Productivity and the Density of Economic Activity

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To explain the large differences in labor productivity across U.S. states we estimate two models—one based on local geographical externalities and the other on the diversity of local intermediate services—where spatial density results in aggregate increasing returns. Both models lead to a relation between county employment density and productivity at the state level. Using data on gross state output we find that a doubling of employment density increases average labor productivity by around 6 percent. More than half of the variance of output per worker across states can be explained by differences in the density of economic activity. (*JEL* R10)

Differences in average labor productivity across U.S. states are large: in 1988, output per worker in the most productive state was two thirds larger than in the least productive state, and output per worker in the top ten productive states was one third larger than in the ten states which ranked at the bottom. Our purpose in this paper is to look at data on the spatial dimension of externalities and increasing returns to examine how they relate to these differences in average labor productivity. To do so we develop two different models—one based on local geographical externalities and the other on the variety of local intermediate services—where the spatial density of economic activity is the source of aggregate increasing returns.

By density we mean simply the intensity of labor, human, and physical capital relative to physical space. Density is high when there is a large amount of labor and capital per square foot. Density affects productivity in several ways. If technologies have constant returns themselves, but the transportation of products from one stage of production to the next involves costs that rise with distance, then the technology for the production of all goods within a particular geographical area will have increasing returns—the ratio of output to input will rise with density. If there are externalities associated with the physical proximity of production, then density will contribute to productivity for this reason as well. A third source of density effects is the higher degree of beneficial specialization possible in areas of dense activity. Although the idea that denser economic activity had advantages from agglomeration was implicit in a large earlier literature, there does not appear to be any earlier work in which density was an explicit element of the theory, nor has there been empirical work based on measures of density.

The finest level of geographical detail in the United States for which reliable data on value added have been assembled appears to be the state level. Thus the observations on output are for the 50 states and the District of Columbia. But the average density of activity for a state is a meaningless concept. Most of the area of the United States supports essentially no economic activity at all. To get a meaningful measure of density, as well as a sensible specification for the geographical extent of the spillovers, we use much more detailed data by county. This work views the unit of production to be the labor, capital, and land present in a county. Estimation involves dealing with the aggregation from the county to the state level. In effect, we create an index of inputs for each

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state, adjusted for density at the county level. The index depends on the extent of increasing returns. The estimate of increasing returns is the one that generates a cross-sectional pattern of the input index that most closely matches the pattern of value added across states.

To summarize our results briefly: capital accounts for some of the differences in productivity across U.S. states but leaves most of the variation unexplained. Estimation of our model of locally increasing returns reveals that accounting for the density of economic activity at the county level is crucial for explaining the variation of productivity at the state level. According to our estimates, a doubling of employment density in a county results in a 6-percent increase of average labor productivity. This degree of locally increasing returns can explain more than half of the variation of output per worker across states.

The paper starts by discussing the related literature. We develop the two models and the corresponding method of aggregating the county employment data to state levels. We then describe how differences in density and productivity can persist in equilibrium. Following a description of the county employment data and state value-added data, we discuss identification and estimation. The main results are summarized in Section VI. Section VII extends the models to account for differences in the availability of public capital, externalities from density at the state level, and externalities from output at the county level.

I. Related Literature

The economics of agglomeration began with Alfred Marshall (1920), who emphasized technological spillovers from one firm to another one nearby. J. Vernon Henderson (1974) formalized Marshall's ideas and demonstrated—building on work by Edwin S. Mills (1967)—that, in an equilibrium, disamenities from agglomeration on the side of households may offset the productivity advantages on the side of firms. A second branch of the literature on agglomeration hypothesizes economies of scale internal to firms. Mills was an early contributor. An essential task with internal increasing returns is to offer a coherent theory of the firm and its market. Mills assumed that all goods are produced by monopolists. More recent papers use a monopolistically competitive market structure to study agglomeration with internal increasing returns to scale. H. M. Abdel-Rahman (1988), Masahisa Fujita (1988, 1989) and Francisco L. Rivera-Batiz (1988) employ the well known formalization of monopolistic competition of A. Michael Spence (1976) and Avinash Dixit and Joseph E. Stiglitz (1977) to demonstrate that nontransportable intermediate inputs produced with increasing returns imply agglomeration. In related models, Paul R. Krugman (1991) demonstrates that agglomeration will result even when transportation costs are small, if most workers are mobile, and Ciccone (1992) argues that patterns of agglomeration are reinforced by endogenous technology adoption. The essence of these models is that when local markets are more active, a larger number of producers of the differentiated intermediate inputs break even. The production of final goods is more productive when a greater variety of intermediate inputs is available.

Empirical studies of agglomeration have focused on city and industry size as determinants of productivity and on technological spillovers as a source of agglomeration economies. Leo A. Sveikauskas (1975), David Segal (1976) and Ronald L. Moomaw (1981, 1985) estimated the effect of city population on productivity. Henderson (1986) found that the productivity of firms increases with the size of the industry as measured by industry employment. But all of these studies are seriously flawed by their reliance on unsatisfactory measures of output from the Census of Manufactures. In addition—following past theoretical and empirical work—they focus on the return to city size. We are not aware of any studies that have examined spatial density directly. We believe that density rather than size is a more accurate determinant. Close calls, such as whether San Francisco and Oakland are the same or different cities, have an important effect in empirical work based on city size, but none at all in our approach based on density. Finally, there appears to be little empirical work investigating the role of geographically localized externalities and increasing returns for explaining the differences in labor productivity across U.S. states. The most closely related work is by Gerald A. Carlino and Richard Voith (1992) who find that total factor productivity across U.S. states increases with urbanization.

An important branch of the empirical literature on agglomeration has studied geographic wage differentials. Wages are higher in cities and other dense areas. No study of wage differentials, though (so far as we know), has considered spatial density directly. We do not attempt to summarize this literature. Edward L. Glaeser and David C. Mare (1994) is an important recent contribution which looks at the wages in many cities. U.S. data on the noncash component wages and salaries are not available by state or other geographic breakdown, as far as we know; differences in cash wages may therefore reflect differences in state taxes and other policies that would affect the split between cash and noncash compensation. Still, exploitation of the rich geographic detail available for cash compensation may be a promising area for future research.

II. Models

A. *Increasing Returns from Externalities*

The ideas of this paper are easiest to understand in models without capital, with land and labor are the factors of production. We begin with a model based on externalities developed to show how density affects productivity and how to aggregate across productive units. The model says nothing new about the sources of agglomeration effects. Let $f(n, q, a)$ be the production function describing the output produced in an acre of space by employing n workers (all space is considered equivalent). The acre is embedded in a larger area (a county, in our empirical work) with total output q and total acreage a. The last two variables describe the density externality in a very general way. We make the assumptions that the externality depends multiplicatively on a particular measure of density, namely output per acre, the elasticity of output with respect to density is a constant, $(\lambda - 1)/\lambda$, and the elasticity of output with respect to employment is also a constant, α:

$$(1) \qquad f(n, q, a) = n^{\alpha}\left(\frac{q}{a}\right)^{(\lambda-1)/\lambda}.$$

The labor employed in a county, n_c, is distributed equally among all the acres in the county. Thus total output in county c is $q_c = a_c(n_c/a_c)^{\alpha}(q_c/a_c)^{(\lambda-1)/\lambda}$. The county-wide joint technology is described by the production function obtained by solving this equation for output:

$$(2) \qquad \frac{q_c}{a_c} = \left(\frac{n_c}{a_c}\right)^{\gamma}.$$

Here γ is the product of the production elasticity, α, and the elasticity of the externality, λ: $\gamma = \alpha\lambda$; α measures the effect of congestion and λ measures the effect of agglomeration. Only the product, γ, is identified in our data. Our empirical results show that the net effect favors agglomeration.

We turn now to aggregating to the state level. Let C_s be the set of counties covering state s. Output in state s is $Q_s = \Sigma_{c \in C_s} n_c^{\gamma} a_c^{-(\gamma-1)}$ and hence average labor productivity in the state is

$$(3) \qquad \frac{Q_s}{N_s} = \frac{\sum_{c \in C_s} n_c^{\gamma} a_c^{-(\gamma-1)}}{N_s},$$

where N_s is the number of workers in state s, and we define the factor density index, $D_s(\gamma)$:

$$(4) \qquad D_s(\gamma) = \frac{\sum_{c \in C_s} n_c^{\gamma} a_c^{-(\gamma-1)}}{N_s}.$$

Letting d_c be employment per acre in county c, D_s be employment per acre in state s, and D be employment per acre in the United States, we can decompose the density index into three components:

$$(5) \qquad D_s(\gamma) = D^{\gamma-1}\left(\frac{D_s}{D}\right)^{\gamma-1} \frac{\sum_{c \in C_s} n_c\left(\frac{d_c}{D_s}\right)^{\gamma-1}}{N_s}.$$

That is, the state density effect is the product of a national effect, a state effect—which depends on the relation of average state density to national density—and a factor that depends on the inequality of density across counties within the state. The last factor is the sum of county density relative to state density raised to the power $\gamma - 1$, weighted by county employment. If, for example, average density in state s is identical to average density in the nation, $D_s = D$, then, relative to the nation, productivity in the state depends on the distribution of employment within the state only.

Under neoclassical conditions, with γ less than one, the density factors would predict lower productivity in states with higher average density, and even lower productivity in states with some particularly dense, congested areas. But if agglomeration effects outweigh congestion effects, density has the opposite effect. States with higher average density and higher inequality of density will have higher levels of productivity.

B. *Increasing Returns from a Greater Variety of Intermediate Products in Denser Areas*

The second model hypothesizes increasing returns in the production of local intermediate goods, as in Abdel-Rahman (1988), Fujita (1988, 1989), and Rivera-Batiz (1988). Our development of this model gives density an explicit role. Let the production function for making the final good on an acre of land be

$$f(m, i) = [m^{\beta} i^{(1-\beta)}]^{\alpha}; \tag{6}$$

here m is the amount of labor used directly in making the final good, i is the amount of a composite service input which cannot be transported outside the acre, α describes decreasing returns to the two variable inputs on the acre (congestion effect), and β is a distribution parameter (agglomeration effect). The service composite, i, is produced from individual differentiated services, $x(t)$, indexed by type t, according to the constant elasticity of substitution production function,

$$i = \left(\int_0^z x(t)^{1/\mu} \, dt \right)^{\mu}. \tag{7}$$

Here z describes the variety of intermediate products produced—types 0 through z are available. The parameter $\mu > 1$ controls the substitutability of the intermediate products. The higher μ is, the less one product substitutes for others and the more monopoly power the producer of that product has. Under the standard Spence-Dixit-Stiglitz assumptions of Bertrand competition that, μ is the markup of price as a ratio to marginal cost that the producer will set in order to maximize profit.

We further assume that it takes $x + v$ units of labor to produce x. With labor paid w, the intermediate product maker will charge a price of μw and make a profit of $(\mu - 1)wx - wv$. With free entry to the intermediate product business, this profit will be pushed down to zero—the fixed cost will just offset the operating profit from market power. The level of output at the zero-profit point is

$$x = \frac{v}{\mu - 1}. \tag{8}$$

Putting this common value for all the service inputs into the production function for the service composite, equation (7), we have

$$i = z^{\mu} x. \tag{9}$$

Production of i uses zx units of intermediate inputs, so the productivity of the i-making process is $z^{\mu-1}$. Because $\mu > 1$, productivity rises with the available variety of intermediate goods. Denser acres have greater variety, because more intermediate services producers can break even. The result is a positive relation between density and productivity.

The Cobb-Douglas specification of the final output technology implies that the share of final output paid to labor employed directly is $\alpha\beta$; hence, $wm = \alpha\beta f(m, i)$. The share paid to land is $(1 - \alpha)$. In a free entry equilibrium all output not paid to land accrues to labor, either directly or indirectly through the intermediate service business. Therefore, $wn = \alpha f(m, i)$, where n, as before, is total labor employed in the acre. Combined, these relationships imply that the equilibrium allocation of labor to direct employment in final goods production is governed by the share parameter,

$$m = \beta n. \tag{10}$$

The remaining share $(1 - \beta)n$ of the labor makes intermediate services. Because we know the total amount of labor devoted to intermediate services and the amount of each one produced, we can solve for the variety of those services:

$$z = (1 - \beta)\frac{\mu - 1}{\mu}\frac{n}{v}. \tag{11}$$

Intermediate product variety, as measured by z, is proportional to density, as measured by the number of workers, n, working on the acre.

Now we can insert the equilibrium value of z into equation (9) to determine i, and then put m and i into the production function for final goods to get the consolidated production function,

$$\phi n^{\gamma}. \tag{12}$$

Here ϕ is a complicated function of the other constants and the elasticity of the production function is

$$\gamma = \alpha[1 + (1 - \beta)(\mu - 1)]. \tag{13}$$

Again, the parameter α describes congestion effects—lower productivity resulting from crowding more workers onto the same acre. To the extent that the differentiated intermediate goods are important ($\beta < 1$) and they are not good substitutes for each other ($\mu > 1$), there is a countervailing effect favoring higher density, because it makes possible a greater variety of the intermediate products. With a high enough μ and a low enough β, the production function could have increasing returns, where the favorable effect of density outweighs the congestion effect.

In this equilibrium the market provision of intermediate inputs is inefficient due to distortions from monopoly pricing. We have worked out the alternative where the quantity and variety of intermediate services is optimal, either because of government intervention or vertical integration. The resulting elasticity of output with respect to total labor is the same as for the monopolistic competition case.

If we normalize the measurement of the quantities to make $\phi = 1$ and assume, as before, that labor is distributed uniformly across the acres of a county, we have the county production function,

$$\frac{q_c}{a_c} = \left(\frac{n_c}{a_c}\right)^{\gamma}. \tag{14}$$

Aggregation to the state level proceeds exactly as before. There are no observational distinctions between the externalities model and the intermediate product variety model. Both provide a theoretical foundation for the same estimation procedure in state data.

C. *Capital and Total Factor Productivity*

Now let the production function describing output produced in an acre of space in county c by employing n_c workers and k_c machines be

$$A_s[(e_c n_c)^{\beta} k_c^{1-\beta}]^{\alpha}\left(\frac{q_c}{a_c}\right)^{(\lambda-1)/\lambda}, \tag{15}$$

where A_s is a Hicks-neutral technology multiplier for state s and e_c is a measure of the efficiency of labor at the county level. As before, the elasticity α is less than one by the amount of the share of land in factor payments. The quantities of labor and capital employed in a county, n_c and k_c, are distributed equally among all the acres in the county. Thus total output in county c is

$$q_c = a_c A_s\left[\left(\frac{e_c n_c}{a_c}\right)^{\beta}\left(\frac{k_c}{a_c}\right)^{1-\beta}\right]^{\alpha} \times \left[\frac{q_c}{a_c}\right]^{(\lambda-1)/\lambda}. \tag{16}$$

Solving for output per acre, we get

$$\frac{q_c}{a_c} = A_s^{\lambda}\left[\left(\frac{e_c n_c}{a_c}\right)^{\beta}\left(\frac{k_c}{a_c}\right)^{1-\beta}\right]^{\gamma}. \tag{17}$$

Again, γ is the product of the production elasticity, α, which is less than one, and the elasticity from the externality, λ, which is greater than one. If γ exceeds one, agglomeration effects dominate congestion.

To deal with capital, we make the assumption that the rental price of capital, r, is the same everywhere. Then we use the factor demand function to substitute the factor price for the factor quantity. That is,

$$(18) \qquad \frac{k_c}{a_c} = \frac{\alpha(1-\beta)}{r}\frac{q_c}{a_c}.$$

Thus the county technology becomes

$$(19) \qquad \frac{q_c}{a_c} = \phi A_s^{\omega}\left(\frac{e_c n_c}{a_c}\right)^{\theta}$$

where ϕ is a constant that depends on the interest rate, and the elasticities for the technology multiplier for the state, ω, and for labor input for the county, θ, are:

$$(20) \qquad \theta \equiv \frac{\gamma\beta}{1-\gamma(1-\beta)}$$

and

$$(21) \qquad \omega \equiv \frac{\theta}{\alpha\beta}.$$

We assume that labor efficiency depends log-linearly on workers' average years of education h_c, $e_c = h_c^{\eta}$, where η is the elasticity of education. Using this relationship in equation (19) and aggregating to the state level, we obtain

$$(22) \qquad \frac{Q_s}{N_s} = \phi A_s^{\omega} D_s(\theta, \eta)$$

where

$$(23) \qquad D_s(\theta, \eta) = \frac{\sum_{c \in C_s} (n_c h_c^{\eta})^{\theta} a_c^{1-\theta}}{N_s}.$$

Under these alternative assumptions, the index of density has the same functional form as before with the elasticity θ in place of γ and efficiency units of labor instead of raw labor. The underlying value of γ can be calculated from equation (20). The relation between γ and θ for $\beta = .7$ is such that for values close to 1, as found in our empirical work, the overstatement of γ associated with the treatment of capital is small. The extension of the intermediate product variety model is analogous.

Regarding the stochastic specification in equation (22), we assume that state productivity A_s is distributed log-normally around an underlying nationwide level. We also allow for mismeasurement in state productivity, assuming that the measurement error has a log-normal distribution with zero mean. Using this stochastic specification in equation (22) and taking logarithms yields

$$(24) \qquad \log \frac{Q_s}{N_s} = \log \phi + \log D_s(\theta, \eta) + u_s.$$

Here u_s is the sum of the measurement error, and it is ω times the deviation of state productivity from the underlying level in the nation. We assume that the errors of different states are uncorrelated.

III. Equilibrium

How can states or counties be in equilibrium with different densities? This question arises if θ exceeds 1. Under neoclassical assumptions, density should be equal everywhere. The marginal product of labor is lower in a denser area, and there are arbitrage profits or a higher standard of living available by moving a worker from a dense area to a less dense one. On the other hand, with θ greater than one, the worker is more productive when moved to a denser area. Absent other considerations, the only equilibrium is for employment to concentrate in a single county.

The simplest answer, and a realistic one, is that some workers prefer to live in areas that turn out to be less dense. These workers are willing to accept the lower wages in those locations. The preference could be, but need not be, a preference for lower density itself. The preference could also take the form of devotion to a location that is not an agglomeration point. Furthermore, as households value land, its price drives a wedge between the product wage and the consumption wage. In equilibrium, there are no incentives to move for either firms or households. The marginal cost of production is equalized across all counties as the decrease in marginal cost associated with higher density is offset by higher product efficiency wages and higher land prices. Households find that differing product wages are counterbalanced by any of the considerations described above.

IV. Data

The data needed for estimation are available for the year 1988. The data cover the private nonproprietary economy. That is, data on labor input at the county level includes only employees, not the self-employed. The corresponding measure of output at the state level is gross state product (GSP) less Proprietors' Income (U.S. Bureau of Economic Analysis, 1985). We use GSP at sellers' prices; Indirect Business Taxes are excluded from the output measure. Data on employment by county are compiled by the Regional Economic Measurement Division (U.S. Bureau of Economic Analysis, 1991). Data on the area of each county are from U.S. Bureau of the Census (1989).

Data on GSP are conceptually far superior to those used in previous work on spatial differences in productivity. Moomaw (1985), Sveikauskas (1975), and Segal (1976) all measure output as the concept of value added or total value of production used in the Census of Manufactures. This concept omits all services either purchased in the market or obtained from corporate headquarters. It is hard to see how the Census of Manufactures value added could be used for any purpose in production economics, but it is a particularly unusable concept for agglomeration issues. Because there is likely to be less vertical integration in big cities or in dense areas, firms in those places are likely to purchase more services than do their counterparts in less dense areas. Moreover, a plant in a dense area is more likely to be close to its corporate headquarters and therefore more dependent on it for transferred services. For both reasons, studies using Census of Manufactures value added will overstate the productivity advantage of cities or dense areas. The research of Henderson (1986) uses total value of production, also from the Census of Manufacturers. Compared to the value-added data this concept has the added disadvantage of double-counting inputs traded within an industry. Our data are based on a careful allocation of purchased and transferred services by industry at the state level. Gross state output is a much more satisfactory measure of output than is the Census of Manufactures concept of value added. Our theoretical formulation assumes that all land is equivalent. Therefore, we use data on state output and county employment which exclude the agricultural and the mining sector. In some states, natural resources are sufficiently important for local economic activity to make our output measure unrealistic—despite our adjustment for mining output at the state level and mining employment at the county level. On this basis, we excluded all states where the mining contributes more than 15 percent of our output measure. These states are Alaska, Louisiana, West Virginia and Wyoming; on average mining stands at 21 percent of our output measure in these states compared to 1 percent in all other states.

Our data on education comes from two sources. At the state level, we have data from the Annual Demographic File of the Current Population Survey (U.S. Bureau of the Census, 1988) for 1988. Our education measure at the state level weighs the workers' years of education by the number of hours worked in 1988. At the county level, we have data from the 1990 Census of Population (U.S. Bureau of the Census, 1992). Our education measure at the county level is average years of education.

V. Identification and Estimation

We make two alternative identifying assumptions. First we assume that the random element of output per worker is uncorrelated with density and average education levels. This assumption amounts to saying that density and education are measured with little error and do not respond to the random element of productivity. Because it appears that much of the noise in productivity across states comes from measurement error, this assumption is not as strong as it may seem at first. Under this identifying assumption, we estimate the returns-to-scale parameter, θ, and the elasticity of average product with respect to education, η, by nonlinear least squares.

If there are true differences in the determinants of productivity across states, it is not realistic to assume that density is uncorrelated with those differences. States with natural features that make them more productive (for example, climate or geographic features suited to transportation) will attract workers because wages will be higher. Our alternative identifying assumption is that there is an exogenous characteristic of states that can function as an instrumental variable for the density index. The corresponding estimator is nonlinear instrumental variables. All of our candidate instruments rest on the hypothesis that the original sources of agglomeration in the United States have remaining influences only on the preferences of workers about where to live; they are not related to modern differences in productivity not explained by our model. The characteristics we use are:

1. Presence or absence of a railroad in the state in 1860, from John F. Stover (1961).
2. Population of the state in 1850, from U.S. Bureau of the Census (1975).
3. Population density of the state in 1880, from U.S. Bureau of Census (1975).
4. Distance from the eastern seaboard of the U.S., from Rand McNally (1993).

The direct measures of agglomeration—population and population density—are eligible as instruments if the main sources of agglomeration in the 18th and 19th centuries are not related to the residuals in our equation. Thus our hypothesis is that the early patterns of agglomeration in the United States did not reflect factors which significantly contribute to productivity today but have a remaining influence mainly through the legacy of agglomeration. Railroads became an important historical factor in agglomeration in the second half of the 19th century. Our hypothesis is that the development of railroads was not driven by modern productivity differences not accounted for by our model. Finally, we include distance from the eastern seaboard as an exogenous determinant of agglomeration in the 18th and 19th centuries.

The nonlinear instrumental variables estimator is discussed by Takeshi Amemiya (1983). The nonlinear model is

$$y_s = f_s(\beta) + u_s \tag{25}$$

where y is productivity, f describes our nonlinear specification, and β is the vector of parameters. Z is a matrix of values of the instrumental variables. The nonlinear instrumental variables estimator minimizes

$$[y - f(\beta)]'Z(Z'Z)^{-1}Z'[y - f(\beta)]. \tag{26}$$

When the number of instruments is the same as the number of parameters, the estimator is just the solution to the orthogonality condition,

$$[y - f(\beta)]'Z = 0. \tag{27}$$

When the instrument is a dummy variable, the estimator is based on grouping. For example, with our instrument for the presence of a railroad in 1860, the estimator of the density parameter is the value that explains the difference in the average level of productivity in states that did have railroads compared to states that did not.

The estimated covariance matrix of β is

$$\hat{\sigma}^2[F'Z(Z'Z)^{-1}Z'F]^{-1} \tag{28}$$

where $\hat{\sigma}$ is the standard error of the residuals and F is the matrix of derivatives of the model with respect to the parameters.

VI. Results

Table 1 gives the results using the county level education data. The least squares estimate of θ is 1.052 with a standard error of .008.[1] The elasticity of labor efficiency with respect to education, η, is .41 with a standard error of .40. The R^2 of the regression is 55 percent. Steven J. Davis (1992), using data on individuals, estimates the elasticity of earnings with respect to education to lie between .80 and 1.35. Our estimate is one standard error away from Davis's lower estimate. The

[1] Some readers have been surprised at the small size of the standard error. It is much smaller, for example, than the standard error of the OLS regression of state productivity on our density index. The reason is that θ predicts zero slope at $\theta = 1$. Small changes in θ correspond to large changes in the density index, so the parameter is correspondingly well estimated.

TABLE 1—ESTIMATION RESULTS

Instrument	Density elasticity, θ (standard error)	Education elasticity, η (standard error)	R^2
None (NLLS)	1.052	0.410	0.551
	(0.008)	(0.396)	
Eastern seaboard	1.055	0.460	0.548
	(0.017)	(0.51)	
Railroad in 1860	1.061	0.330	0.537
	(0.011)	(0.450)	
Population in 1850	1.060	0.350	0.539
	(0.015)	(0.510)	
Population density	1.051	0.530	0.549
in 1880	(0.019)	(0.550)	
All	1.06	0.060	0.536
	(0.01)	(0.82)	

Notes: The equation estimated is (24). The data are value added for 46 states and Washington DC. For the 46 states we have used data on employment and average years of education at the county level.

instrumental variable estimates for θ is 1.06 making joint use of all instruments. This estimate implies that doubling the employment density in a county increases labor productivity by 6 percent. The estimated value for γ is about 1.04 which implies that doubling employment density in a county results in a 4-percent increase of total factor productivity. Using the education data available at the state level, the estimating equation in (24) simplifies to

$$(29) \qquad \log \frac{Q_s}{N_s} = \log \phi + \eta \log h_s + \log D_s(\theta) + u_s$$

where $D_s(\theta)$ is defined in equation (4). The nonlinear least squares estimate of θ and η in equation (25) are 1.051 and .51, with standard errors of .008 and .45 respectively. Using instrumental variables, the estimates are 1.058 and .36 and the standard errors .011 and .49.

Table 2 shows the factor density index $D_s(\theta)$ evaluated at θ = 1.058, average years of education of workers at the state level, and the private gross state product per worker in all sectors except farming and mining. The states are ranked in declining order of density. The densest area for which reliable output data is available is Washington, DC. Not surprisingly, New York ranks second. It is the extreme concentration of employment in New York City that gives the high value of the density measure. In fact, New York City comprises the densest county in the United States, New York county (with a factor density index of 1.94), and three of the 10 next densest counties (Bronx county, Kings county, and Queens county).

Our estimated density index for New York county implies, for example, that workers in New York county are 22 percent more productive than workers in New York state, the state with the highest average productivity in our sample. The other dense states are the highly urbanized states of the northeast plus Illinois and California. The least dense states are the thinly populated states of northern New England, the south, and the southwest. It is important to note that density is not just a measure of the inequality of distribution of the work force across counties—it is also dependent on the actual density in the counties where employment is significant. The third column of Table 2 shows output (in 1988 dollars) per worker by state. Output per worker in New Jersey ($44,488), the most productive state, is two-thirds higher than in South Dakota ($26,196), the least productive state. Average output per worker in the ten most productive states ($38,782) is one-quarter higher than in the ten least productive states ($31,578). The positive partial correlation of density and productivity is immediately apparent from Table 2

TABLE 2—DENSITY, EDUCATION, AND PRODUCTIVITY FOR $\theta = 1.058$

State	Density index	Years of education	Productivity (1988 $)
District of Columbia	1.67	14.0	43,164
New York	1.59	13.3	41,921
New Jersey	1.48	13.4	44,488
Massachusetts	1.47	13.4	37,296
Illinois	1.46	13.2	39,150
Maryland	1.45	13.6	34,439
Rhode Island	1.43	12.7	30,055
Connecticut	1.42	13.5	41,927
California	1.42	12.9	40,723
Pennsylvania	1.40	13.2	34,661
Top 10 average	1.48	13.3	38,782
Ohio	1.40	13.1	36,553
Virginia	1.40	13.2	35,986
Delaware	1.40	13.3	35,223
Michigan	1.39	13.2	39,001
Missouri	1.38	13.1	34,520
Hawaii	1.38	13.3	34,485
Minnesota	1.37	13.2	35,494
Florida	1.36	13.1	30,808
Georgia	1.36	12.7	35,407
Texas	1.36	12.6	36,798
Colorado	1.35	13.6	33,342
Indiana	1.34	12.9	34,721
Wisconsin	1.34	13.2	33,495
Tennessee	1.33	12.6	33,169
North Carolina	1.32	12.8	32,677
Kentucky	1.32	12.7	34,406
Utah	1.31	13.6	32,160
Washington	1.31	13.6	32,661
Nebraska	1.30	13.2	30,323
New Hampshire	1.30	13.4	33,668
Oklahoma	1.29	13.0	33,567
Oregon	1.29	13.3	32,713
South Carolina	1.28	12.8	29,623
Kansas	1.27	13.4	36,223
Alabama	1.27	12.4	32,980
Arizona	1.25	13.2	33,579
Iowa	1.25	13.1	32,318
Average	1.33	13.1	34,071
Maine	1.24	13.1	33,097
Vermont	1.23	13.4	33,733
Arkansas	1.23	12.5	32,150
Mississippi	1.21	12.8	32,707
New Mexico	1.21	12.6	31,249
Nevada	1.20	12.9	36,234
Idaho	1.17	12.7	29,861
South Dakota	1.15	13.0	26,196
North Dakota	1.12	13.3	30,248
Montana	1.10	13.3	30,302
Bottom 10 average	1.19	13.0	31,578

Note: Education is at the state level. The density index as defined in equation (4), uses raw employment at the county level, evaluated at $\theta = 1.058$.

and Figure 1. There are a number of outliers that call for further investigation: most conspicuous is Rhode Island, which is just as dense as its neighbors but has a productivity level of the least dense state. Nevada, on the other hand, has a much higher productivity

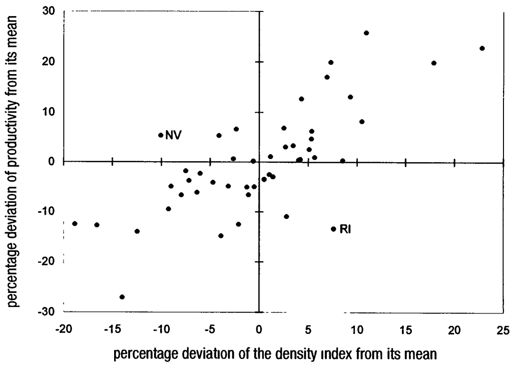

FIGURE 1. DENSITY AND PRODUCTIVITY BY STATE FOR $\theta = 1.058$

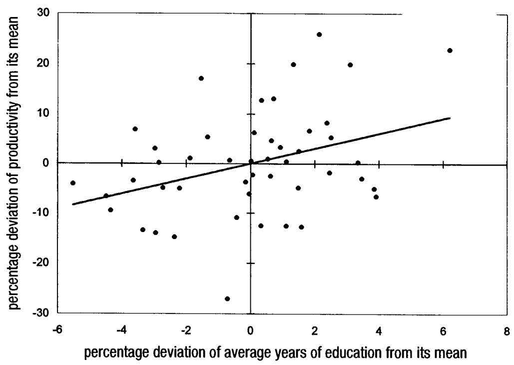

FIGURE 2. EDUCATION AND PRODUCTIVITY BY STATE

level than its low densities would predict. Among the counties with the smallest density indices are Garfield county (Montana), Kimball county (Nebraska), Newton county (Arkansas) and La Paz county (Arizona). We estimate that workers in the 15 counties with lowest density produce on average less than half the output of a worker in New York City.

Figure 2 plots average years of education against productivity at the state level. We have estimated equation (29) without density effects ($\theta = 1$). Using the education data at the state level, we find the elasticity of output per worker with respect to average education in equation (29) is 1.5 with a standard error of .5. The R^2 of the regression is .09. Using the education data at the county level to estimate equation (24) under the same hypothesis, $\theta = 1$, results in an elasticity of labor efficiency with respect to education of 1.2 with a standard error of .6 and a R^2 of .1. Education is a significant determinant of productivity in both cases and the estimate using data at the county level is within the range estimated by Davis (1992).

Figure 3 summarizes the results of our instrumental variables regression of equation (25). It plots estimated output per efficiency unit of labor at the state level against estimated density for $\theta = 1.058$. It is seen that Rhode Island remains an outlier despite the fact that its workers are significantly less educated than workers in neighboring states.

Table 3 decomposes the estimated density index $D_s(1.058)$ as defined in equation (4) into a state effect and a distribution effect along the lines described in equation (5). The column headed *state effect* gives the part of the state effect arising from the average density of the entire state. For example, if the density of employment in Massachusetts fell to the national level, while the distribution of employment over the counties remained unchanged, then this would result in a 15-percent drop in average product. The *distribution effect* measures the part of the state productivity effect attributable to an unequal distribution of employment over counties. For example, productivity in New York would fall by 19 percent if employment were to be allocated uniformly across the area of the state. Colorado, Nebraska, Missouri, Texas and Utah are examples of states with great inequality across counties but low overall density, because their major metropolitan areas have relatively high levels of employment per acre.

VII. Extensions

A. *Public Capital*

Some of the differences in average labor productivity across U.S. states may be due to differences in the amount of public capital that is available. To consider this possibility, we extend the basic model to account for public capital. We assume that the services from the public capital available in state s, g_s, enter county level production with constant elasticity, δ. Then, the county level production function in equation (1) becomes

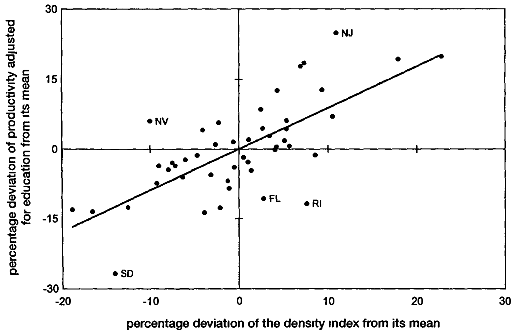

FIGURE 3. DENSITY AND PRODUCTIVITY ADJUSTED FOR EDUCATION, $\theta = 1.058$

$$(30) \quad f(n, q, a) = n^{\alpha}\left(\frac{q_c}{a_c}\right)^{(\lambda-1)/\lambda} g_s^{\delta}.$$

Solving for state level productivity we get that

$$(31) \quad \frac{Q_s}{N_s} = g_s^{\lambda\delta} D_s(\gamma),$$

where $D_s(\gamma)$ is defined in equation (4). Extending the model for exogenous differences in total factor productivity and differences in human and physical capital along the lines of Section II, we get the following estimating equation:

$$(32) \quad \log\frac{Q_s}{N_s} = \psi + \omega\delta \log g_s + \log D_s(\theta, \eta) + u_s$$

where $D(\theta, \eta)$ and ω are defined in equations (23) and (21) above, and ψ is a constant. Table 4 shows the results of estimating equation (32) using the value of public capital in streets and highways in 1988 at the state level from Douglas Holtz-Eakin (1993). The density elasticity falls somewhat, but public capital does not affect productivity at the state level significantly. We have also used other, more inclusive measures of public capital with the same results. Our negative results on public capital are in line with Teresa Garcia-Mila et al. (1994).

B. *State versus Distribution Effects*

As a further exploration of the role played by the state and distribution effects in Table 3 for agglomeration in the United States, we consider an extension of the basic model which allows for influences from density at the state level as well as at the county. The extended model provides a specification test for our earlier model. If there were important state effects not captured by our earlier model it would suggest that we need to consider externalities that cross county boundaries. In the extended model, the production function in equation (1) for output produced in an acre of space becomes

$$(33) \quad f(n, q, a) = n^{\alpha}\left(\frac{q_c}{a_c}\right)^{(\lambda-1)/\lambda}\left(\frac{Q_s}{A_s}\right)^{\kappa},$$

where κ denotes the elasticity with respect to average output per acre in the state, Q_s/A_s. Solving for state productivity along the lines of equations (2) and (3) yields

$$(34) \quad \frac{Q_s}{N_s} = D_s^{\lambda\kappa/(1-\lambda\kappa)} D_s(\gamma)^{1/(1-\lambda\kappa)}.$$

State productivity is determined by average employment density in the state, D_s, and the density index, $D_s(\gamma)$, as defined in equation (4). For $\kappa = 0$ the model reduces to the one discussed at the beginning of Section II.

Extending the model along the lines in Section II results in the following estimating equation:

$$(35) \quad \log\frac{Q_s}{N_s} = \delta + \xi \log D_s + (\xi + 1)\log D_s(\theta, \eta) + u_s,$$

where δ is a common constant across states and ξ is the elasticity of productivity with respect to state density,

$$(36) \quad \xi \equiv \frac{\kappa\omega}{1 - \kappa\omega}.$$

Table 5 shows the results of estimating equation (35). With nonlinear least squares, we obtain an estimate for ξ of 0.005 with a standard

TABLE 3—PRODUCTIVITY, STATE AND DISTRIBUTION EFFECTS FOR θ = 1.058

State	State effect (percent)	Distribution effect (percent)	Productivity (1988 $)
District of Columbia	36	0	43,164
New York	9	19	41,921
New Jersey	16	4	44,488
Massachusetts	15	5	37,296
Illinois	6	13	39,150
Maryland	11	7	34,439
Rhode Island	16	1	30,055
Connecticut	14	2	41,927
California	5	11	40,723
Pennsylvania	7	8	34,661
Top 10 average	8	12	38,782
Ohio	7	7	36,553
Virginia	4	11	35,986
Delaware	10	4	35,223
Michigan	4	10	39,001
Missouri	−1	14	34,520
Hawaii	4	9	34,485
Minnesota	−2	15	35,494
Florida	6	6	30,808
Georgia	2	10	35,407
Texas	−2	14	36,798
Colorado	−6	17	33,342
Indiana	4	6	34,721
Wisconsin	1	9	33,495
Tennessee	2	7	33,169
North Carolina	3	5	32,677
Kentucky	−1	9	34,406
Utah	−9	18	32,160
Washington	−1	9	32,661
Nebraska	−8	16	30,323
New Hampshire	3	3	36,688
Oklahoma	−5	12	34,567
Oregon	−6	12	32,713
South Carolina	1	3	29,623
Kansas	−6	11	36,223
Alabama	−1	5	32,980
Arizona	−6	9	33,579
Iowa	−3	6	32,318
Average	−1	11	34,071
Maine	−4	6	33,097
Vermont	−2	2	33,733
Arkansas	−5	5	32,150
Mississippi	−4	4	32,707
New Mexico	−12	13	31,249
Nevada	−11	10	36,234
Idaho	−12	9	29,861
South Dakota	−13	9	26,196
North Dakota	−13	6	30,248
Montana	−16	7	30,302
Bottom 10 average	−10	9	31,578

Note: The density index as defined in equation (4), uses raw employment at the county level, evaluated at θ = 1.058 decomposed into a state effect and a distribution effect as described in equation (5).

TABLE 4—ESTIMATION RESULTS WITH PUBLIC CAPITAL

Instrument	Density elasticity, θ (standard error)	Education elasticity, η (standard error)	Public capital elasticity, $\omega\delta$ (standard error)
None (NLLS)	1.046	0.570	0.021
	(0.010)	(0.430)	(0.015)
All	1.056	0.480	0.017
	(0.012)	(1.040)	(0.023)

Notes: The equation estimated is (32). The data are value added for 46 states and Washington, DC. For the 46 states we have used data on employment and education at the county level. The data for public capital at the state level is the capital in streets and highways for 1988 from Holtz-Eakin (1993).

error of .015 and an estimate for θ of 1.047 with a standard error of .018. With nonlinear instrumental variables, the estimates for ξ and θ are $-.019$ and 1.084 respectively with standard errors of .023 and .028. Both estimates suggest rather precisely that there is no state effect at all once the county effects are considered. Although we believe that a specification with some cross-county effects would be an improvement over our model, the absence of incremental state density effects indicates that our current specification captures most density effects. This issue is investigated further in Christopher J. Wilkins (1994) who considers different specifications for the geographical extent of externalities.

C. *Size versus Density Effects*

Finally, our framework allows us to consider size versus density effects at the county level. To do so we consider a further extension of the basic model, where the production function (1) accounts for both an externality from output density in the county and output in the county. We assume that the elasticity of firm level output with respect to county output also enters as a constant, ν:

$$(37) \quad f(n, q, a) = n^{\alpha}\left(\frac{q_c}{a_c}\right)^{(\lambda-1)/\lambda} q_c^{\nu}.$$

Solving for state level productivity, we obtain

$$(38) \quad \frac{Q_s}{N_s} = \frac{\sum_{c \in C_s} (n_c^{\gamma} a_c^{-(\gamma-1)})^{1/(1-\nu\lambda)}}{N_s}.$$

When $\nu = 0$, the framework reduces to the basic model with no size externalities at the county level. When $\gamma = 1$, the framework displays no density effects but only size effects. Extending the model for differences in physical and human capital and exogenous total factor productivity, we obtain

$$(39) \quad \log \frac{Q_s}{N_s} = \delta + \log D_s(\theta, \eta, \sigma) + u_s,$$

where $D_s(\theta, \eta, \sigma)$ is defined analogous to equation (38) with efficiency units of labor, $n_c h_c^{\eta}$, in place of raw labor, n_c,

$$(40) \quad D_s(\theta, \eta, \sigma) = \frac{\sum_{c \in C_s} ((n_c h_c^{\eta})^{\theta} a_c^{-(\theta-1)})^{\sigma}}{N_s},$$

where

$$(41) \quad \sigma = \frac{1}{1 - \nu\omega}.$$

Table 6 shows the results of estimating equation (40). The nonlinear least squares estimate of θ is 1.035 with a standard error of 0.013. The county size parameter σ is estimated to be 1.029 with a standard error of 0.019. The R^2 of the regression is 0.58. The nonlinear instrumental variables estimate of the density parameter is 1.046 with a standard error of 0.023 and the county size parameter is 1.026 with a standard error of 0.039. These estimates suggest that density externalities are more important than size externalities at the

TABLE 5—ESTIMATION RESULTS WITH STATE DENSITY EFFECTS

Instrument	County density elasticity, θ (standard error)	Education elasticity, η (standard error)	State density elasticity, ξ (standard error)
None (NLLS)	1.047	0.450	0.005
	(0.018)	(0.420)	(0.015)
All	1.084	0.235	(0.019)
	(0.028)	(0.498)	(0.023)

Notes: The equation estimated is (35). The data are value added for 46 states and Washington, DC. For the 46 states we have used data on employment and education at the county level for the density index $D_s(\theta, \eta)$. Average density D_s is total employment in the state over its area.

TABLE 6—ESTIMATION RESULTS WITH SIZE EFFECTS

Instrument	County density elasticity, θ (standard error)	Education elasticity, η (standard error)	County size elasticity, σ (standard error)
None (NLLS)	1.035	0.259	1.029
	(0.013)	(0.398)	(0.019)
All	1.046	0.140	1.026
	(0.023)	(0.82)	(0.039)

Notes: The equation estimated is (39). The data used is value added for 46 states and Washington, DC. For the 46 states we have used data on employment and education at the county level.

county level. This issue is investigated further in Wilkins (1994) for size externalities at the level of metropolitan statistical areas and standard metropolitan statistical areas.

VIII. Concluding Remarks

Increasing returns to density play a crucial role for explaining the large differences in average labor productivity across U.S. states. We estimate that doubling employment density in a county increases average labor productivity by 6 percent. This degree of locally increasing returns can explain more than half of the variation in labor productivity across U.S. states. Our instrumental variables estimates rest on the hypothesis that the patterns of agglomeration in the 18th and the middle of the 19th century did not reflect factors which significantly contribute to productivity today but have a remaining influence mainly through the legacy of agglomeration. Our estimates control for labor quality at the county level and for differences in the available public capital at the state level. We also compare increasing returns to density with increasing returns to size at the county level and find that increasing returns to density describes the data better than increasing returns to size.

Our work can be extended in several directions. Because our data are available by industry it would be possible to distinguish between economies from localization—which arise from the collocation of firms in the same industry—and economies from urbanization—where favorable effects arise from the general diversity and scale of urban areas. Our approach can also be used to examine dynamic externalities. Work by Edward L. Glaeser et al. (1992) and Henderson (1994) quantifies dynamic externalities by looking at the behavior of industry employment. But small productive externalities may have large employment effects. This makes it difficult to infer the significance of spatial externalities for productivity from data on the spatial concentration of industry employment. Because our data are available since 1962 our approach could be extended to examine the effect of dynamic spillovers on productivity. Our empirical work also suggests that rising density over time may be an important factor in growth. Large U.S. cities are denser now than in earlier centuries, and a much larger fraction of the population is employed in cities or other dense areas. Our estimates could be applied to historical data on the distribution of employment by county to measure the part of total growth that can be associated with rising density.

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