

## Management Science and Engineering 221 Stochastic Modeling

Mondays and Wednesdays, 2:15 PM–3:30 PM  
Building 380, Room 380C  
3 units

*Instructor:*

Ramesh Johari  
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*Office hours:* Tuesdays, 1:30-3 PM

*E-mail policy:*

If you want to e-mail me regarding the class, make sure you include “221” in your subject line. If you forget to do this, I will almost certainly be delayed in responding to you.

*Course staff:*

Siddhartha Banerjee  
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Rui Fu  
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Apaar Sadhwani  
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*Office hours:* Tuesdays, 4-5:30 PM, location TBA

*Lectures and discussion section:*

Per the Stanford Academic Calendar, there are no lectures on Monday, January 19, or Monday, February 16.

In addition, there is no lecture on Monday, January 2; I will schedule an additional makeup lecture for this one once the class has begun.

Finally, there will be discussion sessions on (some subset of) Fridays from 11 AM - 12:15 PM (room TBA). On Friday, February 6, we will hold a review for the take-home midterm.

*Course webpage:*

Course materials will be available via CourseWork: `coursework.stanford.edu`.

*Course description:*

A more detailed title for this course might be “Stochastic Modeling of Time-Dependent Systems.” We will be focusing almost entirely on dynamic models of random phenomena, and in particular, the most well-studied class of such models: *Markov chains*. The majority of the class will focus on Markov models in discrete time; with this background the development of Markov models in continuous time becomes straightforward. Time permitting, we will conclude with extensions and applications of the basic theory. The tentative list of topics (not necessarily in order of presentation):

1. Markov models in discrete time (10 lectures)
  - (a) Basic definitions, calculating transition probabilities
  - (b) Classification of states; recurrence and transience
  - (c) Absorption probabilities and hitting times
  - (d) Invariant distributions
  - (e) Convergence to equilibrium
  - (f) Strong Markov property
  - (g) Regeneration and LLN for Markov chains
  - (h) Reversibility
  - (i) Simulation
  - (j) Markov chain Monte Carlo and the Gibbs sampler
2. Markov models in continuous time (5 lectures)
  - (a) The exponential distribution and the Poisson process
  - (b) Generators
  - (c) Forward and backward equations
  - (d) The jump chain
  - (e) Analogs of discrete time results
  - (f) Queueing models

### 3. Further directions (3 lectures)

- (a) Financial models
- (b) Algorithmic models
- (c) Branching processes and epidemics
- (d) Hidden Markov models

The focus of the course will be on modeling and applications of the theory. Students seeking a more pure mathematical treatment of the subject should consider taking Statistics 217.

## Grading

You are responsible for keeping up with all announcements made in class and for all changes in the schedule that are posted on the class website.

The grade will be based on the following:

- 20% problem sets
- 40% midterm (take home)
- 40% final exam

## Problem Sets

There will be a total of 4 problem sets. Problem sets will be assigned on Wednesdays and *due two weeks later, no later than 5:00 PM, in the homework drop box in Huang Engineering Center*. All assignments will be posted to the course website. Problem sets are assigned and due as follows:

- Problem Set 1 handed out on January 7, due 21
- Problem Set 2 handed out on January 21, due February 4
- Problem Set 3 handed out on February 11, due February 25
- Problem Set 4 handed out on February 25, due March 11

Depending on their length and difficulty, the total number of points in each set might vary. Each part of each problem will be graded from 1 to 3 points.

You can discuss the assignments among yourselves, but everybody must turn in his/her own written solutions in his/her own words. If you do a substantial subset of the work on your problem set with others, document on each assignment the other students that you worked with.

If you are having difficulty, find help right away— *do not wait until you fall even further behind!* There is an obvious temptation to wait until the day before the due date to do all the work on the problem sets, and I can assure you this approach does not correlate well to success in the class.

*Late assignments will receive no credit; no exceptions will be made.*

Please familiarize yourself with the Stanford Honor Code; violations will be prosecuted to the fullest extent of the (Stanford) law.

## **Exam Policy**

The midterm exam will be a take-home exam. It will be assigned on Monday, February 9, 2014, and due on Wednesday, February 11, 2014.

The final exam will be a three-hour exam, held on Monday, March 16 (as per the registrar's schedule).

Except for medical necessity, there will be no alternate exam dates or times. *You should only register for the class if you are certain you can take the exams on these dates.*

## **Matlab**

One of the most valuable features of Markov chain theory in practice is that it has deep connections to linear algebra and matrix analysis. To numerically explore this connection, at least one assignment in the course will make use of Matlab, a widely used software package for linear algebra (among other things); several other assignments will be easier for you if you have some facility with the software. Matlab is available on the all Stanford UNIX/Linux machines, and a short Matlab tutorial is available online. More details on accessing Matlab on the UNIX/Linux machines can be found here:

[www.stanford.edu/services/sharedcomputing/](http://www.stanford.edu/services/sharedcomputing/)  
[www.stanford.edu/group/farmshare/cgi-bin/wiki/index.php/Matlab-interactive](http://www.stanford.edu/group/farmshare/cgi-bin/wiki/index.php/Matlab-interactive)

A useful means of using X Windows programs such as Matlab remotely on Macs (Apple X11) and PCs (VNC Client) is described here:

[itservices.stanford.edu/service/sharedcomputing/moreX](http://itservices.stanford.edu/service/sharedcomputing/moreX)

*Matlab review:* A Matlab review session will be arranged for Friday January 9; time and location TBA.

## **Prerequisites**

This course is intended for master's students and first year Ph.D. students, and is particularly targeted at students who wish to use Markov chains for applied work in operations and management. The main prerequisite for the course is probability at the level of **MS&E 220**; an acceptable substitute is Stat 116. *Note that this is a hard constraint; we will not be reviewing any of the material from these courses in class.* Students without this prerequisite can only enroll with permission of the instructor. The other primary prerequisite for the class is some familiarity with linear algebra and matrices, at the level of Math 51.

If you have taken probability elsewhere and would like to know if you satisfy the prerequisite, contact Prof. Johari via e-mail prior to registering for the course.

## **Textbook**

You will only be tested on material presented in lectures, and learned through the problem sets. Some of the problems and supplementary material will draw on *Introduction to Probability Models* (Tenth Edition), by Sheldon Ross; this is the recommended textbook for the class, and available through Amazon. Most students find it valuable to have the book for an alternate presentation of the same material seen in lecture. Note that older editions of this book are typically available used at a significant discount; these are generally fine as well.

A few other books to consider, for an alternate perspective (presented in increasing order of difficulty):

- Bertsekas and Tsitsiklis, *Introduction to Probability*. This is an excellent introduction to basic probability, at the advanced undergraduate level. It contains two chapters on random processes and (finite state) Markov chains.
- Norris, *Markov Chains*. This is a slightly more mathematical treatment of the subject, but one of the most clearly presented versions of the material available.
- Durrett, *Essentials of Stochastic Processes*. This book is also more mathematical than Ross' book; it is a good place for an introduction to *martingales* that is not very technical.
- Grimmett and Stirzaker, *Probability and Random Processes*. This book is a comprehensive treatment of basic probability and Markov chains, at a more rigorous pace than the books above.