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Physics 41N
Mechanics: Insights, Applications and Advances
Lecture 5: Tidal Forces

Reference: *An Introduction to Mechanics* by D. Kleppner and R. Kolenkow, Example 8.4, pages 348 to 352.

The tidal force is a secondary effect of the force of gravity and is responsible for the tides on earth, tidal heating of satellite objects, tidal locking of the rotational and orbital frequencies of a satellite object, the breaking up of satellites into rings, tidal stripping of one galaxy by another, and many other important phenomena in planetary science, astronomy and astrophysics. Tidal forces arise because the gravitational field due to another body is not constant across a satellite's diameter. (When I use the word "satellite" here, I mean any body orbiting another body; e.g., a moon orbiting a planet, a planet orbiting a star, a star orbiting a black hole, one galaxy orbiting another galaxy...)

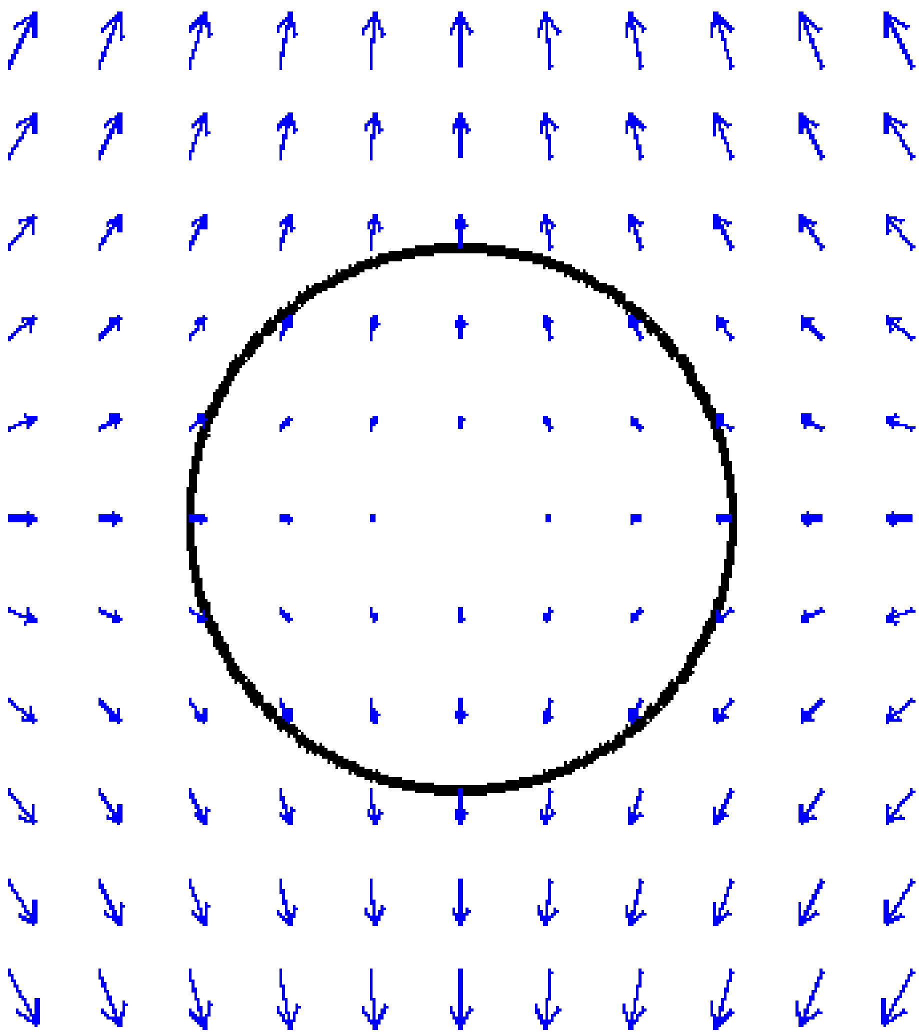
The figure on the next page shows the net tidal force field generated by a mass that is off the bottom of the page. These residual tidal forces are calculated by subtracting the vectorial force at the center of the illustrated satellite. This leaves radial outward forces at the nearest and farthest side of the satellite with respect to the mass off the bottom of the page (i.e., at the top and bottom of the illustrated satellite), and inward radial forces around a circular belt between these points (e.g., at the left-most and right-most points on the illustrated satellite). We used Taylor expansions to show that in the limit that the radius of the satellite is small compared to the distance between the objects, the inward tidal forces around the circular belt are half the outward tidal forces at the nearest and farthest points.

In class, we also showed that

1. the tidal force due to the moon is 2.2 times that due to the sun;
2. the tidal force due to the moon is about 100 parts per billion of the force due to earth's gravity;
3. tides vary by up to a factor of 2.7 depending on the relative alignment of the sun, earth and moon.

In each problem, we calculated a ratio. We did *not* plug in numerical values until we had the simplest possible form for the ratio. If we had plugged in numerical values too soon, we would have lost the opportunity to gain insight into what the ratio really depends on in each case.

Here is a more detailed statement of each problem.



1. Calculate the ratio F_{moon}/F_{sun} where F_{moon} is the maximum tidal force at the earth's surface due to the moon and F_{sun} is that due to the sun.

2. Calculate the ratios F_{moon}/F_g and F_{sun}/F_g where F_{moon} and F_{sun} are the maximum tidal forces at the earths' surface due to the moon and the sun, and F_g is the force due to earth's gravity at the earth's surface.

3. Explain why the strongest tides (called the spring tides) occur when the moon is full or new, and the weakest tides (called the neap tides) occur midway between, at the quarters of the moon. Let ΔF be the difference between the maximum and minimum tidal forces over a 24 hour period. Calculate the ratio of ΔF for spring and neap tides: $\Delta F_{spring}/\Delta F_{neap}$.

We also discussed the *Roche limit* or Roche radius, which is the distance from a celestial body within which a second celestial body held together only by its own gravity will disintegrate due to the first celestial body's tidal forces exceeding the second body's gravitational self-attraction. Inside the Roche limit, orbiting material will tend to disperse and form rings, while outside the limit, material will tend to coalesce.

We talked about *tidal heating*, which occurs due to friction as the satellite is deformed due to tidal forces. Orbital and rotational energy are dissipated as heat in the satellite (e.g., a moon orbiting a planet). A good example of this is Io, a moon of Jupiter, which is the most volcanically active body in the solar system; the energy comes from tidal heating.

Due to resistance of the satellite material to deformation, the tidal bulges on the near and far side of the satellite will lag or lead the points of maximum and minimum force, depending on the rotational frequency of the satellite compared to its orbital frequency. This misalignment of the bulge with the maximum and minimum forces will lead to a torque on the satellite. This torque in turn leads to *tidal locking*, in which one side of an astronomical body always faces another, just as one side of the Earth's Moon always faces the Earth. A tidally locked body takes just as long to rotate around its own axis as it does to revolve around its partner. This synchronous rotation causes one hemisphere to constantly face the partner body. Therefore, the bulges on the satellite are always at the same point on the satellite. Usually, only the satellite becomes tidally locked around the larger planet, but if the difference in mass between the two bodies and their physical separation is small enough, each will become tidally locked to the other. An example of a tidally locked pair is Pluto and its moon Charon.

For more information on the above phenomena, see the wikipedia entries for Roche limit, tidal acceleration and tidal locking.