Model studies of laser absorption computed tomography for remote air pollution measurement

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Laser absorption computed tomography offers the possibility of sensitive remote atmospheric measurements of pollutants over kilometer sized areas with 2-D resolution at modest laser source powers. We present detailed model studies which demonstrate the potential of this new remote sensing technique. The tomographic reconstruction process is studied as a function of measurement signal to noise, laser power, range, and system geometry. The analysis shows that the proposed system is capable of providing 2-D maps of pollutant concentration at ranges and resolutions superior to that attainable from contemporary direct detection laser radars.

1. Introduction

The past decade has witnessed a revolution in the field of medical imaging brought about by the application of reconstructive tomography. From its first applications in radiology, the scope of tomographic imaging has steadily increased to the point where its application is an accepted part of medical practice. The awarding of the 1979 Nobel Prize in medicine to Hounsfield and Cormack is testimony to the importance of tomography in contemporary medical practice.\(^1,2\)

In the late 1970s investigators began to explore the possible uses of tomography outside the field of medicine. There have been many efforts to employ tomographic techniques in such disparate areas as phase-flow diagnostics in reactor cooling loops,\(^3\) determination of geological features with acoustic and electromagnetic waves,\(^4\) in laboratory gas concentration measurements,\(^5,6\) and remote air pollution monitoring.\(^7-11\) There is also been interest in applying laser tomography to fluid mechanics problems as an adjunct to hot-wire anemometry, schlieren imaging, and interferometric holography which are currently used in flow visualization.

In this paper we address the problem of applying tomography using laser sources over kilometer ranges to derive 2-D maps of air pollution concentration. It is shown here that the construction of a system that can generate pollution maps with ranges of 6 km, resolution cells of 200 m, and detection sensitivities of fractions of ppm to ppb is within the current state of the art. Moreover, the use of tomography allows measurements to be made at much less laser power than the direct detection laser light radar systems now in use. There is a corresponding signal-to-noise and measurement range advantage using tomography. We also show that it is possible to obtain pollution information with far fewer line integral measurements than is customary in medical practice. This latter finding is important, because it permits the construction of simpler and less costly laser tomographic systems.

In 1978 Byer and Shepp\(^7\) proposed air pollution tomography measurements using an approach shown in Fig. 1. A laser in the center of the imaging circle of radius \(R\) shines a beam toward mirrors that are on poles that are high enough to be above obstacles such as trees or buildings. The mirror \(M_1\) is shaped to provide a fan beam which illuminates the detectors \(D_j\) on the opposite perimeter of the circle. The required number of fans is generated by rotating the beam of the laser in the imaged plane thus illuminating each mirror in turn. The mirror acts as a virtual source which simulates a fan beam similar to those produced in medical CT scanners. In practice each mirror could be optics mounted on a supporting structure or at a higher cost in required transmitted energy, buildings, trees, or available terrain features. The image can be reconstructed by means of conventional algorithms to provide maps of the laser beam attenuation for the species of interest. The virtual source idea greatly reduces implementation costs and increases system reliability because one laser source...
is located at a central transmitting station and not at each mirror site. This paper presents results of a simulation of pulsed and cw virtual source optical tomography.

In Sec. II we briefly survey the state of the art of laser radar (lidar) which is currently used to measure air pollution concentrations. This discussion provides a basis for comparison with the tomographic method. Section III provides a brief introduction to the subject of tomographic image reconstruction. This part is intended primarily for the reader unfamiliar with the reconstruction process. The next section describes the mathematical model used to simulate the remote air pollution tomographic scanner and discusses a few implementation details of the simulator. Section V discusses the results of the simulation study and treats the questions of required resolution, signal-to-noise, required transmitted energies, and fan taper distortion of the image. A final section presents the conclusions of this modeling study.

II. Laser Radar Technology

There are at present two general approaches for obtaining air pollution measurements: in situ and remote sensing. The in situ methods usually utilize monitoring stations at fixed locations that sample the air at periodic intervals. These stations utilize chemical analysis or spectroscopic methods that require integration times of up to 1 hr. Thus, the temporal resolution of the data is low. A network of these stations is expensive to equip and operate so that the spatial resolution of the data is also limited. This approach is the one in predominant use today.

In the early 1970s laser radar was proposed as a remote measurement approach. During the last decade lidar techniques have evolved such that measurements of many pollutant species over ranges of a few kilometers are possible. However, the development of lidar has been slowed due to the requirement for high power tunable laser sources. The lidar consists of a pulsed or cw laser transmitter, beam forming optics, a receiving telescope that focuses the return signal on the detector, postdetection electronics for gain and signal conditioning, and a computer for data processing and system control. The return signal is derived from either scattering or emission from the species of interest as in Raman or fluorescence lidars, or from absorption with either a man-made or natural retroreflector or particulate backscatter as the reflector.

Lidar measurements using Raman scattering have been shown to lack the sensitivity for kilometer measurement ranges except for N₂ and O₂ determination. Fluorescence lidar measurements have proven useful for measuring alkali metal vapors in the stratosphere but are not useful in the troposphere due to quenching at high gas densities. Lidar measurements by absorption are the method of choice for remote measurement of atmospheric pollutants, humidity, and temperature.

Absorption lidars are conveniently divided into two subcategories: long path absorption using cooperative or topographic retroreflectors and DIAL (differential absorption lidars) systems that derive their return from atmospheric particulates which act as a distributed retroreflector.

The methods differ in the way in which energy is scattered back to the receiving telescope. In the long path case, the transmitted pulse sent to the retroreflector is scattered, and a small fraction of the transmitted energy is collected by the receiver. On both the outward and inward legs of the path, the pulse is attenuated by molecular absorption of the gas species of interest. In the distributed retroreflector case the propagation path is divided into spatial resolution cells and range resolved pollution measurements can be made. In contrast, the long path method allows only a path average measurement to be made with no range resolution. As in the long path case, attenuation is due to molecular absorption on both of the pulse propagation paths.

To make range resolved measurements it is necessary to use DIAL. Unfortunately, the use of DIAL requires high transmitted powers. This problem arises because the backscatter coefficient αᵢ is very small in the infrared spectral region where most of the pollutants have suitable absorption lines. Moreover, the energy penalty increases with wavelength due to the inverse variation of backscattering with wavelength so that there is an increasing penalty as one goes further into the infrared. Typical energies required for kilometer range measurements are in the 0.1–1.0-J/pulse range. It is important to note that long path absorption measurements require 2–5 orders of magnitude less transmitted laser power than the DIAL method.

Currently there is a lack of tunable coherent sources that can produce pulse energies >100 mJ over a wide tuning range, and progress in alleviating this situation has been slow. The laser source problem has delayed wide application of the DIAL technique. DIAL measurements have been made to ranges of 3 km with current tunable source technology. DIAL is most useful
in local enforcement situations such as stack monitoring because the necessary equipment can be placed in vehicles for portability.18

Laser absorption tomography can overcome the limited range of the DIAL technique. The process of collecting projection data in an absorption tomography experiment is equivalent to a series of long path absorption measurements. It should be possible to make range resolved measurements with tomography at a significantly reduced transmitted energy. In addition, since the spatial resolution properties of a tomographic measurement are solely dependent on the geometry of the detector–source grid and not the distribution and backscattering efficiency of the aerosols on which the distributed reflector lidars depend, the size of the resolution cells does not increase with increasing wavelength as in the case for a DIAL system operating at a fixed energy.

The potential for significantly increased atmospheric measurement capability and the potential applications of laser tomography in laboratory and industrial applications provide ample motivation for our model studies.

III. Tomographic Inversion Algorithms

Tomographic reconstruction methods are extensively reviewed in the literature.19–25 In this section we consider image reconstruction from projection data that are obtained via optical absorption measurements in a fan beam geometry. If $I_t$ is the intensity in $\text{W/m}^2$ transmitted from the laser, $P_t$ is the power received on the detector and $\alpha_0$ is the attenuation coefficient. The relation between the transmitted intensity and detector power may be written as

$$P_t = \int_{L_1}^{L_2} \int_{u_1}^{u_2} \int_{v_1}^{v_2} I_t(u,v,\lambda) \cdot \exp \left[-\int_0^R \alpha_0(u,v,\lambda,\rho)\,d\rho\right] du\,dv\,d\lambda,$$

(1)

where $R$ is the distance between the laser and a point on the detector surface, $u$ and $v$ are local coordinates on the detector surface transverse to $R$, and $\lambda$ is the wavelength. The quantities $L_1$, $L_2$, $u_1$, $u_2$, $v_1$, $v_2$, and $u_H$ delimit the spatial extent of the detector.

For a monochromatic beam whose transverse variations dominate the picture or convolution–backprojection methods based on Fourier theory. The iterative methods have the advantage of permitting more freedom in the placement of the virtual sources and detectors, but they have the disadvantages of greatly increased computational burden and possible convergence failure in high input noise situations. The convolution–backprojection method requires less computation, is more stable against noise, and converges more rapidly. It has the disadvantage of requiring a more symmetrical geometry in the measurement system. In this paper we consider only the convolution–backprojection approach and leave the possible application of iterative methods for further study.26

The convolution–backprojection algorithms are based on the theoretical work done on the projection reconstruction problem by Radon early in this century.27 The fan beam geometry of interest to us here is shown in Fig. 2. The source of radiation located at $S$ illuminates an array of detectors along the arc on the opposing side of the circle. The function to be reconstructed is in the polar coordinate system and is denoted as $f(r,\phi)$. $\beta$ is the angle of the projection with respect to the fixed $x, y$ coordinate system, while $\gamma$ is the angle of the detector with respect to the center ray of the projection. $\gamma_m$ represents the maximum half-angle of the fan beam.

The distance from $S$ to the pixel at $(r,\phi)$ is

$$L(\beta, r, \phi) = \sqrt{r^2 + 2rR \sin(\beta - \phi)}.$$

(4)

The angular position of the ray which passes through $(r,\phi)$ in the current projection is given by

$$\gamma(\beta, r, \phi) = \tan^{-1} \left( \frac{P_E}{S_E} \right) = \tan^{-1} \left( \frac{r \cos(\beta - \phi)}{\rho + r \sin(\beta - \phi)} \right).$$

(5)

As shown by Herman and Naparstek,28 the image is related to the fan beam projection

$$f(r,\phi) = \int_0^{\gamma_m} \rho \int_{\gamma-m}^{\gamma} \cos\gamma p_\beta(\gamma) \left\{ \left[ \frac{(\gamma' - \gamma)}{\sin(\gamma' - \gamma)} \right]^2 k(\gamma' - \gamma) \right\} d\gamma'.$$

(6)
This integral can be written as a succession of 1-D integrals. The first is the convolution of the modified projection data $p_{ij}$ with a kernel function. The outer integral represents the contribution of each fan in the reconstruction process and is termed the backprojection integral.

Each of these two integrals may be written in various discrete forms depending on what approximations are made in the sampling and interpolation processes necessary in the approximation procedure. Assuming that sufficiently sampled projection data are available, the convolution–backprojection method provides a relatively fast means for the reconstruction of 2-D images of attenuation. Because of its simplicity and speed advantages, the convolution–backprojection algorithm is the method of choice in medical x-ray CT scanner machines.

### IV. Computer Model

The model used in this study is based on the conservation of laser energy propagating from the actual source to the detector and is similar in spirit to the models commonly used for existing lidar systems.

Figure 3 shows the geometry of the tomographic measurement. The laser beam is emitted from a central laser and illuminates the mirror $M_i$, which becomes the virtual source. The illumination may have a Gaussian intensity distribution as shown in the figure. The radiation may also undergo attenuation along the path from the laser to the mirror. Hence we may write the power incident on mirror $M_i$ as

$$P_{in} = Pt \exp \left[ - \int_0^R \alpha_l(r)dr \right]. \quad (7)$$

Here $P_t$ is the laser source power, $R$ is the distance to $M_i$, and $\alpha_l$ is the attenuation coefficient due to pollutants between the laser and the mirror. In a similar fashion we may write the power received at detector $j$ as

$$P_j = \frac{\eta_i A_j D_{ij} P_{in}}{d_{ij}^2} \exp \left[ - \int_0^{d_{ij}} \alpha_j(r)dr \right]. \quad (8)$$

Here $P_j$ is the power received at detector $j$ due to illumination at mirror $i$, $A_j$ is the effective receiving area of detector $j$, $d_{ij}$ is the distance between $M_i$ and the detector $j$, $\alpha_j$ is the total attenuation coefficient between $i$ and $j$, and $\eta_i$ is the fraction of power incident on $M_i$ that is scattered toward the detectors. $\eta_i$ provides a measurement of how much incident radiation is collected for retransmission to the detector array. Each of the mirrors acts as an antenna and as such has a directivity as defined in antenna theory. That is, $D_{ij}$ is the directivity of the virtual source in the direction of the jth detector. The purpose of the directivity is to take the angular dependence of the mirror scattering into account. The total transmitted power $P_t$ is defined as

$$P_t = \int \Phi(\theta, \phi) d\Omega, \quad (9)$$

where $\Phi$ is the radiation intensity in watts/steradian.

The average power radiated per unit solid angle is

$$\Phi_{av} = \frac{P_t}{4\pi}. \quad (10)$$

The directivity is defined as ratio

$$D_{ij}(\theta, \phi) = \frac{\Phi_{ij}(\theta, \phi)}{\Phi_{av}} = \frac{4\pi \Phi_{ij}(\theta, \phi)}{\int \Phi(\theta, \phi) d\Omega}. \quad (11)$$

The directivity in Eq. (11) is the directivity along the direction $i, j$. Substituting Eq. (7) into Eq. (8) the received power at the detector array becomes

$$P_j = \frac{\eta_i A_j D_{ij} P_t}{d_{ij}^2} \exp \left[ - \int_0^R \alpha_l(r)dr - \int_0^{d_{ij}} \alpha_j(r)dr \right]. \quad (12)$$

As before, the attenuation terms include both scattering and absorption losses. The scattering can be estimated separately or normalized by making two measurements with the laser tuned on and off the absorption line. With the latter method, $\alpha_j$ can be treated strictly as an absorption term. Experimentally, the laser tuning may be reset after each scan at each fan position. In either case we combine tomographic scanning with the normalization of on/off tuning.

In both the transit from the laser to the mirror and from the mirror to the detector, the beam encounters attenuation from pollutant clouds. These clouds are modeled as a superposition of elliptical disks, each of which is described by geometric parameters and an attenuation amplitude. For each disk, the distance of the beam enclosed by the disk is first calculated and then multiplied by the attenuation coefficient of the disk. The total line integral in a given direction is the sum of all the line integrals of all the intersected disks in the same direction. In this manner, phantoms of arbitrary complexity may be constructed limited only by execution time and memory constraints. Similar methods have been employed in simulations of medical tomographic applications.

We must also model the directivity of the scattered beam. Since we assume a laser source, we chose to use an intensity distribution typical of such sources. Most lasers used in Lidars are operated in a near Gaussian intensity mode. When the beam hits the mirror the resulting fan beam also has a Gaussian profile in the vertical and horizontal directions. Since it is desirable to have the scattered radiation concentrated in the horizontal plane, it is advantageous to have different mirror radii of curvature in the vertical and horizontal.
directions. In addition, we specify two angles beyond which the laser intensity is truncated to zero. This gives a model beam which is a product of Gaussian functions in two dimensions.

The parameters required to specify the beam are the two angles of truncation and the two half-intensity angles. This model has the advantage that it permits a fast evaluation of the directivity integral as its denominator is the product of two integrals of the form

\[ 2 \int_0^\alpha \exp(-\beta x^2)dx, \]

where \( \alpha \) is the truncation half-angle and \( \beta \) is the half-power angle. In the program, the kernels of the integrals are contained in two functions which are passed as parameters to a subroutine where they are integrated numerically. The directivity in the desired direction may then be calculated from the expression

\[ D_{ij} = \frac{\pi \Phi_h \exp(-\gamma_h x^2)}{\int_0^{\alpha_h} \exp(-\beta_h x^2)dx \cdot \int_0^{\gamma_h} \exp(-\beta_v y^2)dy}, \]

where the \( h \) subscript denotes angles in the image plane and \( v \) denotes the vertical plane orthogonal to the image plane. \( \Phi_h \) is the maximum intensity at the center of the fan and the line between \( M_i \) and the detector \( j \). This approach provides a simple method of computing the appropriate directivity characteristic since it is assumed that each mirror scatters identically. In some of the simulations presented later, we consider Lambertian scatterers as virtual sources. In this case the directivities are equal to 2 over a hemisphere. The case of Lambertian scattering is important in modeling situations where available topographic features are used as virtual sources.

Once the power incident on the detector is known it can be compared with the noise power. We decided to use a simple phenomenological description for detector noise. In this model there are several noise sources that characterize the detectors. They include thermal noise collected by the receiving optics, dark current noise, and shot noise. The dominant terms depend on the physical nature of the detector and its environment. However, in general, the signal-to-noise may be calculated by the expression

\[ S = P_r(4hnpP \cdot \Delta f/\eta + 2\text{NEP}^2\Delta f)^{-1/2}, \]

where \( P_r \) is the received signal power, \( \Delta f \) is the amplifier bandwidth, \( \nu \) is the frequency, \( h \) is Planck's constant, \( \eta \) is the quantum efficiency, and \( \text{NEP} \) is the noise equivalent power of the detector. The factor of 2 arises from the necessity to take account of the noise resulting from two measurements. The first term in the denominator represents signal dependent noise (shot noise), while the second represents the noise in the absence of signal (dark current and thermal background noise) characterized by the noise equivalent power. Usually one or the other of these terms is dominant. In this paper we consider only infrared systems in which the detectors are background-limited and Eq. (15) reduces to

\[ P_n = \text{NEP}(2\Delta f)^{1/2}, \]  

The dark current noise limit usually applies to photoconductors and other detectors used in the infrared region of the spectrum. The NEP is defined by

\[ \text{NEP} = D^*(\Delta f)^{-1/2}, \]

where \( A \) is the area of the detector's photosensitive surface, and \( D^* \) is the detectivity of the detector.

From the above expressions one may compute the variance of the equivalent Gaussian noise source. On the computer, the Gaussian noise source is simulated from an available system uniform random number generator. The process is repeated for each detector in the array thus simulating a series of noisy line integral measurements which are then input to the reconstruction algorithm. For Monte Carlo simulation, an ensemble of noisy pictures may be generated on which statistics may be collected either on a pixel or spatial average basis.

It should be noted that Eq. (15) only applies for high photon count rates. In some laboratory applications where the count rates are low, the Gaussian noise model becomes unsatisfactory and it is necessary to use Poisson or conditional Poisson statistics as is done in x-ray tomography.

We may summarize the model by stating what is required as input. In a given calculation the model requires that the user supply a scaling parameter that establishes the dimensions of the geometric parameters; a set of geometric and attenuation values of the disks comprising the cloud model; the number of desired fan beams; the number of detectors around the circle and two angles describing the horizontal and vertical extent of the fan; a parameter which specifies what fraction of the laser energy incident on the mirror is reflected back as fan energy; two angles specifying the vertical and horizontal half-power points of the laser intensity reflected back from the virtual source; an effective area that describes the collection properties of the receiving optics; a noise equivalent power and bandwidth to describe the background noise properties of the detectors; and a parameter that specifies how many random pictures to generate in Monte Carlo simulations.

The preceding ideas are incorporated into a series of programs that comprise the optical tomography simulator. There is an input program which creates files that are read by the main program which reconstructs the images and collects statistics. The output of the reconstruction program can be fed into either a program which takes cross sections through the data and grey scales the image for later bitmap display or prints the image directly.

The plots shown in the next section were generated by a Pascal program which reads the grey scale coded data and generates a shaded line printer plot using standard ordered dither techniques. Dither matrices allow one to trade the number of grey levels against the number of pixels in the image. This technique is useful with display devices such as plasma displays and line printers that have high spatial resolution but are only
capable of two levels in amplitude resolution. A series of test pattern tomography programs showed that the optimal trade off in grey scale vs pixel resolution for the available line printers occurred on 50 x 50 pixel grids with 10-16 grey levels. All the reconstructions of this paper use these values. With the exception of the Pascal display code generator and an assembly language routine which controls setting the seed of the random number generator, all code is written in FORTRAN for transportability. This permitted the simulator to be run on a DECSYSTEM-20 at Stanford University for picture display runs and at the NASA Ames Research Center’s CDC 7600 for computation-intensive Monte Carlo runs.

V. Computer Simulations and Reconstructions

A. Introduction

We now present the results of our simulation studies. We chose to model tomography on a kilometer scale as it would apply to air pollution measurements. However, the simulation results also provide a guide to tomographic measurements on meter or centimeter scales which are of interest in combustion diagnostics or flow visualization applications. In Sec. B we give examples of how the simulator can be used to study reconstruction accuracy requirements for a class of images. The simulations show how the global reconstruction accuracy varies as a function of the number of fans used in the reconstruction.

The virtual source geometry gives rise to new possibilities for reconstruction artifacts. The first of these arises from the attenuation of the laser beam on its way to the virtual source mirror. We refer to this artifact as depletion distortion. The second artifact arises from a combination of variations in the laser intensity profiles, range variations of the rays of a given fan, and directivity and apodization effects of the virtual source mirrors. This second artifact source we call directivity distortion because it appears as a nonuniform weighting of the rays of a given fan beam. In Sec. C we briefly discuss depletion distortion but concentrate on directivity distortion as it is more significant in practice. We also suggest corrective measurements for both problems.

Section D treats a simulation of the imaging of a small pollution cloud in detector noise. This simultaneously demonstrates the resolution potential and provides an estimate of the pollution concentrations that can be imaged by virtual source laser tomography.

Section E presents results of Monte Carlo simulations of four different tomographic situations but with reconstructions of the same object. A cw and a pulsed laser source are simulated, each with a directive fan beam and Lambertian scattering virtual source. These simulations compare the input laser power required to obtain a given picture averaged signal-to-noise ratio. This type of simulation permits one to make judgments of the relative advantages and disadvantages of the systems simulated in the light of available laser source technology.

Section F is a similar Monte Carlo study of a system with uniform errors in the line integral measurements such as might be caused by laser peak-to-peak power variations.

We conclude in Sec. G with two reconstructions of a more complicated pollution cloud to illustrate the overall imaging capability of the virtual source concept.

B. Convergence of Approximate Phantoms

First we investigate how the degree of approximation of an artifact affects the reconstruction accuracy. In each of three cases, no noise was added to the projection data. In the first case, a single disk was taken as an approximation to a Gaussian function whose first standard deviation radius was taken to be equal to the radius of the approximating disk. In the second trial, the same Gaussian was approximated by a superposition of thirteen disks. A twenty-three disk approximation was used for the final trial. \( \gamma = 114^\circ \) with twenty detectors/fan were used in the simulation. The results of the simulation are shown in Fig. 4. The relative reconstruction error is calculated by comparing the reconstructed image with the input phantom on a pixel by pixel basis and then summing the squared error at all pixels and normalizing by the number of pixels.

Figure 4 shows that all three reconstruction error curves level off at \(<200\) fans. The curves for both the thirteen- and twenty-three-disk cases taper off at \(<50\) fans and twenty detectors/fan. This suggests that for images with low spatial frequency content, such as these Gaussian functions, acceptable image quality is attainable with \(<100\) fans. This conclusion has important implications on the cost of implementing laser tomography systems for a wide variety of applications in which the expected images are well approximated by Gaussians.

Figure 4 also shows that a single disk function as an approximation to a Gaussian cloud provides a conservative bound on the error since the single disk error is greater than that of the multidisk approximations. As a result, we may use simple phantoms of only a few disks for computation intensive runs. Similar simulations with objects that are nearly circularly symmetric show...
that one may often reconstruct such objects with an accuracy of a few percent with 10 fans or less. The reduced number of fans required for the reconstruction of such objects implies faster data taking and reconstruction times and the use of significantly simplified and less costly collection geometries.

C. Probe Depletion and Beam Tapering Artifacts

The geometric configuration of the virtual source scanner introduces the possibility of two new kinds of artifacts. The first arises because the probe beam may be selectively depleted on its way from the laser to the virtual source. The magnitude of this error on picture reconstructions is highly picture dependent and difficult to characterize in a useful manner. Some simulations were done for small clouds, and it was found that probe depletion error was not significant for these localized clouds until the beam was attenuated by values of $e^{-10}$ or greater. In practice this value represents attenuation that is so strong that the system cannot reconstruct the image due to inadequate SNR. This finding is in accord with those of Shepp and Stein who studied missing fan artifacts in the medical context. They found that the error induced by a few missing fans out of several hundred was small. Since a probe depletion error is the same as a fan that is weak or missing, the lack of sensitivity to probe depletion artifact in the case of localized clouds is anticipated. As long as there is sufficient SNR on the laser virtual source path, it is possible to make power measurements at both ends of the path so that a correction factor for the fan can be estimated. This would permit compensation of the projection data. The distortion induced by depletion error was found to be at least an order of magnitude below that caused by fan tapering error for all problems studied in this work.

A more serious artifact arises from the different weights associated with laser beam intensity profile, virtual source mirror directivity effects, and unequal path lengths of rays of a given fan. This directivity distortion artifact is more important than probe depletion errors because it occurs in all fans of the scan. Simulation runs were made to demonstrate the directivity distortion artifact. An elliptical cloud with axes of 70 and 140 m was reconstructed with the tapering factors of the fan turned on and off. The peak attenuation coefficient of the cloud was 0.5 km$^{-1}$. The diameter of the tomographic scanner was 2 km and there were 160 detectors arranged in the scanning circle. In this simulation the beam illumination truncation angle was chosen equal to the half-power angle so that the fan illumination at the ray extremes would be half of that at the maximum. This image plane angle was chosen to be 112° so that fifty detectors were illuminated by each fan. With this arrangement some of the same detectors were used by adjacent fans of the scan. In the first set of runs the beam taper was adjusted so that the illumination as seen by the detector array was uniform. In the second series of reconstructions the fan beam was tapered as described above.

The pixel plots for these two cases are shown in Figs. 5(a) and (b). A comparison of these two plots reveals an asymmetric distortion introduced as a result of the fan beam error. While this type of error is introduced in all views, the relationship between the taper and the cloud changes with each fan and thus the distortion is generally asymmetric. The reduction in the star artifacting is due to the different grey scale peak pulse amplitude relationship between the two plots. Figures 6(a) and (b) show cross sections taken through the long semiaxis of the ellipse with the artifact present and not present. The peak attenuation values of the clouds are 5.5 and 40 km$^{-1}$, respectively.
We conclude that it is important to know the intensity characteristics of the laser, the mirror directivities, and the ray ranges so that the measurements may be corrected for ray tapering errors.

D. Detection Sensitivity

The purpose of this section is to find the minimum gas concentration that can be measured by the tomographic scanner. The approach is to keep all system parameters fixed except the amplitude of a very small cloud which is simulated by a disk phantom. The amplitude of the disk is lowered until it is deemed that the disk is no longer visible in the detector noise. In these reconstructions it is assumed that background noise in the detector is the dominant statistical instability in the system.

In this series of runs a small cloud, 33 m in diameter, was imaged by a tomographic scanner with the same illumination taper and geometry as that in the preceding section. The size of the cloud was chosen to satisfy the Nyquist sampling criterion in the projection data. As in the preceding section the effective reflectivity of the virtual source is 50% with a 10-cm collecting mirror.

In this case we use a 50-W cw laser operated at a 10.6-μm wavelength. These values are typical for an inexpensive CO₂ laser. We also assume that the detectors have a moderate sensitivity with a NEP of 10⁻¹¹ W/Hz¹/₂ and a bandwidth of 10 kHz.

The cw measurement is similar in concept to a pulsed measurement except the pulse width of the laser as seen by the detector is determined by the illumination time of the virtual source. By a simple geometric argument the width of the pulse is

$$t_p = W t_s / 2 \pi R,$$

where \( t_s \) is the time required for a complete scan of the laser, \( W \) is the angle subtended for the virtual source mirror as seen from the position of the laser source, and \( R \) is the distance between the laser and the mirror. In the noise driven simulations in this paper we set the scan rate so that the specified bandwidth of the detector just resolves the pulse. We require that \( t_p = 2 / B \), where \( B \) is the detector bandwidth. If we assume the values \( R = 1 \) km, \( B = 10^4 \) Hz, and \( W = 10 \) cm, we get a total scan time of 12.6 sec which is well within the capabilities of a mechanical beam rotating device.

The image reconstruction for a peak cloud attenuation amplitude of 10.0 km⁻¹ is shown in Fig. 7(a). Examination of other members of the ensemble for this situation shows that the image defects in the reconstruction are due to deterministic errors intrinsic to the inversion process and not noise fluctuations. Thus we are in a high signal-to-noise regime. Figure 7(b) shows a reconstruction taken for a cloud amplitude of 0.005 km⁻¹. Many points in the reconstruction begin to approach the cloud in intensity but it is still possible to see the cloud. At an amplitude of 0.0025 km⁻¹ shown in Fig. 7(c), it is quite difficult to separate the cloud from the background noise. The output signal-to-noise varies between 1 and 10 over the picture, while the average signal-to-noise of the input projections is ~100. At this point we are on the borderline of useful imaging.

The amplitude of the cloud is the product of \( N \sigma \), the number density of the absorbing gas, and the absorption cross section. The exponential factor for the case shown in Fig. 7(c) is the cloud amplitude times the cloud width \( L \). Its value for the case shown in Fig. 7(c) is 0.008. In the infrared, the molecular absorption cross sections vary between \( 10^{-17} \) cm² for fundamental transitions and \( 10^{-21} \) cm² for overtone transitions. This implies that the range of minimum detectable concentrations for the above system should lie between 0.2 ppb and 2 ppm. Thus tomographic imaging is very sensitive which is an important feature in a number of potential applications.

E. Average Signal-to-Noise Ratios

In this section we discuss the spatial average signal-to-noise ratios obtained for four different imaging configurations. Our purpose is to determine the input power required to produce a given picture SNR. These simulations allow us to assess the realizability of the laser tomographic system with currently available optics, lasers, and detectors. In each case the goal was to
see if each system could reconstruct a 100-m 10-km\(^{-1}\) disk cloud in the same 2-km diam scanning geometry used in the previous section but with seventy and twenty detectors/fan. In the first case, we consider a cw laser system operating at 10.6 \(\mu\)m with a directive fan beam reflected from a mirror which acts as the virtual source. The second case is also a cw 10.6-\(\mu\)m system except the virtual source is a Lambertian reflector which reflects the incident laser energy into 2\(\pi\) sr. The directivity for such a beam is 2. The first case has a directivity of 7900 so that a comparison of the two results shows the advantage of using a directive fan beam. For both cw laser source cases a detector of modest sensitivity is chosen with a NEP of 10\(^{-11}\) W/Hz\(^{1/2}\).

The third and fourth cases consider pulsed laser sources with directive virtual sources and Lambertian scatterers. The directivities are the same as those used in the cw case mentioned above. The primary motivation is to see whether existing objects such as buildings, trees, and hillsides could act as virtual sources. If such objects were available, the implementation cost of the tomographic system could be reduced.

In each of the four cases the simulation was run in Monte Carlo mode with 100 observations for each value of the laser output power. Batch means were used to estimate the pixel and picture wide means and variances used in the SNR calculations. The data were plotted against both the laser output power and the fan average input SNR at the detectors. The purpose here was to compare the spatial average SNR of the image with the signal-to-noise value of 100 which is accepted as a criterion for good imaging. We now look at the average signal-to-noise performance for four laser systems.

The power levels for the cw runs were chosen to be those easily obtainable by cw CO\(_2\) lasers operating in the 9–10-\(\mu\)m region. The pulse power levels are typical of a Nd:YAG pumped tunable source such as a LiNbO\(_3\) parametric oscillator system operating at 4 \(\mu\)m. We assume in each case that we are imaging a 1-km diam area by means of a virtual source detector array arranged on the circumference of a 2-km diam circle. These results may be scaled to other conditions by using Eq. (19) as long as proper caution is observed in interpreting the parameters. In short, these runs were designed to determine if it is possible to image with cw or pulsed lasers in a typical situation and if it is feasible to image with inexpensive retroreflectors and receiver/detector optics.

The results for the fan beam cw laser are plotted in Fig. 8(a). About 3 W of laser power is required to achieve the adequate signal-to-noise ratio. Since there are CO\(_2\) lasers currently on the market which can generate 50–100 W of cw power, we conclude that imaging over the distances of 1–2 km is possible with this system. A similar run was made with the power and directivity each increased by a factor of 10 with the same cloud. In these conditions it is possible to image a 10-km diam circle within a 20-km diam detector circle with an input power of 30 W for a spatial SNR in the image of 100. This is clearly the order of distance required for metropolitan air pollution monitoring applications. It should be pointed out that this is about the most we can expect by increasing the directivity of the beam in the vertical direction because atmospheric turbulence limits the attainable beam divergence angle to values >0.1 mrad.

Figure 8(b) shows the SNR for the Lambertian virtual source case. At no place in the data is the SNR greater than 10. We conclude that it is not possible to image over the stated distance using a Lambertian source with a 30–50-W cw infrared source. However, simulations of this type show that it is possible to image over much
improvement factor lying between $10^5$ and $10^8$ compared to a distributed particulate retroreflector lidar or DIAL lidar measurement.

An additional improvement in directivity can be obtained by using more complicated virtual sources that direct radiation to one detector at a time rather than one fan at a time. For a very concentrated beam the directivity $D \approx 4\pi/\theta_1\theta_2$, where $\theta_1$ and $\theta_2$ are the half-power beamwidths in the horizontal and vertical directions. As previously mentioned, atmospheric turbulence limits beam divergence angles to 0.1 mrad for the beamwidth so that $D = 1.26 \times 10^9$. This represents an improvement of the order of $10^5$ over the fan beam tomographic system modeled above.

F. Systematic Random Variations

The preceding runs were predicted on the assumption that all statistical uncertainties were due to background noise. This neglects such problems as laser amplitude variations as well as reflector and detector variations. Laser amplitude pulse variations in particular are a problem due to the poor pulse stability of many lasers.

smaller distances such as those useful in combustion diagnostics with cw laser sources and Lambertian scattering virtual sources.

Figure 9(a) shows SNR variation with input power for a pulsed laser with fan beam virtual source. Note that we obtain a SNR of $>10^4$ for input peak powers $>10$ kW. Since laser sources are available that can produce peak powers of 10 MW, it is apparent from the standpoint of background noise alone that there is no problem in using such a system for tomographic imaging for 10-km diam image circles.

Figure 9(b) shows that with a pulsed source with 1-MW peak power and Lambertian scattering it is possible to obtain sufficient signal for successful imaging. This indicates that it is possible to utilize topographic targets as virtual sources at a potential savings in optical component and setup costs. In contrast with previous lidar methods, laser tomography concentrates the imaging radiation so that it is utilized efficiently. In the cases discussed in this section, tomography provides an improvement of the order of $10^3$ over a remote retroreflector lidar system. As determined by the discussion in Sec. II, the tomography approach offers an
In theory, this problem can be dealt with by either building more stable laser sources or taking several scans and averaging them.

In this section we simulate random fluctuations in the line integral measurement due to pulse amplitude variations. Here we simulate such a fluctuation by adding a uniform noise sample to each line integral measurement. As an example of this type of study we reconstructed a circular disk cloud 100 m across and an amplitude of 10 km⁻¹ with the same collection geometry as in Sec. E. In this case the taper in the fan beam was omitted, reflecting the assumption of uniform apodization at the detector array.

Uniform random variations were added to the projection data to simulate the desired fluctuations. The fluctuation variances were scaled as a fraction of the power incident on the array to facilitate the computation of SNRs. The abscissa in Fig. 10 represents values of the reconstruction error at the pixel at the center of the phantom while the ordinate specifies the input SNR of the line integrals averaged over all the rays. This SNR was adjusted during the simulation by varying the output power of the laser. Figure 10 shows that the error begins to rise for SNRs <80. This indicates that, for convolution–backprojection algorithms of the type studied here, the systematic input fluctuations should be held to a few percent to maintain reconstruction accuracy.

In practice, laser fluctuation noise will dominate noise in line integral measurements arising from detector, background, and scintillation. In the case of scintillation recent work has shown that previous theoretical estimates of the magnitude of atmospheric scintillation were overly pessimistic. This finding is supported by many experimental lidar measurements where meaningful measurement accuracies were obtained without correcting for scintillation effects. In practice, it is possible to nullify the effects of scintillation by double pulsing the laser transmitter. Thus power fluctuations due to scintillation are not considered a serious limitation in the laser tomographic measurement.

Fortunately, there are several methods of reducing the effects of random fluctuations. First, it is possible to build laser sources with fluctuations of <2%. Second, it is possible to take several scans and average the results, thus smoothing the input data which are then applied to the reconstruction algorithm. Fortunately, the requirement for real-time data acquisition for air pollution monitoring is minimal, a complete picture every few minutes is adequate in practice. Since a complete scan can be taken in a few seconds, many scans may be averaged to obtain a complete picture thus relaxing other system variance requirements.

The convolution–backprojection algorithm is stable with respect to changes in the assumed input noise distribution for equal projection signal-to-noise ratios. One might suspect, from the weak dependence of the output statistics on the input noise distribution that the requirement for the maintenance of reconstruction accuracy stated above, that the input signal-to-noise of the projections >100:1 provides a design rule that is valid for a wide variety of input noises. All the noise simulations that have been run to date support this rule for the convolution–backprojection algorithms used in this study.

G. Imaging of Multicloud Phantoms

The previous simulations have used simple monocloud phantoms. The model is capable of handling a more complicated class of images, however. In this section an example of a reconstruction of a more complicated multicloud phantom is presented. Here we consider three clouds which are approximated by a superposition of nine disks. A 6.30-km-diam system images an area of 3.15-km diam as before with uniform illumination across the detector array. A 500-W cw laser is used as the primary energy source illuminating detectors with a NEP of 10⁻¹¹ W/Hz¹/₂ and a bandwidth of 10 kHz. Note that each pixel diagonal is ~60 m across. The first cloud appears in the first quadrant as a superposition of two disks and is intended to show intermediate resolution capabilities. Each of these clouds is circular with a 315-m diam and a 1.05-km⁻¹ absorption amplitude. The second cloud is composed of two small partially overlapping circular disks with 100-m diam and 1.6-km⁻¹ peak absorption amplitudes. This could be a model for a highly localized pollution source. The small cloud serves as a test of the resolution of the reconstruction. The final artifact is composed of four 2.05-km⁻¹ 315-m diam clouds that are partially overlapped. This configuration was designed to show the type of cloud that would be formed from several point sources in close proximity. This behavior would be typical of emissions from several smokestacks combining into an area source.

In Fig. 11(a) we see a reconstruction for 20 fans. In this image we have assumed that pump depletion and beam taper error are compensated but a 2% projection input noise remains. Note that the star artifacts are clearly visible near the edges of the image. These could lead to the erroneous conclusion that they are real pollution clouds.
In Fig. 11(b) the same phantom is reconstructed with a 100-fan sampling density. The higher rotational sampling rate has the effect of reducing the amplitude of the star artifacts to the point where they are barely noticeable. In addition, the reconstruction accuracy of the clouds is enhanced as is particularly evident in the intermediate sized cloud. In both reconstructions a seventy-element sampling array is assumed. This is sufficient to produce resolution of the order of 180 m out of a resolution circle diameter of 3.15 km. Such spatial resolution over this range is quite impressive in comparison with the DIAL lidar measurement approach.

We see that it is possible to obtain spatial resolution that is more than sufficient at the relatively low cost of seventy line integral measurements rather than the thousands of such measurements that are used in medical practice. This finding is important because of the reduced system complexity that it implies. In medical tomography it is possible to reuse the detector array in each fan of the scan. In the virtual source scheme this is not possible because the detector array is fixed. If the resolution requirements of pollution monitoring were equal to those of medical practice, of the order of $10^4$ rays, the construction costs and low reliability of such a system would be prohibitive. We have seen that reconstruction with sufficient resolution and sensitivity can be obtained with a few hundred line integral measurements which results in lower costs, reduced system complexity, and less computer reconstruction time than would otherwise be the case.

VI. Conclusion

In conclusion we summarize the important points learned from this investigation. First we note that air pollution monitoring requires much lower spatial resolution than medical applications. In medicine, the primary problem is usually one of detecting small tumors situated in complex organ structures and hence fine resolution is emphasized. In the pollution monitoring application, the primary demand is for grid spacings commensurate with the capabilities of micrometeorological models. Since spacings of hundreds of meters over ranges of tens of kilometers are more than sufficient for this purpose, we have seen that arrays of one hundred detectors or so provide sufficient resolution for this application. Current air pollution monitoring grids consist of fixed stations spaced kilometers apart. A coarse tomographic grid of the kind discussed here would bring a tenfold increase in resolution capability. Such a spacing would provide more useful data that would support microclimatological models.

In the case where a nearly circularly symmetric object is imaged, acceptable image quality may be obtained with fewer than ten views. This conclusion is independent of whether virtual source or conventional tomography is employed. This implies that tomographic imaging of flames and nozzle flow situations may be done successfully with relatively few detector arrays.\textsuperscript{40,41}

Another important advantage of laser tomography is that measurements can be made over larger ranges or with a large signal-to-noise advantage over current techniques. We have demonstrated that ranges of up to 20 km are possible with currently available commercial lasers, detectors, and optics, depending on atmospheric burden. Such ranges are beyond the capabilities of direct detection DIAL systems operating at acceptable transmitted laser energies. Even better performance is expected if virtual sources are used that focus all the incident radiation into one ray at a time rather than one fan at a time. We have shown that it is possible to do virtual source tomography with pulsed lasers.
sources by using available topographic features in place of the virtual source mirrors. It is worth noting that this kind of signal-to-noise performance indicates an advantage of applying laser tomographic techniques to laboratory sized problems as well.

We have noted that the performance of distributed reflector lidar systems decreases with increasing wavelength due to the reduced scattering efficiency. Laser tomography has a definite advantage because the resolution attained by the system is dependent solely on the number and arrangement of the projection measurements. If the system utilizes several sources in order to measure many different pollutants, all the measurements can be made with the same spatial resolution permitting the construction of pollution maps with the same resolution for all monitored species. This is not the case for DIAL measurements due to the need to use larger resolution cells with increasing wavelength.

The simulations point out the possibility of image distortion from two types of measurement error. The first type of error occurs because the pollution cloud may intercept the beam on its way from the laser source to the virtual source. This distortion may be minimized by using a sufficient number of fans and measuring the laser power at the virtual source site so that a correction may be applied. The second source of error arises from the transverse variations in the laser beam intensity. This type of distortion may also be remedied by one of two schemes. One could conceive of designing the beam forming optics to provide uniform illumination of the detector field. This approach would probably be difficult to design and keep in alignment. The second technique involves tuning the laser off the absorption line as in a regular DIAL measurement and measuring the taper directly at the detector array so that the values may compensate in the computer prior to reconstruction. This method has the advantage of simplicity because it eases the design constraints on the virtual source optics.

The signal-to-noise simulations indicate that, for the ranges discussed here, background noise is much less important than other random variations in system parameters. As in the DIAL case, the effects of these random disturbances can be minimized by signal averaging and proper system design. Experience from these simulations also points out the need to restrict the fluctuations in the input projections to <5% in order to maintain reconstruction accuracy. For the usual convolution–backprojection algorithms the problem of fluctuations and taper errors suggests that the implementation of a practical system will be largely concerned with the elimination of these fluctuations.

The advantages of laser tomography do not imply that tomography will completely replace DIAL lidar measurements. DIAL has the advantage that it is comparatively compact and can be placed in vehicles such as vans and aircraft. Its short range capabilities are not a handicap in local applications as stack monitoring. In fact, DIAL complements wide area monitoring provided by optical computed tomography. For example, a laser tomography system could provide a local control board with information for area inventories while mobile DIAL systems could be sent to problem areas on a short time basis.

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References


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