

# *III. Stabilization of moduli in string theory II*

- A detailed arguments will be given why stabilization of certain moduli is a prerequisite for string cosmology.
- New ideas about stabilization of moduli via instanton corrections to the superpotential will be presented based on counting of the zero modes of Dirac operator on M5 brane with background fluxes and on D3 brane in IIB string theory

# Outline

**Landscape of String Theory : steep and flat  
Friendly/Unfriendly Dilemma**

- **Flux Compactification, Stabilization of Moduli, Metastable de Sitter Space in String Theory**
- **Dirac equation on M5 brane and instanton corrections**
- **The Index of the Dirac operator on D3 brane with background fluxes**



# Can string theory afford runaway moduli: a dilaton and the total volume?

No-scale supergravity has **non-canonical kinetic terms**

$$K = -\ln(-i(\tau - \bar{\tau})) - 3\ln(-i(\rho - \bar{\rho}))$$

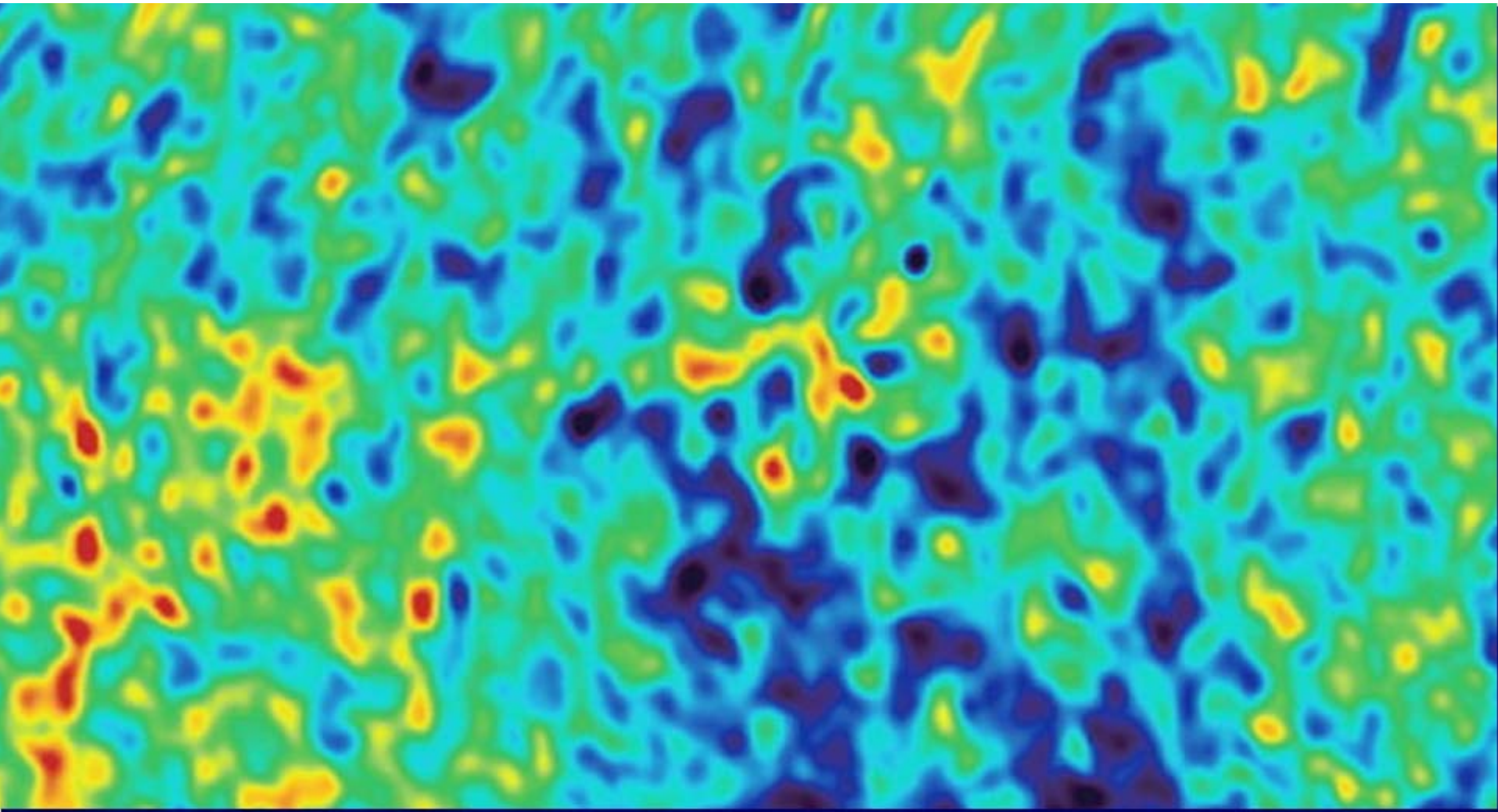
$$V = e^K V_0$$

$$-\frac{\partial\tau\partial\bar{\tau}}{(\tau-\bar{\tau})^2} - 3\frac{\partial\rho\partial\bar{\rho}}{(\rho-\bar{\rho})^2} \quad \tau = a + ie^\phi \quad \rho = \alpha + i\sigma$$

**To compare with observations one should switch to canonical kinetic terms** for the dilaton and the volume, ignore axions for simplicity

$$-\frac{1}{2}(\partial\tilde{\phi})^2 - \frac{1}{2}(\partial\tilde{\sigma})^2 - e^{-\sqrt{2}\tilde{\phi}-\sqrt{6}\tilde{\sigma}}V_0$$

A photographic image of  
quantum fluctuations blown  
up to the size of the universe



# Inflationary slow roll parameters

**in units**  $M_P^2 \equiv 8\pi G_N = 1$

**Primordial slope**  $n_s \equiv 1 - 6\epsilon + 2\eta$

$$\eta[\varphi] = \frac{V''}{V} \sim \text{const} \quad \epsilon[\varphi] = \frac{1}{2} \left( \frac{V'}{V} \right)^2 \sim \text{const}$$

**Derivatives w. r. to canonically normalized fields!**

Observational data

$$n_s = 0.98 \pm 0.02$$

# Compare with observations,

simplest case, 1 scalar field

$$-\frac{1}{2}(\partial\varphi)^2 - e^{-\lambda\varphi}$$

1. For early universe inflation

$$\lambda \leq 10^{-1}$$

2. For dark energy  $\lambda \leq 1$

For the dilaton  $\lambda = \sqrt{2}$

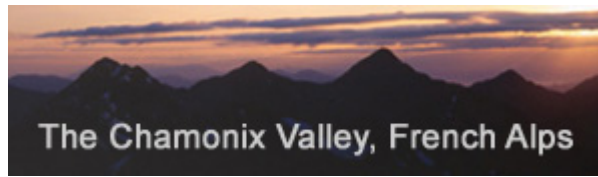
For the total volume  $\lambda = \sqrt{6}$

Both stringy moduli have **very steep potentials incompatible with the data** even for the current acceleration of the universe, particularly the total volume

## The Chamonix Valley, French Alps



*Alpenglow, the Chamonix Aiguilles*



**SLOW-ROLL**



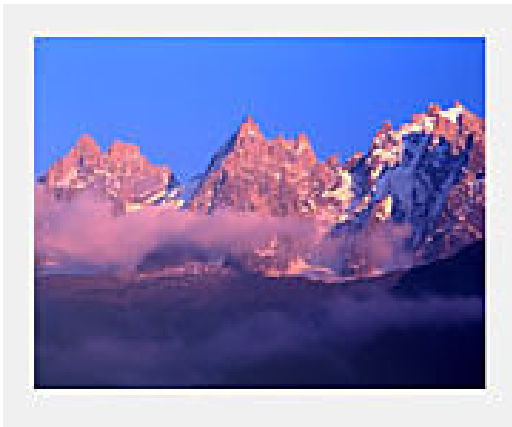
Aiguille du Dru



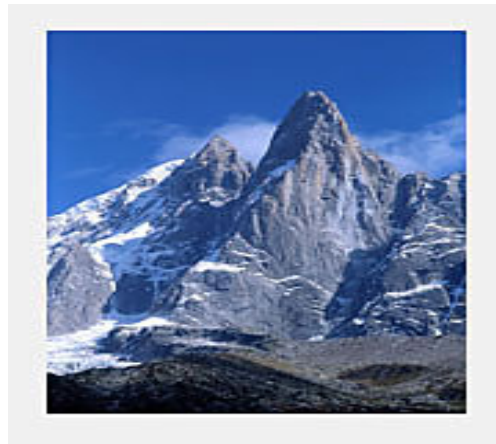
Mt. Dolent, Argentiere Glacier



Aiguille Verte, Afternoon Clouds



Alpenglow, Chamonix Aiguilles



Aiguille du Dru



Aiguille Verte

# Racetrack Inflation

An example of the complex field evolution

Blanco-Pilado, Burgess, Cline, Escoda, Gomes-Reino, Kallosh, Linde, Quevedo

hep-th/0406230

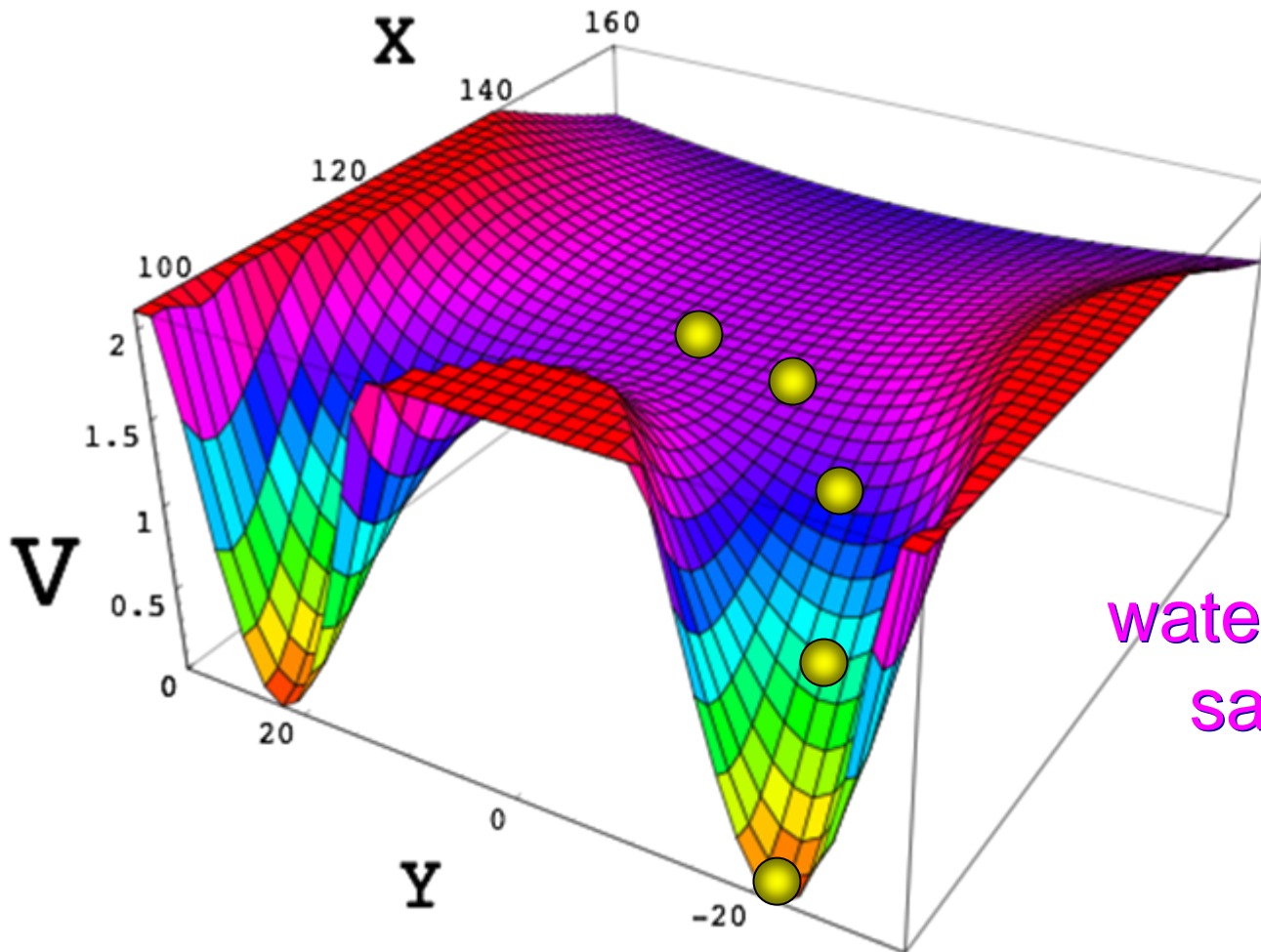
**Superpotential:** 
$$W = W_0 + Ae^{-a\rho} + Be^{-b\rho}$$

**Kahler potential:** 
$$K = -3 \log(\rho + \bar{\rho})$$

Effective potential for a complex field:  
volume of the compact manifold and its axionic partner

**Eternal topological inflation**

# Fine-tuned saddle point



waterfall from the  
saddle point

# **INFLATIONARY PREDICTIONS**

## **for the modular racetrack inflation:**

Flat spectrum of metric perturbations with

$$n_s \sim 0.95$$

Marginally consistent with data, but may  
be ruled out by future observations

Parameters require fine-tuning with accuracy  $O(0.1\%)$ ,  
which may not be a problem if one takes into account

**The string theory landscape**

# String Theory Landscape



# Friendly/Unfriendly Dilemma

Example of most recent studies of

■  $G_2$  holonomy vacua

■ All de Sitter vacua are unstable

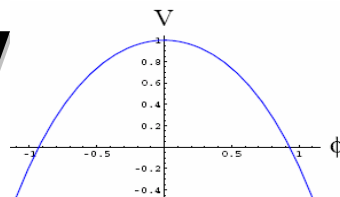
■ All anti de Sitter vacua are unstable

■ Freund-Rubin vacua

■ All de Sitter vacua are unstable

■ Anti de Sitter vacua are unstable mostly but there is a special class of stable adS vacua with negative CC

Potentials mostly unbounded from below: global instability



$\Lambda < 0$

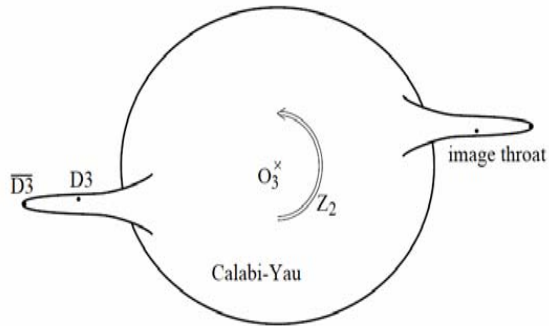
?



$\Lambda > 0$

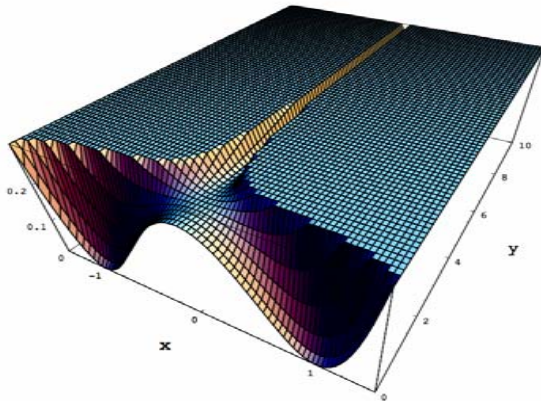
**CAN WE USE THESE NEW VACUA FOR COSMOLOGY?**  
**Can we find slow-roll parameters depending on fluxes?**

# New class of inflationary models in string theory



KKLMMT brane-anti-brane inflation

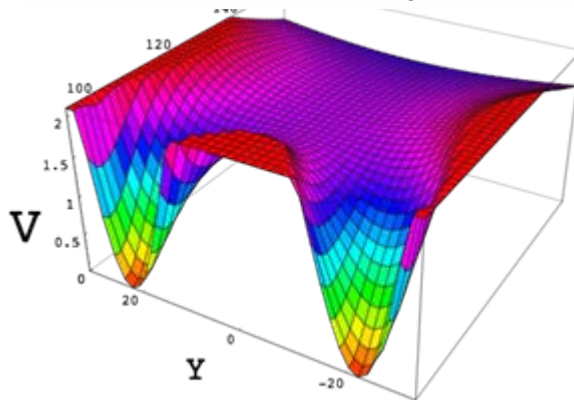
D-term  
inflation



D3/D7 brane  
inflation



Racetrack modular inflation

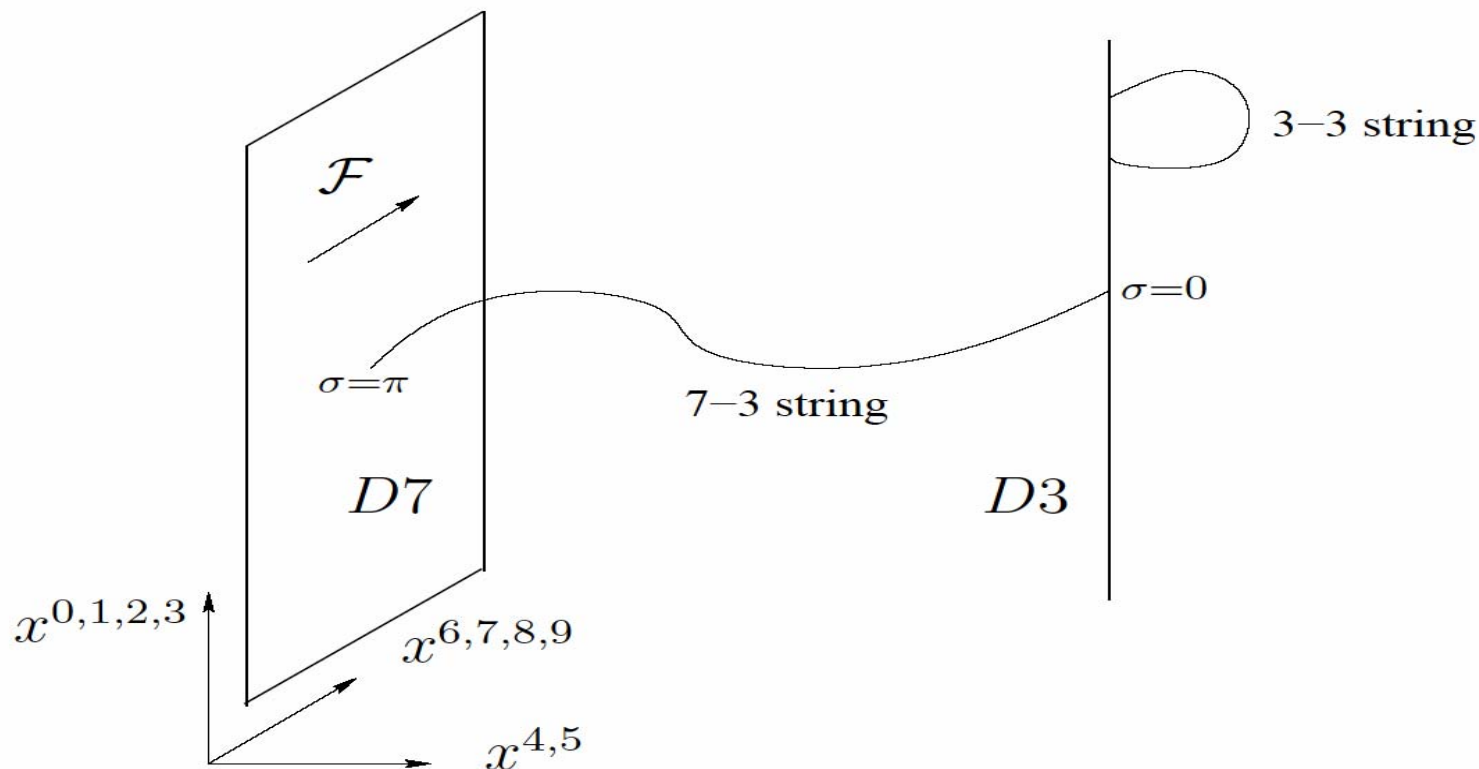


DBI inflation, [Silverstein et al](#)

Grav. Waves  
Non-gaussianity?

**MANY NEW MODELS**

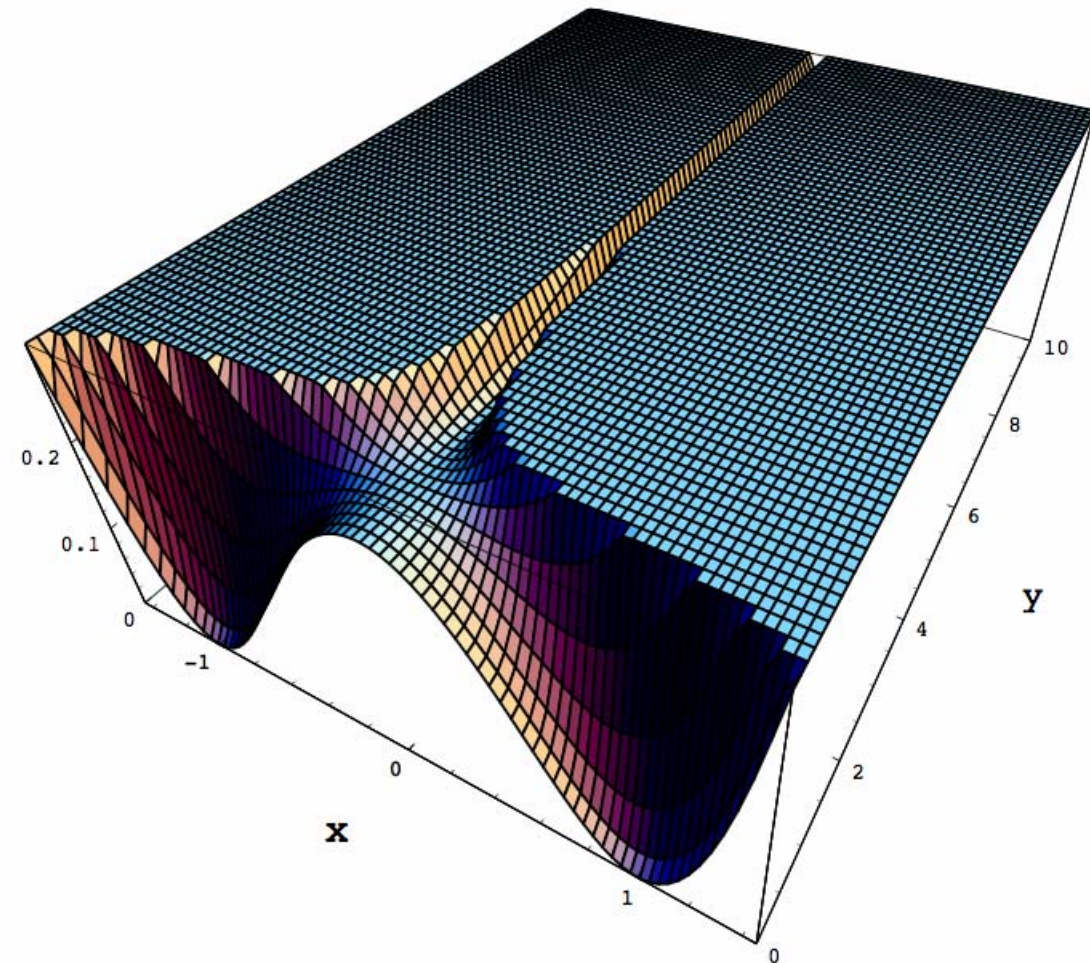
# D3/D7 BRANE INFLATION MODEL



The mass of D3-D7 strings (hypers) is split due to the presence of the anti-self-dual flux on D7

# Hybrid D3/D7 Inflation Model

How to make this model  
valid in string theory  
with the **volume  
stabilization**



**Shift Symmetry?**

# Isometry of the compactified space provides shift symmetry slightly broken by quantum corrections

- Type IIB string theory compactified on

$$K3 \times \frac{T^2}{Z^2}$$

- orientifold with fluxes, mobile D3 branes and heavy D7 brane

$$V = \frac{g^2 \xi^2}{2} \left[ 1 + \frac{g^2}{8\pi^2} \ln \frac{S^2}{S_{cr}^2} \right]$$

# D3/D7 Phenomenology with Stabilized Volume and Inflation

$$V = \frac{g^2 \xi^2}{2} \left[ 1 + \frac{g^2}{8\pi^2} \ln \frac{S^2}{S_{cr}^2} \right]$$

The conditions for successful slow-roll inflation require

$$\xi \sim 1.5 \times 10^{-5}$$

To find other parameters one should use the dictionary between  
brane construction and D-term model

**Dasgupta, Hsu, R.K., Linde, Zagermann**

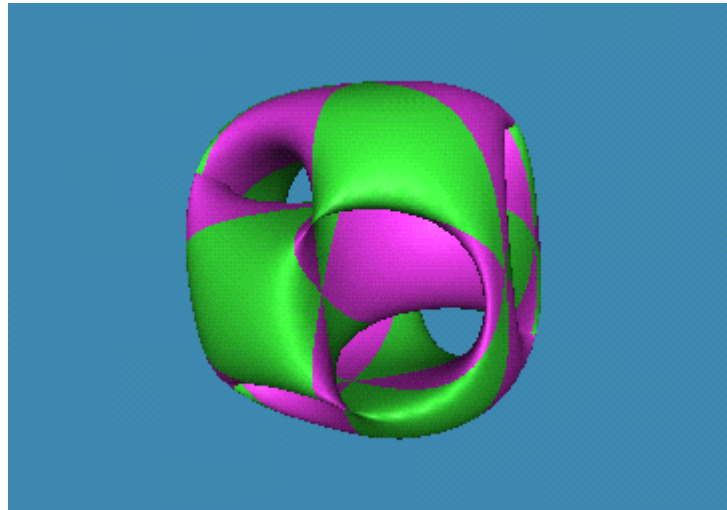
This is possible for quantized fluxes,  
and realistic values of volume and  
string coupling

$$\mathcal{F} \sim 2\pi 10^{-7} \frac{\sigma^3}{g_s} \sim 2\pi$$

$$n_s \sim 0.98$$

# Major problem

The mechanism of volume stabilization in this (and many other models of string theory) does not seem to work.

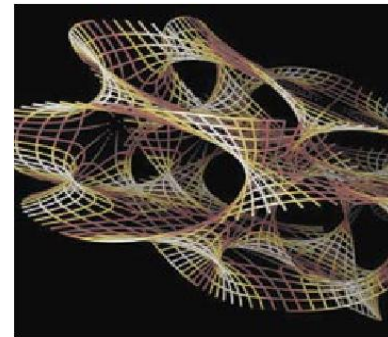


Witten 1996: in type IIB compactifications under certain conditions there can be corrections to the superpotential coming from Euclidean D3 branes. His argument was based on the **M-theory counting of the fermion zero modes in the Dirac operator** on the **M5 brane wrapped on a 6-cycle of a Calabi-Yau four-fold**. He found that **such corrections are possible** only in case that the four-fold admits divisors of arithmetic genus one,

$$\chi_D \equiv \sum (-1)^n h^{(0,n)} = 1$$

In the presence of such instantons, there is a correction to the superpotential which at large volume yields a new term

$$W_{\text{inst}} \sim \exp(-a\rho)$$



In type IIB string theory the leading exponential dependence comes from the action of an Euclidean D3 brane wrapping a 4-cycle.

Until recently, Witten's constraint  $\chi_D = 1$  was a guide for constructing all models in which Kähler moduli are stabilized via superpotentials generated by Euclidean instantons.

- First indication that in presence of fluxes there are more possibilities were given by Gorlich, Kachru, Tripathy and Trivedi: in a case not covered by Witten's analysis, they found examples of generation of non-perturbative superpotentials even in models with branes wrapping divisors of arithmetic genus

$$\chi_D \geq 1$$

- We have performed a systematic study of this issue, (also Saulina)
- Constructed the Dirac operator on M5 brane with background fluxes
- Performed the counting of fermionic zero modes and have found the generalized “index”

# M5 brane

- Dirac action on M5 with background fluxes

$$\Gamma^a \hat{\nabla}_a \theta = 0$$

- Here  $\hat{\nabla}_a$  is a covariant derivative including **torsion when fluxes in the background M theory are present**

$$\Gamma^a (\nabla_a + T_a^{\underline{abcd}} F_{\underline{abcd}}) \theta = 0$$

# New Dirac Equation on the Brane

$$(\tilde{\gamma}^a \nabla_a - \frac{1}{24} \gamma^i \tilde{\gamma}^{abc} F_{abci})\theta = 0$$

- Flux is a 4-form in M-theory. A part relevant to our problem of generation of non-perturbative superpotentials to stabilize the moduli has 3 “legs” on the brane and 1 “leg” in the normal direction
- Witten’s counting of fermionic modes in case  $F=0$

$$\chi_D \equiv \sum (-1)^n h^{(0,n)} = 1$$

- INDEX OF DIRAC OPERATOR : **can flux affect it?**

R. K., Kashani-Poor, Tomasiello

## The equations made explicit in terms of forms

Requiring the preservation of  $N = 1$  supersymmetry imposes the 4-flux is (2,2) and primitive, Becker<sup>2</sup>.

$$D\epsilon_+ = 0, \quad D\epsilon_- = F\epsilon_+$$

The M5 brane is wrapped on a 6-cycle inside the compactifying 4-fold. Following Witten, we identify the bundle  $S$  in which the spinors take values as

$$S = S_+ \otimes (N^{1/2} \oplus N^{-1/2}) = K^{1/2} \oplus (K^{1/2} \otimes \Omega^{0,2}) \otimes (K^{1/2} \oplus K^{-1/2})$$

This is equal to

$$\mathcal{O} \oplus \Omega^{0,2} \oplus K \oplus (\Omega^{0,2} \otimes K).$$

The index thus results to be

$$\chi_D = (-)^p h^{(0,p)}$$

which is called the holomorphic characteristic, or arithmetic genus of the divisor  $D$ .

## **U(1) symmetry in the presence of fluxes and F-chirality: invariance under the structure group of the normal bundle**

The action is invariant and equations of motion covariant under the  $SO(5)$  structure group which is the reflection of the symmetries of 11-dimensional supergravity. The flux  $F_{abci}$  transforms as a vector. In the case of interest the structure group  $SO(5)$  is broken down to  $SO(3) \times SO(2)$ .

In Witten's analysis of instanton generated superpotentials, the  $SO(2) \sim U(1)$  structure group of the normal bundle plays the role of an R-charge for the 3 dimensional effective theory. This R-charge of the superpotential of the 3d theory is related to the number of zero modes of the Dirac equation on the M5 brane. This charge depends on the chirality of the zero modes in the normal direction.

In the presence of flux, the solutions of the Dirac equation no longer have definite chirality in the normal direction, as  $\Gamma$  no longer commutes with the Dirac operator.

$$\Gamma(\not{D} + \Gamma^i F_i) = (\not{D} - \Gamma^i F_i)\Gamma .$$

The action of the  $\mathbb{Z}_2$  subgroup of  $U(1)$  on the Dirac operator in the presence of flux is hence generated by  $\Gamma$  with simultaneous rotation of  $F$  by  $\pi$ , i.e.  $F \rightarrow -F$ . We will refer to this symmetry as F-chirality. F-chirality commutes with the Dirac operator: we can choose a basis of solutions of the Dirac equation with definite charge under F-chirality such that  $\Gamma\epsilon(-F) = \pm\epsilon(F)$ . We refer to the number of solutions to the Dirac equation in the presence of flux weighted by F-chirality as  $\chi_D(F)$ .

## Solving the spinor equations

$$D\epsilon_+ = 0, \quad D\epsilon_- = F\epsilon_+$$

### Ansatz

$$\epsilon_+ = \phi|\Omega\rangle + \phi_{\bar{a}\bar{b}}\Gamma^{\bar{a}\bar{b}}|\Omega\rangle$$

$$\epsilon_- = \phi_{\bar{z}}\Gamma^{\bar{z}}|\Omega\rangle + \phi_{\bar{z}\bar{a}\bar{b}}\Gamma^{\bar{z}\bar{a}\bar{b}}|\Omega\rangle$$

**BRUTE FORCE solution of  
Dirac equation with fluxes**

$$[\partial_{\bar{a}}\phi + 4g^{\bar{b}c}\partial_c\phi_{\bar{b}\bar{a}}]\Gamma^{\bar{a}}|\Omega\rangle = 0$$

$$[\partial_{\bar{a}}^A\phi_{\bar{z}} + 4g^{\bar{b}c}\partial_c^A\phi_{\bar{b}\bar{a}\bar{z}} + 8F_{\bar{a}\bar{z}bc}\phi^{bc}]\Gamma^{\bar{a}\bar{z}}|\Omega\rangle = 0$$

$$[\partial_{\bar{a}}\phi_{\bar{b}\bar{c}}\Gamma^{\bar{a}\bar{b}\bar{c}}]|\Omega\rangle = 0$$

$$[\partial_{\bar{a}}^A\phi_{\bar{b}\bar{c}\bar{z}}]\Gamma^{\bar{a}\bar{b}\bar{c}\bar{z}}|\Omega\rangle .$$

# Solutions of Dirac with fluxes

- Our ansatz reproduced Witten's  $\chi_D = 1$  in absence of fluxes
- ❖ With fluxes present we were able to solve equations and calculate the new index.

# New constraint on zero modes

$$\mathcal{H}(F_{\bar{a}\bar{z}bc}\phi^{bc}dz^{\bar{a}}) = 0$$

- We can interpret this equation as a linear operator  $\mathcal{H}F$  annihilating  $\phi^{bc}$
- For a generic choice of flux the system is of maximal rank, and hence admits no solutions. This kills all of the  $(0, 2)$  forms.

We introduced the projector  $\mathcal{H}$  onto harmonic forms, i.e.  $\mathcal{H}(\omega)$  is a harmonic form (possibly zero) for any form  $\omega$ . This projector gives zero on any exact or co-exact form:

$$\mathcal{H}\bar{\partial}\omega = 0 ; \quad \mathcal{H}\bar{\partial}^\dagger\omega = 0 \quad \forall \omega :$$

exact forms are trivial in cohomology, and co-exact forms are trivial in  $\bar{\partial}^\dagger$  cohomology.

# Counting fermionic zero modes M5 with fluxes

- New computation of the normal bundle U(1) anomaly

$$\chi_D(F) = \chi_D - (h^{(0,2)} - n)$$

- Here  $n$  is the dimension of solutions of the constraint equation which depends on fluxes.
- To have instantons we need  $\chi_D(F) = 1$

$$n = h^{(0,1)} + h^{(0,3)} \quad \chi_D \neq 1$$

# Witten's condition is generalized

- New vacua with  $\chi_D(F) = 1$   $\chi_D \neq 1$   
 $\chi_D = 1$
- it seems the landscape just got another factor  $10^{500}$  bigger
- Examples include “friendly” parts of the landscape



## Example of $K3 \times K3$ compactification

An M5 brane wraps a 6-cycle  $D = K3 \times \mathbb{P}^1$  of the 4-fold  $X = K3 \times K3$ . Since both  $K3$  and  $\mathbb{P}^1$  only have even cohomology, the same is true for the cohomology of the 6-cycle by the Künneth formula. The Dirac equation in the fluxless case hence only has positive chirality solutions

$$\epsilon_+ = \phi|\Omega\rangle + \phi_{\bar{a}\bar{b}}\Gamma^{\bar{a}\bar{b}}|\Omega\rangle.$$

Consider a flux which is a  $(2, 0) + (0, 2)$  form on  $K3$

$$\frac{F}{\pi} = \Omega_1 \wedge \bar{\Omega}_2 + \bar{\Omega}_1 \wedge \Omega_2$$

We need to contract the flux with  $\bar{\Omega}_1$  and project onto the non-harmonic piece. This amounts to contracting  $\Omega_1$  with  $\bar{\Omega}_1$ , which is a number by covariant constancy of the complex structure. We are left with  $\bar{\Omega}_2$ , the harmonic projection of which is itself.  $\bar{\Omega}_1$  hence does not solve the constraint. Thus we lose a zero mode of the Dirac operator, namely  $\phi_{\bar{a}\bar{b}}\Gamma^{\bar{a}\bar{b}}|\Omega\rangle$ , upon turning on flux. In particular,  $\chi_D = 2$ , while  $\chi_D(F) = 1$ .

**Stabilization of Kahler moduli is possible!**

# A Simple Example of Moduli Fixing

Aspinwall, R.K.

We analyze M-theory compactified on  $K3 \times K3$  with fluxes and its F-theory limit, which is dual to an orientifold of the type IIB string on  $K3 \times T^2/Z_2$

**We argue that instanton effects will generically fix all of the moduli.**

Before branes are introduced

**Moduli space is no more**

# The index of the Dirac operator on D3 brane

E. Bergshoeff, R. Kallosh, A. Kashani-Poor,  
D. Sorokin, A. Tomasiello

## Abstract

We study the Dirac equation on a D3 brane in the presence of a background flux. The brane is wrapped on a 4-cycle of the compactifying Calabi–Yau 3-fold/orientifold. The computation of the normal bundle  $U(1)$  anomaly is reduced to the counting of the solutions of a finite-dimensional linear system on cohomology. This system depends on the choice of a flux. We find that some models with  $\chi = 2$  acquire an effective  $\chi = 1$  index with account of fluxes, which makes the instanton corrections possible.

# Dirac equation on D3 with background fluxes

Our starting point is the quadratic Lagrangian for D3 brane fermions (without the DBI field contribution) derived by Marolf, Martucci, Silva and more recently by Martucci, Rosseel, Van den Bleeken, Van Proeyen from an M2 brane Lagrangian by applying a chain of dualities

$$L_f^{D3} = \frac{1}{2} e^{-\phi} \sqrt{-\det g} \bar{\theta} (1 - \Gamma_{D3}) [\Gamma^\alpha \delta\psi_\alpha - \delta\lambda] \theta$$

This form of the action shows that a pull-back to the brane of any solution of the unbroken supersymmetry equations in the background

$$\delta\psi_\alpha \theta = 0, \quad \delta\lambda \theta = 0$$

supplemented with the gauge-fixing condition

$$(1 - \Gamma_{D3})\theta = 0$$

will give us a zero mode of the Dirac operator. However, some other (non-supersymmetric) zero modes may exist for which  $\delta\psi_\alpha \theta \neq 0$  and only  $(\Gamma^\alpha \delta\psi_\alpha - \delta\lambda) \theta = 0$ .

# Orientifolding and the choice of the gauge for fixing $\kappa$ -symmetry

- BASIC EQUATIONS

- COUNTING FERMIONIC ZERO MODES

- Effective  $\chi_D(F) = 1$  for K3 and for  $P^1 \times \frac{T^2}{Z_2}$

- Volume of K3, volume of  $\frac{T^2}{Z_2}$  and other Kahler moduli get stabilizing instanton corrections







Deep valley Lauterbrunnen

