Values that Occasion and Guide Mathematics in the Family[1]

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Summary

In the spirit of deepening our understanding of the social conditions of everyday uses of mathematics, we studied twenty diverse families with a middle school child by interviewing family members together at home about their occasions of mathematics use. Beyond determining that family life is a math-rich learning context, in which mathematics functions as a part of many different situated activity systems (such as home improvement, games, hobbies, sports, shopping, cooking, finance), we observed that identifiable values were both generative of occasions for mathematics use and guiding in the direction of their conduct involving mathematics. We identify in these family stories the recurring value systems (each expressing a dominant *telos)*, describe how the functional role of values in their mathematical activities differs from school math, and discuss the implications of such "value gaps" for mathematics education.

Author Biographies

Roy Pea is Professor of Education and Learning Sciences at Stanford University. His research and writing often examines how innovations in computing and communications technologies can influence learning, thinking, collaboration, and educational systems. He was co-editor of *Video Research in the Learning Sciences* (2007), and co-author of the 2000 National Academy's *How People Learn*. His current work is developing the DIVER paradigm for everyday networked video interactions for learning and collaboration, and examining how informal and formal learning can be better understood and connected, as Co-PI of the LIFE Center funded by the NSF. Other current research includes studies on informal math learning in families, the influence of point of view on video-supported learning and collaboration, large-scale video collaboratories for studying effective teaching, and precollege mobile science inquiry and learning.

Lee Martin is an assistant professor in the School of Education at the University of California, Davis. His research looks across contexts to study how people use tools and resources to help them be adaptive thinkers in the realm of quantitative reasoning. His recent publications include “The Tanda: A Practice at the Intersection of Mathematics, Culture, and Financial Goals” (2009, *Mind, Culture, and Activity*) with Shelley Goldman and Osvaldo Jiménez, and “Prospective Adaptation in the Use of External Representations” (2009, *Cognition and Instruction*) with Daniel Schwartz.

Values that Occasion and Guide Mathematics in the Family

Introduction

Research on math in daily contexts has upended a view of mathematics as a primarily school-based set of tools and practices. Mathematics is embedded in a wide variety of activities—dairy workers filling orders (Scribner, 1984), dieters solving fraction problems in the kitchen or unit price problems when grocery shopping (de la Rocha, 1985; Lave, 1988), Brazilian children figuring profit while selling candy (Nunes et al., 1993; Saxe, 1990), high school basketball players figuring shooting percentages (Nasir, 2000), and nurses calculating drug doses (Hoyles et al., 2001). Throughout these cases, the approaches people take to the problems emergent for them in their practices are not constrained by school algorithms. They instead exploit contextual features of the material and social environments and flexibly integrate the pursuit of non-mathematical goals, such as minimizing effort or time, into problem solving (Pea, 1993). One important contribution of this work is to document when and how mathematics occurs in everyday cultural practices, thereby offering a counterpoint to an educational research focus on schooling and standards-based curricula. For our purposes, an even more valuable contribution of such investigations is their foregrounding of foundational problems in how people learn and employ cultural tools (like mathematics) as they engage in the emergent problems of everyday life. Specifically, this work offers hope for a better understanding of learner orientations to the content and processes of mathematical problem solving.

In our research, we turn to families as unit of analysis (Goldman, 2006; Gonzales et al., 2005), examining their practices to understand family life as a math-rich learning context. Likewise, we investigate families as a context in which mathematics is a meaning-rich enterprise (a situation often hoped for, but difficult to find, in school-based contexts). Our specific focus is on families with middle school children, in the midst of encountering the symbol manipulation and formula-laden activities of algebra. What are the occasions for the use of mathematics in family life? How does mathematics function as a part of the situated activity systems of family life?

Values as Key in Mathematical Activity at Home

In addressing these questions, we have identified *values* as a critical construct for conceptualizing the structure and function of mathematical activity at home. As one of our parents put it succinctly: “value is a very personal thing.” But values are more than simply preferences. We define value as a subjectively experienced quality of something that makes it important for a person, group, family, or community in relation to action. The “something” that is valued can be a process, outcome, event, strategy, resource, or tool. Values express an active relation between subject and object.

We have been guided in this framing by John Dewey (1929) when he defines values as “whatever is taken to have rightful authority in the direction of conduct” (p. 256). He notes the connection between values and enjoyment but rejects the notion that values and enjoyments are equivalent. He instead defines values as “enjoyments which are the consequences of intelligent action” (p. 259) and adds that “enjoyment becomes a value when we discover the relations upon which its presence depends” (p. 259). To illustrate the contrast, he distinguishes “satisfying” from “satisfactory”. To say that something is satisfying denotes that it is enjoyable, but not that it is good in a broader sense. Some enjoyable things have serious and negative consequences. Yet calling something satisfactory "involves a prediction, it contemplates a future in which the thing will continue to serve; it *will* do" (pp. 260-261). Thus, for Dewey, values are not mere preferences, but are intimately tied up with thought, with planning, and with action.

We wish to highlight how making decisions and solving problems—the "work of managers, of scientists, of engineers, of lawyers…that steers the course of society and its economic and governmental organizations" (Simon & Associates, 1986)—has its origins and growth conditions in the learning ecology of specific situations of family life. We focus on values because, although they have been relatively neglected in research, they are important for our understanding of the prospective paths of developmental transformations leading from family problem solving and decision making, to the mathematics experienced in school, and on to the world of societal elites.

The argument that math is guided by, situated in, and generative of family values builds on the ideas, articulated in Lave, Murtaugh and de la Rocha (1984), Lave (1988), and Saxe (1990), that problems emerge out of dilemmas in situations, and that learning arises when means are sought to resolve these dilemmas:

> In the dialectical relation between grocery shopping and the supermarket setting, repeated interactions produce a smooth 'fit' between the activity and setting, streamlining each in relation to the other, and generating expectations about how the activity will proceed. … It is in relation to this expectation that 'problems' take on meaning; they are viewed as snags or interruptions in the smooth process of shopping.
>
> (Lave, Murtaugh, & de la Rocha, 1984, p. 79)

We contend that a deeper accounting of the values that are in play for family members as structuring resources for mathematical problem solving takes us beyond Lave's general idea that "problems and solutions are dialectically co-constituted". We find in family math that people are able to recruit values in the service of simplifying problems down to comfortable levels of complexity, difficulty, risk, and so on. In other words, the utilization of values can be an effective problem solving strategy. Values are instrumental throughout the entire cycle of problem formation and resolution.

Something becomes problematic, as Lave notes, only in relation to an expectation and an associated set of values. Lave and colleagues found that grocery shoppers considered size and brand before price, and performed arithmetic only in 17% of cases, adding that "shoppers compare prices only when they have no strong preference among brands" (Lave, Murtaugh, & de la Rocha, 1984, p. 82). In our family math data, we certainly see examples that match Lave's prototype, where various factors converge to make the doing of what educators would think about as mathematics unnecessary. But we also observe shortly that the Family Math dataset as a whole indicates that the elimination of recognizable mathematics is only one possible outcome of the interactions among values, cultural artifacts, and mathematical activity. Math is not always suppressed by values. Among the stories we have collected resides an example of a family that routinely engages in "unnecessary" mathematics for fun; of family members who do math, even though they dislike it, in order to support a child's efforts to succeed in school; of a family that creates a novel, idiosyncratic economy to consider tradeoffs between consumer goods and interpersonal time with mother; and a family which organized and runs a collective savings plan among friends and family to better meet financial goals while also furthering social relational goals (Martin, Goldman, & Jiménez, 2009).

Our focus on values as central to mathematical problem solving in family contexts is centrally aligned with the purposes of this volume, and we carry forward O'Connor and Penuel's (in press) metaphor of "organizing" as a means of conceptualizing activity systems (also see O'Connor & Allen, this volume) that we focus on in our learning research. We ask, as they do, how it is that actors collectively organize local activity for and with one another by drawing on available resources, and with the meaning of current activity depending upon continual acts of re-organizing. We seek to understand participants' experiences emically – from an insider's perspective (Pike, 1967) - using such constructs as values, purpose, interest, attention, and engagement. Such emic frames are evidenced in their narratives about their experiences, with stories about the self and the family a primary vehicle of expressing their perspectives while also seeking to rhetorically shape the perspectives others have about them (Esmonde et al., 2010).

Context of Our Study on Family Math

Our focus is on mathematics in family life and so we observe how values interact with mathematical problem solving, decision-making, and learning in family activity systems—where there were no teachers formally organizing participant structures. We interviewed 20 families at home, each with a middle-schooler, to elicit family members' narrative accounts of the problem situations within which mathematics arises in the conduct of their lives. The families interviewed represented 74 participants, including 37 children. Our 20 families are more racially and ethnically diverse than the general population of the Bay Area: family members self-identified as African-American (*n*=5), Latino (*n*=23), Asian (*n*=7), Pacific Islander (*n*=8), Multi Racial (*n*=6) and white (*n*=25). The families represent a spectrum of economic diversity, from low income (where children are eligible for receiving free lunch) to upper-middle income. The educational level of family heads was also diverse—high school through graduate school.

Interview Protocol

We interviewed families at home, using a semi-structured protocol for inviting conversations and stories about family members' use of and experiences with mathematics in their activities as part of family life, work, and school. For example, for home improvement, we asked questions like "Have you decorated or made any improvements to the inside or outside of your home? What are they? Can you describe what you did? When you do it? A favorite story about it? Who do you do it with?" The interview was designed for family members to have ample space for detailed accounts of their experiences, while collecting additional information such as education levels, perceived comfort with math, school attainment, number and description of math classes that parents (and students) have taken, and their attitudes towards mathematics in general and towards school math. Videotaped interviews took about 2 hours. The advantage of the home interview setting is that family members often pulled in environmental supports to embellish and illustrate their math activity narratives.

Data Analyses

The analyses of the interviews proceeded through several stages. Three were particularly critical to the analyses of data for this chapter: content logging, identification of talk and interactions that had problem solving as an aspect, and identification of those problem solving entries in our database that evidenced foregrounding of values issues.

All videotapes were first content logged to delineate events, turns in conversation topics, stories, events, and sub-topics of interest inside participants' accounts. Math-relevant segments and other subjectively interesting stories were flagged. In a second data categorization stage, we identified all occasions where family members depicted a situation as creating a problem for them, using Spradley's (1979) 'problem domain' analysis technique for ethnography, that they came to solve using mathematics. These categorization processes resulted in a database of 315 reported and observed math events — events that participants described, where they were solving a problem using one or more types of mathematical practice. Each story was also categorized by activity context (Arts/Crafts, Big Events, Budgeting, Community/Church, Cooking, Games, Hobbies, Home Improvement, Keeping in Touch, Music, Personal Finance, School Stories, Shopping, Sports, Time Management, Travel, and Work Stories), participants involved, types of mathematics used, and what, if any, mediating tools were invoked during their problem solving activities.

Although not our primary emphasis here, it is noteworthy how wide-ranging the mathematical content put to use in families is, including fractions, decimals and percents; ratios and proportions; measurement and conversion; odds and probability; basic geometry; charts and graphs; statistics (such as averages), and statistical comparisons. In other analyses, we have reported on the 20 categories of mathematics utilized in the families[2], and their relative frequencies in the 17 different types of mathematical activities that we classified (Pea et al., 2007).

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[2] These categories, in rank order from most to least frequent in their occurrence in the database, are: Arithmetic, Comparing Magnitudes, Measurement, Estimation, Optimization, Geometry, Proportional Reasoning, Fractions, Data Representation, Formulas, Data Analysis, Unit Conversion, Decimals, Percentages, Logic, Interest Rates, Programming, Algebra, Probability, and Calculus.

We also tagged whether these database entries explicitly or implicitly referenced values involved in the mathematical activities, yielding 100 events in which values were foregrounded. In an important sense, values are involved in all of the uses of mathematics that we identified, in that the action of using mathematics in the family situations was "directing of conduct," as Dewey would characterize it, even if the family members did not explicitly refer to values issues in their occasions of mathematics use. But we reasoned that analyzing the 100 events that we tagged as values-focal in mathematical use for solving problems could provide us with a overarching perspective on central questions such as: What are the kinds of values that were expressed? What is it that is valued? How do the values in the family situations matter in everyday mathematics?

Findings

In this report about our findings of values and their place in occasioning and guiding mathematics in the family, we offer several summary observations:

- Problems lead math;
- Families used a small set of value categories recurrently;
- Values play out interactively in family problem solving and decision making situations using math;
- Values serve as resources in structuring the different phases involved in family problem solving and mathematical thinking.

Problems Lead Math: The Emergent Nature of Mathematics.

In the family, math rarely leads as an end in itself. At times, family members do want to learn or practice math directly, but more often problems were defined and constrained by life imperatives (e.g., we must find a way to pay our credit card or phone bills), or by more casual interests and needs (e.g., let's create a spinner for a board game). As problems emerge, family members must decide how to deal with them—and sometimes mathematical means are perceived to be useful to resolve the problems at hand. The problems that surface can be complex, involve many steps, are commonly nested inside larger problems, and often lead to yet other problems raised in a cascading manner. For example, home improvement projects were common across many families, and spawned varieties of mathematical activities ranging from overall design sketching, to measurement, to scaling of models or drawings , to budgeting and shopping for materials, to layout and work on the home project. Children in the families were often recruited into supporting roles, yielding opportunities to experience how their parents surfaced the problems, considered and employed mathematics, and revealed the productivity of math for local purposes.

A Small Set of Value Categories Were Used Recurrently

When our families spoke about their problem solving strategies and gave examples of the kinds of problems that arose in their lives, their narratives were consistently infused with talk about what they valued as important to them. This carried through discussions of emerging problems and projects, decisions to be made, options considered, and the social interactions and relationships that made family problem solving and decision making possible. Family members described concerns with minimizing effort, fiscal expenditures, and making mistakes—when they mattered to them.

In analyzing the kinds of values expressed in the 100 events across the 20 families we identified exemplars of five general *value systems* that appear to guide the uses of mathematics in family problem solving. We characterize these as value systems since each expresses a dominant *telos*, about what ought to be the aims of their activity together, and towards which their interactions should be directed. Our concern with values is also rooted in the approach to human development of Werner and Kaplan (1963; Kaplan, 1992), in which "development" itself as a value concept is conceived as a means-end relation, not as a characterization of forms alone. We seek to capture the aim of each value system in its name, and provide several categories of exemplars within each system to instantiate its use in our family's mathematical problem solving activities:

A. Minimization

- Of *time* expenditures (e.g., reducing effort/mess with tools by having Home Depot cut lumber for projects)
- Of *money* expenditures (e.g., calculating best buys, from home improvement materials to grocery items for everyday or big party occasions)
- Of *risks* associated with errors, by being as accurate as needed (e.g., wallpapering requires good measurements or time-wasting return trips will be needed to the store; carpentry errors in measurement or cutting are costly in effort, time, money)
- Of *effort* expenditures (e.g., on a trip in the UK, the ease of doubling dollars to pounds for currency conversion rather than using currency exchange rate; in another family, frequent parent-modeling and child-practicing of rounding during mental math)
- Of investment *loss* (e.g., seeking profit among different investment options)

B. Sociality

- *Doing projects together* as a family (whether involving math or not)
- *Having fun together* in doing math as a family
- *Spending time together* as a family rather than making more money

- *Helping* your child with math homework (even if you do not like math yourself)
- Being *socially accountable* to pooling money for a common good (e.g., in the trust-based "tanda" rotating-credit association used by several different families: Martin, Goldman, & Jiménez, 2009; also see Geertz, 1962, Vélez-Ibañez, 1983)

C. Empowerment

- In *controlling* one's own money using the socially-regulated tanda solution
- In defining the *semantics of money* – where not all pots of money are treated the same (e.g., savings; regular budget; unexpected income – one family with two parents in financial services solve family math problems differently based on these value differences)
- In the *agency* fostered from the joy of learning something new (one 8th grader gets excited about creating programs on her programmable calculator when she realizes they can help her do her homework in the car more quickly and without getting as carsick)

D. Beauty

- Making aesthetically pleasing designs (e.g., when measuring for picture hanging to ensure precision; in many home improvement projects)

E. Schoolishness

- Doing math mentally (for practice and building proficiency)
- Positive symbolic value placed on 'school math'

Values Play Out Interactively in Family Problem Solving Situations Using Math

Whether or not, when, and how math is engaged has to do with consideration and interplay of such value issues as minimization, sociality, empowerment, beauty and schoolishness for family members. Values contribute to how family ensembles do or do not divide labor, use tools, pursue precision, devote effort, risk error, and weigh alternatives. Our participants often talked about values in relation to decision making—especially when talking about tradeoffs between alternatives, or about trying to minimize or maximize something in particular (e.g., money spent, or time with family). Family members recruited values for simplifying problems to comfortable levels of complexity, difficulty, risk, relatedness, and so on—and for debating and negotiating what to do about a situation. Certain courses of action have more perceived or imagined friction between intention and goal attainment—in terms of difficulty, effort, cost, risk, relational demands, time demands. In other words, using values can be an efficiency-making problem solving strategy at the individual and family levels of analysis.

There are also differences in values among family members that can further animate problem solving as the values that will dominate a given situation are negotiated during family mathematical problem solving and decision-making. Greeno & Hall (1997; also see Hall & Stevens, 1995) highlight that representational forms in conversations, whether in classrooms or workplaces, serve as "boundary objects" (Star & Griesemer, 1989) that can be interpreted by people from different communities in ways that allow them to share information. We consider the values expressed and reflected upon in the family conversations about their mathematical problem solving situations as boundary objects as well, as they get expressed and challenged. In particular, families often describe their efforts to (a) manage the distribution of the family's resources (e. g., time, money, and physical effort) toward more desirable ends and (b) to control more and less desirable factors (e. g., minimize financial risk, reduce stress, preserve face, or increase time spent with family). These efforts characteristically involve tradeoffs between competing values, either between family members or between different values that the family members share. As in the field of business, where a "value proposition" defines what a customer gets from a good or service that a vendor provides to them in exchange for their money, time or attention, there are alternatives "in play" during family activities that provide different "value propositions" as desirable courses of action or outcomes. We learn about how parents and children try to influence each other's mathematical decision making by highlighting and negotiating priorities among values (e.g., we have a number of allowance/vacation-money examples), and an example in which family tradeoff discussions center on more-time-with-mom vs. the bigger TV screen they could buy with her consulting income (Martin & Goldman, 2010).

King Beach (1999) uses the theoretical construct of "leading activity" to characterize the dominant focus in a learner's activity system at any given time in which multiple activities are co-occurring. Similarly, an individual or group in the family may have a "leading value" that is the dominant focus among a set of co-occurring values by which they are appraising an unfolding mathematics-engaging problem solving situation (see also Levias, 2008). At one point in the development of a problem situation and its solution, "minimizing time" may be the leading value, but it can change to "minimizing error" if a quick effort to estimate has risks affiliated with it that the use of measurement techniques could avoid.

Values Serve as Resources in Structuring the Different Phases Involved in Family Problem Solving and Mathematical Thinking

How do values serve as resources for families in structuring their problem solving? Building upon John Dewey's (1910) influential theoretical account of stages of problem-solving, mathematician George Polya (1957) popularized a six-stage problem solving model that was widely assimilated into cognitive psychology. The six stages are: (1) finding the problem, (2) representing the problem, (3) planning a problem solution, (4) executing the plan, (5) checking the solution, and (6) reflecting to consolidate learning. These are not linear stages flowing top-down, but represent a recursive system where each new set of constraints created by materials the problem solver produces makes new opportunities that can be exploited in further work on the problem (e.g., as in writing: Pea & Kurland, 1987; or in architectural design: Schön, 1983).

Finding the Problem

What is the felt problem that emerges as needing solution? This was one key place in the sequence of problem solving phases for values to make a mark. For example, different economic profiles for the families meant that for some, small dollar amounts had much greater significance and made the time involved in pursuing bargains worth the additional effort. One mother carefully checked weekly supermarket advertisements for sales and only visited markets with good sales. If a sale had a limit per customer, she would "go to the car, put them in the car, and then go back in and get some more," exchanging effort for a good price. How the problem was defined by a family was *shaped* by their values in play: in one illustrative case, a middle-school girl's father was making a birdhouse for her but took the occasion to apprentice her in measurement activities. Solving the birdhouse design and building project alone was not his aim. For a number of families, an important value was solving problems together as a family—not having the most competent mathematics practitioner chunk through solution methods to derive a problem solution. This then provided opportunities for divisions of labor, and learning by peripheral participation (Lave & Wenger, 1991) in a scaffolded way. It is also a place where different individuals in the family may contribute to how the problem is scoped, and what counts as 'success' in the problem solving.

Consider Murtaugh's (1985) findings on supermarket shoppers: "People not only seek satisfactory solutions to problems, they simultaneously seek satisfactory problems as well" (p. 192), that are solvable in the situation with available resources. In Lave et al.'s (1984) studies of dieters and grocery shoppers, the search for a satisfactory problem often led to the elimination of what would be recognizable as 'school math,' in which symbolic mathematical equations are employed and manipulated to solve problems rather than the physical world quantities or other referents that they are about. In our family math data, this was only one possible outcome of the interactions among values and mathematical activities in families; many of the examples family members offered took heed of mathematical concepts and techniques from 'school math.'

Not all the mathematics that families came to use was instrumental in solving problems in the traditional sense. One family routinely engaged in "unnecessary" mathematics because their 'problem to solve' was creating locally-relevant situations for "having fun with math" – one incident involved promoting mental math for their daughter as they asked her to compute whether she could hold her breath for the 1.7 miles of Golden Gate Bridge span within a 45 mph speed limit:

> *Daughter*: We were coming home across the Golden Gate Bridge and I wanted to see if I could hold my breath across it. And our guidebook said it was two miles long, and at 45 miles per hour, so we had to figure out how long I had to hold my

breath and it ended up being three minutes. And that's a long time. So I didn't hold it for the whole entire time, and [my dad] drove a little faster.

Mom (laughing): We drove faster!

Father: We realized we'd have to change some of the physics for this to work because three minutes was too long so we drove faster and shortened the span to just the part that was kind of out there.

Interviewer: Was that something you figured out before you got to the bridge?

Daughter: Yeah.

Father: She figured it out while we were driving. That was her project, to do the math.

Daughter: There are a lot of times in cars when dad has a problemish thing, like "if we're going this fast how long will it take us to get somewhere."

Dad: That's right. And you have to do it in your head. Which is hard, isn't it? But really good practice.

In another family, the sixth grade girl derived particular pleasure in creating over a dozen imaginary businesses, and she enthusiastically enlisted her family in role playing and fantasy fiscal transactions and their associated computations.

Representing the Problem

Representations of the problem to be solved were less often written down, as one sees in classroom mathematics, than they were 'in the air' of a family discourse. Of course, transient representations of aspects of the problem do get created, as in the forms of lists of things to purchase and in what quantities, or in measurements taken in a home improvement project. When the family members felt that they could remember the problem that they were working on, or when projects generated multiple problems that were dealt with as they arose, explicit symbolic representation of the problems to be solved may have been viewed as unnecessary. When precision is needed in computing answers to mathematical computations, explicit representations of the problem may be used. For example, in one family, the older of two daughters enjoyed sewing. She noted the importance of precise measurement in sewing projects, and told us of how she used to struggle with converting between units of measurement, often getting confused or making mistakes. Then she learned a paper and pencil bookkeeping method in school, which quickly improved her ability to be precise and accurate in her work. She excitedly described learning the method, and realizing its applicability to sewing, as "like a door opening."

Planning a Problem Solution

What are the options perceived or considered for closing the gap between the current situation and the desired state? Values came to play here either *implicitly* in habits—routines that embodied the values affiliated with conducting a practice in a specific way—or *explicitly* when family participants noticed resources or prior experiences that they could leverage in the current situation. In an example of planning a condo renovation, the daughter knew not only how to make and use scale models as part of her design problem solving, but also was aware of their limitations. She knew when to switch into "reality mode" and do embodied mathematical reasoning, standing in the space and using her body as a measure of the appropriateness of alternate layouts.

Values thus helped families determine *how* to solve problems once identified *as* problems. When one mother wanted a savings plan to pay down credit card debt, she chose the “tanda” socially-regulated rotating credit association, not a bank. She took on risk in exchange for social commitment and cultural participation. Her description of the tanda and her reason for choosing it extended beyond financial factors to include values such as the importance of social relationships and the associated issues of trust and fairness (Martin, Goldman, & Jiménez, 2009).

Executing the Plan

As work gets underway on the problem solving project that the family or some subset of its members has taken on, there are discoveries to be made. New subgoals get generated as incomplete specifications of the problem in its original formulation become evident. This is another phase where values shape problem solving processes. It can become clear that the project is more complex than imagined, that too ambitious an agenda was established, and the problem is pared back. The goal state thus changes. The burdens involved in executing the plan lead to a relaxation of constraints in what it means to solve “it”.

Checking the Solution

Math in the family also requires participants in the situation to evaluate their own solutions—people have to decide if their solution is correct in terms they care about, if the correct solution is really relevant or appropriate to the situation’s particularities. What are the criteria by which outcomes will be judged? These are rarely pre-determined at the outset of their problem posing, but are discovered in the course of their work as different values come into play and they learn more about the situation and what matters to them as these values surface in that process.

Reflecting to Consolidate Learning

In Polya's model of mathematical problem solving, expressing reflections on how one might generalize from the mathematical problem solved to broader classes of solutions is a highly valued feature of mathematical expertise. In the family situations, the lessons learned were less commonly about the mathematics per se than its utilities.

In summary, values had pervasive influences within family activities in structuring problem solving and mathematical thinking across these phases of Polya's model. Families often had to consider what factors mattered to them, and sometimes to weigh their relative importance, when they were making decisions in solving a problem. Was getting the "best" deal most important? Some shoppers felt so satisfied with the sense of the best deal that they were willing to spend lots of extra time finding it, even for inexpensive items. Was time spent pursuing a solution an important consideration? Sometimes expediency was vital because deadlines loomed. Did social relationships among family participants play a part in how problem solving was structured? Sometimes spending time and learning together on a problem was more important than finding an efficient solution. Answering these questions concerning values often then transformed the problems to be solved by defining the constraints in the problem, the resources used, and even the criteria by which outcomes would be judged.

Discussion and Implications

We have illustrated in this chapter on mathematical problem solving activities within families how the functions and structures of their mathematics at home are dependent upon values, and how interacting values during problem situations came to shape what counted as solutions. These values examples from family math illustrate the complex interconnections between problem solving processes, human relations, cultural practices and tools. Since values are deeply intertwined with how mathematics emerges and is used for problem solving in family situations (and in unassigned math problems that may surface in the classroom: McDermott & Weber, 1998; Stevens, 2000), we wonder whether we might find productive pedagogical models for re-introducing the complexity of values into the framing and conduct of mathematical learning in the classroom? It seems worthwhile to seek ways to leverage learners' experiences with these features of mathematics out of school.

There have been two dominant tendencies in seeking to compare and possibly bridge the informal and formal learning environment experiences of youth. On the one hand we have work since Vygotsky into modern times that highlights the differences in the precision of language use and discourse in everyday and school settings, and the importance of fostering academic language and the more precise frames for inquiry in formal settings. On the other hand we have work from Moll et al. (1992), Civil (2007), de Abreu (1995), Gonzalez (Gonzalez et al., 2001, 2005) and others arguing for the need on the part of educators to focus on the content available in learners' cultural funds of knowledge that can create synergies across the full learning spectrum for learners. What our work suggests is that the issue of values provides a third dimension for consideration—beyond discourse transformation and everyday funds of knowledge.

Since values weigh significantly in how mathematics plays out in the situations of family life and problem solving, we would do well to consider values as a design resource for advancing mathematical learning and understanding. We conjecture that one promising approach to this design route would be role playing of values perspectives in group mathematical activities to help bridge the boundaries of family life and school settings - and yield more portable, sustainable and dependable learning across situations. Instances of the value systems we have identified could be seeded in roles that different students take on in role playing situations where mathematics would surface as an instrumentality to problem resolution. At the same time, sensitivity and participatory design with learners and family members will be required to adequately attend to the multidimensionality of the social, historical, and institutional dimensions accompanying these practices, to avoid simplistic co-opting by "schooling" (Ares, this volume).

Our analyses illuminate how family math opportunities vary from documented structure of school mathematics. We have a growing sense that these differences may help account for some of the challenges for mathematics learned to productively thrive across the transitions from everyday activities into school, and vice versa. Values are not quite accounted for adequately in extant descriptions of problem solving, except insofar as they lie latent inside the construct of the "goal" of the problem and its required means of solution in much of school mathematics. In our family math examples, values are important and they typically need to be shaped to fit the emerging problem, just as the problem needs to be shaped to fit the values. This dynamic alignment process provides important occasions for learning about both values and about mathematics as means. This learning to a significant extent involves cultural reproduction of existing, and we assume, long-standing value systems such as Minimization, Sociality, Empowerment and Beauty – as well as the historically more-recent value system of Schoolishness. In school, mathematics problems typically come pre-packaged with values—with well-defined goals and means—and not taking up those values amounts to missing the point of the problem, and making errors. This “values gap” between everyday family math and school math may hold some of the clues to the alienation that many individuals feel from learning mathematics as a subject in school.

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