needs to drop below its normal level. Although it is tempting to conclude that a firm with constrained output has no benefit from an added worker, factor substitution stands in the way of that simple conclusion. If the firm cannot sell more output, the firm can still substitute away from other inputs when it hires a worker. Material inputs seem the most likely substitution opportunity. No framework yet exists to measure the marginal benefit of labor.

I expect that the hypothesis of excess product supply will continue to be developed. My inclination, however, is to consider the alternative of equilibrium in the product market based on the flexible-unemployment hypothesis.

8 Equilibrium in the Product and Labor Markets under the Flexible Unemployment Hypothesis

The flexible unemployment hypothesis holds that the wage-determination function coincides with the zero-profit condition, equation (1), for a wide range of values of u. In this case, the unemployment rate, based on the DMP principles, is consistent with a range of product-market equilibria. The resulting theory describes a full equilibrium in both the output and labor markets.

In this section I argue that the flexible-unemployment hypothesis implies a plausible view of wage determination. I stop short of demonstrating the inevitability of the hypothesis. It would be a great convenience for an economy if the hypothesis held. It's hard to see how an economy can function without the hypothesis holding. How the economy arrives at the specific wage-determination outcome I leave unexplained.

8.1 Full equilibrium

The standard general definition of equilibrium is the absence of unilateral or bilateral opportunities for one or a pair of agents to improve their payoffs. The solution to the model developed in this paper satisfies this definition. No firm or worker has an unexploited opportunity for self-improvement and all firm-worker and firm-customer relationships are bilaterally efficient.

The product-market-only models that have been the exclusive mode of analysis of the zero lower bound to date make no claim to describe an equilibrium. Rather, they view the bound that keeps the interest rate too high as causing excess supply in the current product market. In the excess-supply view, firms could hire unemployed workers for a wage low enough to make production profitable. For reasons not explained in the models, firms are unable to find customers for the resulting output. The diagnosis that these models describe an outcome that is not an equilibrium would be going too far. Rather, the models lack sufficient description of what is preventing firms from hiring workers or expanding sales to understand how the models relate to the concept of equilibrium. The DMP model is one way to augment the model in a way that is clear about frictions that limit an expansion of employment, but other approaches might answer the question as well.

Adding the flexible-unemployment DMP labor market to a product-market model changes from excess supply to full equilibrium. The DMP model is completely specific about how employers benefit from the availability of unemployed workers. To hire a worker, an employer must incur a recruiting cost. The expected cost of one hire is $\gamma/h(u)$, which falls with rising unemployment. But the benefit of the hire—the difference between the worker's contribution to profit and the worker's wage—is the declining function $J_F(u)$, which offsets the decline in expected hiring cost. The employer has no affirmative incentive to expand employment when a force such as the zero lower bound depresses product demand.

8.2 Considerations making the flexible-unemployment hypothesis plausible

The flexible-unemployment hypothesis implies a modest negative slope of the wage- determination function—when unemployment is high, workers pay less for their jobs. This property strikes me as reasonably intuitive. A worker pays the job value to the employer for the privilege of holding the job. If a higher investment of search time is also required to gain the job, shouldn't the worker pay less for the job?

Figure 11 shows the implications of the job-value function

$$J_F(u) = \frac{\gamma}{h(u)} \tag{43}$$

for hourly compensation, with the marginal revenue product of labor held constant. I assume that the job value is deducted from compensation in equal amounts spread over the duration of the job. To estimate that duration, I take the reciprocal of the total separation rate reported in JOLTS, averaged over the period December 2000 through December 2010. The

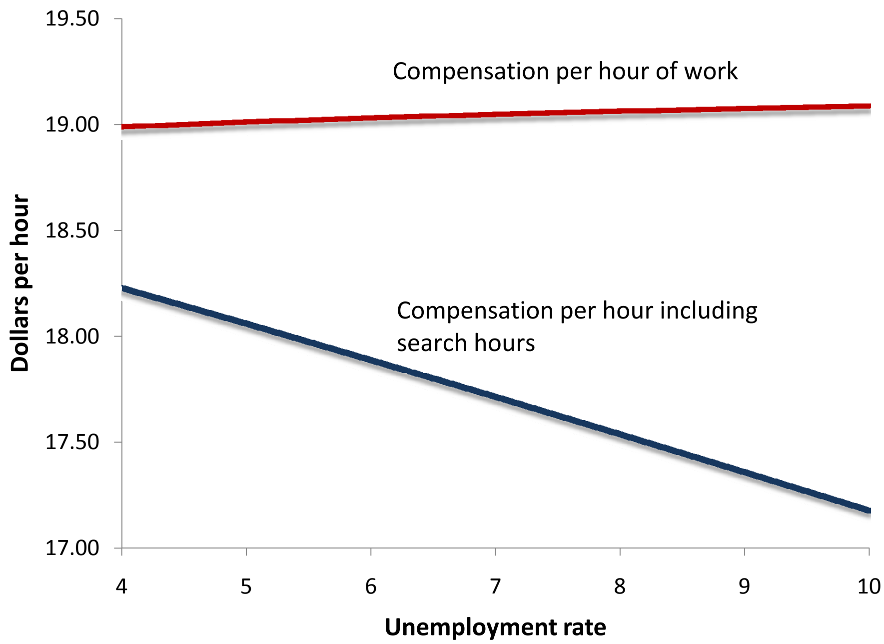

Figure 11: Relation between Hourly Compensation and Unemployment, with Marginal Revenue Product of Labor Held Constant

average rate is 4.16 percent per month, implying an average duration of 24.0 months per job. Average pay in the U.S. economy in January 2011 was \$19.07. I add \$0.23 as the amortized hourly job value, to get an estimate of \$19.30 as the average marginal revenue product of labor. I use the fitted hiring rate function $h(u)$ described earlier. The line labeled *Compensation per hour of work* is

$$\$19.30 - \frac{\gamma}{h(u)}. \tag{44}$$

The line labeled *Compensation per hour including search hours* is compensation per hour multiplied by one minus the unemployment rate.

Compensation per hour rises slightly with unemployment, while compensation per hour including search hours falls substantially. One reason that $J_Z(u)$ rises with u may be that job-seekers get a slight reward to the extra time spent waiting for work to begin when unemployment is higher. That is, job-seekers are willing to pay less for jobs when jobs take longer to find.

The value lost from higher unemployment is vastly greater than the value gained from the decline in the job value. Thus higher unemployment unambiguously lowers the earnings of members of the labor force even though it gives workers a better deal.

Figure 11 understates the rise in pay per hour of work as unemployment rises in the model that follows, because the marginal revenue product of labor rises with declining employment.

8.3 Relation to bargaining theory

The flexible-employment hypothesis specifies a modest negative relation between unemployment and the job value negotiated for new jobs by workers and employers. The job value describes the point along the contract curve resulting from the parties' bargain. Any non-negative job value not exceeding the job-seeker's productivity is in the bargaining set and is therefore consistent with the fundamental principles of bilateral bargaining. The necessary negative relation between unemployment and the job value could arise as an equilibrium selection rule. The other way is that the bargaining protocol—such as alternating-offer bargaining—has a unique equilibrium that happens to match the needed relationship.

Hall (2005) discusses the fundamentals of the wage bargain. The most general view of a bilateral bargain limits the outcome in only one way—if the parties have a joint surplus, they will successfully bargain to a point where each receives a non-negative share of that surplus. The division of the surplus is indeterminate. For example, if the employer and worker engage in the two-sided demand auction of Chatterjee and Samuelson (1983), any wage in the bargaining set is a Nash equilibrium (not to be confused with a Nash bargain). In that auction, the parties submit wage bids simultaneously. If the employer's bid does not fall short of the job-seeker's bid, employment occurs at the average of the two bids. Otherwise, the parties do not contract. The employer's best response to a wage bid from the job-seeker that is no higher than the job-seeker's productivity is to match the wage. The job-seeker's best response to a wage bid that is no lower than the job-seeker's reservation wage is to match the bid. The bargaining set runs from the reservation wage to productivity. Hence any wage in the bargaining set is a Nash equilibrium.

My earlier paper demonstrated that a wide class of state-contingent wages satisfies the condition that the wage is in the bargaining set in every state. All of these state-contingent wage functions are candidate equilibria of the wage bargaining problem. For example, they are all Nash equilibria of the demand auction.

In the setup considered in this paper, the bargaining is over the job value J rather than the wage itself, but the point is the same. The employer's reservation value of J is zero—it is remunerative to hire a worker for any positive job value that the worker pays. The worker's reservation value is the worker's opportunity cost of taking the job. The bargaining set runs from zero to the opportunity cost.

A demonstration that the job-value function converged to the flexible-unemployment form to the exclusion of all other outcomes in the bargaining set would be of great interest, but I have not come up with a framework with that property.

8.4 Relation of the flexible-unemployment hypothesis to sticky wage

The flexible-unemployment condition requires that the wage-determination curve in Figure 6 coincide with the zero-profit curve. The figure shows that this condition requires only a bit more rotation past the horizontal position of the fixed-wage line. In that respect, the flexible unemployment hypothesis is a cousin of the rigid-wage theory. But in another respect, the two approaches differ tremendously. An awkward and unrealistic property of a rigid wage is the sensitivity of unemployment to productivity. The rise in productivity in 2009 would have resulted in a large decline in unemployment with a rigid wage, whereas the flexible-unemployment model is consistent with the observed combination of rising productivity and rising unemployment.

9 Concluding Remarks

Macroeconomics badly needs to resolve the conflict between a theory of unemployment based on excess supply in the current product market and the DMP theory based on equilibrium in the labor market. A regular DMP model—one where a slack labor market improves the bargaining outcome for the employer—cannot coexist with a standard model of excess supply resulting from a binding lower bound on the interest rate. Either (1) the excess-supply model needs to include an element that reduces the benefit to the employer from a new hire or (2) the DMP model needs to include the flexible-employment property that a slack labor market improves the bargaining outcome for the worker.

An extended period of high unemployment in the U.S. economy with the risk-free interest rate pinned at zero has left believers in the DMP model puzzled about the forces that caused such a large change in the labor market. Although some of the rise in unemployment appears to be the associated with an adverse shift of the matching function, most seems to be the result of forces that operate in the product market, much amplified by the inability of the interest rate to fall enough to restore current product demand to normal levels.

Although the ideas in this paper are developed in the context of an economy at the lower bound on the interest rate, they apply as well to an economy in a normal mode with a positive interest rate. There, they have something to say about the effects of the central bank's control over the short-term risk-free rate.

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