CONVEX HULL - PARALLEL AND DISTRIBUTED ALGORITHMS

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Overview

1 Ultimate Planar Convex Hull Algorithm

2 Quick Hull Algorithm
Ultimate Planar Convex Hull

- Recursive algorithm employing the divide and conquer approach
- Computes the upper convex hull and lower convex hull
- Divides the space into two halves and finds the edge of upper (lower) convex hull cutting across the half
Ultimate Planar Convex Hull - Sequential and Parallel

Sequential
- $n$ - number of points, $h$ - number of edges in the convex hull
- Recurrence is $f(n, h) = cn + \max_{h_l + h_r = h} \left( f\left(\frac{n}{2}, h_l\right) + f\left(\frac{n}{2}, h_r\right) \right)$
- Upper Hull - $\mathcal{O}(n \log h)$ - Lower Hull
- Overall Work (Worst Case) - $\mathcal{O}(n \log h)$
- Scales with $n$ and $h$

Parallel
- Recurrence is $f(n, h) = c \log^3 n + \max_{h_l + h_r = h} \left( f\left(\frac{n}{2}, h_l\right), f\left(\frac{n}{2}, h_r\right) \right)$
- Overall Depth (Worst Case) - $\mathcal{O}(\log^4 n)$
Ultimate Planar Convex Hull - Distributed

- Not amenable to distributed scenario
- Divide and conquer paradigm - generally not good for distributed systems
- Involves call to a recursive function inside a recursive function
Quick Hull

- Approach similar to QuickSort
- Recursive algorithm - divides the space into subsets of points
- Removes points which doesn’t belong to the convex hull
Quick Hull - Sequential and Parallel

- **Sequential**
  - Each call performs $O(n)$ work and $h$ such calls
  - Overall Work (Worst Case) - $O(nh)$
  - Scales with $n$ and $h$

- **Parallel**
  - Each call performs $O(\log n)$ work and $h$ such calls
  - Not amenable to parallelization - in the $h$ dimension
  - Overall Work (Worst Case) - $O(h \log n)$
  - Scales with $n$ and $h$
Quick Hull - Distributed

- Communication Pattern
  - All to One and One to All - All Reduce

- Communication Cost
  - $m$ - number of machines, $\frac{n}{m}$ - data per machine
  - In each call, $O(m)$ communications
  - $h$ rounds, so $O(mh)$ total communications
  - Scales only with $h$

- Work - $O\left(\frac{n}{m}h\right)$
- Depth - $O\left(\log\left(\frac{n}{m}\right)h\right)$
The End