Quantifying the Forces Leading to the Collapse of GDP after the Financial Crisis *

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Abstract

The financial crisis of late 2008 shifted household expenditure downward, as financial institutions tightened lending standards and required repayment of outstanding consumer loans. The crisis also raised financial frictions by depleting the equity capital of financial institutions. The result was a severe reduction in business and residential investment expenditure. The zero bound on the interest rate worsened the adverse effects of these developments by limiting the corrective response of monetary policy. In a straightforward and comprehensible macro model, I measure the two financial driving forces by matching the actual and forecasted movements of two key variables, the unemployment rate and the investment/GDP ratio. I then use the model to describe a counterfactual economy over the period 2009 through 2020 in which the same increase in financial frictions occurred but no household deleveraging took place. The comparison of the counterfactual and actual economies reveals the separate effects of the two financial driving forces. Deleveraging had an important but transitory role immediately after the crisis, while high financial frictions account for the long period of high unemployment, depressed GDP, and subnormal investment.

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In the wake of the financial crisis in September 2008, output and employment fell precipitously in the United States. Although economic activity had begun to decline gradually starting in the previous December, the dramatic decline immediately after the crisis suggests that financial events—specifically the tightening of lending standards by surviving financial institutions—had a major role in the deep and prolonged slump in the economy. Events in financial markets stand at the forefront of most explanations of the slump. Commentary has focused on two channels. The first, household deleveraging, emphasizes cutbacks in consumption forced on credit-dependent households by the elimination of opportunities to borrow and by rising requirements to repay existing debt. The second emphasizes the cutback in plant, equipment, and inventory investment resulting from a rising gap between returns earned by savers and the effective cost of funds for credit-dependent businesses and households. That gap—or financial friction—rose both because financial institutions earned higher spreads between their funding costs and their interest charges and because the institutions rationed credit.

In October 2008, the Federal Reserve lowered its policy interest rate to essentially zero, where it remains at this writing. The transition from an earlier policy regime, where the rate responded to current developments, to one that was incapable of any response, was an important feature of the economy in the aftermath of the crisis. I take explicit account of the altered response to driving forces in an economy where monetary policy is disabled by its inability to stimulate the economy beyond the point where the nominal interest rate hits zero.

To investigate the effects of the two driving forces in the post-crisis economy, I focus on two key macroeconomic indicators: the unemployment rate, taken as a measure of overall economic activity, and the ratio of gross investment to output, a variable known to be sensitive to financial frictions. In a reasonably simple and transparent macroeconomic model, I solve for the paths of the driving forces that account exactly for the prolonged period of high unemployment and low investment. My calculation of the financial friction resembles earlier work in the literature on macroeconomic wedges. My calculation of household deleveraging is novel, I believe.

Taking proper account of the behavior of inflation in the post-crisis economy is centrally important in understanding its evolution. It is well established that the decline in inflation after 2008 was far smaller than declines in previous slumps. After the financial crisis of 1929, inflation was deeply negative for four years. The resulting high real interest rates and rising burden of nominal debt are generally thought to have made a major contribution to the severity of the Great Depression of the 1930s. In the aftermath of the major recession of 1981 and 1982, inflation fell by many percentage points from the high level of 1980. Thanks to the headroom for expansionary monetary policy that high earlier nominal interest rates provided, the Fed was able to engineer a rapid expansion starting in late 1982 by cutting its policy rate dramatically. Though the Fed had much less headroom in 2008, the contraction that ended in mid-2009 was far smaller than the one in 1929 to 1933. The remarkable stability of inflation in 2008 and 2009 appears to have saved the economy from the disaster of depression.

Because the relation between inflation and slack appears to have changed so much since the 1980s, I focus on a specification of the relation that matches recent experience, rather than estimating a relation from lengthy time-series data. Obviously the relation needs to be really simple for that strategy to work. But the facts are also really simple. The trend rate of inflation—which I measure as the one-year ahead forecast of CPI inflation—was around 2.5 percent prior to the crisis and fell to about 1.5 percent when unemployment rose into the 9 to 10 percent range. Since then, it has risen slightly as unemployment has declined. The idea that prolonged slack results in cumulation of disinflation—the accelerationist hypothesis—receives no support from recent experience. I emphasize that this conclusion is specific to the situation of recent years. I have no doubt about the wisdom of Milton Friedman's conclusion that a permanent change in policy regime toward high chronic inflation would not keep unemployment low. The stability of recent inflation appears to be the result of the Fed's achievement of stable low inflation over the 15 years prior to the crisis.

1 The Basic Issues in Modeling the Post-Crisis Economy

1.1 Unemployment

One of the major objectives of this paper is to combine a theory of product supply rooted in the Diamond-Mortensen-Pissarides unemployment model with a conventional model of product demand. This section describes the issues in building the combined model in the simplest form. A later section embodies the ideas in a complete dynamic model suited to studying the actual crisis.

Technology is a proportional relation between output y and employment n:

$$y = An. \tag{1}$$

Unemployment is

$$u = 1 - \frac{n}{\bar{n}} = 1 - \frac{y}{A\bar{n}}. \tag{2}$$

The reduced form of the DMP model of unemployment maps productivity A into the unemployment rate u:

$$u = U(A). \tag{3}$$

In principle, the interest rate is also an argument of U, but nothing of importance is lost by neglecting that dependence. For a complete discussion of the relation between productivity and unemployment, with an emphasis on the special case where unemployment is independent of productivity, see Shimer (2010). In brief summary, productivity influences the payoff that an employer gains from hiring a worker. Competition among employers for new workers achieves a zero-profit equilibrium, where the cost of recruiting a worker exhausts the net benefit of the hire. That benefit is the present value of the difference between the worker's marginal revenue product and the wage. For a given wage, a rise in productivity raises the net benefit, encourages added recruiting effort, and so tightens the labor market and lowers unemployment. To the extent that the wage rises when productivity rises, the effect diminishes. Shimer (2005) demonstrated that, with reasonable parameter values, the wage offset essentially cancels the productivity effect, if the wage of a newly hired worker split the surplus roughly evenly between worker and employer and if the fraction accruing to the worker remained roughly constant as productivity changed. Shimer's version of the DMP model came close to being a model of a constant unemployment rate, independent of anything else happening in the economy. Numerous subsequent papers proposed modifications of the model or different values of its parameters, with the effect of strengthening the influence of productivity on unemployment.

Figure 1 shows an index of quarterly total factor productivity with adjustment for factor utilization constructed from data in Fernald (2009). Productivity did decline after the crisis in late 2008, but the decline was nowhere near large enough to account for the rise in unemployment in even the most ambitious model attempting to overturn Shimer's finding.

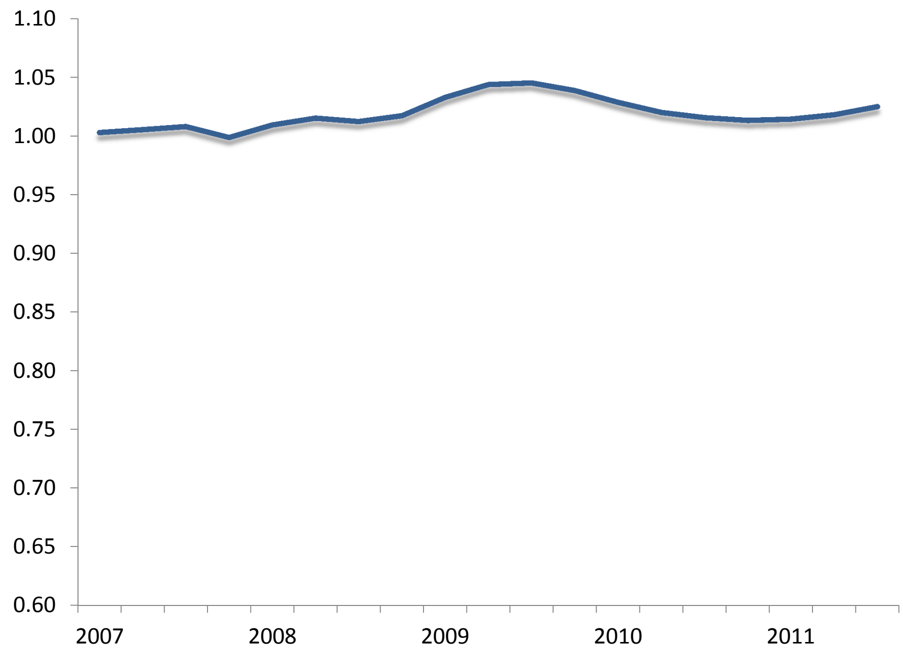

Figure 1: Fernald's Index of Total Factor Productivity with Adjustment for Factor Utilization

1.2 Product demand and equilibrium

On the product demand side of the model, demand is a strictly decreasing function of the real interest rate r:

$$y = D(r). \tag{4}$$

Thus

$$u = 1 - \frac{D(r)}{A\bar{n}}. \tag{5}$$

The equilibrium real interest rate r^* satisfies

$$U(A) = 1 - \frac{D(r^*)}{A\bar{n}}. \tag{6}$$

1.3 Monetary policy

Now I add a central bank, the Fed, to the model. The Fed has the power to set the nominal interest rate, r_N to any level at or above zero. I follow modern mainstream monetary theory in omitting any discussion of how the Fed accomplishes its control of the nominal rate—nothing of importance is added by explicit treatment of money demand and the like (monetary economics should be re-labeled as price-level economics). In the model, the economy finds its equilibrium real rate r^*. Given the Fed's choice of the nominal rate r_N

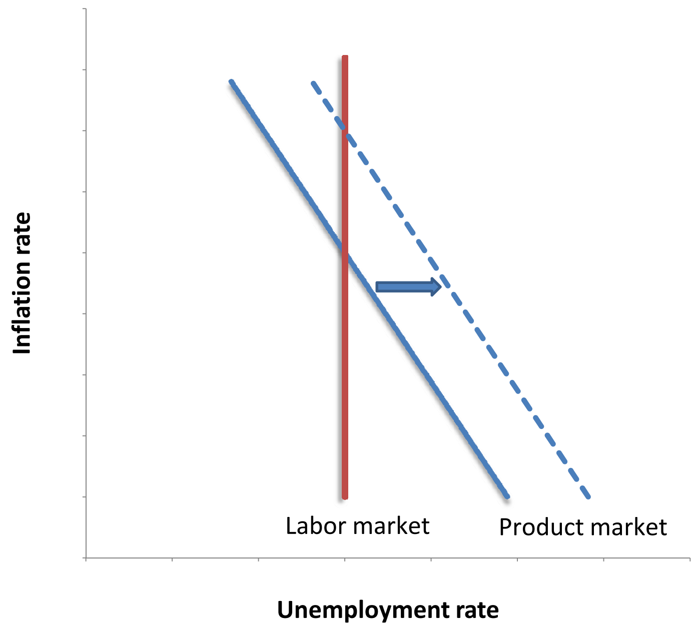

Figure 2: Equilibration through the Inflation Rate

and given the equilibrium real rate r^*, the endogenous variable that clears the product and labor markets is the rate of inflation π.

For most of my discussion, I will take the nominal rate to be at its lower bound of zero, the relevant case for an extended period after the crisis. But the analysis applies to any value the Fed picks.

Figure 2 shows equilibration through the inflation rate when $r_N = 0$. The line labeled "Labor market" shows the left side of equation (6), a constant independent of the rate of inflation. The solid line labeled "Product market" slopes downward because a higher inflation rate corresponds to a lower real interest rate, more output, more employment, and thus less unemployment. The dashed line to its right shows the effect of a decline in current product demand—unemployment is higher for a given level of inflation.

Figure 2 seems completely incapable of accounting for the actual behavior of the economy in times of a binding zero lower bound. First, the decline in product demand leaves unemployment unchanged. Second, the rate of inflation *rises* when the economy softens, contrary to the evidence that inflation slows down when unemployment rises.

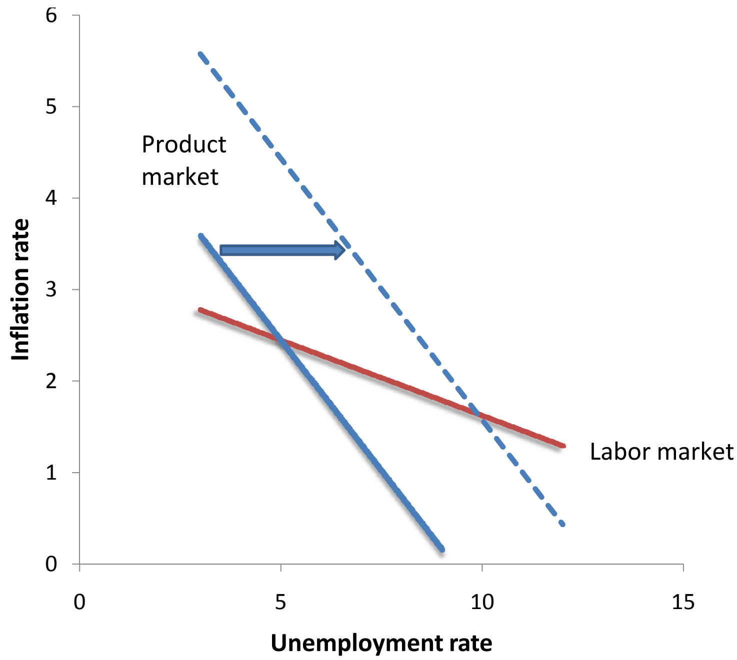

Figure 3: Equilibration through the Inflation Rate with a Negative Dependence of DMP Unemployment on Inflation

1.4 Extending the DMP model to make unemployment depend negatively on inflation

To introduce a class of alternative models with more realistic implications for the effect of a decline in product demand, I extend the DMP model to make unemployment depend on inflation π as well as productivity A:

$$u = U(A, \pi). \tag{7}$$

The dependence is negative. Higher inflation raises employers' incentives to recruit new workers.

Figure 3 illustrates the effect of a decline in product demand in the altered model. The line describing the relation between inflation and unemployment in the DMP part of the model is now flatter than is the unchanged relation from the product-market part of the model. A decline product demand has the expected effect—a large increase in unemployment and a decline in inflation.

1.5 Models with inflation in the DMP labor market

Walsh (2003) first brought a nominal influence into the DMP model. Employers in his

New Keynesian model have market power, so the variable that measures the total payoff to employment is the marginal revenue product of labor in place of the marginal product of labor in the original DMP model. Price stickiness results in variations in market power because sellers cannot raise their prices when an expansive force raises their costs, so the price-cost margin shrinks. Rotemberg and Woodford (1999) give a definitive discussion of the mechanism, but see Nekarda and Ramey (2010) for negative empirical evidence on the cyclical behavior of margins. Hall (2009a) discusses this issue further. The version of the New Keynesian model emphasizing price stickiness suffers from its weak theoretical foundations and has also come into question because empirical research on individual prices reveal more complicated patterns with more frequent price changes than the model implies.

Walsh adopts the Nash wage bargain of the canonical DMP model, which implies that his model may generate low unemployment responses for the reason that Shimer (2005) pointed out. Conceptually, it remains the case that Walsh was the first to resolve the clash between Keynesian models with excess product supply and the DMP model of unemployment.

The second proposal—and the more widely accepted currently—introduces a nominal element into wage determination. The canon of the modern New Keynesian model, Christiano, Eichenbaum and Evans (2005), has workers setting wages that are fixed in nominal terms until a Poisson event occurs, mirroring price setting in older versions of the New Keynesian model. That paper does not have a DMP labor market. Gertler, Sala and Trigari (2008) (GST) embed a DMP labor-market model in a general-equilibrium model, overcoming Shimer's finding by replacing Nash bargaining at the time of hire with a form of wage stickiness. Gertler and Trigari (2009) developed the labor-market specification. A Poisson event controls firm-level wage bargaining, which takes the Nash form. Between bargaining times, the wage of newly hired workers adheres to the most recent bargain. If labor demand turns out to be higher than expected at bargaining time, the part of the surplus captured by the employer rises and the incentive to recruit workers rises. By standard DMP principles, the labor market tightens and unemployment falls. Though the model is Keynesian in the sense of sticky wages, it describes an equilibrium in the labor market in the sense of Hall (2005)—the relation between workers and an employer is privately efficient. GST build a model of the general-equilibrium response to monetary and other shocks in a version of the Gertler-Trigari setup where the wage bargain is made in nominal terms. The GST paper resolves the clash by making the DMP determination of unemployment sensitive to the rate of inflation. It does not treat the zero lower bound on the nominal interest rate explicitly, though it contains all the elements necessary for that analysis.

A key idea in Gertler and Trigari (2009), put to work in the GST paper, is that workers hired between bargaining times inherit their wage terms from the most recent bargain. In principle, this setup could violate the private efficiency criterion by setting the wage too high to deliver a positive job value to the employer or too low to deliver a job value below the job candidate's reservation level, but, again, in practice this is not likely to occur. If it were an issue, the introduction of state-dependent bargaining would solve the problem, at the cost of a more complicated model.

The GST model assumes that the wage bargain is made in money terms, as the traditional Keynesian literature likes to say. The substance of the assumption is that a state variable—the most recently bargained nominal wage—influences the job value for new hires until the next bargain occurs. This assumption has had a behavioral tinge in that literature—the role of the stale nominal wage arises from stubbornness of workers or employers or from money illusion. From the perspective of bargaining theory, however, as long as the stale wage keeps the job value in the bargaining set, that wage is an eligible bargain. See Hall (2005) for further discussion, not specifically in the context of a nominal state variable. There's no departure from strict rationality in the GST model.

The implications of a model linking the current job value to a stale nominal variable are immediate: The more the price level rises from bargaining time to the present, the higher is the job value in real terms. A sticky nominal wage links inflation and unemployment in the way required by Figure 3. Among the modifications of the DMP model that may aid understanding of high unemployment in the zero lower bound, I believe that GST's is the most promising.

1.6 A streamlined but realistic version of the DMP model with inflation

The key idea in GST is that nominal wages depend on stale wages bargained earlier. As a result, the real wage depends on the rate of inflation. The payoff to hiring a worker is higher with higher inflation because inflation erodes the real wage but not the real marginal product of labor. In accord with the principles of the DMP model, when the payoff to a new hire rises, employers recruit more vigorously and the labor market tightens, with a lower

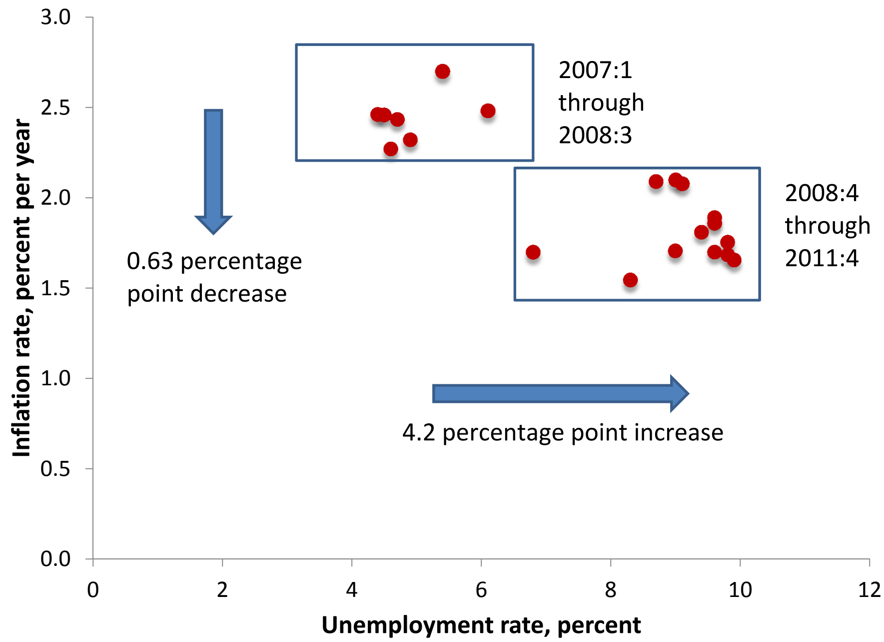

Figure 4: Inflation and Unemployment after the Crisis

unemployment rate.I take the reduced form of this relation to be:

$$u_t = \phi_0 - \phi_1 \pi_t. \tag{8}$$

One can also view this as the economy's Phillips curve,

$$\pi_t = \frac{\phi_0 - u_t}{\phi_1}. \tag{9}$$

Figure 4 shows data for the period just before the crisis and after the crisis, through the end of 2011. Inflation here is the one-year ahead forecast for CPI inflation from the Survey of Professional Forecasters and unemployment is the standard rate from the Bureau of Labor Statistics. The crisis brought an immediate decline of around 0.6 percentage points in the rate of inflation, accompanied by an increase of 4.2 percentage points in the unemployment rate. The ratio of the two is 6, an estimate of the parameter ϕ_1. This value is the instrumental variables estimator of the slope, where the instrument is an indicator variable for the post-crisis quarters.

The slope parameter ϕ_1 is plainly not a deep structural parameter—it is context-dependent. Stock and Watson (2010) show a dramatic decline in the sensitivity of inflation to unemployment in the past decade. That paper also surveys other recent contributions to the literature on the joint behavior of inflation and unemployment. A reasonable hypothesis is that strong anchoring of inflation expectations resulting from an extended period of low and stable inflation has lessened the response of inflation to slack conditions. Stock and Watson also show that the joint movements of inflation and unemployment in the years around the crisis are typical—inflation drops immediately if unemployment rises, but tends to resume its normal level if unemployment stabilizes at a higher level.

2 Is the Nominal Shift in the DMP Model Big Enough to Account for the Bulge in Unemployment?

The GST model appears to me to be the most coherent model that embodies the logic of Figure 3. This section investigates whether the condition derived earlier, that the labor-market curve in that diagram be flatter than the product-market curve, is likely to hold. I consider the two underlying questions: (1) Is the effect of the stickiness of the nominal wage in a GST-type model large enough to twist the labor-market curve enough from its vertical slope in the standard DMP model? and (2) is the product-market curve sufficiently sloped so that the labor-market curve is flatter?

2.1 Slope of the labor-market curve

I have calculated the slope of the price- and wage-adjustment block in GST. I measure the slope by treating a product demand shock—specifically, what they call the monetary shock—as an instrumental variable that moves the model along its price-wage adjustment curve without shifting that curve. The corresponding measure is the ratio of (1) the impulse response function of unemployment to the monetary shock to (2) the impulse response function of inflation to the monetary shock. At four quarters past the shock, the ratio is 3.3 percentage points of increased unemployment per percentage point of decreased inflation. This figure is about double the value found above, based only on crisis-period data. The discrepancy presumably relates to the use of historical data in estimation of the GST model.

2.2 Slope of the product-market curve

I use a similar logic to find the slope of the product-market curve in the GST model. I use the labor bargaining power shock as an instrument for the product market. That shock moves the model along its product-market curve without shifting the curve. There is one further detail—I need to measure the slope with respect to the real interest rate, but the model deals with the nominal rate and the rate of inflation. I compute the slope as

$$\frac{f_{u,\eta}}{f_{r,\eta} - f_{\pi,\eta}}, \tag{10}$$

where $f_{u,\eta}$ is the impulse response function four quarters out for the effect of the wage markup shock η on unemployment u, and similarly for the nominal interest rate r and the rate of inflation π.

The wage-markup shock lowers output, raises inflation, and raises the nominal interest rate by less than the increase in inflation, so the shock lowers the real interest rate. The ratio of the unemployment response to the real-interest-rate response is 0.6, which is substantially less than the 3.3 for the labor-market curve. Thus the GST model easily satisfies the criterion for resolving the clash between the product market and the labor market.

3 U.S. Unemployment and Inflation, 2007 through 2009

Figure 5 summarizes the entire analysis of this paper in terms of the huge rise in unemployment that began in 2007. In December 2007, the unemployment rate was 5.0 percent and the rate of inflation was 2.4 percent, measured by the average one-year-ahead forecast for the Consumer Price Index in the Survey of Professional Forecasters (other measures of inflation were quite similar). In December 2009, inflation was 0.8 percentage points lower at 1.6 percent and unemployment was 4.9 percentage points higher. The figure portrays these two pairs of values as occurring at the intersection of the product-market and labor-market curves of Figure 3.

Figure 5 make the reasonable assumption that no shift occurred in the labor-market curve—the impetus for the contraction came entirely from the adverse developments in the product market. These include the consumption decline resulting from household deleveraging, the collapse of homebuilding, and the cutback in producer and consumer durables purchases resulting from the increase in financial frictions from the crisis. Based on that assumption, I take the labor-market curve to be the line connecting the two observed points. The slope of the line is 6 percentage points of unemployment per percentage point of decreased inflation, substantially flatter than the 3.8 calculated earlier. A reasonable explanation for the difference is that the earlier calculation used GST's estimate of the re-bargaining hazard, inferred from several decades of U.S. history, including times of higher and less stable

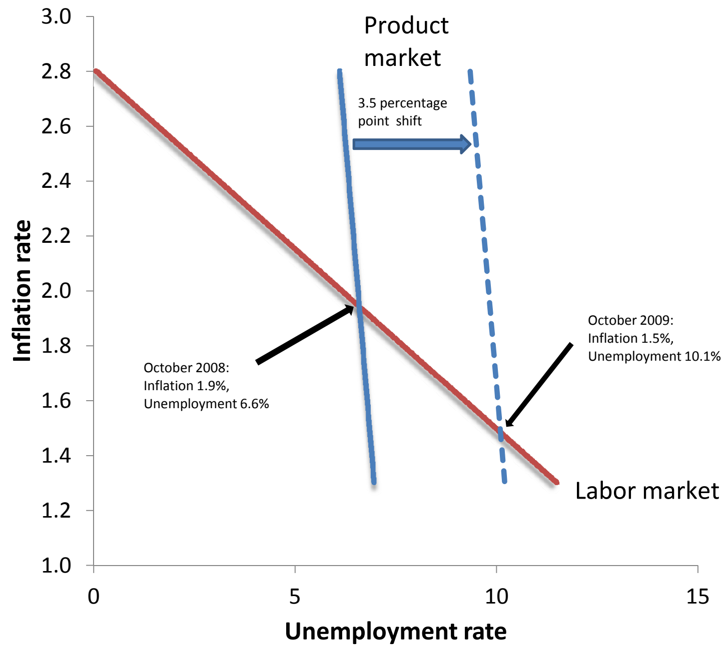

Figure 5: The U.S. Economy in December 2007 and December 2009

inflation. The period that I consider, 2007 to 2009, followed a period of low and stable inflation, so it is reasonable to conclude that the re-bargaining hazard fell, making unemployment more sensitive to a decline in the inflation rate.

I take the slope of the product-market curve to be 0.7 percentage points of unemployment per percentage point increase in the real interest rate, or, in terms of the figure, with a nominal rate pinned at zero, 0.7 percentage points of increased unemployment per percentage point decrease in the rate of inflation. The figure shows the 2007 product-market curve as the solid line with this slope passing through the observed inflation-unemployment point. It shows the 2009 product-market curve as the dashed line with the same slope passing through the 2009 inflation-unemployment point.

The rightward shift of the product-market curve is 4.4 percentage points. If the rate of inflation had remained constant despite the recession, the unemployment rate would have risen from 5.0 percent to 9.4 percent rather than to 9.9 percent. The downward slope of the labor-market curve somewhat amplified the effect of the negative shock to product demand, from 4.4 percentage points of unemployment to 4.9 points.

The notion that expectations of lower inflation amplify negative shocks when the nominal rate is at the zero lower bound has a long history in macroeconomic thought. DeLong and Summers (1986) is a prominent treatment with an extensive discussion of the analysis of

Irving Fisher and others during the Great Depression. Eggertsson (2008) is a more recent discussion of the topic in a New Keynesian framework. According to the calculations in Figure 5, the amplification is quite modest, however. Based on the experience from 2007 to 2009, inflation responds only slightly to increased unemployment in the context of the current U.S. economy. Further, the feedback from the small decrease in inflation to product demand is weak, according to the GST model. I have made similar calculations based on the stronger feedback in the model of Smets and Wouters (2003), but the amplification still remains weak because of the flatness of the labor-market curve inferred from the recent behavior of unemployment and inflation. In an environment of less stable prices, such as the U.S. in 1929 to 1933, the analysis could be altogether different, as Eggertsson has emphasized.

Because the product-market and labor-market curves both slope downward, the economy faces the danger that their slopes might be almost equal, in which case a negative shock would cause a deflationary collapse. The figures in this paper make it clear that the danger is maximal not when inflation is highly responsive to negative shocks, but rather at the point where the labor-market curve is just slightly flatter than the product-market curve.

4 Dynamic Model

The model is a straightforward expression of standard macroeconomic principles. Its core is a standard aggregate technology including capital adjustment costs. The variables in the model are:

- k_t is capital held during period t
- q_t is the price of installed capital, at the beginning of period t
- r_t is the real interest rate for funds loaned at the beginning of period t and returned at the beginning of $t+1$
- $r_{N,t}$ is the nominal interest rate for funds households loan to businesses at the beginning of period t and receive back at the beginning of $t+1$
- y_t is output produced at the end of period t
- $c_{P,t}$ is the consumption of patient households at the beginning of period t

- v_t is the amount that impatient households borrow at the beginning of period t and repay at the beginning of $t+1$
- $c_{I,t}$ is the consumption of impatient households at the beginning of period t
- n_t is employment at the beginning of period t
- u_t is unemployment at the beginning of period t
- π_t is the rate of inflation from t to $t+1$.

My approach to general-equilibrium dynamic modeling differs from the approach embodied in standard New Keynesian DSGE models. That standard involves formulating a model in explicit stochastic form, approximating the model as its log-linear counterpart, and solving for its stationary equilibrium. By stationary, I mean that the solution gives the equilibrium in terms of functions of the state variables but not of time itself. My interest in time-dependent driving forces prevents me from taking that approach.

A second reason to adopt a different approach is proper implementation of the zero lower bound on the nominal interest rate. Judd, Maliar and Maliar (2011) discuss the challenge of calculating the stationary equilibrium of a model subject to the zero lower bound. They find that the procedure generally used, of applying the bound to the solution for a model without a bound, fails to deliver a good approximation to the true equilibrium of the model if it is likely to encounter the bound.

My approach is to calculate the exact equilibrium of a model with time-dependent driving forces under the assumption that the economy has no further uncertainty after a shock launches those driving forces. The equilibrium satisfies the zero lower bound exactly (by exact, I mean with an error in its Euler equations and all other equations no larger than a proportion of 0.0000000000000001). I treat the model as a huge system of equations of dimension equal to the product of the number of variables and the number of observations, and solve for the trajectories of all of the variables simultaneously. Quite elementary non-linear solution methods can solve the system quickly and easily. The only shortcoming of this approach is its neglect of uncertainty about the future once the major shock occurs. Obviously the approach cannot deal with the equity premium and other issues that depend intrinsically on uncertainty. In principle, treating states of the world in the same way that I treat time could overcome this shortcoming, but the dimension of the resulting model might be a challenge for existing computers.

4.1 Technology

At the beginning of period t, the economy has surviving capital $(1-\delta)k_{t-1}$ plus output y_{t-1} from production at the end of the earlier period. These resources are divided among consumption $c_{N,t} + c_{S,t}$, government purchases g_t, gross investment $k_t - (1-\delta)k_{t-1}$, and adjustment cost $(k_t - k_{t-1})^2/(2\kappa k_{t-1})$. Labor input is n_t. The two factors n_t and k_t produce output y_t at the end of the period. Installed capital costs q_t (Tobin's q). The production function is

$$y_t = k_t^{\alpha} n_t^{1-\alpha}, \tag{11}$$

so the marginal product of capital is

$$\alpha \frac{y_t}{k_t}. \tag{12}$$

4.2 Consumption with borrowing limits

Guerrieri and Lorenzoni (2011) describe a natural framework for studying the effects of changes in borrowing limits. Each household has a personal state variable, liquid wealth. Earnings are stochastic and are zero with a significant probability, interpreted as the unemployment rate. Liquid wealth is distributed between a negative level, the borrowing limit, up to a maximum feasible level. Households with positive wealth behave according to the life-cycle principle, with extensive consumption smoothing and low marginal propensities to consume from transitory income. Households with negative wealth near the borrowing limit are intensely precautionary. They have high propensities to consume from transitory income and thus resemble hand-to-mouth consumers. The distribution of wealth across households is a high-dimension state variable of the model.

Rather than trying to incorporate the Guerrieri-Lorenzoni consumption model in my general-equilibrium model by adopting a low-dimension approximation to the wealth distribution, I approximate its behavior in a different way. I hypothesize two groups of consumers, patient and impatient, with discount ratios β_P and β_I, where the second is lower than the first. There is a market for one-period loans where one unit of output loaned by a household to a firm at the beginning of period t returns $1+r_t$ units at the beginning of period $t+1$. In a second market, an impatient household borrows from a patient one at a total borrowing cost of $\frac{\rho}{2}\left([v_{t-1} - b_{t-1}]^+\right)^2$ so the marginal rate is $r_{I,t} = r_t + [v_{t-1} - b_{t-1}]^+$, where v_t is the amount borrowed, b_t is a parameter controlling borrowing opportunities, and $[\cdot]^+$ is the positive part of a number.

The Euler equations of the two types of households are

$$\beta_P \left(\frac{c_{P,t+1}}{c_{P,t}} \right)^{-1/\sigma} (1 + r_t) = 1 \tag{13}$$

and

$$\beta_I \left(\frac{c_{I,t+1}}{c_{I,t}} \right)^{-1/\sigma} (1 + r_{I,t}) = 1. \tag{14}$$

Patient households are the residual claimants on the economy and their budget constraint is implicit in the economy's resource constraint, so it is not necessary to keep track of their wealth. Impatient households are a fixed fraction ν of the population and labor force. At the beginning of each period t after the first, they repay existing debt plus interest and receive v_t from new borrowing. Their budget constraint is

$$c_{I,t} = w_{t-1} n_{t-1} - (1 + r_{t-1}) v_{t-1} - \frac{\rho}{2} \left([v_{t-1} - b_{t-1}]^+ \right)^2 + v_t. \tag{15}$$

Here w_t is the wage, taken to be the marginal product of labor,

$$w_t = (1 - \alpha) \frac{y_t}{n_t}. \tag{16}$$

In period 1, impatient households start with net wealth per person of V_0; their consumption is

$$c_{I,1} = V_0 + v_1. \tag{17}$$

4.3 Financial friction and return to capital

There is a financial friction at rate f_t on the value of capital, payable one period after acquisition. It is modeled as a tax and is returned to unconstrained consumers as a lump sum. If a firm receives a unit of output at the beginning of period t, the firm gets $1/q_t$ units of capital to hold during period $t + 1$, which results in the ability to sell

$$1 + r_t = \frac{1}{q_t} \left[\alpha \frac{y_t}{k_t} + (1 - \delta) q_{t+1} \right] - f_t \tag{18}$$

extra units of output at the beginning of period $t + 1$.

4.4 Tobin's q

One unit of output at the beginning of period t becomes one unit of installed capital held during the period. Installation incurs a cost at the beginning of the period $\kappa(k_t/k_{t-1} - 1)$. Thus the price of a unit of installed capital at the beginning of the period is

$$q_t = \kappa \left(\frac{k_t}{k_{t-1}} - 1 \right) + 1. \tag{19}$$

4.5 Inflation and monetary policy

The price level is p_t. The rate of inflation from t to $t+1$ is π_t: $p_{t+1} = (1+\pi_t)p_t$. One dollar loaned at the beginning of period t becomes $1/p_t$ units of output, which returns $(1+r_t)/p_t$ units of output in period $t+1$, whose nominal value is

$$1 + r_{N,t} = \frac{p_{t+1}(1+r_t)}{p_t} = (1+\pi_t)(1+r_t). \tag{20}$$

A Taylor rule governs monetary policy, as constrained by the zero lower bound:

$$r_{N,t} = [\tau_0 + \tau_\pi \pi_t - \tau_u u_t]^+. \tag{21}$$

4.6 Unemployment

The streamlined version of the sticky-wage Diamond-Mortensen-Pissarides model of Gertler and Trigari (2009) and Gertler et al. (2008) is:

$$u_t = \phi_0 - \phi_1 \pi_t. \tag{22}$$

For simplicity and realism, I take unemployment u_t to be more comprehensive than the standard measure from the Bureau of Labor Statistics. It satisfies the relation in the data,

$$y_t = A k_t^\alpha [(1+\lambda)^t (1-u_t)]^{1-\alpha}. \tag{23}$$

Here λ is the trend rate of labor-augmenting productivity growth and population growth. Fluctuations in my measure of unemployment absorb cyclical fluctuations in productivity, fluctuations in labor-force participation, and fluctuations in hours per employee. This measure of unemployment has substantially higher cyclical volatility than does the standard measure. This approach eliminates the need to model the sources of these cyclical fluctuations separately. I fix the corresponding concept of the labor force at one, so

$$n_t = 1 - u_t. \tag{24}$$

Figure 6 compares the comprehensive rate to the standard BLS rate.

4.7 Resource constraint

The economy's resource constraint is:

$$y_{t-1} + (1-\delta)k_{t-1} = \nu c_{N,t} + (1-\nu)c_{S,t} + g_t + k_t + \kappa \frac{(k_t - k_{t-1})^2}{2k_{t-1}}. \tag{25}$$

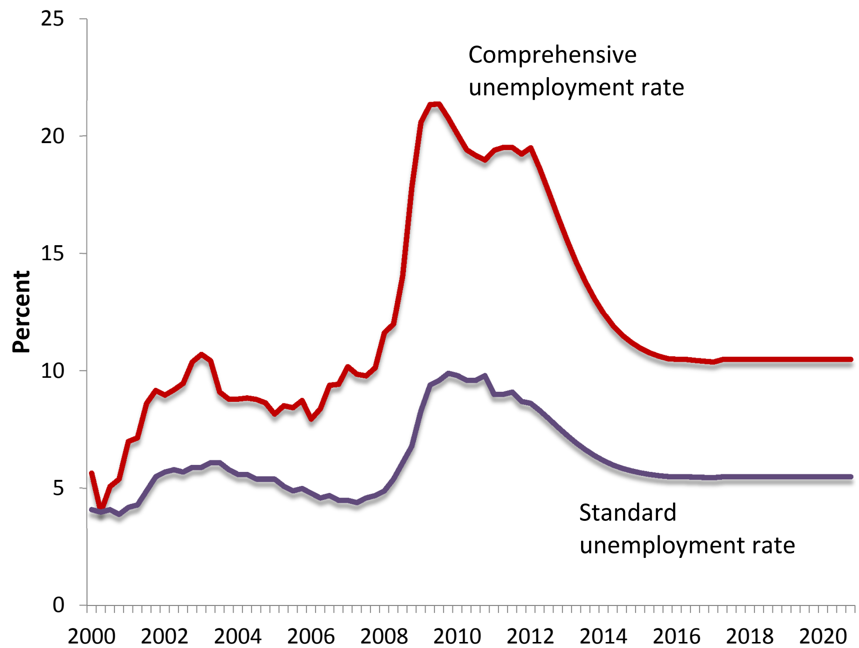

Figure 6: Standard and Comprehensive Unemployment Rates

5 Calculating the Borrowing Limit and the Friction

To calculate the implied values of the borrowing limit and the financial friction, I proceed in the following way: I take comprehensive unemployment u_t and the gross investment/output ratio z_t as inputs. First,

$$n_t = 1 - u_t. \tag{26}$$

The gross investment/capital ratio is

$$z_t = \frac{k_{t+1} - (1-\delta)k_t}{k_t^\alpha n_t^{1-\alpha}}, \tag{27}$$

so I can calculate the implied capital stock from the recursion,

$$k_{t+1} = z_t k_t^\alpha n_t^{1-\alpha} + (1-\delta)k_t. \tag{28}$$

Output is

$$y_t = k_t^\alpha n_t^{1-\alpha}. \tag{29}$$

Inflation is

$$\pi_t = \frac{\phi_0 - u_t}{\phi_1}. \tag{30}$$

The nominal interest rate is, according to the Taylor rule,

$$r_{N,t} = [\tau_0 + \tau_\pi \pi_t - \tau_u u_t]^+. \tag{31}$$

The real interest rate is

$$r_t = \frac{1 + r_{N,t}}{1 + \pi_t} - 1. \tag{32}$$

Tobin's q is

$$q_t = \kappa \left(\frac{k_t}{k_{t-1}} - 1 \right) + 1. \tag{33}$$

The wage is

$$w_t = (1 - \alpha) \frac{y_t}{n_t}. \tag{34}$$

Total consumption is:

$$c_t = y_{t-1} - \left[g_t + k_t - (1 - \delta) k_{t-1} + \kappa \frac{(k_t - k_{t-1})^2}{2 k_{t-1}} \right]. \tag{35}$$

Taking terminal consumption of patient households to be at the stationary level, I compute their consumption in earlier periods from the reverse Euler equation,

$$c_{P,t} = \left(\frac{\beta_P}{1 + r_t} \right)^{\sigma} c_{P,t+1}. \tag{36}$$

Consumption of impatient households is

$$c_{N,t} = \frac{c_t - (1 - \nu) c_{S,t}}{\nu}. \tag{37}$$

From the Euler equation for impatient households,

$$x_t = [v_t - b_t]^+ = \frac{1}{\beta_I} \left(\frac{c_{I,t+1}}{c_{I,t}} \right)^{1/\sigma} - 1 - r_t. \tag{38}$$

Then, from their budget constraint, borrowings of impatient households are:

$$v_t = (1 + r_{t-1}) v_{t-1} + \frac{\rho}{2} x_t^2 - w_{t-1} n_{t-1} + c_{I,t}. \tag{39}$$

The borrowing constraint is

$$b_t = v_t - x_t \tag{40}$$

provided $x_t > 0$ and is not identified (and also irrelevant) if $x_t = 0$. Finally, the implied financial friction is the difference between the rate of return to capital and the real interest rate:

$$f_t = \frac{1}{q_t} \left[\alpha \frac{y_t}{k_t} + (1 - \delta) q_{t+1} \right] - 1 - r_t. \tag{41}$$

This calculation is on the same conceptual footing as the investment wedge in Chari, Kehoe and McGrattan (2007), stated as an interest spread.

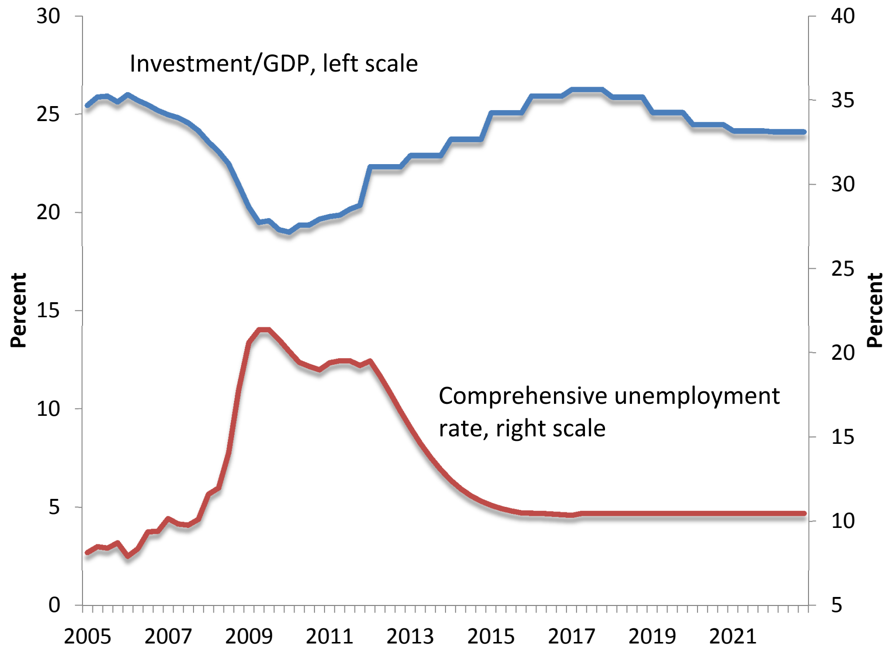

Figure 7: Investment/GDP Ratio and Comprehensive Unemployment Rate, 2005 to 2022

6 Data and Parameters

Figure 7 shows actual values of the two input variables from 2007 through 2011 and projections for 2012 through 2022. The projections are updates of forecasts of the Congressional Budget Office made in January 2012.

Table 1 gives values for the parameters of the model and their sources.

7 Results

7.1 The Implied Values of the Financial Friction and the Borrowing Constraint

Figure 8 shows the values of the financial friction, stated as an annual percent equivalent to a property tax on capital, calculated from equation (41). The friction began at a low value immediately after the crisis, in the first quarter of 2009, rose to a high level in 2012, then is predicted to decline gradually back to normal over the future. Recall that the friction is the difference between the quarterly realized return to capital and the risk-free short-term interest rate. There was an immediate decline in investment after the crisis. Tobin's q began to fall as investment fell, so the return fell at the same time as the short-term interest

Parameter	*Interpretation*	*Value*	*Source*
β	Quarterly utility discount ratio	0.9975	Corresponds to a one-percent annual risk-free interest rate
δ	Quarterly depreciation rate of capital	0.0188	Ratio of depreciation to capital stock, Fixed Asset Tables
α	Elasticity of output with respect to capital	0.354	1 minus labor share of national income, NIPAs
ν	Fraction of consumption in non-saving households	0.5	Survey of Consumer Finances
κ	Capital adjustment cost, quarterly rate	8	Hall (2004)
σ	Intertemporal elasticity of substitution	0.5	Hall (2009)
ϕ_0	Intercept of employment function	0.266	Corresponds to a two-percent annual rate of inflation at normal unemployment
ϕ_1	Negative of slope of employment function	32.2	Estimated from recent data--see text
τ_π	Response of policy interest rate to inflation (Taylor tule)	1.5	Standard value from literature
τ_u	Response of policy interest rate to unemployment (Taylor tule)	0.2405	Corresponds to the standard value of 0.5 applied to conventional unemployment

Table 1: Parameters and Sources

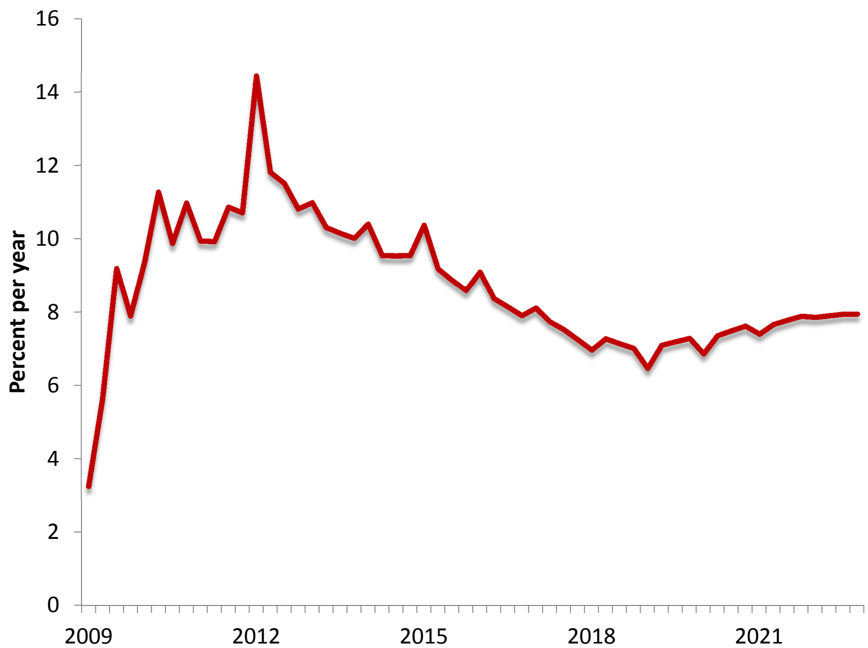

Figure 8: The Implied Values of the Financial Friction

rate, and the gap between them—the measured value of the friction—was small. After q stabilized, the return to capital was closer to normal, but the short-term rate remained low, so the measured friction was high.

Figure 9 shows the estimated amount of household deleveraging, stated as the decrease in the real amount of household debt per person in impatient households as a percent of consumption per person. According to the figure, deleveraging was an important drag on consumption in the first six months of the post-crisis period, in 2009, but not in subsequent quarters, when consumption in non-saver households was able to rise by borrowing permitted by liberalized lending. The calculation forecasts a second episode of deleveraging peaking around 2018. The reason is that the trajectory of the financial friction in Figure 8 implies that the economy, on this account, would go through a boom around 2018 as investment rebounds from an extended slump caused by the friction. The forecast from which I calculated Figure 8 and Figure 9 has no such boom. To explain the absence of the boom, the calculation invokes a period of substantial deleveraging.

The two figures suggest that the financial friction was the leading cause of the depressed levels of output and high levels of unemployment, especially after the middle of 2009. Deleveraging had a large but transitory adverse effect. Most of the lengthy slump results from the financial friction.

Figure 9: Implied Values of the Tightening of the Borrowing Constraint as a Percent of Total Consumption

7.2 Allocating the persistent bulge in unemployment between the financial friction and household deleveraging

Figure 10 shows the path of unemployment attributable to both driving forces—the actual path of unemployment—along with the model's projection of unemployment if only the financial friction had impeded the economy. The difference between the two paths is a measure of the effect of household deleveraging. That effect is a large fraction of the total increase in unemployment in 2009, but declines considerably during 2010 and becomes a fairly small fraction in 2011 and later, corresponding to the decline in deleveraging shown in Figure 9. The effect becomes large again around 2018, for the reason mentioned earlier—according to the model, unemployment would have declined to sub-normal levels between 2014 and 2019. During these years, when the financial fraction had declined recently from high levels, the economy would be rebuilding its capital. The investment boom would depress unemployment as shown by the model solution line in Figure 10. Because the boom does not appear in the CBO's 10-year-ahead forecast for unemployment, the model attributes the lack of the boom to an adverse influence, namely another period of household deleveraging.

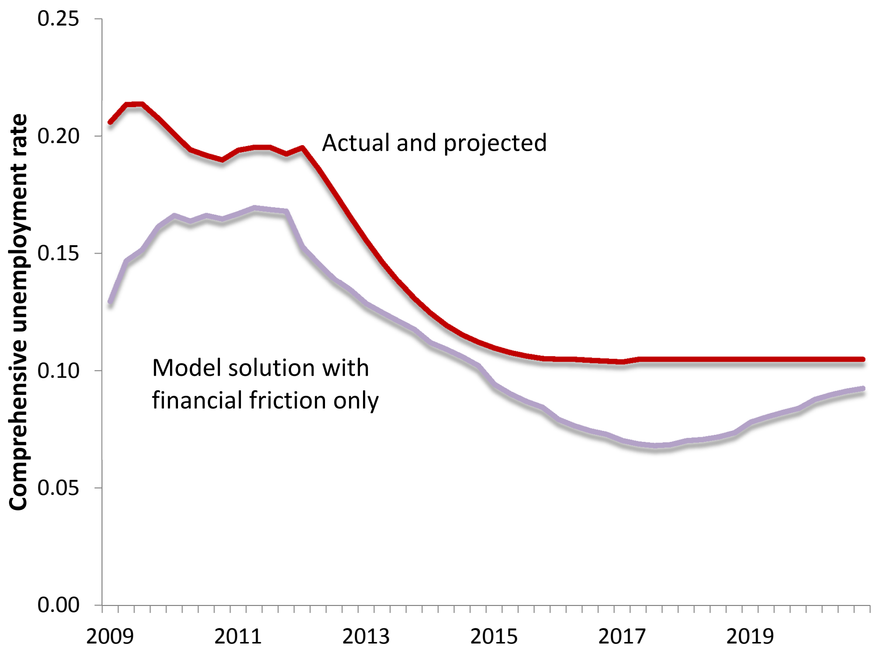

Figure 10: Model Solution with Financial Friction Only

7.3 Direct evidence on deleveraging

The Federal Reserve Board's Flow of Funds data report a large decline in consumer debt—mainly mortgages, car loans, and credit card balances—prior to and after the financial crisis. But the decline in debt outstanding is not a good measure of deleveraging in the sense of cash flows out of households. Debt will decline with defaults, in which case no corresponding cash outflows squeeze consumption. Notwithstanding the name, the Flow of Funds accounts do not report flows of cash out of households—the flow item for consumer debt is literally the first difference in outstanding debt. The same obstacle to measurement of deleveraging, in the sense that I use the term, applies to any research based on loan balances outstanding.

Figure 11 shows a provisional calculation of the deleveraging flow of cash out of households, adjusted for defaults. The measurement of defaults is a challenge. Banks report a concept called *chargeoffs* to account for losses on loans. The amount represents the bank's estimate of the impairment of the value of non-performing loans occurring in a given period. The use of chargeoffs as an offset to loan balance reductions to measure deleveraging is conceptually appealing, because the chargeoff is net of the bank's expected recovery from the sale of the collateral. For example, if a homeowner defaults on a \$120,000 mortgage on a house that sells for \$100,000, the chargeoff is \$20,000. Suppose a new owner buys the house

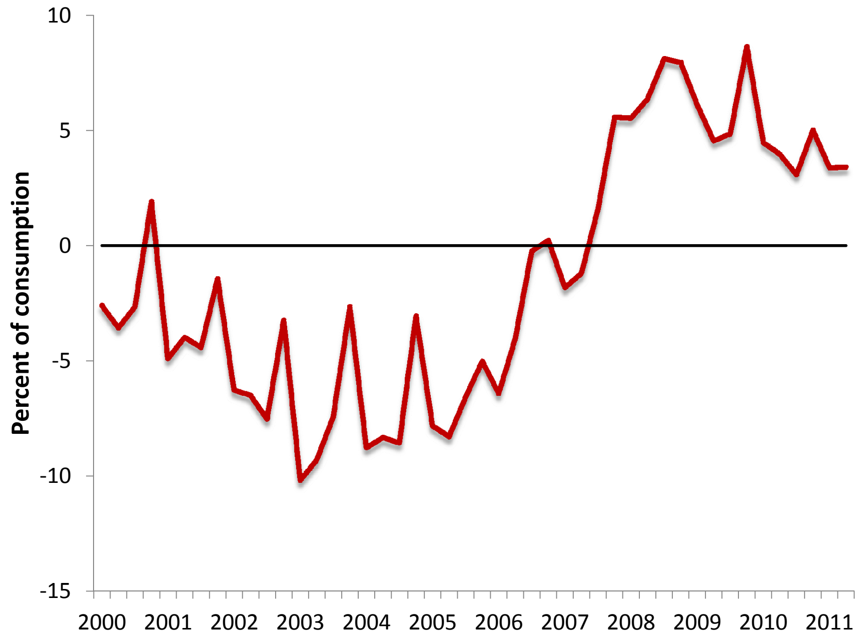

Figure 11: Burden of Deleveraging as a Percent of Consumption

with a no-down-payment loan of $100,000. The household sector has no cash outflow to the financial sector. Outstanding mortgage loans fall by $20,000, the amount of the chargeoff. Subtracting the chargeoff from the decline in outstandings gives the right answer of cash outflow from households of zero. In the case of unsecured credit-card lending, it is immediately apparent that cash outflows to lenders from households is net of chargeoffs.

Relying on banks' estimates of chargeoffs may distort the timing of estimated cash outflows from deleveraging. Saulny (2012) reports that it is common for banks to leave defaulted homeowners in their homes to act as caretakers. Whether banks report full chargeoffs for houses in this situation is not known—the low market value of banks with large mortgage portfolios relative to the book values of those portfolios suggest that there may be lags in updating book values. The book value of a loan declines each time a chargeoff is reported on the loan.

Figure 11 agrees with Figure 9 that there was a spike in cash moving from households to financial institutions around the time of the crisis. But Figure 9 shows deleveraging continuing for several years after the crisis, not subsiding quickly as in Figure 11.

8 Concluding Remarks

A reasonably clear story about the aftermath of the crisis emerges from studying recent U.S. data through the lens of a macro model that embodies the driving forces that have been prominent in non-quantitative discussions of the extended period of high unemployment. Household deleveraging squeezed consumption immediately after the crisis. Rising financial frictions account for most of the persistence of that high unemployment.

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