Risk Premium Shocks Can Create Inefficient Recessions

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Abstract

We develop an equilibrium theory of business cycles driven by spikes in risk premiums that depress business demand for capital and labor. The economy is hit by aggregate shocks that increase firms’ uninsurable idiosyncratic risk and raise risk premiums. We show that risk shocks can create quantitatively realistic business cycles, with contractions in employment, consumption, and investment. Economic fluctuations are inefficient—output and employment fall too much during recessions, compared to the constrained-efficient allocation, and consumption should be countercyclical. The optimal allocation requires subsidizing labor and taxing capital during recessions.

1 Introduction

Market economies experience recurrent recessions with sharp contractions in economic activity. Why do employment, investment, and consumption suddenly fall, and what is the appropriate policy response? We argue that recessions are periods of heightened economic uncertainty when firms and investors shrink from risk. We propose a model of business cycles driven by spikes in risk premiums that depress business demand for labor and capital.

The premise of our model is that businesses face significant uninsurable idiosyncratic risk. Aggregate shocks raise idiosyncratic risk and drive risk premiums up. We show that risk shocks can create quantitatively realistic business cycles, with comovement among employment, consumption, and investment. Furthermore, these economic fluctuations are inefficient—output and employment fall too much during recessions, compared to the constrained-efficient allocation, and consumption should have a countercyclical behavior.

A long tradition attributes business cycles to time-varying risk premiums, dating back, at least, to chapter 12 of the General Theory (Keynes [1936]). The comovement pattern of macroeconomic aggregates during business cycles, however, poses a challenge to this view. It is very hard to explain why employment, consumption, and investment all simultaneously
contract. As Barro and King [1984] noted, in standard models contractions in investment demand must produce expansions in consumption. As a result, most of the macro literature invokes productivity shocks (RBC) and nominal rigidities (New Keynesian) to explain the comovement pattern. In contrast, our model features only risk premium shocks and no nominal rigidities. The only departure from the standard neoclassical growth model is that firm insiders cannot fully insure against the idiosyncratic risk in their firms. This friction enables the model to generate the comovement pattern that characterizes business cycles.

The mechanism can be understood in terms of risk premiums and precautionary saving, and the different duration of capital and labor plays a central role. Since the marginal products of capital and labor are uncertain, demand for them carries a risk premium to compensate for uninsurable idiosyncratic risk. As a result, in partial equilibrium risk shocks depress the demand for capital and labor symmetrically.

In contrast, in general equilibrium risk shocks affect capital and labor asymmetrically because of their different durations. Higher idiosyncratic risk creates a precautionary saving motive that depresses interest rates. Because capital has a long duration, lower interest rates stimulate investment, compensating the effects of the higher risk premium. Labor, on the other hand, has short duration, precisely zero in the model with spot labor markets. It trades off disutility for output intratemporally, and is therefore unaffected by lower interest rates.

The result is a large countercyclical labor wedge, and a small procyclical capital wedge. This configuration of wedges enables the model to deliver realistic business cycles. A labor wedge—like a labor tax—reduces employment and output, and thereby consumption and investment. It creates a recession. The distribution of the decline in output between lower consumption and lower investment depends on intertemporal substitution, the persistence of the shock, and capital adjustment costs.

A capital wedge—like a capital tax—reduces investment and employment, but increases consumption. To the extent that risk premium shocks show up as a capital wedge, they can’t produce business cycles. This property is the essence of the Barro and King [1984] problem, and is the main reason that the business-cycle literature has focused so much on technology shocks or a combination of nominal rigidities and inefficient monetary policy as drivers of business cycles. The insight in our paper is that the precautionary motive damps and overturns the effect of the risk premium on the capital wedge, but not on the labor wedge, because of the different durations of capital and labor. As a result, risk shocks generate business cycles with realistic comovement among employment, consumption, and investment.

To evaluate the quantitative importance of the mechanism, we calibrate the model to US data. Idiosyncratic risk is highly countercyclical, both in terms of establishment-level
TFP or stock returns.\footnote{See Christiano et al. [2014], Gilchrist et al. [2014], Herskovic et al. [2016], and Bloom et al. [2018].} We find that the model can reproduce the business cycle-behavior in the data. The model is stylized in the interest of theoretical clarity, but our claim is that the theoretical mechanism we describe is quantitatively important and a promising way to understand business cycles.

We also pursue a sufficient statistic approach that allows us to quantitatively evaluate the relationship between the labor and capital wedges in terms of measurable equilibrium objects, without having to specify many structural details of the environment. The capital wedge $\omega_{kt}$ can be expressed as the labor wedge $\omega_{lt}$, equal to the idiosyncratic risk premium, times a dampening factor that accounts for the countervailing effect of the precautionary savings motive. When we plug in data from the US we get a steady state capital wedge $\omega_{k,ss} \approx \omega_{t,ss} \times (-0.1) < 0$. The precautionary motive slightly dominates and the capital wedge is small and negative in steady state. The dampening factor is relatively stable, so while the labor wedge is countercyclical, the capital wedge is procyclical. Besides giving us a quantitative sense of the mechanism, the sufficient statistic also allows us to see how it would change under different scenarios.

We then study the efficiency of the competitive equilibrium. In the first best there is perfect risk sharing and risk shocks have no effects. We solve for the constrained-efficient allocation that respects the incomplete risk-sharing environment. The main takeaway is that the competitive equilibrium is inefficient. The planner would create a small and procyclical labor wedge, and a large and countercyclical capital wedge—the opposite of the competitive equilibrium.

The inefficiency ultimately arises from the presence of hidden savings in the environment. Private agents don’t realize that increasing aggregate consumption, by increasing employment and reducing investment, improves idiosyncratic risk sharing. This externality is particularly strong after risk shocks.

As a result, in the planner’s allocation risk shocks depress investment but produce an expansion in consumption, and output and employment fluctuations are smaller than in the competitive equilibrium. Instead of recessions, risk shocks should produce the pattern described in Barro and King [1984]. The constrained-efficient allocation can be implemented by subsidizing labor and taxing capital during recessions.

### 1.1 Related Literature

Our paper expands on a large literature that highlights the role of time-varying risk premiums in business cycles, such as Cochrane [2011]. Higher risk premiums act like a negative demand shock to investment. But these models are typically unable to deliver the co-movement pattern between consumption, investment, and employment that occurs over the business cycle. This is the essence of the problem identified in Barro and King [1984],
and the reason that the wedge accounting analysis in Chari et al. [2007] minimizes the role of investment wedges. For example, Christiano et al. [2014], Gilchrist et al. [2014], and Bloom et al. [2018] study the impact of aggregate shocks to uninsurable idiosyncratic risk on investment dynamics. In the absence of nominal rigidities in general equilibrium, this leads to countercyclical spikes in consumption. On their own, risk shocks cannot produce realistic recessions in these models. There is also a large literature on financial frictions, starting with Kiyotaki and Moore [1997] and Bernanke et al. [1999], where tighter financial constraints reduce asset prices and investment.\(^2\) However, in the absence of movements in TFP or nominal rigidities, consumption must move countercyclically in these models.

Hall [2017] treats employment as a form of investment, in the context of search models. Higher risk premiums act like a negative demand shock for labor, and has the potential to deliver recessions. Kilic and Wachter [2018] apply this idea in a rare-disasters asset pricing model, and Kehoe et al. [2018] incorporate persistent human capital accumulation to overcome the puzzle of low employment volatility uncovered in Shimer [2005]. These models largely abstract from investment, however, and often treat prices as exogenous. This limitation is important because, while a higher risk premium may depress job creation, it also reduces the demand for investment. In the short run, consumption may well move countercyclically in models that include investment and endogenous prices. Kilic and Wachter [2018] briefly describe results for an extension of their model to include investment, but in that extension, a financial shock induces a substantial jump in consumption.

Arellano et al. [2016] study the role of uninsurable idiosyncratic risk for demand for labor in an open economy with financial frictions. In their model higher risk increases the probability of default, and, through higher credit spreads, reduces demand for labor. They abstract from investment. Jermann and Quadrini [2012] incorporate investment in a model with borrowing constraints where labor must be paid in advance, as working capital. When financial conditions tighten, firms that are up against the constraint must reduce employment. But this mechanism also affects investment, and in general equilibrium consumption spikes on in a recession. While the role of tight borrowing constraints is related to the role of the idiosyncratic risk premium in our model, the general equilibrium response is different because there is no precautionary motive.

In contrast to these papers, our paper focuses on the general equilibrium response to higher risk premiums and the resulting comovement of consumption, investment, and employment. Higher idiosyncratic risk creates both a risk premium and a precautionary saving motive. Risk shocks reduce demand for capital and labor symmetrically in partial equilibrium. However, the general equilibrium response of interest rates, thanks to the precautionary savings motive, affects them differentially. Because capital has a long duration, lower interest rates stimulate demand, dampening the effects of the risk premium. The

\(^2\)See also Brunnermeier and Sannikov [2014] and He and Krishnamurthy [2012], for example.
difference in duration between capital and labor plays a central role in generating realistic comovement.

We use a simple neoclassical model with a competitive spot market for labor. Uninsurable idiosyncratic risk is the only friction. While we abstract from search frictions, we believe that incorporating them into the framework is a promising approach for future work. The focus of our paper is on uninsurable idiosyncratic risk on the business side, an issue highlighted by Angeletos [2006] and Meh et al. [2004]. We also abstract from uninsurable labor income risk, which is the topic of a large literature, for example, Aiyagari [1994].

We focus on the effect of time-varying idiosyncratic risk as in Di Tella [2017]. This choice is in the interest of being concrete. A time-varying price of risk would have similar effects. This could be a result of habits as in Campbell and Cochrane [1999] (exploited by Kehoe et al. [2018]), heterogenous risk aversion as in Longstaff and Wang [2012], Gârleanu and Panageas [2015], or Kekre and Lenel [2019], or balance sheet effects as in He et al. [2015].

2 Setting

There are two types of agents, workers and entrepreneurs. A representative worker supplies labor, and entrepreneurs use capital \( k_{i,t} \) and labor \( \ell_{i,t} \) to produce goods. Each entrepreneur is exposed to idiosyncratic risk. The net output flow for entrepreneur \( i \) is

\[
dY_{i,t} = f(k_{i,t}, \ell_{i,t})dt + f(k_{i,t}, \ell_{i,t})\sigma_t dB_{i,t},
\]

where \( B_{i,t} \) is a Brownian motion specific to this entrepreneur, and \( f(k, \ell) = k^\alpha \ell^{1-\alpha} \) is the usual Cobb-Douglas production function. Idiosyncratic risk washes out in the aggregate, so aggregate output flow is \( f(k_t, \ell_t)dt \), as usual. As a result, the aggregate resource constraints are

\[
c_t + \phi(x_t)k_t = f(k_t, \ell_t)
\]

and

\[
dk_t = (x_t k_t - \delta k_t)dt,
\]

where \( c_t = c_{w,t} + c_{e,t} \) is aggregate consumption, \( c_{e,t} = \int c_{i,t}di \) is total consumption by entrepreneurs, and \( x_t - \delta \) is the rate of growth of the capital stock. The function \( \phi(x) \) captures adjustment costs. We assume it takes the form \( \phi(x) = (\exp(\epsilon(x - \delta)) - 1)/\epsilon + \delta \), so that in steady state \( \phi(\delta) = \delta \) and \( \phi'(\delta) = 1 \).

The level of idiosyncratic risk \( \sigma_t \) follows a diffusion

\[
d\sigma_t = \theta(\bar{\sigma} - \sigma_t)dt + \sqrt{\sigma_t}v_\sigma dZ_t
\]
driven by an aggregate Brownian motion $Z$ that captures risk shocks. This is the only source of aggregate risk in this economy and the only exogenous driving force.

Markets are complete for aggregate risk $Z$, but idiosyncratic risk $B_i$ cannot be shared. This constraint is the only friction in the economy.

The representative worker has impatience $\rho_w$ and Frisch elasticity of labor supply $\psi$. The worker’s problem is to choose a plan $(c_w, \ell)$ for consumption and labor to solve

$$\max_{c_w, \ell} \mathbb{E} \left[ \int_0^\infty e^{-\rho_w t} \left( \log(c_{w,t}) - \ell_{t+1}^{1+1/\psi}/(1+1/\psi) \right) dt \right],$$

subject to the budget constraint

$$\mathbb{E} \left[ \int_0^\infty \xi_t c_{w,t} dt \right] \leq n_{w0} + \mathbb{E} \left[ \int_0^\infty \xi_t \ell_t w_t dt \right],$$

where $\xi_t$ is the pricing kernel, $d\xi_t/\xi_t = -r_t dt - \pi_t^Z dZ_t$.

Entrepreneurs have impatience $\rho_e > \rho_w$. We take entrepreneurs to be more impatient than workers as a device to obtain a stationary wealth distribution. An entrepreneur’s problem is to choose a plan for consumption and purchases of inputs $(c_i, k_i, \ell_i, x_i, \sigma_{n,i})$ to solve

$$\max_{c_i, \ell_i, x_i, \sigma_{n,i}} \mathbb{E} \left[ \int_0^\infty e^{-\rho_e t} \log(c_{i,t}) dt \right],$$

subject to the law of motion

$$dn_{i,t} = (n_{i,t} r_t + \pi_t^Z n_{i,t} \sigma_{n,i,t} + f(k_{i,t}, \ell_{i,t}) - \phi(x_{i,t}) k_{i,t} + \alpha_{i,t} q_t k_{i,t} - w_t \ell_{i,t} - c_{i,t}) dt$$

$$+ \sigma_{i,t} f(k_{i,t}, \ell_{i,t}) dB_{i,t} + \sigma_{n,i,t} n_{i,t} dZ_t$$

and the solvency constraint $n_{i,t} \geq 0$, where $\alpha_t = x_{i,t} - \delta + \mu_{q,t} - r_t - \pi_t^Z \sigma_{q,t}$ is the ex-dividend risk-adjusted excess return of capital. Notice that if idiosyncratic risk could be shared, the $\sigma_{i,t} f(k_{i,t}, \ell_{i,t}) dB_{i,t}$ term would vanish.

Total wealth is $n_{e,t} + n_{w,t} = q_t k_t$, where $n_{e,t} = \int n_{i,t} di$ is the total wealth of entrepreneurs. For a given initial distribution of wealth, a competitive equilibrium is a process for prices $(r, \pi^Z, w, q)$, aggregate capital $k$, a plan for the representative worker $(c_w, \ell)$, and a plan for each entrepreneur $(c_i, k_i, \ell_i, x_i, \sigma_{n,i})$ such that every agent optimizes taking prices as given; the aggregate resource constraints, equations (1) and (2) hold; and markets clear: $\int \ell_{i,t} di = \ell_t$, $\int k_{i,t} di = k_t$ and $n_{e,t} + n_{w,t} = q_t k_t$.

2.1 Discussion of assumptions

Technology. A crucial feature of the environment is that the marginal products of capital and labor are locally uncertain. When an entrepreneur makes $k$ and $\ell$ decisions, he has a
probability distribution of how this will affect his profits, but he doesn’t know for sure. For example, a car company can increase employment to produce more cars without knowing just how many extra cars will be made or how valuable they’ll be. The realized marginal product is uncertain when the factor quantity decision is made. The central premise of the paper is that economic activity involves uninsurable idiosyncratic risk of this type.

To clarify the technology, consider an entrepreneur who employs a constant amount of capital and labor for one year. The net output produced over this year has a normal distribution:

\[ Y_{i,1} = \int_0^1 f(k, \ell) \, dt + \int_0^1 f(k, \ell) \sigma dB_{i,t} = f(k, \ell)(1 + \sigma B_{i,1}), \]

where \( B_{i,1} \) is distributed as standard normal. The continuous-time formulation says that this uncertain output is revealed gradually and the entrepreneur can continuously adjust labor and capital, but the marginal product is locally uncertain. This formulation allows us to map the entrepreneurs’ problem into standard portfolio-choice theory.

Net output over a year can be negative, \( Y_{i,1} < 0 \). Economic activity may actually destroy resources—a factory could burn down. The probability depends on \( \sigma \). However, because entrepreneurs can continually adjust \( k \) and \( \ell \), their net worth remains always positive, and has a lognormal distribution in equilibrium.

Idiosyncratic shocks are iid, which yields considerable tractability. Introducing persistent shocks would require keeping track of the cross-sectional joint distribution of productivity and net worth. But the crucial property remains that the marginal products are locally uncertain.

**Aggregation.** The setting with two types of agents is tractable because it won’t be necessary to keep track of the distribution of wealth among entrepreneurs. Homothetic preferences, iid idiosyncratic shocks, and a linear budget constraint yield linear policy functions that can be easily aggregated. Crucially, entrepreneurs face uninsurable idiosyncratic risk, but don’t have non-tradable labor income. The representative worker has non-tradable labor income, but doesn’t face idiosyncratic risk. This formulation omits the role of uninsurable labor income risk, which is the focus of an extensive literature. The focus in this paper is on the role of uninsurable idiosyncratic risk on the entrepreneur’s side, and the resulting risk premium on labor and capital.

**Entrepreneurs and risk sharing.** Entrepreneurs in the model should be interpreted as insiders who must retain a significant stake in the firm for incentive reasons. In the case of privately owned firms or startups, this means the founders, owners, and officers. In the case of large, publicly traded corporations, it means significant investors, directors, and officers, who have a substantial exposure to the idiosyncratic outcome of the firm through
stock ownership, options, and bonuses. In the model workers do not take idiosyncratic risk because they don’t have the same incentive problems as these insiders. The type of risk sharing that the first-best allocation requires, however, does not involve workers, but rather risk sharing across entrepreneurs.

3 Solving the model

As noted above, entrepreneurs have homothetic preferences and linear budget constraints, so their policy functions are linear in their net worth \(n_{i,t}\). An entrepreneur’s HJB equation is

\[
\rho \left( \frac{1}{\rho} \log n + A_t \right) = \max_{\hat{c}, \hat{k}, \hat{\ell}, x, \sigma} \log \hat{c} + \log n + \frac{1}{\rho} \left( \mu_{nt} - \frac{1}{2} \sigma_n^2 - \frac{1}{2} (\sigma_t f(\hat{k}, \hat{\ell}))^2 \right) + \mu_{At},
\]

where \(\mu_{nt} = r_t + \pi_t^2 \sigma_n + f(\hat{k}, \hat{\ell}) - \phi(x) \hat{k} + \alpha_t q_t \hat{k} - w_t \hat{\ell} - \hat{c}\), and a hat denotes the variable is normalized by the entrepreneur’s wealth, e.g. \(\hat{c}_{it} = c_t/n_{it}\). Their demands for capital and labor are:

\[
w_t = (1 - \alpha)(k_t/\ell_t)^\alpha \left( 1 - \frac{k_t^{\alpha \ell_t^{1-\alpha}}}{n_{e,t} \sigma_t^2} \right),
\]

and

\[
R_t = \alpha \frac{n_{e,t}}{(k_t/\ell_t)^{\alpha-1}} \left( 1 - \frac{k_t^{\alpha \ell_t^{1-\alpha}}}{n_{e,t} \sigma_t^2} \right),
\]

where \(R_t\) is the rental rate of capital. With perfect idiosyncratic risk sharing, we would get the usual expressions where the wage and the rental rate of capital are equal to the marginal products of each factor. With incomplete risk sharing a risk premium emerges to compensate entrepreneurs for the uninsurable idiosyncratic risk they face when using capital and labor. The risk premium can be written \(\tilde{\sigma}_{c,e,t} \times \sigma\), where \(\tilde{\sigma}_{c,e,t} = f(k_{t,e,t})/n_{e,t}\) is entrepreneurs’ exposure to idiosyncratic risk in their consumption (or their net worth, \(\tilde{\sigma}_{c,e,t} = \tilde{\sigma}_{n,e,t}\)). Marginal utility is \(c_{it}^{-1}\), so its idiosyncratic risk is \(-\tilde{\sigma}_{c,e,t}\), while \(\sigma_t\) is the idiosyncratic risk of the marginal product of capital or labor. The marginal product of labor and capital is discounted with a risk premium that captures their covariance with the entrepreneur’s marginal utility.

It will be useful to work with the state variable \(z_t = c_{e,t}/c_{w.t}\). Using the fact that entrepreneurs’ consumption is \(c_{e,t} = \rho_t n_{e,t}\), and using the resource constraint, equation (1),

\[^3\text{See Angeletos [2006], Meh et al. [2004], Moskowitz and Vissing-Jørgensen [2002], and Himmelberg et al. [2004].}\]
we can re-write the risk premium as,

$$\tilde{\sigma}_{c,e,t} \sigma_t = \frac{(k_t / \ell_t)^{\alpha - 1}}{(k_t / \ell_t)^{\alpha - 1} - \phi(x_t)} \frac{1 + z_t}{z_t} \rho_t \sigma_t \times \sigma_t.$$ 

Working with $z_t$ as a state variable is useful because its law of motion is particularly simple, as we’ll show below. It’s not exposed to aggregate shocks, $\sigma_{z,t} = 0$. Notice that $z_t$ eliminated entrepreneurs’ net worth as a determinant of the risk premium. This means that, while their net worth is important, movements in entrepreneurs’ net worth do not play an important role in the response of the economy to the aggregate shock. This is a property of log preferences. The focus of this paper is not on the balance sheets of entrepreneurs—the topic of a large recent literature— but rather on the time-varying risk premium on capital and labor.

Market clearing in the labor market yields

$$\ell_t^{1 / \psi} c_{w,t} = (1 - \alpha) (k_t / \ell_t)^{\alpha} (1 - \tilde{\sigma}_{c,e,t} \sigma_t).$$

Replacing $c_w = c_t / (1 + z_t)$ and using the resource constraint again, we find the equilibrium condition in the labor market:

$$\ell_t^{1 / \psi} ((k_t / \ell_t)^{\alpha - 1} - \phi(x_t)) k_t / (1 + z_t) = (1 - \alpha) (k_t / \ell_t)^{\alpha} (1 - \tilde{\sigma}_{c,e,t} \sigma_t).$$ (6)

This is one of the main equilibrium equations.

We now need to determine the price of capital $q_t$ and therefore investment $x_t$, given by the optimality condition $\phi'(x_t) = q_t$, which implies

$$x_t = \frac{\log q_t}{\epsilon} + \delta \text{ and } \phi(x_t) = \frac{q_t - 1}{\epsilon} + \delta.$$ 

To pin down the price of capital, we use the entrepreneurs’ first-order condition for $k$ to obtain an asset-pricing equation:

$$\alpha (k_t / \ell_t)^{\alpha - 1} (1 - \tilde{\sigma}_{c,e,t} \sigma_t) - \phi(x_t) + (x_t - \delta + \mu_{q,t} - r_t - \pi_t^Z \sigma_{q,t}) q_t = 0.$$ (7)

We can solve out $r_t$ and $\pi_t^Z$. First, we know that

$$\pi_t^Z = \sigma_{c,e,t} = \sigma_{c,w,t} = \sigma_{c,t}.$$ 

The first and second equalities come from agents’ optimal aggregate risk sharing. The last equality comes from $c_t = c_{w,t} + c_{e,t}$.

For the interest rate, we use the Euler equation for workers and entrepreneurs, $r_t = \frac{1}{\ell_t} (k_t / \ell_t)^{\alpha} (1 - \tilde{\sigma}_{c,e,t} \sigma_t).$
\[ \rho_w + \mu_{c,w,t} - \sigma_{c,w,t}^2 \] and \[ r_t = \rho_e + \mu_{c,e,t} - \sigma_{c,e,t}^2 - \tilde{\sigma}_{c,e,t}^2. \] Weighing by their consumption shares, \(1/(1+z_t)\) and \(z_t/(1+z_t)\), we obtain

\[
\begin{align*}
    r_t &= \tilde{\rho}_t + \mu_{ct} - \sigma_{c,t}^2 - \frac{z_t}{1+z_t} \tilde{\sigma}_{c,e,t}^2 \\
    &= \text{perfect risk sharing} - \text{lower interest rate} - \text{prec. mot.}
\end{align*}
\] (8)

where \(\tilde{\rho}_t = \frac{1}{1+z_t} \rho_w + \frac{z_t}{1+z_t} \rho_e\) is the consumption-weighted impatience rate. The first part is the expression for the real interest rate in a model with perfect risk sharing. With incomplete risk sharing, entrepreneurs’ precautionary motive for idiosyncratic risk depresses the real interest rate, weighted by their consumption share.

Now we use the resource constraint to obtain an expression for \(\mu_{c,t}\) and \(\sigma_{c,t}\) in terms of the drift and volatility of \(q_t\) and \(\ell_t\):

\[
\mu_{c,t} = \mu_{k,t} + \frac{(1-\alpha)(k_t/\ell_t)^{\alpha-1}}{(k_t/\ell_t)^{\alpha-1} - \phi(x_t)} \left( \mu_{q,t} - \frac{\alpha \sigma_{q,t}^2}{2} - \frac{(q_t/\ell_t)\mu_{q,t}}{\sigma_{q,t}} \right)
\]

and

\[
\sigma_{c,t} = \frac{(1-\alpha)(k_t/\ell_t)^{\alpha-1}}{(k_t/\ell_t)^{\alpha-1} - \phi(x_t)} \left( \sigma_{q,t}^2 - (q_t/\ell_t)\sigma_{q,t} \right)
\]

Plugging into equation (7), we get a backward stochastic differential equation system for \(q\) and \(\ell\).

We look for a recursive equilibrium: a pair of \(C^2\) functions for the price of capital \(q\) and employment \(\ell\) as functions of the state variables \((k, \sigma, z)\). We already know the laws of motion of \(k\) and \(\sigma\). For \(z\), optimal aggregate risk sharing, \(\sigma_{c,e,t} = \sigma_{c,w,t}\), implies \(\sigma_{z,t} = 0\). The drift \(\mu_{z,t}\) is pinned down by the Euler equations:

\[
\mu_{z,t} = \mu_{c,e,t} - \mu_{c,w,t} = (\rho_w - \rho_e) + \left( \frac{(k_t/\ell_t)^{\alpha-1}}{(k_t/\ell_t)^{\alpha-1} - \phi(x_t)} \frac{1}{\ell_t} \frac{1+z_t}{z_t} \rho_e \sigma_t \right)^2.
\]

The last term is \(\tilde{\sigma}_{c,e,t}^2\), entrepreneurs’ precautionary saving motive. Here we can see that if entrepreneurs and workers had the same impatience rate, \(\rho_e = \rho_w\), entrepreneurs would accumulate all the wealth in the economy \((z_t \to \infty)\) because of their precautionary saving motive for idiosyncratic risk. We assume \(\rho_e > \rho_w\) to obtain a stationary distribution for \(z\).

Now we use Ito’s lemma to compute the drift and volatility of \(q\) and \(\ell\):

\[
\mu_q = \frac{q'k(x-\delta)k + q'\theta(\delta - \sigma) + q'\mu z + \frac{1}{2} q''v_{\sigma}(\sqrt{v_{\sigma}})^2}{q}, \quad \sigma_q = \frac{q'v_{\sigma}}{q},
\]

\[
\mu_\ell = \frac{\ell'k(x-\delta)k + \ell'\theta(\delta - \sigma) + \ell'\mu z + \frac{1}{2} \ell''v_{\sigma}(\sqrt{v_{\sigma}})^2}{\ell}, \quad \sigma_\ell = \frac{\ell'v_{\sigma}}{\ell}.
\]
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<td>2SD shock doubles $\sigma_t$</td>
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Table 1: Parameters

We can then plug this into equation (7) to obtain a second order partial differential equation for $q$ and $\ell$. Equation (6) gives us an algebraic constraint. Together they pin down $q(k, \sigma, z)$ and $\ell(k, \sigma, z)$. We solve this system numerically using Smolyak interpolation, as described in Judd et al. [2014].

4 Results

Parameter values. We solve the model with the following parameters, summarized in Table 1. The Cobb-Douglas exponent $\alpha = 1/3$ and the Frisch elasticity of labor supply $\psi = 3$ are standard in the macro literature. For the elasticity of the investment function, we use $\epsilon = 5$, which helps determine how lower output is split between lower investment and lower consumption.

We set workers’ impatience $\rho_w = 0.035$ and the depreciation rate $\delta = 0.07$ to obtain the steady state ratios $k_{ss}/y_{ss} = 3$ and $c_{ss}/y_{ss} = 0.8$, which play an important role in the model, as we describe below in section 4.2. We set $\rho_e = \rho_w + 0.0625$ to obtain a steady state idiosyncratic volatility of net worth $\bar{\sigma}_{n,ss} = \sqrt{\rho_e - \rho_w} = 25\%$, corresponding to the idiosyncratic risk in stock values.

For the stochastic process for $\sigma_t$ we use $\bar{\sigma} = 0.1$, in line with evidence in Bloom et al. [2018] on idiosyncratic productivity risk at the establishment level; $\theta_\sigma = 0.692$ so risk shocks have a half-life of one year; and $v_\sigma = \frac{1}{2}\sqrt{\bar{\sigma}} = 0.158$ so that a two-standard deviation shock doubles idiosyncratic risk. We aim to capture transitory fluctuations in idiosyncratic risk at the business cycle frequency. These numbers are broadly in line with evidence in Herskovic et al. [2016] for idiosyncratic risk in the stock market. The implied steady state consumption ratio is $z_{ss} = 5\%$.

Main results. Figure 1 shows the impulse response to a one-standard deviation risk shock, starting from the steady state (the long-run if shocks don’t realize). The first panel shows the behavior of idiosyncratic risk $\sigma_t$, which spikes by 5.5 percentage points on impact and
Figure 1: Impulse response to a risk shock

Figure 2: Sample path simulation
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</table>

Table 2: Business cycle moments. All variables are at the quarterly frequency and in logs. Data is HP filtered.

then returns to its long-run value of 10 percent, and the behavior of the idiosyncratic risk of entrepreneurs’ net worth or consumption, \( \tilde{\sigma}_{c,e,t} \), which spikes by 10 percentage points on impact and then returns to its long-run value of 25 percent. The idiosyncratic risk premium, \( \tilde{\sigma}_{c,e,t} \sigma_t \) therefore displays the same behavior. It spikes on impact by 3 percentage points, and then returns to its long-run value of 2.5 percent.

Risk shocks create recessions in the model. The second panel shows the responses of consumption, investment, and employment. They all fall on impact and slowly recover afterwards. Investment falls by roughly 5.5 percent, consumption by 1 percent and employment by 3 percent. This is broadly in line with stylized facts about US business cycles.

The third panel shows the behavior of real interest rates and wages. Wages fall on impact by around 2 percent, reflecting weaker labor demand, while interest rates have a small hump-shaped increase, reflecting the interaction of an expected recovery of consumption and a larger precautionary motive. The fourth panel shows the capital and labor wedges generated by uninsurable idiosyncratic risk, which we will discuss in the next section in detail.

The model generates comovement patterns that resemble business cycles in the data. Figure 2 shows a stochastic simulation of the economy—a sample path. Table 2 summarizes key moments in the model and compares them to US data. The model can successfully replicate business cycle behavior in the data. It is stylized in the interest of tractability and theoretical clarity, but our takeaway is that the mechanism we propose is quantitatively important and seems like a promising approach to understanding business cycles.

4.1 Wedges

During downturns idiosyncratic risk rises, making both capital and labor less attractive. The result is lower employment, consumption, and investment. The mechanism involves subtle general equilibrium effects. These can be understood from the wedges created by time-varying uninsurable idiosyncratic risk, in the spirit of Chari et al. [2007], where we take as the benchmark the efficient allocation with perfect risk sharing.
Risk shocks create time-varying labor and capital wedges. While higher risk reduces demand for labor and capital symmetrically, in general equilibrium lower interest rates affect labor and capital asymmetrically. The source of asymmetry is the long duration of capital and the short duration of employment. There is in addition a wedge in the law of motion of the consumption ratio $z_t$. While this plays an important role in the long run, it’s a slow moving state variable does not respond on impact to risk shocks, and therefore does not play an important role in business cycles.

We start with the labor wedge $\omega_{\ell,t}$. From the market clearing condition for labor, equation (6), we write

$$\ell_t^{1/\psi} \frac{c_t}{1 + z_t} = (1 - \alpha)(k_t/\ell_t)^{\alpha}(1 - \omega_{\ell,t}),$$  \hspace{1cm} (9)$$

where the labor wedge $\omega_{\ell,t}$ is equal to the risk premium for idiosyncratic risk,

$$\omega_{\ell,t} = \tilde{\sigma}_{c,e,t} \sigma_t.$$  \hspace{1cm} (10)$$

Higher risk $\sigma_t$ raises the risk premium on labor, making employment less attractive.

With $\omega_{\ell,t} = 0$, equation (9) is the equilibrium condition for employment in an economy with perfect risk sharing. If $\omega_{\ell,t} > 0$ the economy behaves as if labor income was taxed. Increases in this type of wedge reduces equilibrium employment and output, as well as consumption and investment. The extent of consumption-smoothing and the magnitude of capital adjustment costs determine the distribution between lower investment and lower consumption.

It’s important to highlight that the $1 + z_t$ term on the left side of equation (9) reflects only agent heterogeneity. We would have this even with perfect risk sharing, because only some agents actually provide labor. When $z_t$ is large, workers’ share of aggregate consumption is small so the income effect induces them to work more. Crucially, however, $z_t$ does not react to the risk shocks, $\sigma_{z,t} = 0$. The direct impact of a risk shock on the equilibrium condition for labor is purely through the labor wedge $\omega_{\ell,t}$.

Second, there is a capital wedge, as if capital income was taxed or subsidized. Combining both agents’ Euler equations and the asset pricing equation for capital, (7), we obtain

$$\alpha(k_t/\ell_t)^{\alpha-1}(1 - \omega_{k,t}) - \phi(x_t) - \delta + \mu_{q,t} - (\tilde{p}_t + \mu_{c,t} - \sigma^2_{c,t}) - \sigma_{c,t} \sigma_{q,t} = 0,$$  \hspace{1cm} (11)$$

where $\omega_{k,t}$ is the capital wedge. With $\omega_{k,t} = 0$, equation (11) yields the equilibrium condition for investment in an economy with perfect risk sharing. If $\omega_{k,t} > 0$ it’s as if capital income is taxed, reducing incentives for investment. In contrast to the labor wedge, this wedge does not create a recession. By itself, a lower $\omega_{k,t}$ reduces investment but increases consumption. This is the classic problem pin-pointed in Barro and King [1984], and the rea-
son the business cycle accounting of Chari et al. [2007] finds a minimal role for investment wedges.

The capital wedge is
\[
\omega_{k,t} = \left( \sigma_{c,e,t} \right) \times \left( 1 - \frac{1}{\alpha} \frac{\rho_c k_t q_t}{c_t} \right).
\]
(12)

It has the same form as the labor wedge, but the risk premium is dampened by the \(1 - \frac{1}{\alpha} \frac{\rho_c k_t q_t}{c_t} < 1\) factor.

Two forces operate on the capital wedge. Higher risk \(\sigma_t\) raises the risk premium on capital, making investment less attractive, as shown in (5). However, it also creates a precautionary saving motive, which depresses the interest rate, as shown in (8), and makes investment more attractive.\(^4\) Which force dominates depends on the sign of \(1 - \frac{1}{\alpha} \frac{\rho_c k_t q_t}{c_t}\). In general this term could be positive or negative.

To explain this, it’s useful to go over a well known benchmark. In a simple AK economy with \(\alpha = 1\), and \(\rho_e = \rho_w\), we know that \(c_t = \rho_c k_t\), and so \(1 - \frac{1}{\alpha} \frac{\rho_c k_t q_t}{c_t} = 0\). In this case the intertemporal wedge \(\omega_{k,t} = 0\) for any \(k_t, \sigma_t\), and \(z_t\). The risk premium and the precautionary motive always exactly cancel out. This is in fact a well known property of log preferences.

If, still in the AK environment, the intertemporal elasticity is greater than 1 (relative risk aversion below 1), then the risk premium dominates. Higher risk depresses investment and raises consumption on impact. If instead intertemporal elasticity is below 1 (relative risk aversion greater than 1), then the precautionary saving motive dominates. Higher risk raises investment and depresses consumption.

As we move away from the simple AK environment, we lose the simple characterization of the sign of the capital wedge. Angeletos [2006] studies this issue in an economy with uninsurable idiosyncratic risk on capital, but safe labor, and finds that the risk premium dominates if the intertemporal elasticity is greater than \(\alpha\). In our environment intertemporal elasticity is 1 (log preferences) but labor is also exposed to uninsurable idiosyncratic risk, and we don’t have an analytical characterization. We therefore turn to numerical solutions, and below in section 4.2 we provide a sufficient statistic in terms of measurable equilibrium objects.

The fourth panel of figures 1 and 2 show the labor and capital wedge, as an impulse response and as a stochastic simulation—a sample path. The main takeaway is that time-varying idiosyncratic risk creates a large countercyclical labor wedge and a small procyclical capital wedge. This constellation of wedges is what is needed to generate business cycles where consumption, investment, and employment co-move. The average labor wedge is 2.7 percent and has a standard deviation of 2.2 percent, while the average capital wedge is 0.35 percent and has a standard deviation of 0.25 percent. Their correlation is \(-0.9\).

\(^4\)To be clear, the precautionary motive reduces the interest rate conditional on the behavior of aggregate consumption, which will move because of the labor wedge.
Finally, there is also a wedge in the law of motion of the consumption ratio $z_t$. With perfect risk sharing, $\mu_{z,t} = \rho_w - \rho_e$ and in the long-run entrepreneurs’ consumption share $z_t/(1 + z_t) \to 0$. With incomplete risk sharing entrepreneurs have a precautionary motive for saving, so $\mu_{z,t} = \rho_w - \rho_e + \tilde{\sigma}_{c.e,t}^2$. As a result, their consumption share $z_t/(1 + z_t)$ has a non-degenerate ergodic distribution. However, because of perfect aggregate risk sharing $z_t$ does not respond on impact to risk shocks, $\sigma_{z,t} = 0$. As a result, while $z_t$ plays an important role in the long run, its role in business cycles is secondary.

*Asymmetry between capital and labor.* It’s natural to wonder why the labor and capital wedges are affected differently by an increase in idiosyncratic risk. The first thing to notice is that the risk-adjusted marginal product of capital and labor are affected symmetrically by the risk premium. In this sense capital and labor are symmetrical.

The asymmetry comes from the long duration of capital and the short duration of the employment relationship (zero because of spot labor markets). Investment depends on the present discounted value of the marginal product of capital. Here the risk premium and precautionary saving motive enter with opposite effects. While the risk premium depresses investment, the precautionary motive drives the risk free interest rate down. Because capital has a long duration, lower interest rates stimulate investment. As we’ve seen, the capital wedge is dampened relative to the labor wedge, and in the quantitative calibration is negative and procyclical.

To show this, we write the capital wedge as:

$$
\omega_{k,t} = \frac{q_t}{\alpha(k_t/\ell_t)^{\alpha-1}} \frac{z_t}{1 + z_t} \frac{\tilde{\sigma}_{c.e,t}^2}{\tilde{\sigma}_{c.e,t}}. 
$$

The first term captures the role of the risk premium on capital, which depresses demand for capital. This role is symmetric to labor. The second term captures the compensating role of lower interest rates because of entrepreneurs’ precautionary saving motive for idiosyncratic risk, weighted by entrepreneurs’ consumption share $z_t/(1 + z_t) = c_{e,t}/c_t$. The price-dividend ratio of capital plays a central role. If capital had a zero duration, its price dividend ratio would be zero, and we would recover $\omega_{k,t} = \tilde{\sigma}_{c.e,t} \sigma_t$, as for labor.

We can write the labor wedge analogously to equation (13), but because employment has zero duration, the price-dividend ratio is zero and we recover $\omega_{l,t} = \tilde{\sigma}_{c.e,t} \sigma_t > 0$. Because employment has zero duration in this model, lower interest rates have no effects on incentives to employ workers. We conjecture that if employment relationships had a short but strictly positive duration, such as in a search model, then the precautionary saving motive would also dampen the labor wedge, but to a lesser extent than the capital wedge because of the shorter duration of employment (reflected in a lower price-dividend ratio). The proper
duration in these models is not the expected length of an employment relationship, but the duration of the dividends from a marginal increase in recruiting activity—e.g. finding a worker 2 or 3 months earlier than otherwise.

There is also a technological asymmetry between labor and capital, in that capital supply is inelastic in the short-run, while labor has an elastic labor supply. This is why employment can fall on impact, while capital use is fixed. We conjecture that with variable capital utilization, capital use will also fall on impact. But this asymmetry is relevant to the different response of capital and labor to given wedges. It does not explain why the wedges themselves are different.

**Summary.** Risk shocks create time-varying labor and capital wedges. The labor wedge depresses employment, output, consumption, and investment. The capital wedge, instead, twists the economy by depressing investment and raising consumption. While higher risk reduces demand for labor and capital symmetrically, the lower interest rates produced by the precautionary saving motive affects them differentially because of their different duration. Because capital has a long-duration, lower interest rates dampen the capital wedge and may even stimulate investment. In contrast, employment has a short duration, so lower interest rates don’t affect it. The result is a large, countercyclical labor wedge and a small procyclical capital wedge, precisely what is needed to produce business cycles.

### 4.2 A sufficient statistic for the sign of the capital wedge

The capital wedge arises out of two conflicting forces. The risk premium reduces demand for capital, while the precautionary motives stimulates it. While we don’t have an analytical characterization of the sign of the capital wedge, we can express it in terms of meaningful measurable equilibrium objects that help us get a sense of its value. The sign of the capital wedge depends on whether the following expression is above or below 1:

\[
\frac{1}{\alpha} \frac{\rho_c q_t}{c_t} = \rho_c \times \frac{y_t}{c_t} \times \frac{k_t q_t}{y_t}.
\]

(14)

We know that \(\alpha = 1/3\), and that in steady state the capital income ratio, \(k_{ss} q_{ss}/y_{ss} = 3\) and the consumption-income ratio \(c_{ss}/y_{ss} = 0.8\), approximately. We can understand this formula in light of equation (13). Notice that \(\frac{1}{\alpha} \times \frac{k_t q_t}{y_t} = \frac{q_t}{\sigma_{g_t} k_t} \) is the price-dividend ratio of capital at the aggregate level (around 9 in steady state), and \(\rho_c y_t / c_t = \frac{\bar{\sigma}_{c,e,t}}{\sigma_{c,e,t} \sigma} \times \frac{1}{1+z_t} \) is the ratio of precautionary motive to the risk premium, weighted by entrepreneurs’ consumption share \(z_t/(1 + z_t) = c_{e,t}/c_t\).

The only remaining parameter is \(\rho_c\). The model pins down its value in terms of workers’ impatience rate \(\rho_w = 0.035\) and the steady state idiosyncratic risk of entrepreneurs’ net
worth (or consumption), $\tilde{\sigma}_{n,e,t} = 25$ percent, which is a core object of interest in this model:

$$\rho_e = \rho_w + \tilde{\sigma}_{n,e,t}^2 = 0.0975$$ (15)

This expression is easy to recover from the law of motion of the consumption ratio, $\mu_z = \rho_w - \rho_e + \tilde{\sigma}_{c,e,t}^2 = 0$. Essentially, entrepreneurs’ consumption will grow faster than workers’ because of their idiosyncratic precautionary motive, until their exposure to risk $\tilde{\sigma}_{c,e,t} = \tilde{\sigma}_{n,e,t}$ falls to the point where it just compensates their higher impatience. In other words, expression (15) tells us how much more impatient entrepreneurs must be if we observe them in a steady state, given their exposure to idiosyncratic risk. The value of $\rho_e$ we recover is high but not unreasonable.

These numbers yield a steady state value of $\frac{1}{\alpha} \frac{\rho_w k_{ss} q_{ss}}{c_{ss}} = 1.097$. The precautionary motive is slightly larger than the risk premium. In response to shocks, the capital-income ratio $k_t q_t / y_t$ and the consumption-income rate $c_t / y_t$ will move a little, but not much. So we get a small procyclical capital wedge, as shown in Figures 1 and 2.

While we have a good idea of how to discipline $\rho_w$, $k_{ss} q_{ss} / y_{ss}$, and $c_{ss} / y_{ss}$, the value of $\rho_e$ is more uncertain because it’s pinned down by the model in a roundabout way. However, this gives us a model-consistent benchmark we can use to understand the mechanism and think of comparative statics. For example, if we reduce $\rho_e$ while still maintaining the $k_{ss} / y_{ss}$ and $c_{ss} / y_{ss}$ ratios, the precautionary motive will become weaker relative to the risk premium and the capital wedge will become positive. It will always be dampened (more negative) compared to the labor wedge, $\omega_{\ell,t} > \omega_{k,t}$.

4.3 Excess return of capital, markups, and labor share

Here we explore implications of our model for some salient equilibrium objects that have received significant attention in the macro literature.

*Excess return of capital.* Our model creates a time-varying excess return of capital. Part of it corresponds to a risk premium for aggregate risk, which can be identified with the equity premium. But the incomplete idiosyncratic risk sharing produces a further excess return, above the risk premium for aggregate risk. We can re-write the equilibrium condition for capital, (7), using the expected or average marginal product of capital,

$$f'_k(k_t, \ell_t) - \phi(x_t) + (x_t - \delta + \mu_{q,t} - r_t) q_t = q_t \times \left( \pi_t^2 \sigma_{q,t}^{2} + f'_q(k_t, \ell_t) \times \tilde{\sigma}_{c,e,t} \sigma_t \right).$$ (16)

The difference with the capital wedge in equation (12) is that here we are using the equilibrium interest rate $r_t$, while the wedge $\omega_{k,t}$ is defined using the interest rate in the model.
with perfect risk sharing. That is, the wedge helps us understand the effect of incomplete idiosyncratic risk sharing in terms of capital and labor taxes in a model with perfect risk sharing, while the excess return in (16) highlights the failure of the perfect-risk-sharing asset-pricing equation at equilibrium prices, ignoring that the equilibrium interest rate \( r_t \) is lower than what it would be with perfect risk sharing given the same allocation.

The advantage of equation (16) is that it’s more directly related to the data. Farhi and Gourio [2018] points out that since the return to capital has remained roughly constant over the past decades, while interest rates have gone down, the excess return on capital has become larger. A rising risk premium is one possible explanation, together with rising market power and intangibles, but measures of the equity premium suggest it’s stable or even decreasing. Although our model is not designed to address secular trends, the presence of an excess return above the equity premium is consistent with our mechanism.

Quantitatively, however, the total excess return attributable to idiosyncratic risk is small, 0.3 percent on average, with a standard deviation of 0.23 percent and a correlation with output of −0.84. The equity premium produced by the model is also small, around 0.017 percent, with a standard deviation of 0.02 percent. Our model is essentially a consumption CAPM with relative risk aversion equal to 1, where the equity premium is equal to the covariance of aggregate consumption and stock returns. It is well known that such models cannot explain the equity premium.

The asset pricing literature has explored several avenues to explain the equity premium puzzle, such as habits as in Campbell and Cochrane [1999] (exploited by Kehoe et al. [2018]), heterogenous risk aversion as in Longstaff and Wang [2012], Gárleanu and Panageas [2015], or Kekre and Lenel [2019], or balance sheet effects as in He et al. [2015]. Our model does not have any of these ingredients, but we believe that exploring the interaction of asset pricing and business cycles is a fruitful area for future research.

**Markups and labor share.** Following Rotemberg and Woodford [1999], a common approach to business cycles is to focus on the cyclical properties of markups. Our model produces countercyclical markups. The marginal cost of goods is \( w_t / f'_t(k_t, \ell_t) = (1 - \tilde{\sigma}_{c,e,t}) \), so we get markups over marginal cost

\[
\mu_t = \frac{1}{1 - \tilde{\sigma}_{c,e,t} \sigma_t} - 1 \approx \tilde{\sigma}_{c,e,t} \sigma_t.
\]

Alternatively we can compute the procyclical labor share of income,

\[
\eta_t = \frac{w_t \ell_t}{f'_t(k_t, \ell_t)} = (1 - \alpha)(1 - \tilde{\sigma}_{c,e,t} \sigma_t).
\]

It’s worth stressing that the mechanism in our paper does not reduce to a time-varying markup because the precautionary saving motive for idiosyncratic risk also depresses the
interest rate relative to the model with perfect risk sharing and a time-varying markup. Taking prices as given, the average user cost of capital is \( q_t \times (r_t + \pi_t^Z \sigma_{q,t} + \delta + \phi(x_t) - x_t - \mu_{q,t}) = f'_k(k_t, \ell_t) \times (1 - \sigma_{c,e,t} \sigma_t), \) which implies a markup of \( f'_k(k_t, \ell_t)/\text{user cost of } k = 1/(1 - \sigma_{c,e,t} \sigma_t) \approx \sigma_{c,e,t} \), the same as using the labor margin. The capital share of income is also procyclical.\(^5\) But we have to remember that the equilibrium interest rate is lower than what it would be with perfect risk sharing and a variable markup because of the precautionary saving motive for idiosyncratic risk. As a result, instead of a common capital and labor wedge (as we would get from adding markups to a perfect risk sharing model), our model delivers a large countercyclical labor wedge and a small procyclical capital wedge, which as we’ve seen delivers business cycles in response to risk shocks.

The average markup in the model is 2.7 percent, with a standard deviation of 2.2 percent and a correlation with output of –0.82. The average labor share is roughly 64 percent, with a standard deviation of 1.4 percent, and a correlation with output of 0.82. In the data, an average markup of 15 percent is common in the literature.\(^6\) While the presence of markups is consistent with our model, quantitatively our model cannot explain such large markups. Common ingredients such as imperfect competition and distortionary taxes are required to account for markups in the data.

### 4.4 Sensitivity analysis

To understand how the model works, we consider perturbations to the benchmark calibration. We focus on the standard deviations of output, consumption, investment, and employment. Since the objective is to understand how the model works, we don’t recalibrate the model when changing each parameter. Table 3 summarizes results.

**Frisch elasticity \( \psi \).** A successful model of business cycles requires a relatively large elasticity of labor supply. With a smaller Frisch elasticity around \( \psi = 1 \), the effects of risk shocks would be dampened, but would still look like business cycles. The second column of table 3 shows the standard deviations of all variables would be smaller.

In the limiting case with \( \psi = 0 \) employment and output would be fixed and could not respond to risk shocks. We would still get a countercyclical capital wedge, so risk shocks would create a small spike in investment and a small contraction in consumption, with no movement in employment or output on impact.

There is a large literature on the appropriate value of \( \psi \) for macro models, to which we

\(^{5}\)When computing markups and capital and labor shares of income, we are counting the profits obtained by entrepreneurs, \( f(k_t, \ell_t)\sigma_{c,e,t} \sigma_t \), as neither labor nor capital income. The pure profit share, \( \sigma_{c,e,t} \sigma_t \), is countercyclical. In the data, depending on exactly how these profits are distributed, a fraction might show up as labor income (e.g. bonuses) or capital income (e.g. accounting profits for private firms).

\(^{6}\)See Edmond et al. [2018], Hall [2018]. Recent work by De Loecker and Eeckhout [2017] finds an average markup of 60 percent. However, this is a sales-weighted markup. Edmond et al. [2018] report a cost-weighted markup using the same data of 25 percent.
don’t have much to add. Rogerson [1988] introduces lotteries to capture the extensive margin for labor supply. Hall [2009] suggests that an elasticity around 3 is a reasonable working approximation for employment fluctuations in an economy with a search and matching setup and realistic equilibrium wage stickiness, in the sense of Hall [2005].

Persistence $\theta/\sigma$ and capital adjustment costs $\epsilon$. The mean reversion parameter $\theta/\sigma$ and the curvature of the adjustment cost function $\epsilon$ are important in determining how lower output is split into lower consumption and lower investment in response to a larger labor wedge. The elasticity of intertemporal substitution would also play a role here, but it’s pinned at one with log preferences.

We calibrate a transitory risk shock with a half-life of one year. Agents are averse to fluctuations in consumption in response to such transitory shocks, so investment must take the brunt of the adjustment. A curved capital adjustment cost function reduces fluctuations in investment and increases fluctuations in consumption.

We calibrate the model with $\epsilon = 5$ to obtain a ratio of standard deviation of consumption to investment roughly in line with the data. To put this number in context, $\epsilon = 5$ means that a contraction in investment of 10 percent is accompanied by a reduction in the marginal cost of capital of 2.5 percent. Without adjustment costs, the effect on the marginal cost of capital would be zero.

The second and third columns of table 3 show standard deviations for smaller adjustment costs $\epsilon = 3$ and for more persistent risk shocks $\theta_v = 0.462$, which implies a half-life of two years. With smaller adjustment costs the standard deviation of consumption becomes smaller, and that of investment larger. With more persistent shocks all standard deviations become larger.

Volatility of risk shocks $\sigma_v$. Risk shocks are the only exogenous driving force in the model. With a smaller volatility of idiosyncratic risk, the standard deviation of all variables would become smaller. The fourth column of table 3 shows standard deviations for a smaller $\sigma_v = 0.1$.

Steady state level of idiosyncratic risk $\bar{\sigma}$. The effects of risk shocks become larger with the long-run level of idiosyncratic risk. The effect of risk enters the model through second moments, so if idiosyncratic risk is very small, a small increase has only second order effects. We calibrate the long-run level of idiosyncratic risk $\bar{\sigma} = 10\%$ in line with Bloom et al. [2018]. A lower long-run level of idiosyncratic risk $\bar{\sigma} = 5\%$ reduces the effects of risk shocks, and therefore reduces the standard deviation of all variables, as shown in the fifth column of table 3.
Table 3: Sensitivity analysis

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</tr>
</tbody>
</table>

*Steady state idiosyncratic risk in entrepreneurs’ net worth σ̄_n,ss*. The effects of risk shocks also depend on the level of idiosyncratic risk in entrepreneurs’ consumption or net worth. The sixth column of table 3 shows standard deviations for a smaller target for this idiosyncratic risk, σ̄_n,ss = 15%. Hitting this target requires a lower impatience rate of entrepreneurs, ρ_e = ρ_w + σ^2_n,ss = 0.0575.

5 Efficiency

To study the efficiency of the competitive equilibrium, we consider a planner who can use taxes on capital and labor to distort the capital and labor wedges in the competitive equilibrium. Because the role of risk shocks can be understood in terms of time-varying labor and capital wedges, this allows us to ask if the planner would like a different macroeconomic behavior in response to risk shocks. The planner has to live with the fundamental frictions in the model and cannot eliminate incomplete risk sharing or prevent agents from saving and sharing aggregate risk.

This planner’s problem can be fully microfounded in an environment with a moral hazard problem with hidden trade, as in Di Tella and Sannikov [2016] or Di Tella (2019). An entrepreneur’s idiosyncratic shock, as well as the resulting capital and labor decisions and hidden savings cannot be observed. The optimal private contract takes the form of the reduced-form incomplete risk sharing problem we’ve used this far. We can then ask what is the best that a planner can do subject to the same contractual frictions. The resulting planner’s problem coincides with the planner’s problem in this section. In the interest of simplicity, we focus here on the reduced-form problem where the planner can use taxes to distort wedges, but the contractual microfoundation is useful to understand why this is the right planner problem to consider, and where the source of inefficiency lies.

The main takeaway from this section is that the response of the competitive equilibrium to a risk shock is inefficient. Output and employment fluctuations are too large, and consumption should move countercyclically. In the competitive equilibrium, risk shocks create a small and procyclical capital wedge, and a large countercyclical labor wedge. As a result, risk shocks create recessions with positive comovement in output, employment,
investment, and consumption. The planner, instead, wants a large and countercyclical capital wedge, and a small and procyclical labor wedge. The result is the typical Barro and King [1984] pattern with negative comovement. Output, employment, and investment fall, but consumption goes up. Because consumption and investment are negatively correlated, fluctuations in output and employment are smaller.

5.1 The planner’s problem

We consider the case where the planner can use labor and capital taxes with lump-sum rebates. In this case, the planner can effectively control employment \( \ell_t \), investment \( x_t \), and therefore aggregate consumption \( c_t \). However, in addition to the resource constraints, equations (1) and (2), and the law of motion of the exogenous states \( \sigma_t \), equation (3), the planner must take as given entrepreneurs’ idiosyncratic risk. Computing \( \tilde{\sigma}_{c_t} \), we obtain:

\[
\tilde{\sigma}_{c_t} = f(k_t, \ell_t) \rho_e \sigma_t = \frac{k^\alpha \ell^{1-\alpha}}{c_t} \left( 1 + \frac{z_t}{z_t} \right) \rho_e \sigma_t. \tag{17}
\]

In addition, because all agents have access to the same financial market, the planner must respect their Euler equations and their aggregate risk sharing. Combining them, we get a law of motion for the consumption ratio, \( z_t = c_{e,t}/c_{w,t} \):

\[
\mu_{z,t} = (\rho_w - \rho_e) + \sigma^2_{c_t}, \quad \sigma_{z,t} = 0. \tag{18}
\]

This means that \( z_t \) will be a state variable for the planner (\( z_0 \) is chosen optimally). If the planner could control agents’ access to the financial market, preventing entrepreneurs and workers from trading intertemporally or across aggregate states, then \( z_t \) would not be a state variable. It’s important to note that while the planner must respect agents’ Euler equations, this does not restrict the control of aggregate consumption, which is a choice variable, not a state variable. The reason is that the Euler equation involves the real interest rate \( r_t \), which the planner does not take as given. For any behavior of aggregate consumption, as long as we respect the law of motion of \( z_t \), there is a process for the real interest rate \( r_t \) that satisfies agents’ Euler equations. Likewise, for any response of aggregate consumption to risk shocks, there is a process \( \pi^Z_t \) that satisfies their aggregate risk sharing equations.

The planner’s objective function is a weighted average of the utility of the representative worker and entrepreneurs, \( \gamma U_w + (1 - \gamma) U_e \). Strictly speaking, we should specify the Pareto weight on each entrepreneur, \( \gamma_i \). However, entrepreneurs’ allocations differ only in their scale, so for aggregate allocations we only need to know the Pareto weight on entrepreneurs as a whole.

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7In the moral hazard microfoundation, the IC constraint requires \( \sigma_{U_{it}} = \frac{f(k_{it}, \ell_{it})}{c_{it}} \sigma_t \), and consumption is \( c_{it} = \hat{c}_i \exp(\rho_i U_{it}) \), where \( \hat{c}_i \) depends only on the aggregate states. Computing \( \tilde{\sigma}_{c_{it}} \), we obtain (17).
It is useful to write an individual entrepreneur’s utility as

$$\mathbb{E} \left[ \int_0^\infty e^{-\rho_w t} \log c_{i,t} dt \right] = \mathbb{E} \left[ \int_0^\infty e^{-\rho_w t} \left( \log c_{e,t} - \frac{1}{2} \rho_w \bar{\sigma}_{c,e,t}^2 \right) dt \right].$$

The planner’s objective function is therefore

$$\mathbb{E} \left[ \int_0^\infty \gamma e^{-\rho_w t} \left( \log c_{w,t} - \ell_t^{1+1/\psi} / (1 + 1/\psi) \right) + (1 - \gamma) e^{-\rho_w t} \left( \log c_{e,t} - \frac{1}{2} \rho_w \bar{\sigma}_{c,e,t}^2 \right) dt \right].$$

Notice that because entrepreneurs are more impatient than workers, $\rho_e > \rho_w$, in the first best with perfect risk sharing they would receive zero consumption in the long-run, $\lim_{t\to\infty} z_t = 0$. Here because entrepreneurs can save on their own and have a precautionary motive, their consumption will not vanish in the long run. The consumption ratio $z_t$ will have an ergodic distribution, and will converge to a steady state in the absence of shocks. However, it is still true that entrepreneurs’ utility vanishes from the objective function in the long run. As we’ll see below, there is a fundamental disagreement between the planner and private entrepreneurs about their consumption profile. They save according to their Euler equations, but the planner would like them to follow an Inverse Euler equation, that eliminates the precautionary saving motive for idiosyncratic risk. Because we care about the properties of the planner problem in the ergodic distribution, we can ignore entrepreneurs’ utility and maximize only workers’ utility subject to a given $z_t$. The Pareto weight $(1 - \gamma)$ matters for how we choose $z_0$ and how we converge to the ergodic distribution.

The planner’s HJB equation is therefore

$$\rho_w V(k, \sigma, z) = \max_{c,x,\ell} \log c + \log \left( \frac{1}{1 + z} \right) - \frac{\ell_t^{1+1/\psi}}{1 + 1/\psi} + V'_{k} k(x - \delta) + V'_{\ell} \ell(x - \delta) + V'_{\sigma} \sigma(\sigma - \bar{\sigma}) + \frac{1}{2} V''_{\sigma} \sigma^2$$

subject to

$$c + \phi(x)k = k^{\alpha} \ell^{1-\alpha}$$

and (17). Notice that, in this formulation, the planner does not care directly about en-

---

8 An individual entrepreneur’s consumption follows $dc_{i,t}/c_{i,t} = \mu_{e,e,t} dt + \sigma_{e,e,t} dB_{i,t} + \sigma_{e,e,t} dZ_{i,t}$, so $c_{i,t} = (c_{i,0}/c_{e,0}) \times c_{e,t} \times \exp \left( \int_0^t \sigma_{e,e,t} dB_{i,t} - \frac{1}{2} \sigma_{e,e,t}^2 dt \right)$. His utility is

$$\mathbb{E} \left[ \int_0^\infty e^{-\rho_e t} \log c_{i,t} dt \right] = \log \left( c_{i,0}/c_{e,0} \right) + \mathbb{E} \left[ \int_0^\infty e^{-\rho_e t} \left( \log c_{e,t} - \int_0^t \frac{1}{2} \sigma_{e,e}^2 ds \right) dt \right]$$

$$= \log \left( c_{i,0}/c_{e,0} \right) + \mathbb{E} \left[ \int_0^\infty e^{-\rho_e t} \left( \log c_{e,t} - \frac{1}{2} \rho_e \bar{\sigma}_{c,e,t}^2 \right) dt \right].$$

The first term captures the initial inequality among entrepreneurs. If they all had the same Pareto weight we would pick $c_{i,0} = c_{e,0}$ for all entrepreneurs.

9 This approach has significant computational advantages, but comes at a cost. The utility gain from switching to the planner’s allocation, conditional on the state, is only a lower bound on the utility gain from implementing the optimal allocation. This is the opposite to the problem that would arise if we evaluate the utility gains in the steady state.
trepreneurs’ consumption or exposure to idiosyncratic risk. The planner only cares because if he exposes them to risk $\tilde{\sigma}_{c,e,t}$ they will save (higher $z_t$ in the future), leaving less consumption for workers, $c_{w,t} = c_t/(1 + z_t)$.

Instead of working with the HJB equation, it’s easier and more revealing to work with the co-states, $m_k = V'_k$, $m_\sigma = V'_\sigma$, $m_z = V'_z$. Taking first-order conditions, we have

$$\frac{1}{c} (1 - 2m_z z \tilde{\sigma}_{c,e}^2) = \lambda$$  \hspace{0.5cm} (20)

$$(1 - \alpha)(k/\ell) \left( \lambda + 2m_z z \frac{\tilde{\sigma}_{c,e}^2}{k^{\alpha} \ell^{1-\alpha}} \right) = \ell^{1/\psi}$$  \hspace{0.5cm} (21)

$$\lambda \phi'(x) = m_k,$$  \hspace{0.5cm} (22)

where $\lambda$ is the Lagrange multiplier on the resource constraint. The first condition says that giving consumption not only delivers utility, but also relaxes entrepreneurs’ risk sharing, $\tilde{\sigma}_{c,e}$. This determines the marginal value of goods $\lambda$. The second condition equates the marginal disutility of labor, $\ell^{1/\psi}$, to its marginal product, taking into account that more labor increasing entrepreneurs’ exposure to idiosyncratic risk. The third condition says that the marginal value of more capital should be equated to its marginal cost. Together with the resource constraint we can solve for the controls $c$, $x$, $\ell$, and $\lambda$ as a function of the states and co-states. It’s hard to obtain closed form expressions, but it’s easy to eliminate $c$ and $\lambda$ and obtain a system of two equations for $x$ and $\ell$.

Now we differentiate the HJB equation with respect to each state to obtain a law of motion for $m_k$ and $m_z$ ($m_\sigma$ is not directly used):

$$\rho_w m_k = m_k (x - \delta) + 2m_z z \tilde{\sigma}_{c,e}^2 \frac{\alpha}{k} + \lambda \left( \alpha (k/\ell)^{\alpha-1} - \phi(x) \right) + \mu_{m_k} m_k$$  \hspace{0.5cm} [m_k] \hspace{0.5cm} (23)

$$\rho_w m_z = -\frac{1}{1 + z} - 2m_z z \frac{\tilde{\sigma}_{c,e}^2}{1 + z} + m_z (\rho_w - \rho_e + \tilde{\sigma}_{c,e}^2) + \mu_{m_z} m_z$$  \hspace{0.5cm} [m_z]. \hspace{0.5cm} (24)

The equation for $m_k$ resembles an asset-pricing equation for capital. Capital delivers dividends net of new investment, transformed into utility using $\lambda$. It grows at rate $x - \delta$ but it’s discounted more heavily because it exposes entrepreneurs to idiosyncratic risk. With this interpretation, the first-order condition for investment, $x$, can be understood as a Tobin’s $Q$ expression, properly taking into account the role of idiosyncratic risk. The equation for $m_z$ captures the fact that higher $z$ means that a smaller fraction of consumption goes to workers (who are all the planner cares about in the long-run formulation). On the other hand, higher $z$ reduces entrepreneurs’ exposure to idiosyncratic risk, and therefore the future value of $z$.

We look for a pair of $C^2$ functions $m_k(k, \sigma, z)$ and $m_z(k, \sigma, z)$. Using Itô’s lemma we transform equations (23) and (24), together with the first-order conditions (20) and the resource constraint, into a system of two second order PDEs and two algebraic constraints,
Table 4: Standard deviation of main variables under competitive equilibrium and social planner’s allocation.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Comp. Eq.</th>
<th>Social Planner</th>
<th>Comparison</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>1.7%</td>
<td>0.62%</td>
<td>↓</td>
</tr>
<tr>
<td>( c )</td>
<td>0.88%</td>
<td>0.96%</td>
<td>=</td>
</tr>
<tr>
<td>( i )</td>
<td>5.8%</td>
<td>5.5%</td>
<td>=</td>
</tr>
<tr>
<td>( \ell )</td>
<td>2.2%</td>
<td>0.56%</td>
<td>↓</td>
</tr>
<tr>
<td>( \omega_{\ell t} )</td>
<td>2.2%</td>
<td>0.9%</td>
<td>↓</td>
</tr>
<tr>
<td>( \omega_{k t} )</td>
<td>0.3%</td>
<td>24%</td>
<td>↑</td>
</tr>
</tbody>
</table>

using the resource constraint and first FOC to eliminate \( c \) and \( \lambda \). We can solve this differential-equation system using numerical methods analogous to those used to solve for the competitive equilibrium.

**Implied prices, wedges, and taxes.** After finding the optimal allocation, we can back out prices and wedges. Set \( q_t = \phi'(x_t) \), \( w_t = \ell_t^{1/\psi} c_t/(1 + z_t) \), \( r_t = \bar{\rho}_t + \mu_{c,t} - \sigma_{c,t}^2 - \frac{z_t}{1 + z_t} \tilde{\sigma}_{c,e,t}^2 \), and \( \pi_t^2 = \sigma_{c,t}^2 \). The implied labor and capital wedges, \( \omega_{\ell,t} \) and \( \omega_{k,t} \) are defined by:

\[
\ell_t^{1/\psi} \frac{c_t}{1 + z_t} = (1 - \alpha)(k_t/\ell_t)^\alpha (1 - \omega_{\ell,t})
\]

(25)

\[
\alpha(k_t/\ell_t)^{\alpha-1}(1 - \omega_{k,t}) - \phi(x_t) + q_t (x_t - \delta + \mu_{q,t} - (\bar{\rho}_t + \mu_{c,t} - \sigma_{c,t}^2) - \sigma_{c,t}\sigma_{q,t}) = 0.
\]

(26)

The optimal allocation can be implemented with a labor and capital tax, \( \tau_{\ell,t} \) and \( \tau_{k,t} \). Entrepreneurs’ after-tax expected marginal product of labor is \( f'_{\ell}(k_t, \ell_t)(1 - \tau_{\ell,t}) \); for capital it’s \( f'_k(k_t, \ell_t)(1 - \tau_{k,t}) \). The tax does not affect the idiosyncratic risk, it’s levied before the idiosyncratic shock realizes, so that it does not interfere with the incomplete risk sharing problem.

\[
\tau_{\ell,t} = \omega_{\ell,t} - \left( \frac{(k_t/\ell_t)^{\alpha-1} + z_t}{(k_t/\ell_t)^{\alpha-1} - \phi(x_t)} \frac{1}{z_t} \rho_e \sigma_{\ell_t}^2 \right)
\]

and

\[
\tau_{k,t} = \omega_{k,t} - \left( \frac{(k_t/\ell_t)^{\alpha-1} + z_t}{(k_t/\ell_t)^{\alpha-1} - \phi(x_t)} \frac{1}{z_t} \rho_e \sigma_{\ell_t}^2 \right) \left( 1 - \frac{1}{\alpha} \frac{\rho_{c,k} q_t}{c_t} \right).
\]

5.2 Numerical results

The main result is that the recession pattern of the responses of capital, employment, and consumption in the competitive equilibrium is inefficient. In the constrained efficient allocation, output and employment fluctuations are smaller, and consumption moves countercyclically. Figure 3 shows the impulse response to a risk shock in the planner’s allocation.
Figure 3: Impulse response to a risk shock in the planner’s solution

Figure 4: Sample path simulation of the planner’s solution
While employment and investment fall, consumption expands. This is the typical Barro and King [1984] pattern. As a result, employment falls less than in the competitive equilibrium.

We can probe these results by looking at the implied labor and capital wedges. The competitive equilibrium featured a small procyclical capital wedge and large countercyclical labor wedge. In contrast, the planner wants a large countercyclical capital wedge and a small procyclical labor wedge. The capital wedge twists the economy in the direction of less investment but more consumption. In contrast to the competitive equilibrium, employment and total output fall because with more consumption, the income effect reduces labor supply. This effect is reflected in an increase in real wages. In contrast, in the competitive equilibrium the labor wedge reduced demand for labor, so wages fell on impact.

The real interest rate falls on impact. Lower expected consumption growth and a larger precautionary saving motive push interest rates down. In contrast, in the competitive equilibrium agents expected a higher consumption growth as the economy recovered, but this was compensated by a larger precautionary motive.

Figure 4 shows a simulated sample path, for the same realization of shocks as in Figure 2, where the negative covariance between investment and consumption is apparent. Table 4 compares the competitive equilibrium with the constrained efficient allocation. In the ergodic distribution the standard deviation of consumption and investment are roughly the same as in the competitive equilibrium, but their negative correlation (−0.88) means that the standard deviations of output and employment are lower. The average labor wedge (4.5 percent) and capital wedge (−8.4 percent) are larger than in the competitive equilibrium, but while the labor wedge becomes less volatile, the capital wedge becomes much more volatile.

The planner’s allocation can be implemented with a combination of (1) a countercyclical labor tax—lower labor taxes, or subsidize labor, during recessions to eliminate the labor wedge; and (2) a procyclical capital tax—raise the capital tax during recessions to reduce investment and free more resources for consumption. Figure 5 shows the impulse response to a risk shock and a sample path simulation. In the ergodic distribution $\tau_{\ell,t}$ has a mean of 1.7 percent (a tax on labor) with a standard deviation of 2.6 percent. It’s positive in the steady state, but becomes negative after risk shocks. In contrast, $\tau_{k,t}$ has a mean of −8 percent but a very large standard deviation of 24 percent. It’s negative in the steady state but becomes very positive after risk shocks.

5.3 Understanding the inefficiency

To understand why the competitive equilibrium is inefficient, it’s useful to go over the role of hidden savings, and then use that to understand why the planner wants different wedges than in the competitive equilibrium.
The role of hidden savings. The inefficiency ultimately arises from the presence of hidden savings in the environment. Without hidden savings, the planner would like agents to follow the Inverse Euler equation, which eliminates the precautionary saving motive. Understanding the optimal allocation without hidden savings helps us see what the planner is hoping to accomplish in the environment with hidden savings.

To see this issue in detail, consider the environment without hidden savings, so $z_t$ is not a state variable. The planner can decide how much consumption goes to workers and entrepreneurs separately. The first-order conditions for consumption for finite $t$ are

$$\gamma e^{-\rho t} c_{w,t}^{-1} = \lambda_t = (1 - \gamma) e^{-\rho t} c_{e,t}^{-1} (1 + \frac{1}{\rho e} \tilde{\sigma}_{c,e,t}^2).$$

Let $\lambda_t$ be the pricing kernel (up to a constant), so workers satisfy their Euler equation. Rearranging, we get that $\lambda_t e^{\rho t} c_{e,t} / (1 + \frac{1}{\rho e} \tilde{\sigma}_{c,e,t}^2)$ must be a constant, and since an individual entrepreneur’s $c_{i,t}$ is equal to $c_{e,t}$ times an idiosyncratic martingale, we get that $\lambda_t e^{\rho t} c_{i,t} / (1 + \frac{1}{\rho e} \tilde{\sigma}_{c,e,t}^2)$ is a martingale. The term $c_{i,t} / (1 + \frac{1}{\rho e} \tilde{\sigma}_{c,e,t}^2)$ is the marginal cost of delivering utility to the entrepreneur, so this expression says that the discounted marginal cost of utility must be a martingale. This is the essence of the Inverse Euler equation. In the standard setting, the marginal cost of utility is the inverse of the marginal utility of consumption, $c_{i,t}^{-1}$. Here it’s modified to take into account that higher consumption reduces the entrepreneur’s exposure to idiosyncratic risk, as captured by (17). In a steady state without aggregate shocks, however, $\tilde{\sigma}_{c,e,t}$ is constant, so we recover the traditional Inverse Euler equation: $e^{(\rho e - r_t) t} c_{i,t}$ is a martingale, or in flow form,

$$\mu_{c,e,t} = r_t - \rho e. \quad (27)$$

The crucial difference with the Euler equation is that it’s missing the precautionary savings term, $\tilde{\sigma}_{c,e,t}^2$. The planner would like to eliminate entrepreneurs’ precautionary saving motive, and to front-load entrepreneurs’ consumption relative to workers’ (raise $z_t = c_{e,t}/c_{w,t}$) when idiosyncratic risk $\tilde{\sigma}_{c,e,t}$ is large, to relax idiosyncratic risk sharing. But when agents have
access to hidden savings, he is forced to respect their Euler equation and aggregate risk sharing, captured by (18).

Labor wedge. Coming back to the setting with hidden savings, we can study the inefficiency in terms of the implied labor and capital wedges, and how they compare to those in the competitive equilibrium. As in the competitive equilibrium, the difference in duration plays a central role in generating the asymmetric response of capital and labor. It is easier to focus first on the labor wedge, which does not involve dynamic considerations, and then extend the analysis to the capital wedge.

We can re-write the first-order condition for labor to obtain an expression for the labor wedge,

\[
1 - \omega_{t, t} = \frac{\text{improve risk sharing} > 0}{\text{worsen risk sharing} < 0} \left(1 - \frac{(2m_{z,t}z_t\tilde{\sigma}^2_{c,e,t})}{1 + z_t y_t} \times \frac{c_t}{y_t}\right). 
\]

(28)

The forces can be split into those that are primarily about the long-run, and those that are important for business cycles.

Business cycles. The numerator of (28) captures the impact of employment on idiosyncratic risk. On the one hand, more employment increases entrepreneurs’ exposure to idiosyncratic risk. Private agents also realize this, which is why they demand a risk premium to compensate them. But the planner also realizes that more aggregate consumption \(c_t\) relaxes the idiosyncratic risk sharing problem, as can be seen in equation (17). Essentially, if aggregate consumption is higher, entrepreneurs’ consumption must also be higher, given the consumption ratio \(z_t\). Because their consumption policy is \(c_{e,t} = \rho_e n_{e,t}\), this means that their net worth is higher and their exposure to idiosyncratic risk, \(\tilde{\sigma}_{c,e,t} = \tilde{\sigma}_{n,e,t} = f(k_t, \ell_t)\), is lower, other things equal.

These countervailing forces create a countercyclical labor wedge. Consider increasing employment by a small amount \(d\ell > 0\), and using the extra output to increase aggregate consumption. Besides the usual considerations (marginal product vs. marginal rate of substitution between labor and consumption), this will improve idiosyncratic risk sharing. We can write entrepreneurs’ exposure to idiosyncratic risk

\[
\tilde{\sigma}_{c,e,t} = \frac{f(k_t, \ell_t)}{c_t + f'(k_t, \ell_t)d\ell} 1 + z_t - \rho_e \sigma_t. 
\]

Because \(y_t = f(k_t, \ell_t) > c_t\), increasing output and consumption by the same amount reduces \(\tilde{\sigma}_{c,e,t}\). Instead of increasing idiosyncratic risk, as private agents think, more employment actually improves idiosyncratic risk sharing. In consequence, the planner wants a lower,
and even negative, labor wedge after risk shocks when \( \sigma_t \) is large. He wants to subsidize labor.

To formalize this, notice that the marginal value of an extra unit of aggregate consumption is not only that it delivers marginal utility, \( c_t^{-1} \), but also the marginal improvement in idiosyncratic risk sharing, \(-2c_t^{-1}m_{z,t}z_t\sigma_{c,e,t}^2 > 0\). This is reflected in the expression for \( \lambda_t \), equation (20). It’s useful to write

\[
\lambda_t = \frac{1}{c_t}M_t,
\]

where \( M_t = 1 - 2m_{z,t}z_t\sigma_{c,e,t}^2 > 1 \) captures the extra value of consumption from improving risk sharing.

When the planner considers the marginal value of labor, he takes into account that it adds to idiosyncratic risk, just like agents in the competitive equilibrium. That’s what the term \( 2m_{z,t}z_t\sigma_{c,e,t}^2/(k_t^{\alpha_t}y^{1-\alpha_t}) \) in equation (21) captures. But he also realizes that the extra output relaxes idiosyncratic risk sharing through higher consumption. That’s why the marginal product of labor is weighted with \( \lambda_t \). The numerator in the labor wedge equation (28) captures both these considerations. The two forces go in opposite directions and become larger after risk shocks, but it’s clear that the improvement in idiosyncratic risk sharing dominates because \( c_t/y_t < 1 \). This gives us a countercyclical labor wedge.

**Long-run.** The denominator of (28) eliminates the \( 1 + z_t \) denominator in the definition of the labor wedge, equation (25), which captures an income effect on labor supply. The planner doesn’t care about entrepreneurs’ utility in the long-run because they are more impatient. He would like to give them zero consumption, \( z_t = 0 \), but can’t prevent them from saving following their Euler equation, which includes an inefficient precautionary saving motive, as explained above.

As a result, we have \( z_t > 0 \) and only a fraction \( 1/(1 + z_t) \) of the extra consumption goes to workers. But the planner gives workers only part of aggregate consumption not because he puts less than complete weight on them, in which case they would also be made to work more, but because of a constraint. He must give entrepreneurs some consumption, but he is not forced to make workers work more. The result is a positive labor wedge that undoes the income effect on labor supply.

However, it’s important to notice that because \( \sigma_{z,t} = 0 \), the denominator does not respond on impact to risk shocks, so this force does not play an important role in business cycles.

**Capital wedge.** The same forces apply to the capital wedge, but because capital is a long-duration asset, investment involves intertemporal tradeoffs. First, notice that \( m_{k,t} = \lambda_t\phi'(x_t) = \lambda_tq_t \) is the marginal cost of new capital in utility terms. In standard models, it would be \( c_t^{-1}q_t \), reflecting the forgone utility from consumption. Here, instead, the lower
aggregate consumption also worsens idiosyncratic risk sharing, which is taken into account by \( \lambda_t = c_t^{-1}M_t \). So the cost of investment for the planner is larger than what private agents realize, and moves with risk shocks. On the other hand, the marginal product of capital will not only be risky, as private agents realize, but also improve risk sharing through increased aggregate consumption, analogously to labor. So it will also be weighted by \( \lambda_t \) in the future.

Now we can use the law of motion for \( m_k \), equation (23), and rewrite it in terms analogous to the competitive equilibrium condition, equation (7),

\[
\alpha \left( \frac{k_t}{\ell_t} \right)^{\alpha-1} \left( 1 - (m_{z,t}z_t 2\tilde{\sigma}_{c,e,t}^2)(1 - c_t/(k_t^{\alpha}\ell_t^{1-\alpha})) \right) - M_t \phi(x_t),
\]

\[
+ M_t q_t \left( x_t - \delta + \mu_{q,t} - (\bar{\rho}_t + \mu_{c,t} - \sigma_{c,t}^2) - \sigma_{c,t} \sigma_{q,t} \right),
\]

\[
+ M_t q_t \left( \mu_{M,t} + \sigma_{M,t} (\sigma_{q,t} - \sigma_{c,t}) + (\bar{\rho}_t - \rho_w) \right) = 0
\]

The differences from equation (7), help explain the inefficiency. First, the marginal product of capital is multiplied by \( (1 - (2m_{z,t}z_t \tilde{\sigma}_{c,e,t}^2)(1 - c_t/(k_t^{\alpha}\ell_t^{1-\alpha})) \) instead of \( (1 - \tilde{\sigma}_{c,e,t} \sigma_{t}) \). The planner internalizes that the marginal product of capital is risky but also helps improve idiosyncratic risk sharing through higher aggregate consumption. This is symmetric with labor.

Second, the cost of creating new capital \( \phi(x_t) \), and the value of capital in the future \( q_t \), is weighted by \( M_t \) because those resources could also be used to relax idiosyncratic risk sharing. If \( M_t \) is high today—for example, if \( \sigma_t \) is large—but is expected to be lower in the future (\( \mu_{M,t} < 0 \))—today is a bad time to invest. In this case, investment uses goods when their value from relaxing idiosyncratic risk sharing is high, and will deliver goods when their value relaxing idiosyncratic risk sharing is expected to be low. The term \( \sigma_{M,t} (\sigma_{q,t} - \sigma_{c,t}) \) captures the covariance between \( M_t \) and the marginal-utility-weighted value of capital \( c_t^{-1}q_t \) to properly incorporate aggregate risk. In a steady state without aggregate shocks \( M_t \) is constant, \( \mu_{M,t} = \sigma_{M,t} = 0 \), and these dynamic issues vanish.

The last term, \( \bar{\rho}_t - \rho_w > 0 \), reflects that the planner doesn’t care directly about entrepreneurs (in the long-run), and so would like to discount the future using \( \rho_w \) rather than \( \bar{\rho}_t = \frac{z_t}{1+z_t} \rho_e + \frac{1}{1+z_t} \rho_w \). The planner would like to give entrepreneurs zero consumption, \( z_t = 0 \), in which case \( \bar{\rho}_t = \rho_w \), but cannot because they save to prevent this. This issue is analogous to the role of the \( 1 + z_t \) denominator in the expression for the labor wedge, (28). Ultimately, the disagreement reflects the inefficiency of entrepreneurs’ precautionary saving motive, as we explained above. However, while this inefficiency may be important in the steady state, it does not play a role in the impact of risk shocks, because \( z_t \) and therefore \( \bar{\rho}_t \) do not respond on impact.
We can use equation (29) to obtain an expression for the capital wedge:

\[
1 - \omega_{k,t} = \left( \frac{1}{1 - \left(2m_{z,t}z_t\sigma_{c,e,t}^2\right)} + \left(2m_{z,t}z_t\sigma_{c,e,t}^2\right) \times \frac{\sigma_{c,e,t}}{\mu_{M,t} + \sigma_{M,t}\sigma_{q,t} - \sigma_{M,t}\sigma_{c,t} + (\rho_t - \rho_w)} \right)
\]

If we compare the expressions for the labor and capital wedge, equations (28) and (30), we see that if the price-dividend ratio for capital, \(q_t\), was zero, the dynamic part of the capital wedge would disappear. We wouldn’t quite obtain the labor wedge \(1 - \omega_{\ell,t}\), however, because instead of \(1 + z_t\) in the denominator we have \(M_t\). The difference is that investment requires goods which could be used to relax idiosyncratic risk sharing, captured by \(M_t\), while labor requires only forgoing workers’ utility from leisure, which cannot relax idiosyncratic risk sharing. In the ergodic distribution, most of the variation in the capital wedge comes from the second term in equation (30). The first term has a standard deviation of 2.5 percent, compared to 25 percent for the total capital wedge.

The role of \(\rho_e > \rho_w\). We assumed entrepreneurs are more impatient than workers to obtain a stationary distribution of consumption and wealth, in both the competitive equilibrium and the planner’s allocation. It may seem that this assumption is driving the result that in the long-run, the planner does not care about entrepreneur’s utility directly. That is, their utility vanishes from the planner’s objective functions (19).

Here we’ll argue that \(\rho_e > \rho_w\) is not essential to this issue. The fundamental issue is that the planner and private entrepreneurs disagree on their appropriate consumption profile. The planner would like to eliminate the precautionary saving motive for idiosyncratic risk and front-load their consumption, so in the long run the only reason he cares about their consumption and risk sharing is to satisfy the hidden savings constraint.

An analogous situation would arise with \(\rho_e = \rho_w\). In this case, because of the precautionary saving motive for idiosyncratic risk, entrepreneurs would accumulate all the wealth and consumption in the long-run, \(\lim_{t \to \infty} z_t = \infty\), both in the competitive equilibrium and the planner’s allocation. The planner would be forced to give almost all of the consumption to entrepreneurs because of the hidden savings constraint. So while the utility of entrepreneurs would not vanish from the objective function (19), the allocation would be so far away from the distribution of consumption that optimizes the objective function in the absence of the hidden savings constraint, that on the margin the planner would not value entrepreneurs’ utility directly (it vanishes relative to the utility of workers). The only
motive to give entrepreneurs consumption or reduce their exposure to risk would come from the hidden savings constraint, as in the $\rho_e > \rho_w$ case.

**Summary.** In the planner’s allocation the labor wedge is procyclical because, while the marginal product of labor has idiosyncratic risk, the planner internalizes that the extra output can also be used to improve idiosyncratic risk sharing through higher aggregate consumption. Both forces go in opposite directions and both become larger after risk shocks, but the improvement in idiosyncratic risk sharing dominates, so the labor wedge is procyclical.

The same logic applies to investment, but the resources used to produce capital could also be used to improve idiosyncratic risk sharing through higher aggregate consumption. Periods when idiosyncratic risk is high, and especially if it’s expected to be lower in the future, are particularly bad periods for investment. The planner does not want to use resources when their value relaxing idiosyncratic risk sharing is high, and get more output or capital in the future when the value of relaxing idiosyncratic risk sharing is low. So we get a large countercyclical capital wedge.

The inefficiency arises from the presence of hidden savings, which are important in the microfoundation of the incomplete idiosyncratic risk-problem. The planner would like entrepreneurs to follow an Inverse Euler equation which front-loads their consumption, especially when idiosyncratic risk is high. However, he must respect their Euler equations. This creates inefficiency both in the long-run and in response to risk shocks.

### 6 Conclusions

In this paper we propose a theory of business cycles driven by spikes in risk premiums that act like negative demand shocks for capital and labor. In this view, recessions are periods of heightened uncertainty, when businesses and investors shrink from risky economic activity. There is a long tradition that attributes business cycles to time-varying risk premiums, going back to chapter 12 of the *General Theory* (Keynes [1936]). However, the comovement pattern of recessions poses a long-standing challenge to this view. It is very hard to explain why employment, consumption, and investment contract simultaneously. This is the essence of the Barro and King [1984] problem, and essentially the reason why the macro literature has focused on productivity shocks and monetary shocks with nominal rigidities as drivers of business cycles. In this paper we aim to provide a theoretical framework to explain business cycles without productivity shocks or nominal rigidities.

Our first result is that spikes in risk premiums can generate quantitatively realistic business cycles, with comovement between employment, consumption, and investment. The mechanism hinges on the interaction of risk premiums and the precautionary saving motive,
and the different duration of capital and labor plays a central role. Our second result is that
the resulting business cycles are not efficient. Employment and output fluctuations are too
large, and consumption should move countercyclically. The constrained-efficient allocation
can be implemented by subsidizing labor and taxing capital during recessions.

Our model is stylized in the interest of theoretical clarity. The only departure from the
neoclassical growth model is incomplete idiosyncratic risk sharing on the business side. Let
us mention here some avenues for further research that seem promising to us. First, our
model of the risk premium is simple, driven by spikes in idiosyncratic risk. We conjecture
that a spike in the price of risk should have similar effects. We know from the asset
pricing literature that habits, heterogenous preferences, ambiguity aversion, and recursive
preferences can play an important role. Second, we abstracted from search frictions in
the labor market to focus on the underlying general equilibrium forces. But incorporating
unemployment into the framework seems like a natural step.
References


