The impact of Einstein’s theory of special relativity on particle accelerators

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Abstract

We describe the consequences of the theory of special relativity on particle accelerators and present a historical overview of their evolution and contributions to science and the present limitations of existing accelerator technology. We report recent results of our experiment where we succeeded in accelerating relativistic electrons with visible light in vacuum. The experimental demonstration is the first of its kind and is the proof of principle for future linear laser-driven particle acceleration schemes in vacuum that may lead to the realization of electron–positron colliders beyond the TeV scale.

Introduction

Einstein’s initial motivation for postulating the theory of special relativity was to explain the observed and, at the time, puzzling behaviour of electromagnetic phenomena under transformation of moving coordinate systems. But he succeeded in formulating his theory in a far more general manner and thus revolutionized the entire body of physics. As a consequence, he modified the basic classical laws of mechanics formulated by Newton and for the first time equated mass to energy.

The predictions made by this new theory are counterintuitive to our natural thinking of space and time and are at first difficult to accept as real physical phenomena. It is difficult to abandon the so-appealing concept of Galilean relativity and the simple addition of velocities. However, one century since its inception numerous physical phenomena are known to us that can only be explained by Einstein’s theory.

One very striking example of special relativity is the observed kinematics of accelerating particles in particle accelerators. Einstein already predicted that accelerating electrons gain arbitrarily large kinetic energies as they approach the speed of light, which he argued they can never reach [1]. Thus, once a particle’s speed is close to c, its kinetic energy increases by its apparent increase of mass and not by a change in speed. In terms of its total energy the
The particle’s velocity is

\[ \beta = \sqrt{1 - \frac{1}{\gamma^2}} \]  

(1)

where \( \beta \) is the velocity of the particle normalized to the speed of light \( c \) and \( \gamma \) is the energy of the particle normalized to its rest mass energy. For example, the speed of a 1 MeV electron is \( \sim 86\% \) \( c \), 10 MeV electrons correspond to \( v \sim 99.8\% \) \( c \), and for a 50 GeV electron \( v \sim 99.99999995\% \) \( c \). This shows that from the laboratory frame the speed of an electron at a few MeV is almost equal to the speed of an electron at GeV or TeV kinetic energies. Clearly, the electrons very quickly approach \( c \) and for practical purposes are almost travelling at \( c \) but never quite reach it.

**History of particle accelerators**

Particle accelerators are primarily employed as sources of high-energy particles for collision experiments that have helped reveal the structure of matter. The challenge is to provide a continuous force on the particle beam that brings it up to the desired energy. The first particle accelerators employed static electric fields [2] and were limited by high-voltage breakdown to a few MeV. The next generation of accelerators overcame the static voltage breakdown limit by employing alternating current accelerating fields. They came in two flavours: cyclotrons and linear drift tubes. A cyclotron consists of two opposing semicircular D-shaped cavities that support an alternating electric field at the gap between them [3]. This alternating potential at the gap is responsible for the acceleration of a particle beam inside the cyclotron. A strong uniform magnetic field forces a particle beam into a circular orbit the radius of which grows as the particle beam gains energy with each passage through the gap. At low particle energies the period of the circular orbit is constant but, as predicted by Einstein, the observed mass of the particle changes with increasing energy, which results in a de-phasing of the particle from the acceleration field in the cyclotron gap. Hence, the cyclotron is a particle accelerator that is not scalable to arbitrarily large particle energies. In comparison, linear drift tubes keep the particles in a linear orbit and have the length of the accelerator sections matched to the increasing speed of the accelerating particle (the Wideröe [4] and the Alvarez drift tube [5]). These first linear accelerators worked at frequencies in the MHz range and would have required forbiddingly large accelerator sections (of the order of \( \lambda/2 \sim 100 \) m) to accelerate relativistic particles travelling at velocities close to \( c \). This type of accelerator was suitable for the acceleration of heavy ions but unsuitable for imparting energy to particles like electrons travelling at relativistic speeds.

Microwave technology came to the rescue. In 1939 the Varian brothers invented the Klystron [6], a very powerful source of microwaves and in 1947 Ginzton and Hansen invented a travelling microwave drift tube [7] that could use powerful microwaves for accelerating relativistic particles travelling very close to \( c \). In essence, the particles are ‘surfing’ a travelling microwave whose phase and spatial profile was determined by the geometry of the waveguide structure. This design was innovative in that the electric field of the microwave is moving with the particles and hence delivering a continuous force for as long as it stays in phase with the particles. By simple cascading of individual accelerator sections arbitrarily high particle energies can be reached. Improvements in Klystron technology, accelerator structure design and in material science to fabricate structures capable of surviving the very powerful microwave fields have allowed the first 6 MeV, 1 m long accelerator from 1947 to evolve to a 3 km long 50 GeV linear accelerator in a matter of a few decades. The evolution of the linear accelerator is shown in figure 1, with W W Hansen and his team at Stanford University holding their 1 m long accelerator tube nicknamed the ‘Mark I’ [8].
In essence, an exponential growth of the particle’s energy has been observed in accelerators over the years [9], a trend that was first recognized by Livingston [10]. The trend observed by Livingston is general and also applies to proton accelerators. As can be observed in figure 2, each particular technology matures and eventually reaches a limit, and it is the introduction of new accelerator technologies that has allowed for the observed steady near-exponential growth in particle energy. The reasons for these limits are numerous and depend on the particular technology, and for existing particle accelerators employing microwave-based technology it is their sheer size, electricity consumption and operating costs. A comprehensive, more in-depth recent review on particle accelerators and detectors has been published by Panofsky and Breidenbach [11].

A TeV electron collider is a highly desirable tool for exploring the frontier of physics. Proposals for such a 1 TeV $e^+e^-$ linear collider based on existing RF-based accelerator technology call for a 30–40 km long structure that will consume more than 200 MW of electricity and will cost several billion dollars to construct [12]. A TeV facility of this kind can only be conceived as a global international effort, such as the proposed International Linear Collider (ILC) [13] and even then it is not clear if it will make it past the myriad of hurdles and become a reality.

**What about ring accelerators to produce 1 TeV $e^+e^-$?**

One of the advantages of ring accelerators like FERMILAB Tevatron or Large Hadron Collider at CERN compared to linear accelerators is that the particle beam can travel hundreds of thousands of times through the ring as it gains kinetic energy. However, basic electromagnetic theory predicts that accelerated charges produce electromagnetic radiation, known as synchrotron radiation, and hence lose a fraction of their initial kinetic energy. For
relativistic particles accelerating in a straight orbit this is not a serious effect since their speed increase is minimal as they gain further energy and approach $c$. However, particles in a circular orbit experience an acceleration that increases as they acquire larger kinetic energies. In a circular accelerator that keeps the particle at a constant energy the energy lost to synchrotron radiation in one turn scales as $U_{\text{loss}} = \gamma^4/R$ where $\gamma$ is the time dilatation constant that is also the ratio of the particles' energy to its rest mass energy $\gamma = E/m$ and $R$ is the radius of the circular accelerator. For the 14 TeV protons in the 4.2 km radius LHC ring at CERN the power lost to synchrotron radiation per turn is about 60 eV, and thus completely insignificant compared to the 14 TeV beam energy. In comparison, for 1 TeV electrons stored in such a ring the energy lost per turn would be about 20 TeV. Even in a ring the size of the earth the energy loss per turn would be a considerable 13 GeV. Clearly, there is no choice for TeV electron accelerators but to keep the particles in a linear orbit.

Although not suitable for acceleration above 100 GeV, circular electron accelerators have found many practical uses. Synchrotron radiation is emitted in a well-defined radiation cone with an aperture angle inversely proportional to the time dilatation constant $\gamma$, which at a few GeV of electron-beam energy is on the order of $10^4$. This is once again a direct consequence of Einstein’s theory of special relativity. Specialized synchrotron facilities around the world take advantage of this effect and are dedicated sources of very bright and collimated x-rays.
The impact of special relativity on laser-driven particle accelerators that have allowed for profound discoveries in the basic sciences and have also found important applications for industry. Science examples include the structure determination of proteins like the human DNA repair protein AGT to better understand the mechanics of DNA replication and the occurrence of mutations [14], or the structure determination of disease agents like neurotoxins [15], anthrax or the influenza virus [16]. Industrial applications include x-ray fluorescence techniques to analyse trace contamination on the surface of silicon wafers [17] and structural studies of industrial catalysts [18]. Figure 3 shows two examples of structures that have been determined with the help of x-ray diffraction from synchrotron radiation.

Furthermore, under certain conditions synchrotron radiation can be produced as a coherent light beam. Such devices are known as free-electron lasers that until recently have existed as the only useful sources of coherent far-infrared and THz radiation. FELs are becoming increasingly appealing as sources for coherent x-ray radiation and two separate hard x-ray FEL facilities are planned for construction. These are the x-ray FEL at DESY [19] and the Linac Coherent Light Source (LCLS) at SLAC [20]. These facilities will use a 10–20 GeV electron beam and produce coherent hard x-ray pulses of unprecedented brightness and pulse durations orders of magnitude shorter than conventional synchrotron radiation that will allow for the study of matter at spatial and time scales not accessible before.

Why do we want 1 TeV electron beams in the first place?

From the very beginning experiments involving particle collisions have been extremely valuable for understanding the nature of matter. In 1911, Rutherford discovered the nucleus of the atom [21] and rendered the previous atomic model of Thompson obsolete. His experiment involved simple bombardment of a thin gold foil with alpha particles from a radioactive source and interpreting the observed scattering pattern. Although experiments of this nature were initially performed with natural sources of high-energy particles, very soon particle accelerators became an ideal substitute for these natural sources that allowed for brighter, more collimated beams of controllable energy. In 1953 Hofstadter employed Mark III, an improved 400 MeV, 64 m version of Hansen’s initial 1 m 6 MeV accelerator, to uncover the inner structure of the nucleus [22] by interpreting the scattering of the high-energy electrons from the nuclei. The next step was the construction of the 3 km SLAC accelerator that allowed for scattering experiments with 10 GeV electrons incident on a fixed target. The scattering experiments helped uncover the inner structure of nucleons and observe their quark structure [23]. In 1983, the carriers of the electroweak force (the W and Z bosons) were observed experimentally at CERN [24]. At present, important experiments include careful studies of CP violation to gain a better understanding of matter–antimatter asymmetry. In the near future the large hadron collider will be commissioned at CERN to perform a dedicated search for the so-far illusive Higgs particle by colliding two proton beams with energy of 14 TeV. The drawback of employing high-energy protons in the Large Hadron Collider experiments is the variety of unwanted side reactions that obscure the sought events. However, electrons and positrons, being elementary particles, provide for ‘cleaner’ collisions and thus a 1 TeV electron–positron accelerator is an attractive tool for exploring physics beyond the standard model [25].

Advanced accelerator technologies

A variety of new accelerator technologies have been under investigation for a number of years. The main goal is to impart a larger amount of kinetic energy to the particle in a shorter distance of travel or, as it is referred to in the accelerator community, ‘to increase the energy gradient
of the particle accelerator. This is achieved by increasing the accelerating force, that is, the electromagnetic field acting on the particle.

For microwave disc-loaded structure accelerators larger radio frequency (RF) fields lead to increased gradients, but eventually the structure fails due to a variety of possible breakdown mechanisms caused by the excessively large electromagnetic fields. The accelerator community has gradually increased the breakdown limit by developing materials, surface treatment methods and careful choice of geometries that avoid localized high fields (like sharp edges or rough material surfaces). But even after decades of such improvements, the maximum reliable energy gradients that these structures can sustain is on the order of 50 MeV m\(^{-1}\) for room temperature accelerators [26] and about 25 MeV m\(^{-1}\) for superconducting accelerators [27]. RF accelerator technology has reached a limit and to increase the energy of particle colliders new accelerator technologies will have to emerge.

The advent of relatively inexpensive and reliable high peak power lasers in the past decade has prompted the particle accelerator community to explore the potential for high-power laser sources for future laser-driven particle accelerators. Several different laser-driven acceleration technologies have been explored and demonstrated so far. Each of these new laser acceleration technologies presents advantages for special applications. For example, laser-driven wake plasma field acceleration has shown very large gradients [28] and inverse-FEL (IFEL) acceleration has demonstrated controlled, multi-stage acceleration of relativistic electrons in the few MeV range [29]. These new technologies could be suitable for the injector and initial acceleration stages of a future high-energy collider. However, for a scalable, extended accelerator they present some intrinsic drawbacks; in a plasma-based accelerator the high-energy beam will inevitably suffer from scattering and beam degradation due to collisions with the atoms or ions in the plasma if such an accelerator is to be employed for an extended distance. The IFEL accelerator forces the particles into a lateral motion and hence suffers from severe synchrotron radiation losses once the beam reaches multiple GeVs of kinetic energy.

**The Laser–Electron Accelerator Project**

In the same spirit as Ginzton’s original microwave-based disc-loaded structure, we have sought a laser–baser acceleration technology that is truly scalable. That is, it will work for any energy once the particle is relativistic, and does not suffer from the energy or scattering or synchrotron radiation problems found at high particle energies. In 1995, our group investigated a dielectric-based vacuum accelerator structure that employs crossed Gaussian laser beams to synthesize a longitudinal electric field inside the vacuum space of the accelerator structure that is suitable for particle acceleration [30]. This conceptual accelerator structure was predicted to sustain 1 GeV m\(^{-1}\) accelerator gradients by taking advantage of the very high damage threshold of dielectric materials under ultra-short near-infrared laser pulses. One kilometre of such a structure could deliver a 1 TeV electron beam and easily fit in an existing high-energy collider facility.

The promise of particle acceleration by visible light motivated us to initiate the Laser–Electron Accelerator Project (LEAP), where we sought to demonstrate the acceleration of electrons from a single interaction with a linearly polarized laser beam in vacuum [31]. The LEAP experiment was carried out at the linear accelerator in the Hansen Experimental Physics Laboratory (HEPL) facility at Stanford University. The set-up of the LEAP experiment and a drawing of the first accelerator cell are shown in figure 4(a). Also shown in figure 4(b) is a photograph of the HEPL-based superconducting accelerator.

In our proof-of-principle experiment the electrons interact with the laser field in the vacuum space of the accelerator cell. The optical phase velocity of the laser field in vacuum
is greater than the relativistic electron beam, and therefore there can only be a nonzero energy transfer if the laser–electron interaction distance is finite. This is known as the Lawson–Woodward theorem [32]. To limit the interaction distance to a length where the electron beam stays in phase with the laser beam we employ an accelerator cell with reflective walls that act as a boundary for the laser field. Figure 5 illustrates the physics for particle acceleration from
crossed laser beams in vacuum. The electron bunch is on the order of 2 ps of duration and thus the individual electrons in the 2 ps bunch have a random phase to the optical field and have an equal probability of experiencing an accelerating or a decelerating laser field. This causes the electron bunch to increase its energy spread, which is recorded with a high-resolution energy spectrometer located downstream of the accelerator cell.

In our initial effort, we sought to observe the laser acceleration effect in a dielectric-based accelerator cell, and for several years we worked with a fused silica-based accelerator cell structure having a 1 mm accelerator gap with very narrow 10 µm wide entrance and exit slits. In the experiment, we encountered numerous challenges, the most difficult being to find the spatial and temporal overlap between a very low brightness ps electron bunch and the laser pulse in a very noisy environment. Over the years, we developed a number of diagnostics located in the vicinity of the accelerator cell that allowed us to achieve spatial overlap and to reduce the temporal uncertainty between the laser and the electron beam to within 100 ps [33]. However, the very tight alignment tolerances for the proper functioning of the accelerator cell and its low laser damage threshold prevented us from applying laser pulses intense enough to produce a signal that was above the noise.

We realized that laser damage of the fused silica accelerator cell was a hard limit that would prevent us from observing a signal in the noisy environment, and this prompted us to rethink our experiment from scratch. Instead of two laser beams we decided to employ a single laser beam that is truncated by a very thin, disposable boundary. We elected to use a gold-coated kapton tape. This geometry eliminated many components and had much broader alignment tolerances for the electron and the laser beams, and furthermore it allowed us to exceed the damage threshold limit. By continuously moving the tape we could apply all the laser power at our disposal to maximize and find the electric-field-induced acceleration that had been so elusive. Each laser shot would see a new tape surface and for the duration of the laser pulse the high reflector surface remained in place, in spite of the laser fluence being above the damage threshold. Immediately upstream of the tape we placed a very compact IFEL that was set to have almost equal alignment conditions between the laser and the electron beam. This IFEL, also the first of its kind for visible wavelengths, showed a very strong modulation signal that allowed us to drastically narrow the timing overlap uncertainty to within a few ps. Indeed, immediately after finding the correct timing between the laser and the electron with...
the IFEL, and ensuring spatial overlap of the two beams at the tape surface, we succeeded in observing an unequivocal laser-induced energy modulation of the electron beam due to linear laser-driven acceleration in vacuum. Figure 6 illustrates the improved laser–electron interaction scheme that we recently employed and enabled us to observe the acceleration of electrons with visible light. Both the laser and the electron pulses had a 2 ps pulse duration. To find the temporal overlap between the two pulses the laser beam was scanned in time in sub-ps increments and was randomly toggled on and off. Thus, at the correct temporal overlap an energy width increase of the laser-on data was clearly visible. Ironically, we simplified the experiment to a set-up that employs a single laser beam proposed by Pantell two decades earlier [34].

The main results for the proof-of-principle experiments are shown in figure 7. The upper left-hand graph shows a series of observed energy profiles of the electron beam recorded at the spectrometer with the laser-on and the laser-off at the condition of temporal overlap between the laser and the electron beam. The upper right-hand graph shows the full-width half-maximum values of the energy profiles of the electron beam as a function of the laser time. The red dots correspond to the laser ‘on’ data and the blue dots to the laser ‘off’ data. Due to timing jitter and slow timing drifts of the different components in the facility our data were collected in the form of these laser time scans. Each laser time scan allowed us to find the maximum energy modulation as a function of the experimental parameters that we kept fixed during the scan. By taking a sequence of scans with a varying parameter enabled us to study the laser energy modulation effect. The graph on the lower left corner of figure 7 illustrates the dependence of the energy modulation effect on the laser polarization angle and the lower right-hand graph shows the average energy modulation strength versus the laser peak electric filed. The data are in good agreement with the expected cosine dependence of the laser polarization angle and the linear dependence of the electric field strength represented by the solid green curves that are fits to the data.

Furthermore, we took laser time scans with the absence of the tape boundary and as expected from the Lawson–Woodward theorem described earlier observed no energy modulation. This also confirmed that our data were not contaminated by a possible effect from the IFEL located upstream of the tape. The graphs in figure 7 are a preliminary analysis...
of the data\textsuperscript{1,2}. Still, they show that the energy modulation effect scales linearly with the electric field of the laser, that it is sensitive to the laser polarization as predicted by theory \textsuperscript{35} and that it requires a boundary that terminates the laser field, also as predicted from theory.

**Future directions**

Our next objective in the near future is to demonstrate a net acceleration by optically bunching the electron beam before it enters the laser accelerator. We plan to accomplish this by using an upstream IFEL that modulates the electron beam and bunches it as the beam drifts downstream into the second accelerator cell \textsuperscript{36}. This scheme has been employed successfully in particle acceleration with two IFELs \textsuperscript{37}. Also, multiple-cell accelerator structures are presently under investigation. Interesting candidates are photonic fibre or photonic bandgap structures

\textsuperscript{1} A definitive analysis of the data presented in figures 7(c) and (d) is in preparation.

\textsuperscript{2} A formal discussion of the residual lateral deflection forces from the single laser beam and the residual energy broadening from the tape is in preparation.
[38] that could serve as waveguides for the optical laser pulse, in much the same way as the existing disc-loaded structures do with microwaves.

Similar to what occurred with klystrons, lasers suitable for particle accelerators will have to be developed. Of special importance will be the ability to phase-lock mode-locked lasers to within one degree of optical phase angle and this will involve newly developed technology for laser comb stabilization [39, 40] and sub-fs laser pulse stabilization as well as interferometric optical path correction methods.

Laser-driven particle acceleration is a young research field, and as can be appreciated, it faces enormous laser engineering, material science and other technological challenges. Nonetheless the fruits of the progress in this field will not only lead to improve future generation particle accelerators but will almost certainly benefit other fields of science and engineering.

Conclusions

Einstein’s theory of special relativity has a profound impact on many aspects of particle accelerators, from the kinematics of high-energy particles travelling close to the velocity of light to time dilatation effects that can be exploited to store short-lived particles.

One century after Einstein’s theory of relativity and the birth of quantum mechanics, physics finds itself again at a major crossroads. Mysteries such as the nature of dark matter, the connection between gravity and quantum mechanics, or the possibility of further spacetime dimensions remain unanswered today and present the new frontier of science for the 21st century. The new generation of proposed future TeV particle accelerators will provide tools to address a number of these questions and may lead to the discovery of new phenomena and have an impact of equal or greater magnitude to the body of physics as the breakthroughs made by Einstein and his contemporaries 100 years ago.

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