Risk Premium Shocks Can Create Inefficient Recessions*

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Abstract

We develop a simple flexible-price model of business cycles driven by spikes in risk premiums. Aggregate shocks increase firms' uninsurable idiosyncratic risk and raise risk premiums. We show that risk shocks can create quantitatively plausible recessions, with contractions in employment, consumption, and investment. Business cycles are inefficient—output, employment, and consumption fall too much during recessions, compared to the constrained-efficient allocation. Optimal policy involves stimulating employment and consumption during recessions.

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1 Introduction

Market economies experience recurrent recessions with sharp contractions in economic activity. In this paper we explore a risk-premium view of business cycles—recessions are periods of heightened economic uncertainty when firms shrink from risk.

We propose a simple flexible-price model of business cycles driven by spikes in risk premiums. The premise of our model is that businesses face significant uninsurable idiosyncratic risk, and demand a risk premium as compensation. Idiosyncratic risk rises during downturns and drives risk premiums up. We show that risk shocks can create business cycles, with employment, consumption, and investment declining together in recessions. We go on to show that these economic fluctuations are inefficient—output, employment, and consumption fall too much during recessions, compared to the constrained-efficient allocation. Optimal policy calls for stimulating employment and consumption during recessions.

A long tradition attributes business cycles to time-varying risk premiums, dating back, at least, to the General Theory (Keynes (1936)), and focusing on the negative impact of higher risk premiums on investment demand. We make two observations that lead us to emphasize, instead, the negative impact of higher risk premiums on labor demand. First, employing workers is a risky endeavor carrying a countercyclical risk premium that acts like a tax on labor. Second, in contrast to labor, capital is a long-duration store of value, so while the risk premium depresses investment demand, a concurrent precautionary saving motive depresses interest rates and stimulates investment. We derive a sufficient statistic to evaluate these two forces on investment demand, and conclude that they roughly cancel out when calibrated to US data. Risk depresses labor demand but leaves investment demand unaffected because of the different duration of labor and capital. Recessions are times when businesses reduce their demand for risky labor. In general equilibrium, the decline in labor demand leads to contractions in employment, consumption, and investment.

The view of business cycles we propose has important policy implications. We characterize the constrained-efficient allocation that respects the key incompleteness in idiosyncratic risk sharing. The competitive economy responds inefficiently to risk shocks, with an excessive contraction in employment and consumption. The inefficiency can be understood in terms of an externality—contractions in aggregate consumption during downturns aggravate the risk sharing problem. Private agents consume according to their Euler equations, taking interest rates as given, without an incentive to internalize the impact of their consumption on idiosyncratic risk sharing. The planner subsidizes employment and consumption to improve idiosyncratic risk sharing during downturns with elevated idiosyncratic risk.

We calibrate our model to US data, and find that the mechanism we propose can produce quantitatively plausible economic fluctuations. We don't claim these quantitative results as definitive. The model is stylized in the interest of theoretical clarity, but we think it provides a promising way to understand business cycles.

Overview of the model. Our baseline model is the neoclassical growth model with uninsurable idiosyncratic risk on the firm side. The economy is populated by two types of agents, workers and entrepreneurs. Both have log preferences over consumption, and workers also supply labor elastically. Entrepreneurs rent capital and hire labor in competitive spot markets, but production involves idiosyncratic risk proportional to output. The only aggregate shock is that the cross-sectional dispersion of idiosyncratic shocks follows a mean-reverting process. The only friction is that idiosyncratic shocks cannot be insured. Workers and entrepreneurs trade Arrow securities contingent on aggregate shocks, but not contingent on the idiosyncratic shocks. There are no TFP shocks or nominal rigidities.

The assumption that entrepreneurs cannot insure their idiosyncratic shocks plays a central role—with insurance, there would be no aggregate fluctuations because full risk sharing would occur. With incomplete risk sharing, a risk premium emerges to compensate entrepreneurs for the uninsurable idiosyncratic risk they face. A crucial feature of our model is that the marginal products of capital and labor are locally uncertain, exposed to firm-specific risk. When entrepreneurs hire labor or rent capital, they don't know the realization of their marginal products for sure. For example, a contractor who uses equipment and workers to build office space does not know with much precision how long the project will take and what the ultimate cost and value of the building will be. In other words, using capital and labor to produce is a risky activity.

Entrepreneurs hire workers and rent capital in competitive markets before the realization of the idiosyncratic shocks, so they bear the idiosyncratic risk as residual claimants. As a result, the marginal product of capital and of labor is discounted by a risk premium that captures their covariance with the marginal utility of the entrepreneur. Uninsurable idiosyncratic risk also creates a precautionary saving motive for entrepreneurs, which stimulates investment because capital is a store of value. The precautionary motive counteracts the negative effect of the risk premium on investment, but not on labor, which is not a store of value. In general equilibrium, employment, consumption, and investment fall in response to risk shocks. Investment falls not because risk shocks directly reduce demand for investment, but rather because they reduce employment and output. Because agents want to smooth consumption, in general equilibrium investment declines more than consumption.

Uninsurable idiosyncratic risk. Uninsurable idiosyncratic risk plays a central role in our model. A large literature shows that firms face a large amount of idiosyncratic risk which rises in recessions, both in terms of establishment-level productivity and demand shocks and in stock returns.[1] Figure 1 shows idiosyncratic risk in stock returns, HP-filtered to highlight the business-cycle fluctuations. Spikes in idiosyncratic risk are visible during recessions, especially during the Great Depression and the 2008 financial crisis.

[1] See Christiano et al. (2014), Gilchrist et al. (2014), Herskovic et al. (2016), and Bloom et al. (2018).

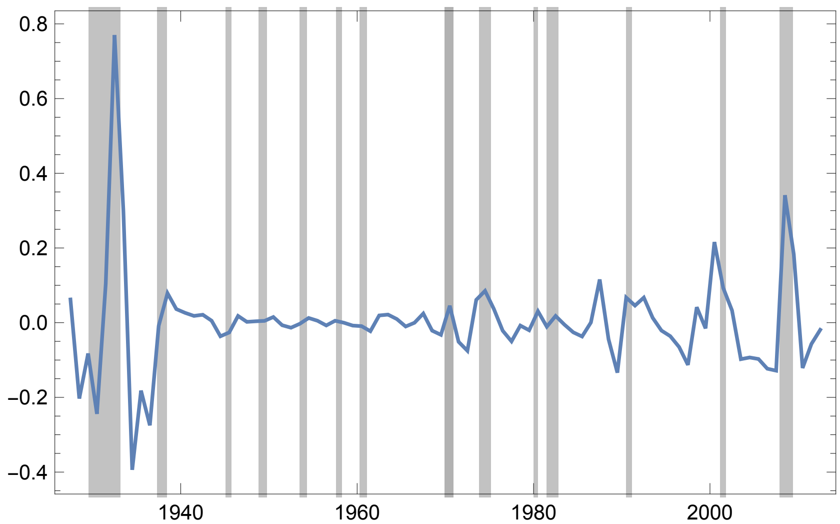

Figure 1: HP-filtered annualized idiosyncratic risk in daily stock market returns, after extracting five principal components, from Herskovic et al. (2016). Post-war mean is 0.28.

We take a Knightean view of entrepreneurs as risk-takers (Knight (1921)). Firms are run by risk-averse entrepreneurs, insiders who must retain an exposure to their firm's idiosyncratic risk for incentive purposes. Kihlstrom and Laffont (1979) develop a theory of the firm based on this idea, and Angeletos (2007) and Meh and Quadrini (2004) study the effect of entrepreneurial uninsurable idiosyncratic risk on long-run capital accumulation. We deploy this idea to explain business cycles.

Our model applies most directly to private firms, not traded in public markets, where entrepreneurs and other insiders often have substantial equity holdings. Private firms account for a significant fraction of output and employment in the US economy. Asker et al. (2015) report that private U.S. firms account for 69 percent of private employment and 59 percent of sales. In contrast, public firms have a more diversified ownership. However, large investors and upper management often retain large risk exposures through equity, bonuses, and stock options. Himmelberg et al. (2004) report that the median inside-ownership fraction at public firms is 19 percent in the US. This share is naturally smaller in the largest firms. But even in the case of large firms, there are some salient examples with concentrated insider ownership, such as Amazon, Facebook, and Alphabet.

Although we focus on countercyclical shocks in the quantity of idiosyncratic risk in the interest of concreteness, we believe that fluctuations in the price of risk could be part of the same story. Many asset pricing explanations for time-varying risk premiums, such as habits (Campbell and Cochrane (1999)), or heterogenous agents (Longstaff and Wang (2012), Gârleanu and Panageas (2015)), boil down to time-varying risk aversion. These asset pricing models deal with risk premiums for aggregate risk, but variations in risk aversion will also affect risk premiums for idiosyncratic risk. We believe there are large returns to incorporating more sophisticated asset pricing theories in models of macroeconomic fluctuations.

Relationship to other literature. Our paper fits within and extends a literature that we call the risk-premium view of business cycles, that highlights fluctuations in either the quantity of risk or in its price as drivers of business cycles. Our model rests on a set of choices that we believe lead business-cycle modeling to interesting, realistic, and novel conclusions: Our model is derived from the neoclassical growth model with labor and capital; it embodies a risk-related driving force of aggregate fluctuations; it describes procyclical movements of investment, employment, and consumption; it does not rely on unrealistic procyclical movements of TFP as a driving force; it does not rely on the New Keynesian propagation mechanism. Here we discuss additional contributions to the literature relevant to our paper.

Uninsurable idiosyncratic risk. Our paper emphasizes that entrepreneurs and other firm insiders face uninsurable idiosyncratic risk for which they require compensation. This point is made by Kihlstrom and Laffont (1979) who use it to develop a theory of the extent of the firm. Meh and Quadrini (2006) and Angeletos (2007) study the impact of uninsurable idiosyncratic risk on capital returns for long-run capital accumulation. These papers do not study business cycles. Goldberg (2014, 2019), Williamson (1987), and Di Tella (2017) study the impact of fluctuations in uninsurable idiosyncratic risk on business cycles and financial crises. In these models, idiosyncratic risk only affects capital returns, so consumption expands on impact. To address this issue, authors studying fluctuations in uninsurable idiosyncratic risk in capital returns such as Christiano et al. (2014) and Caballero and Simsek (2018) incorporate nominal rigidities in a New Keynesian framework where monetary policy does not reproduce the flexible-price allocation. Also within a New Keynesian framework, Ilut and Schneider (2014) study the effect of changes in ambiguity about TFP. More generally, Basu and Bundick (2017) observe that flexible-price general-equilibrium models driven by uncertainty shocks have trouble replicating the procyclical pattern of investment, employment, and consumption that characterizes recessions. They conclude, "we view this macroeconomic comovement as a key minimum condition that business-cycle models driven by uncertainty fluctuations should satisfy". They also propose nominal rigidities. Fernández-Villaverde and Guerrón-Quintana (2020) obtain procyclical aggregate consumption in response to aggregate risk shocks, but obtain countercyclical consumption for workers. We aim to provide an account of business cycles with parallel movement among macroeconomic aggregates that does not hinge on productivity shocks or nominal rigidities.

Financial frictions and irreversible decisions. Our model abstracts from bankruptcy and non-convexities and emphasizes the precautionary saving motive. Arellano et al. (2019)

introduce costly bankruptcy for firms that cannot insure against idiosyncratic risk, so hiring workers is risky for firms. Buera and Moll (2015) show that a credit crunch reduces labor demand if it shifts resources away from firms with more efficient recruiting. Both models abstract from capital and investment, which plays a central role in our model, and thus their models do not embody a precautionary motive. Papers that do consider capital include Jermann and Quadrini (2012), who introduce working-capital requirements in an RBC model with borrowing constraints that tighten after financial shocks. Occhino and Pescatori (2015) model a debt-overhang problem which becomes worse during downturns. Bloom et al. (2018) model non-convex adjustment costs for labor and capital, which create a real-option channel through which higher idiosyncratic risk reduces investment. Gilchrist et al. (2014) add costly bankruptcy and incomplete financial markets, which allows them to address the behavior of credit spreads and other financial data. These papers include capital but abstract from the precautionary motive, so the same mechanism that reduces labor demand also has a large negative effect on investment, leading to countercyclical consumption in response to risk or financial shocks. Authors including Bloom et al. (2018) address this by adding concurrent TFP shocks, which we avoid.

Labor search frictions. Our paper is related to the labor-search literature that portrays employment matches as assets subject to financial shocks. Hall (2017) introduces exogenous shocks to discount rates in a model with labor search frictions to explain unemployment fluctuations. Kilic and Wachter (2018) build on this by modeling the asset pricing side with disaster risk, while Kehoe et al. (2019) introduce on-the-job human capital accumulation, which allows them to make progress on the Shimer puzzle without inefficient wage setting. These papers abstract from investment, and emphasize the intertemporal dimension of employment decisions, which blurs the asymmetry between labor and capital that plays an important role in our mechanism. We make the point that both labor and capital are risky, but only capital is a significant store of value. In an extension of their model, Kilic and Wachter add investment and accept countercyclical consumption, while Kehoe et al. (2019) add investment in an extension with TFP shocks.

2 Setting

Our baseline model is the neoclassical growth model extended to include uninsurable idiosyncratic risk on the firm side. There are two types of agents, workers and entrepreneurs. A representative worker supplies labor, and entrepreneurs use capital and labor to produce goods. They have the same log preferences over consumption, and workers have separable disutility from labor with Frisch elasticity ψ. In obvious notation, preferences are:

$$U^w(c_w, \ell) = \mathbb{E}\left[\int_0^\infty e^{-\rho_w t}\left(\log(c_{wt}) - \frac{\ell_t^{1+1/\psi}}{1+1/\psi}\right)dt\right],$$

and

$$U^e(c_i) = \mathbb{E}\left[\int_0^\infty e^{-\rho_e t}\log(c_{it})dt\right].$$

We assume entrepreneurs are more impatient that workers, $\rho_e > \rho_w$, to obtain a stationary wealth distribution.

Each entrepreneur is exposed to idiosyncratic risk. The output flow for entrepreneur i is

$$dY_{it} = f(k_{it}, \ell_{it})dt + f(k_{it}, \ell_{it})v_t dB_{it}. \tag{1}$$

The expected output flow is $f(k, \ell) = k^\alpha \ell^{1-\alpha}$, a standard Cobb-Douglas production function. But in addition the entrepreneur is exposed to idiosyncratic risk B_i, a Brownian motion specific to entrepreneur i. The risk is proportional to output, and v_t captures the level of idiosyncratic risk. Idiosyncratic risk washes out in the aggregate, so aggregate output flow is $f(k_t, \ell_t)dt$, as usual. As a result, the aggregate resource constraints are

$$c_t + i_t = f(k_t, \ell_t) \tag{2}$$

and

$$dk_t = (i_t - \delta k_t)dt, \tag{3}$$

where $c_t = c_{wt} + c_{et}$ is aggregate consumption, $c_{et} = \int c_{it} di$ is total consumption by entrepreneurs, i_t is investment and δ is the rate of depreciation of capital. In the numerical solution we will add standard convex adjustment costs to capital. Appendix A has the details.

The level of idiosyncratic risk v_t follows a mean-reverting diffusion

$$dv_t = \theta_v(\bar{v} - v_t)dt + \sqrt{v_t}\sigma_v dZ_t \tag{4}$$

driven by an aggregate Brownian motion Z that captures risk shocks. This is the only source of aggregate risk in this economy and the only exogenous driving force for business cycles.

Markets are complete for aggregate risk—agents can trade Arrow securities contingent on the realization of Z. However, entrepreneurs cannot insure their idiosyncratic risk B_i for incentive reasons—they cannot trade Arrow securities contingent on the realization of B_i. This is the only friction in the economy.

The representative worker's problem. The worker's problem is to choose processes for consumption c_w, labor ℓ, and risk sharing σ_{nw}, to solve

$$\max_{c_w,\ell,\sigma_{nw}} U^w(c_w,\ell) \tag{5}$$

$$s.t. \quad dn_{wt} = (n_{wt}r_t + n_{wt}\sigma_{nwt}\pi_t + w_t\ell_t - c_{wt})dt + n_{wt}\sigma_{nwt}dZ_t, \tag{6}$$

and the natural borrowing limit. r_t is the risk-free interest rate, π_t is the price of aggregate risk Z, and w_t the wage rate. We treat σ_{nw} as a choice variable because there are complete markets for aggregate risk. Workers can use Arrow securities to choose their exposure to aggregate risk σ_{nw}, and get a reward for taking aggregate risk $n_{wt}\sigma_{nwt}\pi_t$.[2]

Entrepreneurs' problem. An entrepreneur's problem is to choose processes for consumption, production, and risk sharing $(c_i, k_i, \ell_i, \sigma_{ni})$ to solve

$$\max_{c_i,\ell_i,k_i,\sigma_{ni}} U^e(c_i) \tag{7}$$

$$st: \quad dn_{it} = (n_{it}r_t + n_{it}\sigma_{nit}\pi_t - c_{it} + f(k_{it},\ell_{it}) - w_t\ell_{it} - R_t k_{it})dt + n_{it}\sigma_{nit}dZ_t + f(k_{it},\ell_{it})v_t dB_{it}, \tag{8}$$

and the natural borrowing limit $n_{it} \geq 0$. R_t is the rental rate of capital. Capital itself is priced by arbitrage and can be held by both entrepreneurs and workers. Entrepreneurs can use Arrow securities to choose their exposure to aggregate risk σ_{ni} independently of other choices, and get a reward $n_{it}\sigma_{nit}\pi_t$. But they cannot share their idiosyncratic risk. If they could, they would perfectly insure and eliminate the $f(k_{it},\ell_{it})v_t dB_{it}$ term. This is the only friction in this economy.

Competitive equilibrium. Total wealth is $n_{et} + n_{wt} = k_t$, where $n_{et} = \int n_{it}di$ is the total wealth of entrepreneurs. For a given initial distribution of wealth, a *competitive equilibrium* is a process for prices (r, π, w, R), aggregate capital k, a plan for the representative worker (c_w, ℓ), and a plan for each entrepreneur $(c_i, k_t, \ell_i, \sigma_{ni})$ such that every agent optimizes taking prices as given; the aggregate resource constraints (2) and (3) hold; and markets clear: $\int \ell_{it}di = \ell_t$, $\int k_{it}di = k_t$, and $n_{et} + n_{wt} = k_t$.

[2] The budget constraint for the representative worker is equivalent to $\mathbb{E}\left[\int_0^\infty \xi_t c_{wt}dt\right] \leq n_{w0} + \mathbb{E}\left[\int_0^\infty \xi_t \ell_t w_t dt\right]$, where ξ_t is the pricing kernel, with law of motion $d\xi_t/\xi_t = -r_t dt - \pi_t dZ_t$. The risk-free rate r_t is the drift of ξ, and the price of risk π_t its loading on Z.

3 Characterizing the Competitive Equilibrium

The main departure of our model from the neoclassical growth model is time-varying uninsurable idiosyncratic risk. We express its effects in terms of a risk premium, which depresses demand for capital and labor, and a precautionary motive for idiosyncratic risk which reduces interest rates.

The representative worker's problem is completely standard because they do not face idiosyncratic risk. Entrepreneurs face uninsurable idiosyncratic risk, but they do not have a labor supply decision, so their problem can be mapped into a standard consumption-portfolio problem with a well known solution. Homothetic preferences and linear budget constraints imply that policy functions are linear in wealth, which avoids the need to keep track of the distribution of wealth across entrepreneurs. We focus here on the main economic relationships. Appendix A provides details.

Idiosyncratic risk. Entrepreneurs' exposure to idiosyncratic risk plays a central role in our model. Using the budget constraint (8) and the fact that each entrepreneur uses capital and labor proportional to their net worth, they all have the same idiosyncratic risk in their net worth $v_{net} = f(k_{it}, \ell_{it})/n_{it} \times v_t = y_t/n_{et} \times v_t$. With log preferences consumption is $c_{it} = \rho_e n_{it}$, so idiosyncratic risk in entrepreneurs' consumption is $v_{cet} = v_{net}$. Define $\eta_t = c_{et}/c_t$, the consumption share of entrepreneurs, which is a state variable. Replacing $n_{et} = c_{et}/\rho_e$ and using $c_{et} = \eta_t c_t$, we obtain an expression for idiosyncratic risk in entrepreneurs' consumption,

$$v_{cet} = \frac{k_t^\alpha \ell_t^{1-\alpha}}{c_t} \rho_e \eta_t^{-1} v_t. \tag{9}$$

Risk premium for idiosyncratic risk. From entrepreneurs' problem we obtain Marshallian demand functions for labor and capital,

$$w_t = \overbrace{(1-\alpha) k_t^\alpha \ell_t^{-\alpha}}^{\text{perfect risk sharing}} \times \Big(1 - \overbrace{v_{cet} v_t}^{\text{risk pr.}}\Big) \tag{10}$$

and

$$R_t = \alpha k_t^{\alpha-1} \ell_t^{1-\alpha} \times \Big(1 - v_{cet} v_t\Big). \tag{11}$$

With perfect idiosyncratic risk sharing, $v_{cet} = 0$, we would get the usual expressions where the wage and the rental price of capital are equal to the marginal products of each factor. With incomplete risk sharing a risk premium emerges to compensate entrepreneurs for the uninsurable idiosyncratic risk they face when using capital and labor. The risk premium $v_{cet} v_t$ captures the covariance of the marginal product of capital or labor with the entrepreneur's marginal utility, c_{it}^{-1}, and reduces demand for capital and labor symmetrically.

Precautionary saving motive for idiosyncratic risk. Uninsurable idiosyncratic risk also shows up as a precautionary saving motive for entrepreneurs, which depresses equilibrium interest rates. Using the worker's and entrepreneur's Euler equations, weighted by the consumption share η_t, we obtain the equilibrium interest rate

$$r_t = \overbrace{\bar{\rho}_t + \mu_{ct} - \sigma_{ct}^2}^{\text{perfect risk sharing}} - \overbrace{\eta_t \times \underbrace{v_{cet}^2}_{\text{prec.}}}^{\text{lower interest rate}}, \tag{12}$$

where $\bar{\rho}_t = \eta_t \rho_e + (1 - \eta_t)\rho_w$ is the consumption-weighted impatience rate. The first part of (12) is the expression for the real interest rate in a model with perfect risk sharing. With incomplete risk sharing, entrepreneurs' precautionary motive for idiosyncratic risk v_{cet}^2 depresses the real interest rate, weighted by their consumption share η_t. A lower interest rate makes capital more attractive and stimulates investment.

Labor and capital markets. The two main equilibrium conditions come from the labor and capital markets. The worker's labor supply is given by $\ell_t^{1/\psi} = c_{wt}^{-1} w_t$. Plugging in (10) and using $c_{wt} = c_t(1 - \eta_t)$, we obtain the equilibrium condition in the labor market:

$$\overbrace{(1-\alpha)k_t^{\alpha}\ell_t^{-\alpha}(1 - v_{cet}v_t)}^{w_t} = \ell_t^{1/\psi} \times c_t(1 - \eta_t). \tag{13}$$

Capital is priced by arbitrage, $R_t = r_t + \delta$. Plugging in (11), we obtain the equilibrium condition in the capital market,

$$\overbrace{\alpha k_t^{\alpha-1}\ell_t^{1-\alpha}(1 - v_{cet}v_t)}^{R_t} = r_t + \delta. \tag{14}$$

Equations (13) and (14) govern aggregate employment and consumption/investment. Uninsurable idiosyncratic risk reduces labor demand through the risk premium $v_{cet}v_t$, captured by (10), and therefore equilibrium employment, output, consumption, and investment. It also reduces demand for capital and investment, captured by (11), but this effect is counterbalanced by the precautionary motive v_{cet}^2 that depresses the interest rate r_t conditional on the behavior of aggregate consumption, captured by (12), and therefore stimulates investment. As it turns out, this second force slightly dominates and a risk shock acts like a tax on labor but a subsidy to capital. As we will show, this combination of forces is essential for the risk premium view of business cycles.

Aggregate risk sharing and law of motion of η_t. Finally, to complete the characterization of the competitive equilibrium, we note that complete aggregate risk sharing means that the consumption of entrepreneurs and workers has the same exposure to aggregate risk. From the optimality conditions for aggregate risk sharing for entrepreneurs and workers, we obtain

$$\pi_t = \sigma_{cet} = \sigma_{cwt} = \sigma_{ct}. \tag{15}$$

The Euler equations and aggregate risk sharing conditions give us a law of motion for entrepreneurs' consumption share, $d\eta_t = \mu_{\eta t} dt + \sigma_{\eta t} dZ_t$, with

$$\mu_{\eta t} = \eta_t(1-\eta_t)(\rho_w - \rho_w + v_{cet}^2), \qquad \sigma_{\eta t} = 0. \tag{16}$$

Because aggregate risk sharing is unconstrained, we know from (15) that entrepreneurs' and workers' consumption move together in response to aggregate shocks, so we obtain $\sigma_{\eta t} = \eta_t(1-\eta)(\sigma_{cet} - \sigma_{cwt}) = 0$. If entrepreneurs and workers had the same impatience rate, $\rho_e = \rho_w$, entrepreneurs would eventually account for all the consumption in the economy, $\eta_t \to 1$, because of their precautionary saving motive for idiosyncratic risk v_{cet}^2. We assume entrepreneurs are more impatient, $\rho_e > \rho_w$, to obtain a stationary distribution for η_t. Equation (16) says that the consumption of workers and entrepreneurs fall together in response to risk shocks, but subsequently entrepreneurs' consumption recovers faster than workers', because of their temporarily elevated precautionary motive v_{cet}^2.

Competitive equilibrium. The competitive equilibrium is a solution to the entrepreneurs' idiosyncratic risk (9); prices (10), (11), (12), and (15); the equilibrium conditions for labor and capital, (13) and (14); the resource constraint and laws of motion of k_t, v_t, and η_t, given by (2), (3), (4), and (16). Appendix A shows how the competitive equilibrium can be described by a PDE and solved numerically.

3.1 Risk shocks can create business cycles

Our main result is that risk shocks can generate business cycles in which consumption, investment, and employment all decline in recessions. To fix ideas, Figure 2 shows the impulse response to a one-standard deviation risk shock, in a numerical solution. The economy starts in its long-run configuration and is hit by a rapid increase in v_t, and then all further realizations of aggregate shocks are zero. Relative to the baseline model, we only add standard convex adjustment costs to capital, an addition that does not affect the essence of the economic mechanism. Appendix B presents the details of the calibration. The focus for the moment is purely on illustrating the economic mechanism behind business cycles.

The first panel shows the behavior of idiosyncratic risk v_t, which spikes by 5.5 percentage points on impact and then returns to its long run value of 10 percent, and the behavior

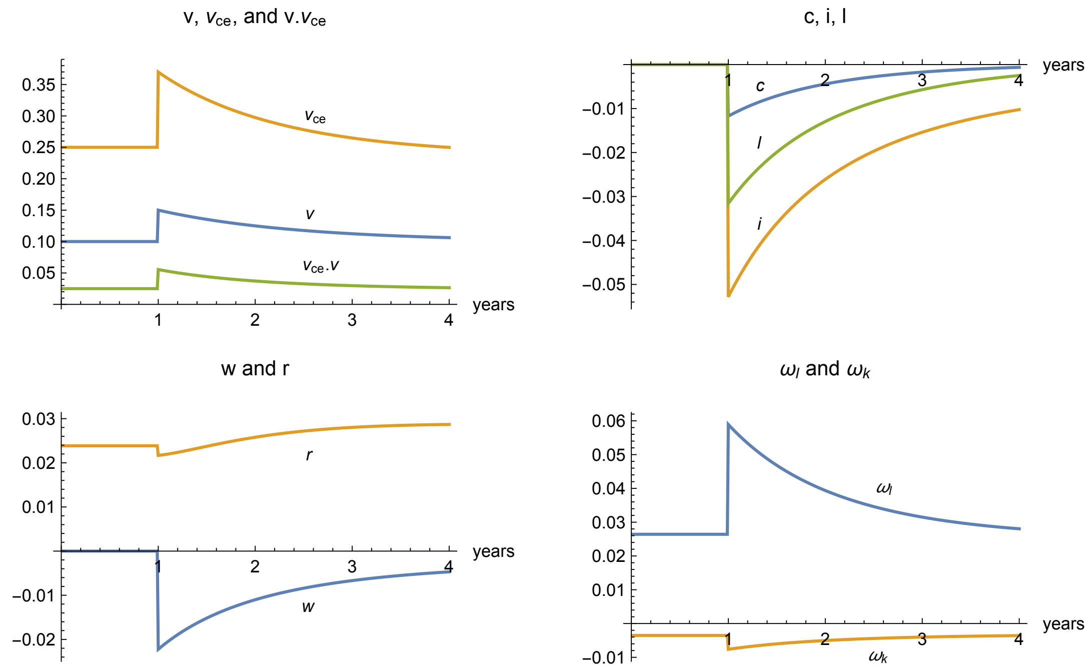

Figure 2: Impulse Response to a Risk Shock. *Note:* c, i, ℓ, and w are expressed in log deviations.

of the idiosyncratic risk of entrepreneurs' wealth or consumption, v_{cet}, which spikes by 10 percentage points on impact and then returns to its long run value of 25 percent. The idiosyncratic risk premium, $v_{cet}v_t$, displays the same behavior. It spikes on impact by 3 percentage points, and then returns to its long run value of 2.5 percent.

The second panel shows the responses of consumption, investment, and employment. They all fall on impact and slowly recover thereafter. Risk shocks cause a contraction in labor demand that reduces employment, and therefore consumption and investment. Since agents prefer to smooth consumption, the contraction in investment is larger. Consumption falls by 1 percent, employment by 3 percent, and investment by 5 percent. This is broadly in line with stylized facts about US business cycles. The consumption share of entrepreneurs, η_t, does not respond on impact. The consumption of entrepreneurs and workers both fall on impact, but entrepreneurs' consumption subsequently recovers faster.

The third panel shows the behavior of interest rates and wages behind these fluctuations in quantities. Wages fall on impact by around 2 percent, reflecting weaker labor demand—entrepreneurs demand a larger risk premium to compensate for uninsurable idiosyncratic risk, and the economy moves along workers' labor supply curve. Interest rates also fall on impact, but this is not a robust property across calibrations. It is the result of two opposing forces. On the one hand, a larger precautionary saving motive lowers interest rates for a given behavior of aggregate consumption. This eliminates the effect of the risk premium on investment demand, and is the reason why risk shocks do not act like a tax on capital. On the other hand, the transitory contraction in labor demand reduces employment and output. Equilibrium is achieved by raising interest rates, which induce agents to reduce consumption and investment. The fourth panel shows the capital and labor wedges generated by uninsurable idiosyncratic risk, to which we now turn.

3.2 Understanding the business cycle in terms of wedges

This subsection explores the way risk shocks produce business cycles with parallel movements of consumption, investment, and employment. To explain the effects of uninsurable idiosyncratic risk, we carry out a wedge exercise in the spirit of Chari et al. (2007), where we take as the benchmark the Pareto-efficient allocation with perfect risk sharing. In this benchmark, risk shocks have no effect because idiosyncratic risk is fully insured, and the economy gradually converges to a steady state following standard growth-model dynamics.

With uninsurable idiosyncratic risk, risk shocks create time-varying labor and capital wedges. The main takeaway is that an increase in idiosyncratic risk can be understood as a *tax* on labor and a *subsidy* to capital. While the risk premium depresses demand for capital and labor symmetrically, the precautionary saving motive makes capital more attractive because it provides a store of value, while employment does not. This asymmetry is key to obtaining business cycles with parallel movement of employment, consumption, and investment. There is an additional wedge in the law of motion of the consumption share η_t. While this wedge plays an important role in the long run, it is a slow-moving state variable that does not respond on impact to risk shocks, and therefore does not play an important role in business cycles, so we will leave its analysis to the end.

The labor and capital wedges, $\omega_{\ell t}$ and ω_{kt}, are defined as follows

$$(1-\alpha)k_t^{\alpha}\ell_t^{-\alpha} \times (1-\omega_{\ell t}) = \ell_t^{1/\psi} \times c_t(1-\eta_t) \tag{17}$$

$$\alpha k_t^{\alpha-1}\ell_t^{1-\alpha} \times (1-\omega_{kt}) = (\bar{\rho}_t + \mu_{ct} - \sigma_{ct}^2) + \delta. \tag{18}$$

With zero wedges, $\omega_{\ell t} = \omega_{kt} = 0$, we have the equilibrium conditions for the first-best with perfect risk sharing. Recall that $\bar{\rho}_t + \mu_{ct} - \sigma_{ct}^2$ is the risk-free rate with perfect risk sharing, $c_{wt} = c_t(1-\eta_t)$ is workers' consumption, and the marginal product of labor and capital correspond to the wage and rental rate with perfect risk sharing. The labor and capital wedges enter the economy as taxes on labor and capital.

Labor wedge. Comparing (17) with the equilibrium condition in the labor market (13), we see that the labor wedge is the risk premium for idiosyncratic risk,

$$\omega_{\ell t} = v_{cet} v_t. \tag{19}$$

Employing workers is a risky activity, and the risk premium acts like a tax on labor—a labor wedge. A higher labor wedge creates a recession. Entrepreneurs reduce their demand for labor, which leads to lower employment and output, and therefore consumption and investment. Because agents prefer smooth consumption, and the risk shock is transitory, investment falls significantly more than consumption. Capital adjustment costs, which we introduce in the numerical solution, smooth out fluctuations in investment.

A countercyclical labor wedge is essential for obtaining business cycles with parallel movements of consumption, investment, and employment. With a constant labor wedge, consumption c_t and employment ℓ_t will always move in opposite directions, as can be seen from (17). In our model, the marginal product of labor is risky and therefore commands a time-varying risk premium that shows up as a countercyclical labor wedge. We note that New Keynesian models with sticky nominal wages also create a countercyclical labor wedge. In this respect, our model works in a way similar to New Keynesian models.[3]

Capital wedge. The challenge for the risk premium view of business cycles is that a higher risk premium also shows up as a tax on capital—a capital wedge ω_{kt}. In contrast to the labor wedge, a larger capital wedge does not produce a recession. Conditional on the labor wedge, the capital wedge reduces investment but raises consumption. This is why models of risk premium shocks typically fail to produce recessions with falling consumption.

The precautionary motive acts in the direction opposite to the risk premium. By lowering equilibrium interest rates, it acts like a subsidy to capital, which is a long-duration store of value, but not to employment because of its short duration. This is precisely what is needed to obtain business cycles with parallel moments of employment, consumption, and investment. It prevents consumption from rising in response to a spike in risk premiums, while preserving the negative effect on employment that characterizes recessions.

So far we have considered two forces acting in opposite directions on the capital wedge, the risk premium and the precautionary motive. We can obtain a *sufficient statistic* for the capital wedge that allows us to evaluate the total effect. Comparing (18) with the equilibrium condition in the capital market (14), we obtain an expression for the capital wedge,

$$\omega_{kt} = \overbrace{v_{cet} v_t}^{\omega_{\ell t}} \times \left(1 - \rho_e \times \frac{y_t}{c_t} \times \frac{1}{\alpha} \times \frac{k_t}{y_t}\right). \tag{20}$$

The capital wedge is equal to the risk premium, like the labor wedge, but multiplied by a damping factor that captures the interaction of the risk premium and the precautionary motive. This factor only involves easily measurable equilibrium objects, the consumption-income ratio c_t/y_t, the capital income-share α, and the capital-income ratio k_t/y_t. Only

[3]Our model produces a labor wedge on the firm-demand side, so in this sense our model is closer to a New Keynesian model with sticky prices. A recent paper (Karabarbounis (2014)) suggest the wedge on the household supply side is more important. Adapting the model to incorporate this fact is beyond the scope of this paper.

the impatience rate ρ_e must be calibrated.

In (20), the terms $\rho_e \times y_t/c_t = \eta_t \times v_{cet}^2/(v_{cet}v_t)$ capture the ratio of the precautionary motive, weighted by entrepreneurs' consumption share η_t, to the risk premium. The smaller is consumption as a fraction of total output, the stronger is the precautionary motive, and the capital wedge is more likely to be negative—a subsidy to capital. The terms $1/\alpha \times k_t/y_t$ form the price-dividend ratio for capital. They capture the success of capital as a store of value. A large price-dividend ratio for capital means the reduction in the equilibrium interest rate caused by the precautionary motive has a large positive effect on capital, and the wedge is more likely to be negative—a subsidy to capital.

We use (20) to determine if the net capital wedge is positive or negative. Expression (20) is true after any history, and the equilibrium objects in the factor are relatively stable. For the US economy, $c_t/y_t \approx 0.8$, $\alpha \approx 1/3$, $k_t/y_t \approx 3$. For the impatience rate we use $\rho_e = 0.0975$, which as we explain in Appendix B, is consistent with steady state consumption for entrepreneurs who face idiosyncratic risk. We obtain a negative capital wedge: $\omega_{kt} \approx \omega_{\ell t} \times (-0.1)$. This means that the precautionary motive dominates the risk premium, and an increase in risk v_t acts like a tax on labor but a subsidy to capital.

The asymmetry in duration between labor and capital. The asymmetry between capital and labor plays a key role in generating business cycles. Capital is a long-duration store of value, so lower interest rates induced by the precautionary motive stimulate investment. In contrast, employment has short duration. In our model with spot labor markets, it is a purely intra-temporal decision with zero duration. As a result, lower interest rates do not stimulate employment. Taking a broader view, in models with search frictions employment can be regarded as a positive duration asset, since initial recruiting costs generate a stream of surplus for some time. In that case lower interest rates will also stimulate employment somewhat. But the point remains that, even with search frictions, the duration of employment will still be short compared to capital, so we will still obtain asymmetric labor and capital wedges.

To see the importance of the asymmetry in duration between capital and labor, notice that in order to understand why consumption falls in equilibrium, it is not enough to observe that the precautionary motive causes entrepreneurs to postpone consumption. That is only a partial equilibrium effect. In general equilibrium, interest rates will fall in response. As they do, they will stimulate investment, but not employment. If employment somehow had the same long duration as capital, lower interest rates would also stimulate employment, and the labor and capital wedge would both be negative, a subsidy to both labor and capital. This illustrates the importance of the asymmetry in duration between capital and labor. Our mechanism rests on the observation that both labor and capital are risky, but only capital is a significant store of value.

Meaning	*Parameter*	*Value*
Capital share	α	1/3
Frisch elasticity of labor supply	ψ	3
Capital adjustment cost	ϵ	4
Depreciation	δ	0.073
Impatience rate, workers	ρ_w	0.035
Impatience rate, entrepreneurs	ρ_e	0.0975
long run idiosyncratic risk	$\bar{v}$	0.10
Mean-reversion of idiosyncratic risk	θ_v	0.693
Aggregate volatility of idiosyncratic risk	σ_v	0.16

Table 1: Parameter Values

Consumption share. There is also a wedge is the law of motion of the consumption share η_t. In the first-best with perfect risk sharing, the consumption share η_t follows a deterministic trend given by the difference in impatience between workers and entrepreneurs, $\mu_{\eta t} = \eta_t(1-\eta_t)(\rho_w - \rho_e)$. With uninsurable idiosyncratic risk, the drift of η_t depends on risk shocks through the precautionary motive v_{cet}^2, that is $\mu_{\eta t} = \eta_t(1-\eta_t)(\rho_w - \rho_e + v_{cet}^2)$. However, because of complete aggregate risk sharing, η_t is not exposed to aggregate shocks, $\sigma_{\eta t} = 0$. The consumption levels of entrepreneurs and workers fall together in response to risk shocks, but subsequently entrepreneurs' consumption recovers faster because of their elevated precautionary motive. As a result, the consumption share η_t is a slow-moving state variable that matters for the long-run, but plays a limited role in business cycles. For example, η_t appears in the equilibrium condition for the labor market (13), capturing the income effect on workers' labor supply, and in the expression for the interest rate (12) through $\bar{\rho}_t$, but because it does not react on impact to risk shocks, it does not play an important role in business cycles.

3.3 Quantitative evaluation

We solve the model numerically to illustrate the mechanism and evaluate its quantitative plausibility. As explained above, relative to the baseline model, we only add standard convex adjustment costs to capital, which do not affect the essence of the economic mechanism. Table 1 shows the parameter values we use. Appendix B has the details of the calibration and quantitative work.

First, we compute standard business cycle moments from the model and compare them to US data. Table 2 summarizes key moments in the model and the data. It shows the standard deviation and correlation structure of cyclical output, consumption, investment,

Variable		*St. dev., percent* Model	Data	*Rel. st. dev.* Model	Data	*Corr. w/output* Model	Data	*Autocorr.* Model	Data
Output	y	1.4%	1.6%	1	1	1	1	0.88	0.85
Consumption	c	0.7%	1.1%	0.5	0.6	0.95	0.77	0.84	0.82
Investment	i	4.3%	6.4%	3	4	0.98	0.87	0.90	0.82
Employment	ℓ	2%	1.9%	1.4	1.2	0.96	0.88	0.86	0.90

Table 2: Business Cycle Moments from the Model Compared to HP-Filtered (1600) Data, at Quarterly Frequency, Per-Capita, and in Logs, for the Period 1948-2018.

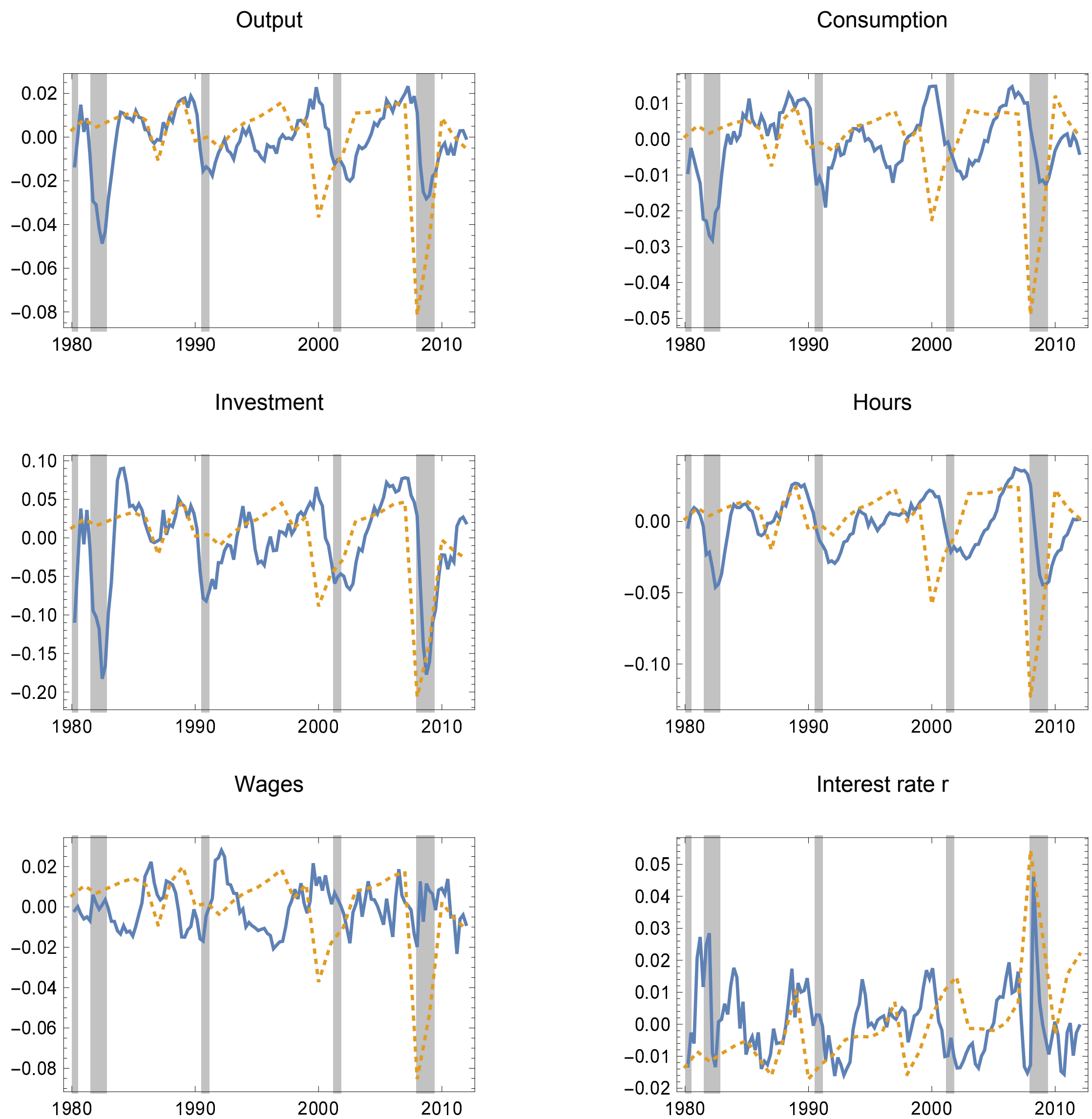

Figure 3: Data vs. Model simulation. *Note:* Output, Consumption, Investment, Hours, Wages and the Interest Rate in the Model (dotted) Compared to HP-Filtered (1600) Data (solid) Per-Capita and in Logs (Except for the Interest Rate), from 1980q1-2012q4.

and hours. The model can generate reasonable business-cycle moments.

Second, we back out the realization of risk shocks to match the behavior of idiosyncratic risk in stock returns from Figure 1. We start the model at the steady state in 1980, and calculate an annual series for idiosyncratic risk v_t to match the time series for idiosyncratic risk in stock returns in Figure 1 (adding a mean of 0.25), which corresponds to $v_{cet} = v_{net}$ in the model. The model then produces time series for output, consumption, investment, hours, wages and interest rates that we compare to US data in Figure 3.

The model does a decent job explaining the fluctuations in the data, with the exception of the 1980-82 recessions. The following three recessions are captured fairly well by the model, except for real wages. Idiosyncratic risk in stock returns spikes early in the recession, and it seems to take a short time for real variables to respond.

Our takeaway is that the mechanism we propose is quantitatively plausible and seems like a promising approach to understanding business cycles. We do not claim these quantitative results as definitive. Our model is stylized in the interest of tractability and theoretical clarity, and one can reasonably question many parameter values. In Appendix B we explore the sensitivity of these results to alternative parameterizations. Our claim is more limited—that the mechanism we propose for explaining some business cycles is plausible and deserves consideration. We note that we do not consider any sources of movements of output not associated with the business cycle, such as variations in productivity growth and in the size of the labor force. Our data does not include the recession that began in early 2020—its driving force is plainly the pandemic, not the shock incorporated in our model.

4 Efficiency

To study the efficiency of the competitive equilibrium, we consider a planner who can use taxes on capital and labor to manipulate the capital and labor wedges. The planner rebates the taxes with lump-sum transfers. We assume that the planner has to live with the fundamental frictions in the model and can neither create insurance against idiosyncratic risk nor prevent workers and entrepreneurs from saving or sharing aggregate risk by trading with each other.

This planner's problem could be microfounded in an environment with a moral hazard problem with hidden trade.[4] An entrepreneur's idiosyncratic shock is private information, which allows diversion of resources to a private account. In addition, workers and entrepreneurs can trade consumption claims contingent on the aggregate shock in financial markets, and entrepreneurs can trade capital and labor in competitive labor and rental markets. The optimal private contract takes the form of the reduced-form incomplete risk

[4] See Di Tella and Sannikov (2016) or Di Tella (2019). These results are in the spirit of Cole and Kocherlakota (2001).

sharing problem we described earlier in connection with the competitive equilibrium. We can then ask, what is the best that a planner can do subject to the same contractual limitations? The resulting planner's problem coincides with the planner's problem we use here, that is, the best allocation can be implemented with time-varying taxes on capital and labor. We use the formulation of the planner's problem with time-varying taxes because it is simpler to interpret, but it may be reassuring that there is a microeconomic foundation involving hidden trade to describe the ultimate source of inefficiency.

The main takeaway from this section is that the response of the competitive-equilibrium economy to a risk shock is inefficient. Employment and output fall too much, and consumption should rise instead of falling. In the competitive equilibrium, risk shocks show up as a tax on labor and a subsidy to capital, and create a recession with lower employment, consumption, and investment. In the planner's solution, instead, the optimal policy response to risk shocks is to raise the subsidy on labor and raise the tax on capital (or lower the subsidy). Output, employment, and investment fall, but consumption goes up. Because consumption and investment are negatively correlated, fluctuations in output and employment are smaller than in the competitive equilibrium, and consumption is countercyclical.

4.1 The planner's problem

The planner uses labor and capital taxes with lump-sum transfers. In this way the planner can control employment ℓ_t, investment i_t, and consumption c_t, and, as we will show below, we can back out the taxes on labor and capital implied by the constrained-efficient allocation. The planner respects the impracticality of idiosyncratic risk sharing and the ban on interfering with trading among agents. To concentrate on the basic issues, we use the baseline model without adjustment costs, and focus on the main economic relationships. For the numerical solution we add capital adjustment costs, as we did with the competitive equilibrium. Appendix C has the extension with adjustment costs and all derivation details.

The planner maximizes the weighted utility of workers and entrepreneurs $\gamma U^w + (1 - \gamma)U^e$. The planner chooses an allocation (c, i, ℓ, k, η) to solve

$$\max_{c,i,\ell,k,\eta} \mathbb{E}\left[\int_0^\infty \gamma e^{-\rho_w t}\left(\log(c_t(1-\eta_t)) - \frac{\ell_t^{1+1/\psi}}{1+1/\psi}\right) + (1-\gamma)e^{-\rho_e t}\left(\log(c_t\eta_t) - \frac{1}{2}\frac{1}{\rho_e}v_{cet}^2\right)dt\right] \tag{21}$$

subject to the resource constraints (2) and (3), the law of motion of v_t (4), and

$$v_{cet} = \frac{k_t^\alpha \ell_t^{1-\alpha}}{c_t}\rho_e \eta_t^{-1} v_t \tag{22}$$

$$\mu_{\eta t} = \eta_t(1-\eta_t)(\rho_w - \rho_w + v_{cet}^2), \qquad \sigma_{\eta t} = 0, \tag{23}$$

Equation (22) captures the impracticality of idiosyncratic risk sharing, and corresponds to equation (9) in the competitive equilibrium. Equation (23) captures agents' Euler equations and aggregate risk sharing, and corresponds to equation (16) in the competitive equilibrium. As a result, η_t is a state variable for the planner, just as in the competitive equilibrium (η_0 is chosen optimally). If the planner could control agents' access to the financial market, preventing entrepreneurs and workers from trading over time or across aggregate states, then η_t would not be a state variable and we could ignore (23) and choose c_e and c_w separately. If the planner could provide idiosyncratic risk sharing, we would further ignore (22) and obtain the first-best allocation.

Once we find the optimal planner's allocation (c, i, ℓ, k, η), we can back out the implied prices w_t, R_t, r_t, π_t, and the taxes on labor $\tau_{\ell t}$ and capital τ_{kt} that support it as a competitive equilibrium with taxes. The wage and rental rate of capital are given by (10) and (11), $w_t = (1-\alpha)k_t^{\alpha}\ell_t^{-\alpha}(1 - v_{cet}v_t)$ and $R_t = \alpha k_t^{\alpha-1}\ell_t^{1-\alpha}(1 - v_{cet}v_t)$. The interest rate and price of risk are given by (12) and (15), $r_t = \bar{\rho}_t + \mu_{ct} - \sigma_{ct}^2 - \eta_t v_{cet}^2$ and $\pi_t = \sigma_{ct}$. Finally, we set taxes $\tau_{\ell t}$ and τ_{kt} to satisfy the equilibrium conditions in the labor and capital markets, corresponding to (13) and (14),

$$w_t(1 - \tau_{\ell t}) = \ell_t^{1/\psi} c_t (1 - \eta_t)$$

$$R_t(1 - \tau_{kt}) = r_t + \delta.$$

This ensures that we have a competitive equilibrium with a labor and capital tax. Verifying that it is an equilibrium only requires checking that agents' transversality conditions hold.

4.2 Planner's response to a risk shock

We solve the planner's problem numerically. As in the competitive equilibrium, we add convex adjustment costs. Appendix C gives the details of the solution.

Figure 4 shows the impulse-response function of the planner's solution to the same shock as in the competitive equilibrium. Investment still falls, and by a similar amount. But employment falls considerably less, 1 percent compared to 3 percent in the competitive equilibrium, and consumption actually goes up on impact instead of falling. The response of the planner's solution to a risk shock does not look like a recession at all. In fact, employment does not fall because of a contraction in labor demand from entrepreneurs, as was the case in the competitive equilibrium. It falls because with higher consumption workers' income effect reduces labor supply. As a result, post-tax wages actually go up on impact.

The real interest rate falls on impact more than in the competitive equilibrium. In the competitive equilibrium there were two opposing forces. The precautionary motive pushed the interest rate down, but the transitory contraction in consumption pushed it back up.

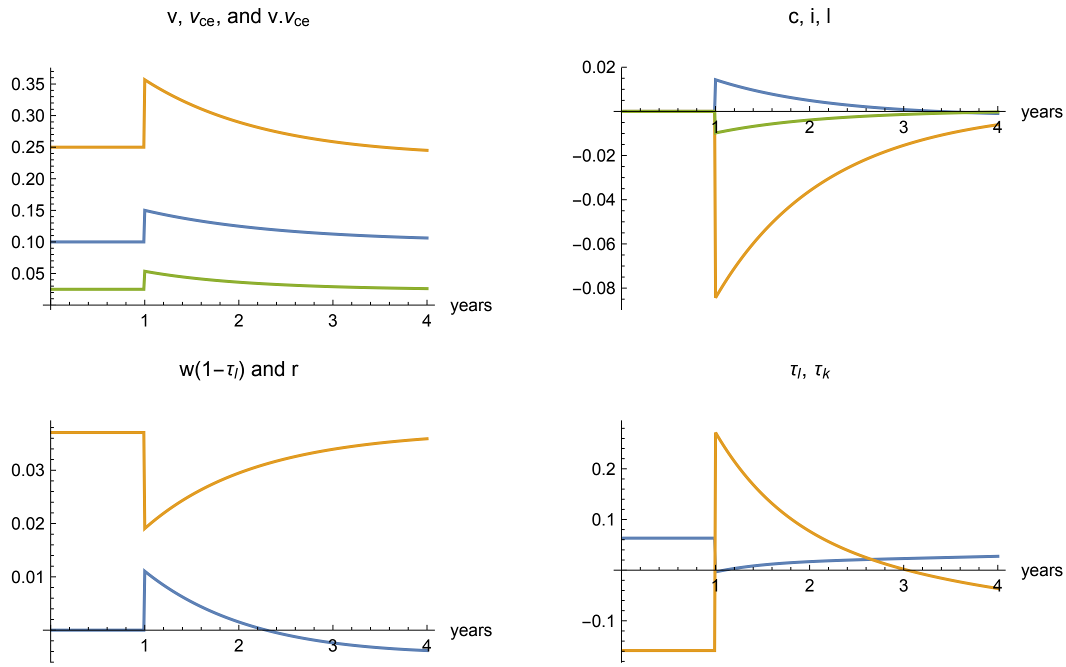

Figure 4: Impulse Response to a Risk Shock in the Planner's Solution. *Note:* c, i, ℓ, and w are in logs.

In the planner's solution, instead, consumption is temporarily elevated, so both forces push the interest rate down.

To summarize, in the planner's solution the interest rate drops and drives consumption up, and employment falls a little from the resulting contraction in labor supply, reflected in higher wages. The only commonality with the competitive equilibrium is the large reduction in investment. The negative correlation of consumption and investment means that standard deviation of output and employment are lower.

The planner's allocation can be implemented by lowering labor taxes during recessions to stimulate employment and raising the capital tax to reduce investment and free more resources for consumption. This is equivalent to a temporary subsidy to consumption. Figure 4 shows the impulse response of the labor and capital tax to a risk shock. The labor tax is positive in steady state and falls after a risk shock to stimulate employment. The capital tax is negative in steady state, but rises after a risk shock to reduce investment.

4.3 An aggregate consumption externality

The environment features an externality that plays a central role in the planner's response to a risk shock. In the expression for idiosyncratic risk-sharing (22), higher output $y_t = k_t^\alpha \ell_t^{1-\alpha}$ raises idiosyncratic risk v_{cet}. On this the planner and private entrepreneurs agree, and that is why they demand a risk premium. But the planner also raises aggregate consumption c_t to improve idiosyncratic risk sharing, while private agents lack an incentive to respond to this externality.

The externality makes the response of the competitive equilibrium to risk shocks inefficient (the competitive equilibrium is always inefficient). Risk shocks raise the risk premium and reduce labor demand and output, and therefore consumption. Lower consumption in turn makes risk sharing even worse and raises the risk premium even more, further reducing employment and output in a negative feedback loop. The planner aims to stop this vicious cycle by stimulating consumption, and does so by raising employment and reducing investment.

Why is this an externality? It comes from agent's access to hidden trade, which is known to be a source of inefficiency.[5] Individual agents follow their Euler equations and aggregate risk sharing equations, which take the interest rate r_t and the price of risk π_t as given, without an incentive to consider the fact that their consumption also affects idiosyncratic risk sharing v_{cet}. The planner, in contrast, does not take r_t and π_t as given. As long as the planner respects constraint (23), for any aggregate consumption path c_t there are prices r_t and π_t that satisfy agents' Euler and risk sharing equations. So the planner can improve idiosyncratic risk sharing by raising aggregate consumption. An equivalent way of seeing this is to write idiosyncratic risk in terms of the net worth of entrepreneurs, $v_{cet} = f(k_t, \ell_t)/n_{et} \times v_t$ (recall that $n_{et} = c_{et}/\rho_e$) and notice that private agents take n_{et} as given, without taking into account how their actions affect net worth and therefore risk sharing.

To see how the planner responds to risk shocks, consider a planner who is thinking about raising employment by a small amount, $d\ell_t > 0$. The effect on idiosyncratic risk sharing is

$$v_{cet} \downarrow = \frac{f(k_t, \ell_t) + f'_\ell(k_t, \ell_t) d\ell_t \uparrow}{c_t + f'_\ell(k_t, \ell_t) d\ell_t \uparrow} \rho_e \eta_t^{-1} v_t.$$

The extra employment raises output and therefore raises idiosyncratic risk v_{cet}. On this private agents and the planner agree, and it is captured by the larger numerator. The planner also exploits the fact that the extra output lowers interest rates and therefore raises aggregate consumption c_t (it raises n_{et}), and this *reduces* entrepreneurs' exposure to idiosyncratic risk v_{cet} through the larger denominator. This second effect dominates because aggregate output is larger than aggregate consumption, $y_t > c_t$. A symmetric analysis applies to capital. For entrepreneurs, using more labor and capital means taking on *more* idiosyncratic risk. For the planner, by contrast, raising everyone's use of labor and capital exposes them to *less* idiosyncratic risk.

As in the competitive equilibrium, the difference between capital and labor is that capital is a long-duration asset that involves intertemporal tradeoffs. Consider a planner who is thinking about raising investment by a small amount, $di_t > 0$. The effect on idiosyncratic

[5] See Farhi et al. (2009), Kehoe and Levine (1993), Di Tella (2019).

risk sharing us

$$v_{cet} \uparrow = \frac{f(k_t, \ell_t)}{c_t - di \uparrow} \rho_e \eta_t^{-1} v_t.$$

Raising investment requires diverting resources from consumption (mediated by higher interest rates and lower net worth n_{et}) and making risk sharing worse. Investment is therefore particularly unappealing when idiosyncratic risk v_t is high, because it makes idiosyncratic risk sharing worse, and yields extra capital (which improves idiosyncratic risk sharing) in the future when idiosyncratic risk v_t is expected to be lower.

5 Discussion

This section takes up a number of topics related to the model that extend and clarify the earlier material.

5.1 Technology

Locally uncertain marginal products. A crucial feature of our model is that the marginal products of capital and labor are locally uncertain. A firm making decisions about k and ℓ has a probability distribution of how this will affect profits, but doesn't know for sure. The realized marginal product is uncertain when the factor quantity decision is made. The alternative, which is common in the literature as a modeling device, is to assume that the firm first learns its productivity for the period and then hires capital and workers, or perhaps just workers (for example, Angeletos (2007) and Meh and Quadrini (2004)), so that using workers to produce is a risk-free activity. In contrast, our model takes seriously the idea that using capital and labor to produce involves risk. Our emphasis on the uncertainty of the marginal product of labor is new to the literature, as far as we know.

This view of risky productive activity arises naturally in models with adjustment costs or search frictions, but these formulations usually entangle the risk and time dimensions (for example, Hall (2017) or Kehoe et al. (2019)), a distinction that is central to our mechanism. Our continuous-time formulation says that uncertain output is revealed gradually and the entrepreneur can continuously adjust labor and capital (so bankruptcy never occurs in equilibrium, for example), but the realized marginal product remains locally uncertain. In a short period of time risk is small, but so is the expected output flow—variance and mean scale linearly with time. As a result, in our model employment is a risky but intratemporal decision. This is an abstraction that helps us make a stark distinction between labor and capital, and clarify what is essential to the mechanism. The online appendix develops a discrete-time version of the model in the body of the paper. While less tractable, it has the advantage that it can be easily solved using standard computational packages such as Dynare.

Persistence of idiosyncratic shocks. Our formulation of technology implies that idiosyncratic shocks are iid, which yields considerable tractability. Entrepreneurs' policy functions are linear in their net worth, so we do not need to keep track of the endogenous joint distribution of productivity and net worth. Idiosyncratic shocks have persistent effects, however, through the net worth of the entrepreneur. Financial losses today lead to lower output in the future, because they reduce the firm's net worth.

Adding more realistic persistence to idiosyncratic shocks themselves is an interesting direction for future work. We conjecture that it will amplify the mechanism in our model. To see why, recall that the risk premium captures the covariance between the marginal product of labor or capital and the marginal utility of the entrepreneur c_{it}^{-1}. If risk aversion is greater than one, persistent shocks have a larger effect on marginal utility because, beyond causing financial losses on impact, they also worsen the entrepreneur's investment opportunities looking forward, so entrepreneurs will demand a larger risk premium. Of course, in general equilibrium other things will adjust in response to higher persistence. Extending the model to capture a realistic persistence of idiosyncratic shocks is a natural next step, but comes at the cost of significant loss of tractability.

Cross-sectional distribution. Our model has an ergodic distribution for aggregates—output, investment, employment, and consumption of workers and entrepreneurs. Thanks to the linearity of entrepreneurs' policy functions, we do not need to keep track of the cross-sectional distribution of entrepreneurs' net worth or consumption, which is well defined at each point in time, but does not converge to an ergodic distribution in the long-run. Suppose we remove aggregate shocks, so that the distribution of the idiosyncratic shocks is constant over time and the economy settles on a steady state in the long-run. Because an entrepreneur's net worth follows a geometric Brownian motion, the cross sectional distribution is log normal, with constant mean (because it is a steady state) but variance that increases proportionally with $e^{v_{ce}^2 t} - 1$. This is a well-known property of geometric Brownian motions. A common way to obtain an ergodic cross-sectional distribution is to posit that entrepreneurs die with Poisson probability, and are restarted at some initial level. This yields a Pareto distribution, which has proven useful in applied work. But because our paper is not focused on the cross-sectional distribution, we prefer to avoid introducing unnecessary ingredients.

5.2 Incomplete idiosyncratic risk sharing

Our model rests on the assumption that firm insiders are exposed to firm-specific idiosyncratic risk. We treat the incomplete idiosyncratic risk sharing as a primitive of the model. What we have in mind is that insiders need to be exposed to idiosyncratic risk in their firm for incentive reasons, and this exposure distorts the firm's production decisions. This is not a corporate-governance failure. Diversified outside investors who only care about the present value of their payoffs would agree to distort production decisions to take insiders' risk aversion into account. We could microfound the incomplete risk sharing in our paper with a moral hazard problem with hidden trade, as in Di Tella and Sannikov (2016).

This mechanism-design approach also shows why workers are not exposed to the firm's idiosyncratic risk. It is not optimal to expose workers to outcomes unless they can influence those outcomes through hidden actions. The first-best risk-sharing arrangement in our setting entails risk sharing among entrepreneurs, not risk sharing between entrepreneurs and workers. In reality most workers in fact receive compensation that is relatively insensitive to firm outcomes, and are instead exposed to significant worker-specific risk such as unemployment and promotions. We abstract from workers' exposure to idiosyncratic risk in their labor income to focus on the core mechanism we propose, and because uninsurable idiosyncratic labor income risk is already the subject of an extensive literature. But incorporating workers' countercyclical labor income risk is a natural next step.

5.3 Cross-sectional implications and evidence

The cross-sectional variation in the amount of uninsurable idiosyncratic risk across firms and industries provides an opportunity to test our mechanism. Our model implies that an increase in uninsurable idiosyncratic risk at the firm level should lead to a contraction in its employment and investment.

In support of this proposition, Panousi and Papanikolaou (2012) show that investment by a publicly traded firm falls when its idiosyncratic risk rises. Importantly for our mechanism, the effect is significantly larger when managers own a larger fraction of the firm. A drawback of these results, for our purposes, is that they do not study the response of employment, and only cover publicly traded firms. Bloom et al. (2018) and Leahy and Whited (1996) also show that higher idiosyncratic risk at the industry level is associated with lower output and investment. However, they do not study the relation to insiders' ownership share, so their results are consistent with a real options channel and risk neutral firms. These findings provide suggestive, though imperfect, empirical support for our mechanism.

Fully exploring the cross-sectional implications of our mechanism requires a multisectoral structural model, which goes beyond the aims of this paper. Firms are linked through input/output relationships. Suppliers to firms or industries with highly countercyclical uninsurable idiosyncratic risk will perceive that demand for their products contracts after risk shocks, and thus contract themselves, even if their own risk is unaffected. A similar logic applies to firms producing complementary goods to, or using inputs from, a sector with highly countercyclical uninsurable idiosyncratic risk.

5.4 Excess return of capital, markups, and labor share

Here we explore implications of our model for some salient equilibrium objects that have received significant attention in the macro literature.

Excess return to capital. The excess return is the difference between the expected return to capital and the return to a risk-free claim. Our model creates a time-varying excess return of capital, as compensation for uninsurable idiosyncratic risk for entrepreneurs. It does not show up as an equity premium because outside investors diversify—their portfolios have infinitesimal holdings of an infinite set of claims to different pieces of capital. We can re-write the equilibrium condition for capital (14), using the expected or average marginal product of capital,

$$f_k'(k_t, \ell_t) - (r_t + \delta) = \underbrace{f_k'(k_t, \ell_t) \times v_{cet} v_t}_{\text{excess return}}. \tag{24}$$

The difference with the capital wedge in equation (18) is that here we are using the equilibrium interest rate r_t, while the wedge ω_{kt} is defined using the interest rate in the model with perfect risk sharing. That is, the wedge helps us understand the effect of incomplete idiosyncratic risk sharing in terms of capital and labor taxes in a model with perfect risk sharing, while the excess return in (24) highlights the failure of the perfect-risk-sharing asset-pricing equation at equilibrium prices, ignoring that the equilibrium interest rate r_t is lower than what it would be with perfect risk sharing given the same aggregate allocation.

The advantage of equation (24) is that it is more directly related to the data. Farhi and Gourio (2018) point out that because the return to capital has remained roughly constant over past decades, while interest rates have gone down, the excess return to capital has risen. A rising risk premium is one possible explanation, together with rising market power and intangibles. Although our model is not designed to address secular trends, the presence of an excess return is consistent with our mechanism. Quantitatively, however, the total excess return attributable to idiosyncratic risk is small, 0.3 percent on average.

Markups and factor shares. Following Rotemberg and Woodford (1999), a common approach to business cycles is to focus on the cyclical properties of markups. In our model, markups rise in recessions. These markups are not signs of market power, but rather compensation for entrepreneurs' risk taking. The marginal cost of goods is $w_t/f_l'(k_t, \ell_t) = (1 - v_{cet} v_t)$, so the markup is

$$\mu_t = \frac{1}{1 - v_{cet} v_t} - 1 \approx v_{cet} v_t, \tag{25}$$

equal to the risk premium for idiosyncratic risk. We obtain the same markup if we use the capital margin. The average user cost of capital is $r_t + \delta = f_k'(k_t, \ell_t) \times (1 - v_{cet} v_t)$, which implies a markup of $f'_k(k_t, \ell_t)/(r_t + \delta) - 1 = 1/(1 - v_{cet}v_t) - 1 \approx v_{cet}v_t$. The markup arises because entrepreneurs take into account the required exposure to idiosyncratic risk as part of the marginal cost. When markups arise from market power, the value of an additional unit of output is discounted because the marginal revenue product of a factor is less than the value of the marginal product. The two sources of discount are analogous in Rotemberg and Woodford (1999)'s analysis.

We can also compute the labor and capital share of income,

$$\theta_{\ell t} = \frac{w_t \ell_t}{y_t} = (1-\alpha) \times (1 - v_{cet}v_t) \tag{26}$$

$$\theta_{kt} = \frac{R_t k_t}{y_t} = \alpha \times (1 - v_{cet}v_t). \tag{27}$$

We are counting the profits obtained by entrepreneurs, $f(k_t, \ell_t)v_{cet}v_t$, as neither labor nor capital income. The profit share, $v_{cet}v_t$, is countercyclical. These profits are not true economic profits, which would take the value of the revenue of the firm to be $1 - v_{cet}v_t$, to account for risk.

We stress that the mechanism in our paper does not reduce to a time-varying markup because the precautionary saving motive for idiosyncratic risk also depresses the interest rate relative to the model with perfect risk sharing and a time-varying markup. As a result, instead of a common capital and labor wedge (as we would get from adding markups to a perfect risk sharing model), our model delivers a large increase in the labor wedge and a small reduction in the capital wedge after risk shocks.

While the presence of markups is consistent with our model, quantitatively our model cannot explain such large markups. The average markup in the model is 2.8 percent. The average labor share is roughly 64 percent. In the data, an average markup of 15 percent is common in the literature.[6] Ingredients such as imperfect competition and distortionary taxes are required to account for markups in the data.

5.5 Relationship to models of nominal rigidities

In our model risk shocks operate as a tax on labor, a labor wedge. This is the same way that monetary shocks operate in New Keynesian models with sticky nominal wages.[7] In this sense both views of business cycles are complementary, although the mechanism behind the labor wedge itself is different. In our view, even if central banks succeeded in targeting inflation and reproducing the flexible-price allocation, this would not eliminate business cycles. An important distinction is that models with wage rigidities create fluctuations in

[6] See Edmond et al. (2018), Hall (2019). Recent work by De Loecker and Eeckhout (2017) finds an average markup of 60 percent. However, this is a sales-weighted markup. Edmond et al. (2018) report a cost-weighted markup using the same data of 25 percent.

[7] See for example Rotemberg and Woodford (1999).

involuntary unemployment. Our paper abstracts from any frictions in the labor market, so all fluctuations in employment are voluntary. We believe that incorporating labor market frictions into our setting, either in terms of search and matching or in terms of wage setting, can help bring our model closer to the data and policy debates.

Our model also delivers policy recommendations that have a sense of familiarity with those derived from New Keynesian models, so it's worth understanding how they are different. In standard models of nominal rigidities, the optimal policy aims to reproduce the flexible-price allocation, which is typically achieved with a suitable inflation target ("divine coincidence"). In our paper, in contrast, prices are flexible but it is the flexible-price allocation that is inefficient. Optimal policy calls for stimulating employment during recessions, which can be achieved in the presence of nominal rigidities through monetary stimulus. But optimal policy in our model also requires direct stimulus to consumption. If we only stimulated employment, most of the extra output would be devoted to investment, while the planner would like the extra output to go mostly to consumption. In this sense, even with nominal rigidities monetary stimulus alone cannot achieve the constrained-efficient allocation. An interesting open question is what would be the optimal monetary policy if we incorporated nominal rigidities into our environment. How close to the constrained-efficient allocation can we get using only monetary policy?

6 Concluding Remarks

In this paper we explore a risk-premium view of business cycles. Recessions are periods of heightened uncertainty when businesses shrink from risky economic activity. We propose a simple model of business cycles driven by spikes in risk premiums that compensate entrepreneurs for uninsurable idiosyncratic risk. The only deviation from the neoclassical growth model is uninsurable idiosyncratic risk on the business side.

The traditional risk-premium view focuses on the negative impact of higher risk premiums on investment demand. We flip the emphasis and focus on the impact of higher risk premiums on risky labor demand. We highlight that employing workers to produce is a risky activity, and that because capital is a long-duration store of value, a concurrent precautionary saving motive eliminates the negative effect of higher risk premiums on investment demand. In our view recessions are essentially times when businesses reduce their demand for risky labor, which in general equilibrium leads to simultaneous contractions in employment, consumption, and investment.

The view of business cycles we propose has important policy implications. In the competitive equilibrium employment, output, and consumption fall too much during recessions. The inefficiency reflects an aggregate consumption externality. Lower aggregate consumption during recessions drives risk premiums higher, deepening the downturn. Optimal policy involves stimulating employment and consumption during recessions.

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Appendixes

A Competitive equilibrium with adjustment costs to capital

Here we extend the model in the body of the paper to incorporate costs of capital adjustment, show how to characterize the equilibrium in detail, and describe a numerical solution method.

A.1 Setting

Technology. The output flow for entrepreneur i is

$$dY_{it} = f(k_{it}, \ell_{it})dt + f(k_{it}, \ell_{it})v_t dB_{it}. \tag{28}$$

The aggregate resource constraint includes costs of adjusting capital:

$$c_t + i_t = f(k_t, \ell_t) = k_t^{\alpha} \ell_t^{1-\alpha} \tag{29}$$

$$dk_t = (x_t k_t - \delta k_t)dt, \tag{30}$$

where

$$i_t = \phi(x_t)k_t = \left(\frac{\exp(\epsilon(x_t - \delta)) - 1}{\epsilon} + \delta\right) k_t. \tag{31}$$

The marginal cost of expanding capital is $\phi'(x_t) = \exp(\epsilon(x_t - \delta))$. In steady state we have $\phi(\delta) = \delta$ and $\phi'(x_t) = 1$. In the limit with $\epsilon \to 0$, we get the case of no adjustment cost, as in the body of the paper, with $\phi(x_t) = x_t$ and $\phi'(x_t) = 1$.

The level of idiosyncratic risk v_t follows a mean-reverting diffusion

$$dv_t = \theta_v(\bar{v} - v_t)dt + \sqrt{v_t}\sigma_v dZ_t \tag{32}$$

driven by an aggregate Brownian motion Z that captures risk shocks.

Markets and agents' problems. There are competitive markets for goods, and spot markets for labor with wage w_t, and capital with rental R_t. Capital is rented each period by entrepreneurs, and can be held by anyone as an asset with price q_t, and is priced by arbitrage. Aggregate investment is set to maximize profits $\max_x xq_t - \phi(x)$, which implies $\phi'(x_t) = q_t$. Agents can trade Arrow securities contingent on the realization of Z. However, entrepreneurs cannot insure their idiosyncratic risk B_i for incentive reasons (they cannot trade Arrow securities contingent on the realization of B_i). The pricing kernel ξ_t follows

$$d\xi_t/\xi_t = -r_t dt - \pi_t dZ_t,$$

where r_t is the implied interest rate and π_t the price of aggregate risk.

The representative worker's problem is

$$\max_{c_w,\ell,\sigma_{nw}} \mathbb{E}\Big[\int_0^\infty e^{-\rho_w t}\Big(\log(c_{wt}) - \frac{\ell_t^{1+1/\psi}}{1+1/\psi}\Big)dt\Big], \tag{33}$$

$$s.t. \quad dn_{wt} = (n_{wt}r_t + n_{wt}\sigma_{nwt}\pi_t + w_t\ell_t - c_{wt})dt + n_{wt}\sigma_{nwt}dZ_t, \tag{34}$$

and the natural borrowing limit.

An entrepreneur's problem is

$$\max_{c_i,\ell_i,k_i,\sigma_{ni}} \mathbb{E}\Big[\int_0^\infty e^{-\rho_e t}\log(c_{it})dt\Big] \tag{35}$$

$$st: \quad dn_{it} = (n_{it}r_t + n_{it}\sigma_{nit}\pi_t - c_{it} + f(k_{it},\ell_{it}) - w_t\ell_{it} - R_t k_{it})dt + n_{it}\sigma_{nit}dZ_t + f(k_{it},\ell_{it})v_t dB_{it}, \tag{36}$$

and the solvency constraint $n_{it} \geq 0$, which is the natural borrowing limit.

Competitive equilibrium. Total wealth is $n_{et} + n_{wt} = q_t k_t$, where $n_{et} = \int n_{it}di$ is the total wealth of entrepreneurs. For a given initial distribution of wealth, a *competitive equilibrium* is a process for prices (r, π, w, R, q), aggregate capital k and investment $i_t = \phi(x_t)k_t$, a plan for the representative worker (c_w, ℓ), and a plan for each entrepreneur $(c_i, k_t, \ell_i, \sigma_{ni})$ such that every agent optimizes taking prices as given; aggregate investment is optimal $q_t = \phi'(x_t)$; the aggregate resource constraints (29), (30) and (31) hold; and markets clear: $\int \ell_{it}di = \ell_t$, $\int k_{it}di = k_t$, and $n_{et} + n_{wt} = q_t k_t$.

A.2 Characterizing the competitive equilibrium

Worker's problem. The representative worker's problem is standard, and can be written with an intertemporal budget constraint

$$\mathbb{E}\left[\int_0^\infty \xi_t c_t dt\right] \leq n_{w0} + \mathbb{E}\left[\int_0^\infty \xi_t w_t \ell_t dt\right]. \tag{37}$$

The FOCs are

$$e^{-\rho_w t}c_{wt}^{-1} = \kappa_w \xi_t$$

$$e^{-\rho_w t}\ell_t^{1/\psi} = \kappa_w \xi_t w_t.$$

From these we obtain that

$$\ell_t^{1.\psi} = c_{wt}^{-1} w_t, \tag{38}$$

and that the workers' consumption c_{wt} follows

$$dc_{wt}/c_{wt} = \underbrace{(r_t - \rho_w + \sigma_{cwt}^2)}_{\mu_{cwt}} dt + \sigma_{cwt} dZ_t \tag{39}$$

with

$$\sigma_{cwt} = \pi_t. \tag{40}$$

For optimality it is sufficient that (37) holds with equality.

Entrepreneurs' problem. An entrepreneur's problem can be mapped into a standard consumption-portfolio problem. First, optimize over k_{it} and ℓ_{it} within a period, conditional on $y_{it} = f k_{it}, \ell_{it})$. There exists a process λ_t such that

$$\lambda_t y_{it} = \max_{k_{it}, \ell_{it}} k_{it}^{\alpha} \ell_{it}^{1-\alpha} - w_t \ell_{it} - R_t k_{it}$$

$$s.t. \quad k_{it}^{\alpha} \ell_{it}^{1-\alpha} = y_{it}.$$

FOCs for optimality in this sub-problem are

$$\alpha y_{it}/k_{it}(1 - \lambda_t) = R_t \tag{41}$$

$$(1 - \alpha) y_{it}/\ell_{it}(1 - \lambda_t) = w_t. \tag{42}$$

We can then write the entrepreneurs dynamic budget constraint

$$dn_{it} = (n_{it} r_t + n_{it} \sigma_{nit} \pi_t - c_{it} + \lambda_t y_{it}) dt + n_{it} \sigma_{nit} dZ_t + y_{it} v_t dB_{it},$$

where y_{it} is now a choice variable that has replaced k_{it} and ℓ_{it}. This is a standard portfolio problem with a well known solution. To characterize the solution, we can first integrate the dynamic budget constraint into an intertemporal budget constraint

$$\mathbb{E}\left[\int_0^\infty \tilde{\xi}_{it} c_{it} dt\right] \leq n_{i0}, \tag{43}$$

where $\tilde{\xi}_t$ follows

$$d\tilde{\xi}_{it}/\tilde{\xi}_{it} = -r_t dt - \pi_t dZ_t - (\lambda_t / v_t) dB_{it}.$$

The FOC is

$$e^{-\rho_e t} c_{it}^{-1} = \kappa_i \tilde{\xi}_{it}.$$

From this we get

$$dc_{it}/c_{it} = \underbrace{(r_t - \rho_e + \sigma_{cit}^2 + v_{cit}^2)}_{\mu_{cit}} dt + \sigma_{cit} dZ_t + v_{cit} dB_{it}, \tag{44}$$

with

$$\sigma_{cit} = \pi_t \tag{45}$$

$$v_{cit} = \lambda_t / v_t. \tag{46}$$

We can also check that $c_{it} = \rho_e n_{it}$ satisfies these conditions. It is sufficient to check that (43) holds with equality.

Equilibrium. Entrepreneurs' plans are proportional to their net worth, so we need not keep track of the distribution of wealth across entrepreneurs. The state variables are k_t, v_t, and $\eta_t = c_{et}/c_t$.

First, all entrepreneurs get the same $v_{cit} = v_{cet}$. Use (46) to write

$$\lambda_t = v_{cet} v_t.$$

Because all entrepreneurs use the same k_{it}/ℓ_{it} ratio, we obtain from (41) and (42) that

$$R_t = \alpha k_t^{\alpha-1} \ell_t^{1-\alpha} \times (1 - v_{cet} v_t) \tag{47}$$

$$w_t = (1-\alpha) k_t^{\alpha} \ell_t^{-\alpha} \times (1 - v_{cet} v_t). \tag{48}$$

Use $c_{it} = \rho_e n_{it}$ and the budget constraint (36) to write

$$v_{cet} = \frac{k_t^{\alpha} \ell_t^{1-\alpha}}{n_{et}} v_t = \frac{k_t^{\alpha} \ell_t^{1-\alpha}}{c_{et}} \rho_e v_t = \frac{k_t^{\alpha} \ell_t^{1-\alpha}}{c_t} \eta_t^{-1} \rho_e v_t. \tag{49}$$

From (40) and (45), and using $c_t = c_{et} + c_{wt}$, we get that

$$\sigma_{cit} = \sigma_{cet} = \sigma_{cwt} = \sigma_{ct} = \pi_t. \tag{50}$$

Now compute the growth rate of aggregate consumption

$$\mu_{ct} = \eta_t \mu_{cet} + (1 - \eta_t) \mu_{cwt}$$

and use optimality conditions (39) and (44) to obtain

$$\mu_{ct} = \eta_t (r_t - \rho_e + \sigma_{cet}^2 + v_{cet}^2) + (1 - \eta_t)(r_t - \rho_w + \sigma_{cwt}^2)$$

$$\mu_{ct} = (r_t - \bar{\rho}_t + \sigma_{ct}^2) + \eta_t v_{cet}^2,$$

and reorganizing

$$r_t = (\bar{\rho}_t + \mu_{ct} - \sigma_{ct}^2) - \eta_t \times v_{cet}^2, \tag{51}$$

where $\bar{\rho} = \eta_t \rho_e + (1 - \eta_t)\rho_w$.

The law of motion for $\eta_t = c_{et}/c_t$ is

$$d\eta_t = \mu_{\eta t} dt + \sigma_{\eta t} dZ_t, \tag{52}$$

with

$$\mu_{\eta t} = \eta_t (1 - \eta_t)(\mu_{cet} - \mu_{cwt}) = \eta_t (1 - \eta_t)(\rho_w - \rho_e + v_{cet}^2)$$

$$\sigma_{\eta t} = \eta_t (1 - \eta_t)(\sigma_{cet} - \sigma_{cwt}) = 0.$$

From the workers' FOC for labor (38) we get

$$\ell_t^{1/\psi} = ((1 - \eta)c_t)^{-1} \times w_t. \tag{53}$$

Capital is priced by arbitrage. Formally, use agents' budget constraint and the equilibrium conditions to obtain

$$R_t - \phi(x_t) + q_t(\mu_{qt} + x_t - \delta) = q_t(r_t + \sigma_{qt}\pi_t) \tag{54}$$

$$q_t = \phi'(x_t). \tag{55}$$

Equilibrium can then be characterized with equations for entrepreneurs' idiosyncratic risk (49), prices (47), (48), (50), and (51), equilibrium in labor and capital markets (53) and (54), and optimal investment (55), the resource constraint (29), and law of motion for aggregate states (30), (32), (52). Once aggregates have been pinned down, we can recover $c_{et} = \eta c_t$, $c_{wt} = (1 - \eta_t)c_t$, and each entrepreneur's plan given the initial distribution of wealth.

Wedges. We define the labor and capital wedge

$$\ell_t^{1/\psi} \times (1 - \eta)c_t = (1 - \alpha)k^\alpha \ell^{-\alpha} \times (1 - \omega_{\ell t}) \tag{56}$$

$$\alpha k_t^{\alpha-1} \ell_t^{1-\alpha} \times (1 - \omega_{kt}) - \phi(x_t) + q_t(\mu_{qt} + x_t - \delta) = q_t((\bar{\rho}_t + \mu_{ct} - \sigma_{ct}^2) + \sigma_{qt}\sigma_{ct}). \tag{57}$$

Comparing (56) and (57) with equilibrium conditions (53) and (54), we obtain expressions for the capital and labor wedges,

$$\omega_{\ell t} = v_{cet} v_t$$

$$\omega_{kt} = v_{cet} v_t \times \left(1 - \rho_e \times \frac{y_t}{c_t} \times \frac{1}{\alpha} \times \frac{q_t k_t}{y_t}\right).$$

A.3 Recursive equilibrium

We can solve for the competitive equilibrium by setting it up in recursive form, and then solving a system of partial differential equations. The state variables are k, v, η. We look for a pair of C^2 functions, $q(k, v, \eta)$ and $\ell(k, v, \eta)$. In what follows we will typically suppress the states to avoid clutter.

With these two functions we can obtain

$$x = \delta + \frac{1}{\epsilon} \log q,$$

$$i = \phi(x)k = \frac{q-1}{\epsilon}k + \delta k$$
$$c = k^\alpha \ell^{1-\alpha} - \phi(x)k$$
$$v_{ce} = \frac{k^\alpha \ell^{1-\alpha}}{c}\eta^{-1}\rho_e v.$$

From this we already have the laws of motion of the states. We just need to compute the law of motion of aggregate consumption, $dc_t/c_t = \mu_{ct}dt + \sigma_{ct}dZ_t$, with

$$\mu_c = \frac{1}{c}\left[\alpha k^{\alpha-1}\ell^{1-\alpha}(x-\delta)k + (1-\alpha)k^\alpha \ell^{-\alpha}\mu_\ell \ell - \frac{1}{2}\alpha(1-\alpha)k^\alpha \ell^{-\alpha-1}\sigma_\ell^2 \ell^2 \right.$$
$$\left. -(\frac{q-1}{\epsilon} + \delta)k(x-\delta) - qk\frac{1}{\epsilon}\mu_q \right]$$

$$\sigma_c = \frac{(1-\alpha)k^\alpha \ell^{-\alpha}\sigma_\ell \ell - qk\frac{1}{\epsilon}\sigma_q}{c}.$$

These are in terms of q, ℓ, their drift and volatility μ_q, μ_ℓ, σ_q, σ_ℓ, and objects derived from q and ℓ.

Now we can get prices

$$r = \eta\rho_e + (1-\eta)\rho_w + \mu_c - \sigma_c^2 - \eta v_{ce}^2$$

$$\pi_t = \sigma_c$$

$$R = \alpha k^{\alpha-1}\ell^{1-\alpha}(1 - v_{ce}v)$$

$$w = (1-\alpha)k^\alpha \ell^{-\alpha}(1 - v_{ce}v).$$

Then we obtain expressions for the drift and volatility of q and ℓ:

$$\mu_q = \frac{q_k}{q}(x-\delta)k + \frac{q_v}{q}\theta_v(\bar{v} - v) + \frac{q_\eta}{q}\eta(1-\eta)(\rho_w - \rho_e + v_{ce}^2) + \frac{1}{2}\frac{q_{vv}}{q}\sigma_v^2 v \tag{58}$$

$$\sigma_q = \frac{q_v}{q}\sigma_v\sqrt{v} \tag{59}$$

$$\mu_\ell = \frac{\ell_k}{\ell}(x-\delta)k + \frac{\ell_v}{\ell}\theta_v(\bar{v}-v) + \frac{\ell_\eta}{\ell}\eta(1-\eta)(\rho_w - \rho_e + v_{ce}^2) + \frac{1}{2}\frac{\ell_{vv}}{\ell}\sigma_v^2 v \tag{60}$$

$$\sigma_\ell = \frac{\ell_v}{\ell}\sigma_v\sqrt{v}. \tag{61}$$

Finally, we can plug all of this into the two main equilibrium conditions:

$$\ell^{1/\psi} = ((1-\eta)c)^{-1}w \tag{62}$$

$$R - \frac{q-1}{\epsilon} + q(\mu_q + x - \delta) = q(r + \sigma_q \pi). \tag{63}$$

This is a system of equations with two unknowns, the functions q and ℓ, where (63) is a PDE and (62) is an algebraic constraint.

A.4 Numerical solution

The system (62)-(63) can be solved numerically with different methods. We use a projection method with Smolyak interpolation, and a false transient.

i. We build a grid of (k, v, η) points according to the Smolyak method. We use the Mathematica package for Smolyak interpolation by Gary Anderson.[8]

ii. We pick an initial guess for q and ℓ on these points, making sure that the algebraic constraint (62) holds exactly. This requires a numerical solution, but it only happens once.

iii. Given values on the grid, we can compute the first and second derivatives of q and ℓ at each grid point.

iv. We will now "solve backwards" from this initial guess, but we need to make sure (62) always holds at every point. So we differentiate (62) with respect to "time"

$$\frac{\partial}{\partial \ell}[\ell^{1/\psi} - ((1-\eta)c(q,\ell))^{-1}w(q,\ell)]\ell_t' + \frac{\partial}{\partial q}[\ell^{1/\psi} - ((1-\eta)c(q,\ell))^{-1}w(q,\ell)]q_t' = 0, \tag{64}$$

   where we write $c(q,\ell)$ and $w(q,\ell)$ to emphasize the these are functions of q and ℓ. This ensures that as we update our guess, the algebraic constraint (62) always holds with equality.

v. We add a time derivative $\frac{q_t'}{q}$ to (58) and $\frac{\ell_t'}{\ell}$ to (60). These will show up in the PDE (63) through μ_q and r. As a result, we obtain a linear system of two equations for two unknowns, q_t' and ℓ_t', for each point on the grid, which can be easily solved.

[8] https://github.com/es335mathwiz/mathSmolyak

vi. We have now a first order ODE in the "time" dimension. We solve it backwards from the initial guess:

$$q_{new}(k, v, \eta) = q_{old}(k, v, \eta) - q_t'(k, v, \eta)dt$$

$$\ell_{new}(k, v, \eta) = \ell_{old}(k, v, \eta) - \ell_t'(k, v, \eta)dt.$$

We write the state (k, v, η) to emphasize that this happens at every point of the grid. We use a Runge-Kutta 4 integrator to compute this step.

vii. When we converge to $q_t' = \ell_t' = 0$, we have found a solution to our original system of equations (62)-(63).

Once we have a solution, we can reconstruct the stochastic processes that constitute a competitive equilibrium. For example, k_t and η_t solve the SDEs $dk_t = (x(k_t, v_t, \eta_t)k_t - \delta k_t)dt$ and $d\eta_t = \mu_\eta(k_t, v_t, \eta_t)dt + \sigma_\eta(k_t, v_t, \eta_t)dZ_t$, the consumption of each entrepreneur solves $dc_{it}/c_{it} = (r(k_t, v_t, \eta_t) - \rho_e + \sigma_c^2(k_t, v_t, \eta_t) + v_{ce}(k_t, v_t, \eta_t)^2)dt + \sigma_c(k_t, v_t, \eta_t)dZ_t + v_{ce}(k_t, v_t, \eta_t)dB_{it}$, and so on. Once we do this, how do we know we actually have a competitive equilibrium? Because the FOCs and resource constraints are already incorporated into our system of equations, we only need to verify that the resulting allocation is feasible and that agents' plans are indeed optimal. So need to check (1) that q and ℓ are always positive; (2) that the resulting process for k is always positive and the resulting process for η remains in $(0, 1)$; (3) that given the equilibrium process for $r_t = r(k_t, v_t, \eta_t)$, $\pi_t = \pi(k_t, v_t, \eta_t)$, and $v_{cet} = v_{ce}(k_t, v_t, \eta_t)$, agents' intertemporal budget constraints hold with equality, $\mathbb{E}\left[\int_0^\infty \xi_t c_t dt\right] = n_{w0} + \mathbb{E}\left[\int_0^\infty \xi_t w_t \ell_t dt\right]$, and $\mathbb{E}\left[\int_0^\infty \tilde{\xi}_{it} c_{it} dt\right] = (c_{i0}/c_{e0}) \times n_{e0}$, where $n_{e0} = c_{e0}/\rho_e$ and $n_{w0} = q_0 k_0 - n_{e0}$, and where $d\tilde{\xi}_{it}/\tilde{\xi}_{it} = -r_t dt - \pi_t dZ_t - v_{cet} dB_{it}$ and $d\xi_t/\xi_t = -r_t dt - \pi_t dZ_t$. This last step ensures agents' plans are indeed optimal.

B Quantitative exploration

B.1 Calibration and data

Here we describe how we calibrate the model for our numerical results in the body of the paper. To the baseline model in Section 2, we add convex costs of capital adjustment, as described in Appendix A.

We solve the model with the parameter values shown in Table 3. We use an informal combination of calibration to existing research and rough matching of observed moments to arrive at these values. Our objective is to evaluate the quantitative plausibility of the theoretical mechanism we propose. In B.2 we perform a sensitivity analysis to understand how results change under different parameter specifications.

The Cobb-Douglas capital elasticity, $\alpha = 1/3$, is standard. The Frisch elasticity of labor supply, $\psi = 3$, is standard in the macro literature when employment volatility is an

Meaning	*Parameter*	*Value*
Capital share	α	1/3
Frisch elasticity of labor supply	ψ	3
Capital adjustment cost	ϵ	4
Depreciation	δ	0.073
Impatience rate, workers	ρ_w	0.035
Impatience rate, entrepreneurs	ρ_e	0.0975
long run idiosyncratic risk	$\bar{v}$	0.10
Mean-reversion of idiosyncratic risk	θ_v	0.693
Aggregate volatility of idiosyncratic risk	σ_v	0.16

Table 3: Parameter Values

issue. Hall (2009) suggests that an elasticity around that value is a reasonable working approximation for employment fluctuations in an economy with a search and matching setup and realistic equilibrium wage stickiness, in the sense of Hall (2005). Our value of 3 is well above the findings of microeconomic studies of the wage elasticity of labor supply, because it includes effects on unemployment, a component of employment fluctuations not included in micro studies, so far as we know. For the elasticity of the adjustment costs, which helps determine how lower output is split between lower investment and consumption, we use $\epsilon = 4$ to match the observed split roughly.

We set workers' impatience $\rho_w = 0.035$ and the depreciation rate $\delta = 0.073$ to match the ratios $k/y = 3$ and $c/y = 0.8$, which play an important role in the model, as we describe below in Section 3.2. In the ergodic distribution we have $\mathbb{E}[k_t/y_t] = 3.01$ and $\mathbb{E}[c_t/y_t] = 0.78$. We set $\rho_e = \rho_w + 0.0625$ to obtain a steady-state idiosyncratic volatility of wealth $v_{net} = v_{cet} = \sqrt{\rho_e - \rho_w} = 25$ percent, roughly corresponding to the idiosyncratic risk in the stock market as in Herskovic et al. (2016).

For the stochastic process for v_t we use $\bar{v} = 0.1$, in line with evidence in Bloom et al. (2018) on idiosyncratic productivity risk at the establishment level; $\theta_\sigma = 0.692$ so risk shocks have a half-life of one year; and $\sigma_v = \frac{1}{2}\sqrt{\bar{v}} = 0.158$ so that a two-standard deviation shock doubles idiosyncratic risk. We aim to capture transitory fluctuations in idiosyncratic risk at the business cycle frequency. These numbers are broadly in line with evidence in Herskovic et al. (2016) for idiosyncratic risk in the stock market. The implied steady-state ratio of entrepreneurial consumption to worker consumption is $\eta_{ss} \approx 5$ percent.

Figure 3 compares the model to US data. For the data we use output, consumption of nondurables and services, investment (including durables), hours and real compensation per hour in the non-farm business sector, all quarterly, in logs, and per capita terms. For the interest rate we use quarterly time series on interest rates with one year maturity from Schneider (2019), in levels. All series are HP-filtered with parameter 1600.[9]

B.2 Sensitivity Analysis

To describe how the model works, we study perturbations to the benchmark parameter values—we change one parameter value at a time while holding the other parameters at their baseline values. We focus on the effects of each perturbation on the standard deviations of output, consumption, investment, and employment. Table 4 presents the results.

Frisch elasticity ψ. A successful model of business cycles requires a relatively large elasticity of labor supply. With a smaller Frisch elasticity around $\psi = 1$, more consistent with micro evidence, the effects of risk shocks are dampened, but would still look like business cycles. The second column of table 4 shows that the standard deviations of all variables would be smaller.

In the limiting case with $\psi = 0$, employment and output would be fixed and would not respond to risk shocks. We would still get a countercyclical capital wedge, so risk shocks would create a small spike in investment and a small contraction in consumption, with no movement in employment or output on impact.

Persistence θ_σ and capital adjustment costs ϵ. The mean reversion parameter θ_v and the curvature of the adjustment cost function ϵ are important in determining how a decline in output is split into lower consumption and lower investment in response to a larger labor wedge. The elasticity of intertemporal substitution would also play a role here, but it is pinned at one with log preferences.

We calibrate a transitory risk shock to have a half-life of one year. Agents are averse to fluctuations in consumption in response to such transitory shocks, so investment takes the brunt of the adjustment. A convex capital adjustment cost function reduces fluctuations in investment and increases fluctuations in consumption.

The second and third columns of table 4 show standard deviations for smaller adjustment costs $\epsilon = 3.5$ and for more transitory risk shocks $\theta_v = 1.32$, which implies a half-life of two years. With smaller adjustment costs the standard deviation of investment becomes larger. With more transitory shocks all standard deviations become smaller.

Volatility of risk shocks σ_v. Risk shocks are the sole exogenous driving force in the model. With a smaller volatility of idiosyncratic risk, the standard deviation of all variables would be smaller. The fourth column of table 4 shows standard deviations for a smaller $\sigma_v = 0.1$.

[9] We thank Andres Schneider for sharing his data with us.

	Benchmark⇒	$\psi = 3$	$\epsilon = 4$	$\theta_v = 0.693$	$\sigma_v = 0.16$	$\bar{v} = 0.10$	$\rho_e = .0975$
	Variant⇒	$\psi = 1$	$\epsilon = 3.5$	$\theta_v = 1.38$	$\sigma_v = 0.10$	$\bar{v} = 0.05$	$\rho_e = 0.0575$
y	1.4%	1%	1.5%	1%	1%	1.3%	1.2%
c	0.7%	0.7%	0.7%	0.5%	0.45%	0.9%	1%
i	4.3%	2.5%	4.6%	3%	3.1%	2.7%	2.5%
ℓ	2%	1.2%	2.1%	1.4%	1.3%	1.4%	1.2%

Table 4: Log Standard Deviations of Key Variables at Quarterly Frequency in the Benchmark Case and Variants

Steady-state level of idiosyncratic risk $\bar{v}$. The effects of risk shocks become larger with the long run level of idiosyncratic risk, $\bar{v}$. The effect of risk enters the model through second moments, so if idiosyncratic risk is on average small, an increase has only small effects. We calibrate $\bar{v} = 10$ percent in line with Bloom et al. (2018). A lower long run level of idiosyncratic risk $\bar{v} = 5$ percent reduces the effects of risk shocks, and therefore reduces the standard deviation of all variables, as shown in the fifth column of table 4.

Steady-state idiosyncratic risk in entrepreneurs' consumption v_{ce}^{ss}. The effects of risk shocks also depend on the level of idiosyncratic risk in entrepreneurs' consumption or wealth. The sixth column of table 4 shows standard deviations for a smaller target for this idiosyncratic risk, $v_{ce}^{ss} = 15$ percent. Hitting this target requires a lower impatience rate of entrepreneurs, $\rho_e = \rho_w + (v_{ce}^{ss})^2 = 0.0575$.

C Planner problem

Here we study the planner's problem in more detail. We also incorporate adjustment costs as in Appendix A. The case with $\epsilon \to 0$ recovers the setting in the body of the paper without adjustment costs.

The planner maximizes agents' utility, with weight γ on the representative worker and $(1-\gamma)$ on entrepreneurs as a whole. An individual entrepreneur's consumption follows $dc_{it}/c_{it} = \mu_{cet}dt + v_{cet}dB_{it} + \sigma_{cet}dZ_t$, so $c_{it} = (c_{i0}/c_{e0}) \times c_{et} \times \exp\left(\int_0^t v_{cet}dB_{i,t} - \frac{1}{2}v_{cet}^2 dt\right)$, so the associated utility is

$$\mathbb{E}\left[\int_0^\infty e^{-\rho_e t}\log c_{it}dt\right] = \log(c_{i0}/c_{e0})/\rho_e + \mathbb{E}\left[\int_0^\infty e^{-\rho_e t}\left(\log c_{et} - \int_0^t \frac{1}{2}v_{cet}^2 ds\right)dt\right]$$
$$= \log(c_{i0}/c_{e0})/\rho_e + \mathbb{E}\left[\int_0^\infty e^{-\rho_e t}\left(\log c_{et} - \frac{1}{\rho_e}\frac{1}{2}v_{cet}^2\right)dt\right].$$

The first term captures the initial inequality among entrepreneurs. If they all had the same Pareto weight we would pick $c_{i0} = c_{e0}$ for all entrepreneurs. In any case, it is just a constant. So the planner's objective is

$$\max_{c,i,\ell,k,\eta} \mathbb{E}\left[\int_0^\infty \gamma e^{-\rho_w t}\left(\log c_{wt} - \frac{\ell_t^{1+1/\psi}}{1+1/\psi}\right) + (1-\gamma)e^{-\rho_e t}\left(\log c_{et} - \frac{1}{2}\frac{1}{\rho_e}v_{cet}^2\right) dt\right].$$

Notice that, because entrepreneurs are more impatient than workers, $\rho_e > \rho_w$, in the first best with perfect risk sharing, they would receive zero consumption in the long run, $\lim_{t\to\infty} \eta_t = 0$. Because entrepreneurs can save on their own and have a precautionary saving motive v_{cet}^2, their consumption will not vanish in the long run. The consumption ratio η_t will have an ergodic distribution, and will converge to a steady state in the absence of shocks, just as in the competitive equilibrium. For this reason, entrepreneurs' utility vanishes from the objective function in the long run —it is infinitesimal relative to workers'. As we show below, there is a fundamental disagreement between the planner and private entrepreneurs about their consumption profiles. They save according to their Euler equation, which places a high marginal value on consumption in the future, to self insure, but the planner would like them to follow an inverse Euler equation, which places a lower value on consumption in the future because it eliminates the precautionary saving motive.

With this in mind, and because our concern is about the properties of the planner's problem in the ergodic distribution, we can ignore entrepreneurs' utility in the objective function ($\gamma = 1$), and maximize only workers' utility subject to a given η_0, which captures entrepreneurs' past savings, which must be respected. In the long run the planner does not care directly about entrepreneurs' consumption c_{et} or exposure to idiosyncratic risk v_{cet}. The planner only cares because, if exposed to risk, they will save to self-insure (raise η_t) and leave less consumption for workers in the future, $c_{wt} = c_t(1-\eta_t)$. The planner's problem therefore is

$$\max_{c,i,\ell,k,\eta} \mathbb{E}\left[\int_0^\infty e^{-\rho_w t}\left(\log(c_t(1-\eta_t)) - \frac{\ell_t^{1+1/\psi}}{1+1/\psi}\right) dt\right] \tag{65}$$

subject to the resource constraints and law of motion of v_t, (29)-(32), and

$$v_{cet} = \frac{k_t^\alpha \ell_t^{1-\alpha}}{c_t}\rho_e \eta_t^{-1} v_t \tag{66}$$

$$\mu_{\eta t} = \eta_t(1-\eta_t)(\rho_w - \rho_w + v_{cet}^2), \qquad \sigma_{\eta t} = 0, \tag{67}$$

with η_0 given.

C.1 Recursive formulation

The states are k, v, and η, just as in the competitive equilibrium. The planner's value function is $V(k, v, \eta)$, and the associated HJB equation is

$$\rho_w V = \max_{c,x,\ell} \log c + \log(1-\eta) - \frac{\ell^{1+1/\psi}}{1+1/\psi} + V_k' k(x-\delta) \tag{68}$$
$$+V_\eta' \eta(1-\eta)\left(\rho_w - \rho_e + \left(\frac{k^\alpha \ell^{1-\alpha}}{c}\rho_e \eta^{-1} v\right)^2\right) + V_v' \theta_v(\bar{v} - v) + \frac{1}{2} V_{vv}' v \sigma_v^2$$

subject to

$$c + \phi(x)k = k^\alpha \ell^{1-\alpha}. \tag{69}$$

Instead of working with the HJB equation, it is easier and more revealing to work with the co-states, $m_k = V_k'$, $m_v = V_v'$, $m_\eta = V_\eta'$. Taking first-order conditions, we have

$$c^{-1}\left(1 - 2m_\eta \eta(1-\eta) v_{ce}^2\right) = \lambda \qquad [c] \tag{70}$$
$$(1-\alpha)k^\alpha \ell^{-\alpha}\left(\lambda + 2m_\eta \eta(1-\eta)\frac{v_{ce}^2}{k^\alpha \ell^{1-\alpha}}\right) = \ell^{1/\psi} \qquad [\ell] \tag{71}$$
$$\lambda \phi'(x) = m_k, \qquad [x], \tag{72}$$

where λ is the Lagrange multiplier on (69). The first condition says that giving consumption not only delivers utility, but also relaxes entrepreneurs' risk sharing, v_{ce}. This determines the marginal value of goods λ. The second condition equates the marginal disutility of labor, $\ell^{1/\psi}$, to its marginal product, taking into account that more labor increases entrepreneurs' exposure to idiosyncratic risk. The third condition says that the marginal value of more capital should be equated to its marginal cost. Together with the resource constraint we can solve for the controls c, x , ℓ, and for λ as a function of the states and co-states. It is hard to obtain closed-form expressions, but easy to eliminate c and λ and obtain a system of two equations for x and ℓ.

Now we differentiate the HJB equation with respect to each state to obtain a law of motion for m_k and m_η (m_v is not directly used):

$$\rho_w m_k = m_k(x-\delta) + 2m_\eta \eta(1-\eta)v_{ce}^2 \frac{\alpha}{k} + \lambda\left(\alpha k^{\alpha-1}\ell^{1-\alpha} - \phi(x)\right) + \mu_{m_k} m_k \qquad [m_k] \tag{73}$$
$$\rho_w m_\eta = -\frac{1}{1-\eta} - 2m_\eta \eta(1-\eta)\eta^{-1} v_{ce}^2 + m_\eta(1-2\eta)(\rho_w - \rho_e + v_{ce}^2) + \mu_{m_\eta} m_\eta \qquad [m_\eta]. \tag{74}$$

The equation for m_k resembles an asset-pricing equation for capital. Capital delivers dividends net of new investment, transformed into utility using λ. It grows at rate $x - \delta$ but it is discounted more heavily because it exposes entrepreneurs to idiosyncratic risk. With this interpretation, the first-order condition for investment, x, can be understood as a

Tobin's Q expression, properly taking into account the role of idiosyncratic risk. The equation for m_η captures the fact that higher η means that a smaller fraction of consumption goes to workers, who are all the planner cares about in the long run formulation. On the other hand, higher η reduces entrepreneurs' exposure to idiosyncratic risk, and therefore the future value of η.

C.2 Optimality conditions

To understand the optimality conditions we to start with the planner's evaluation of the marginal products of labor and capital,

$$\tilde{w}_t = (1-\alpha)k_t^\alpha \ell_t^{-\alpha} \times \overbrace{(1 - 2m_\eta \eta(1-\eta)v_{cet}^2(1 - c_t/y_t))}^{>1} \tag{75}$$

$$\tilde{R}_t = \alpha k_t^{\alpha-1}\ell_t^{1-\alpha} \times (1 - 2m_\eta \eta(1-\eta)v_{cet}^2(1 - c_t/y_t)). \tag{76}$$

The marginal product of capital and labor are distorted symmetrically to account for their contribution to idiosyncratic risk in entrepreneurs' consumption v_{cet} through (22). If entrepreneurs face higher idiosyncratic risk, their precautionary motive v_{cet}^2 raises η_t through (23). $m_\eta < 0$ is the co-state associated with η_t, and measures the cost for the planner of devoting a larger fraction of consumption to entrepreneurs—recall that, in the long run, the planner only gives them consumption because they have access to hidden savings, but would prefer to give all consumption to workers.

In contrast to the competitive equilibrium, this distortion *raises* the marginal product of capital and labor above their true value, that is, the term in parenthesis is larger than 1. This reflects the consumption externality described in Section 4.3. While higher output raises entrepreneurs' exposure to idiosyncratic risk, higher consumption improves risk sharing and reduces their exposure to idiosyncratic risk, and this second effect must dominate because output is greater than consumption, $y_t > c_t$.

The consumption externality also shows up in the valuation of output at different points in time or states. The marginal value of a unit of output at time t is

$$\lambda_t = c_t^{-1} \times \overbrace{(1 - 2m_\eta \eta(1-\eta)v_{ce}^2)}^{M_t>1} = c_t^{-1} M_t \tag{77}$$

In the first-best equilibrium with perfect risk-sharing, it would be c_t^{-1}, as usual. But the planner realizes that extra consumption not only delivers utility, but also relaxes idiosyncratic risk sharing. Hence, the $M_t > 1$ factor. The planner only cares about the consumption of workers, $c_{wt} = c_t(1-\eta_t)$, but aggregate consumption c_t appears in (77) because the planner must give a fraction η_t to entrepreneurs, so the marginal value is $(1-\eta_t) \times c_{wt}^{-1} = c_t^{-1}$.

We can now express the optimality conditions for employment and investment in an analogous way to (13) and (14). First, use (71) and plug in λ from (70),

$$\ell^{1/\psi} = (1-\alpha)k^{\alpha}\ell^{-\alpha}\left(c^{-1}\left(1-2m_\eta\eta(1-\eta)v_{ce}^2\right)+2m_\eta\eta(1-\eta)\frac{v_{ce}^2}{k^{\alpha}\ell^{1-\alpha}}\right)$$

$$\ell^{1/\psi}c = (1-\alpha)k^{\alpha}\ell^{-\alpha}\left(\left(1-2m_\eta\eta(1-\eta)v_{ce}^2\right)+2m_\eta\eta(1-\eta)v_{ce}^2\frac{c_t}{k^{\alpha}\ell^{1-\alpha}}\right)$$

$$\ell^{1/\psi}c = \underbrace{(1-\alpha)k^{\alpha}\ell^{-\alpha}\times\left(1-2m_\eta\eta(1-\eta)v_{ce}^2(1-c/y)\right)}_{\tilde{w}}. \tag{78}$$

Second, let $\lambda = c^{-1}\times M$ and use (73):

$$\rho_w m_k = m_k(x-\delta)+2m_\eta\eta(1-\eta)v_{ce}^2\frac{\alpha}{k}+\lambda\left(\alpha k^{\alpha-1}\ell^{1-\alpha}-\phi(x)\right)+\mu_{m_k}m_k$$

$$\rho_w m_k = m_k(x-\delta)+c^{-1}\times\underbrace{\alpha k^{\alpha-1}\ell^{1-\alpha}\times\left((1-2m_\eta\eta(1-\eta)v_{ce}^2)(1-c/y)\right)}_{\tilde{R}}-c^{-1}M\phi(x)+\mu_{m_k}m_k$$

$$\rho_w c^{-1}M\phi(x) = c^{-1}M\phi(x)(x-\delta)+c^{-1}\times\tilde{R}-c^{-1}M\phi(x)+\mu_{m_k}c^{-1}M\phi(x)$$

$$\rho_w M\phi'(x) = M\phi'(x)(x-\delta)+\tilde{R}-M\phi(x)+\mu_{m_k}M\phi'(x).$$

Reorganizing, we get

$$\tilde{R} = M\phi'(x)\left(\rho_w-\mu_{m_k}+\frac{\phi(x)}{\phi'(x)}-x+\delta\right). \tag{79}$$

In the case without adjustment costs, $\epsilon = 0$, we have $\phi(x) = x$ and $\phi'(x) = 1$, and we can write $\mu_{m_k} = -\mu_c+\sigma_c^2+\mu_M-\sigma_M\sigma_c$, so we obtain

$$\tilde{R} = M\left(\rho_w+(\mu_c-\sigma_c^2-\mu_M+\sigma_M\sigma_{ce})+\delta\right), \tag{80}$$

which is easier to interpret.

The optimality condition for labor (78) is relatively straightforward. The one for investment (80) has, on the right-hand side, the opportunity cost of capital, taking into account that the alternative, consumption, not only delivers utility, but also relaxes risk sharing, so it is multiplied by M_t. The term $(\mu_{ct}-\sigma_{ct}^2-\mu_{Mt}+\sigma_{Mt}\sigma_{ct})$ is minus the drift of λ_t, which measures the marginal value of output at a point in time. In the first best it would be $\mu_{ct}-\sigma_{ct}^2$. Relative to this, we have to take into account the dynamic behavior of M_t, which reflects the discussion in Section 4.3. Suppose M_t is large today but is expected to go down in the future, $\mu_{Mt} < 0$. The opportunity cost of investment is especially high. It requires diverting resources from consumption when the value of improving idiosyncratic risk sharing through higher consumption is particularly high, only to obtain output in the future when its value relaxing idiosyncratic risk sharing is expected to be lower. The term $\sigma_{Mt}\sigma_{ct}$ captures (minus) the covariance between the marginal utility of consumption c_t^{-1}

and M_t. The full expression with adjustment costs (79) captures the same tradeoffs, but taking into account the marginal cost of producing capital.

We can now describe the planner's response to a risk shock that raises idiosyncratic risk v_t. Relative to the first best, the planner wants to raise consumption c_t to improve idiosyncratic risk sharing, and accomplishes this by raising employment ℓ_t and reducing investment i_t. In terms of wedges, we get a subsidy to labor (a lower labor wedge) and a tax on capital (a higher capital wedge).

Finally both optimality conditions have a distortion relative to the first-best, related to η_t, but this is a slow-moving state variable that does not react on impact to risk shocks, and therefore plays a secondary role in the response of the optimal allocation to risk shocks. First, the income effect on workers' labor supply in (78) depends on aggregate consumption c_t, rather than workers' consumption $c_{wt} = c_t(1-\eta_t)$. The reason is that a fraction η_t of the extra consumption must be given to entrepreneurs and wasted, so the income effect is $(1-\eta_t) \times c_{wt}^{-1} = c_t^{-1}$. Second, the impatience rate in (79) is workers' ρ_w, rather than the average $\bar{\rho}_t = \eta_t \rho_e + (1-\eta_t)\rho_w$. While the planner must give some consumption to entrepreneurs, the planner actually only cares about workers, so intertemporal tradeoffs are evaluated using their impatience rate ρ_w. But while these effects are important in the long run, they play only a secondary role in business cycles because η_t does not react on impact and moves only slowly.

C.3 Numerical solution

We construct C^2 functions $m_k(k,v,\eta)$, $m_\eta(k,v,\eta)$, $\ell(k,v,\eta)$, and $x(k,v,\eta)$. Using Ito's lemma we transform equations (73) and (74), together with the first-order conditions (70) and the resource constraint, into a system of two second-order PDEs and two algebraic constraints, using the resource constraint and first FOC to eliminate c and λ. We can solve this differential-equation system using numerical methods analogous to those used to solve for the competitive equilibrium.

To this end, we start with $m_k(k,v,\eta)$ $m_\eta(k,v,\eta)$, $\ell(k,v,\eta)$, $x(k,v,\eta)$, and build

$$c = k^\alpha \ell^{1-\alpha} - \phi(x)k$$

$$v_{ce} = \frac{k^\alpha \ell^{1-\alpha}}{c} \rho_e \eta^{-1} v$$

$$\lambda = c^{-1}(1 - 2m_\eta \eta(1-\eta)v_{ce}^2).$$

The FOCs for ℓ and x give us two algebraic constraints:

$$\ell^{1/\psi} c = (1-\alpha)k^\alpha \ell^{-\alpha} \times \left(1 - 2m_\eta \eta(1-\eta)v_{ce}^2(1-c/y)\right) \tag{81}$$

$$m_k = \lambda \phi'(x). \tag{82}$$

Then we compute the drift of the co-states

$$\mu_{m_k} = \frac{\partial_k m_k}{m_k} k(x-\delta) + \frac{\partial_v m_k}{m_k}\theta_v(\bar{v}-v) + \frac{\partial_\eta m_k}{m_k}\eta(1-\eta)(\rho_w - \rho_e + v_{ce}^2) + \frac{\partial_{vv} m_k}{m_k}\sigma_v^2 v$$

$$\mu_{m_\eta} = \frac{\partial_k m_\eta}{m_\eta} k(x-\delta) + \frac{\partial_v m_\eta}{m_\eta}\theta_v(\bar{v}-v) + \frac{\partial_\eta m_\eta}{m_\eta}\eta(1-\eta)(\rho_w - \rho_e + v_{ce}^2) + \frac{\partial_{vv} m_\eta}{m_\eta}\sigma_v^2 v.$$

Plugging this into the co-state equations (73) and (74) we obtain two PDEs, which together with (81) and (82) define a system of equations that we can solve numerically with analogous methods to those used for the competitive equilibrium.

C.4 Understanding the inefficiency: the role of hidden trade

To show why the competitive equilibrium is inefficient, it is useful to discuss briefly the moral-hazard microfoundation for the incomplete idiosyncratic risk-sharing problem. What we have in mind is that individual shocks are private information of entrepreneurs, so, under a scheme of insurance based on their reports, they can misreport advantageously. An entrepreneur must be exposed to idiosyncratic risk to overcome this problem—this is the rationale for constraint (66).

There is also hidden trade. Entrepreneurs and workers can trade claims in financial markets contingent on aggregate shocks, and entrepreneurs can also access labor and capital rental markets to trade capital and labor among themselves.[10] This means that all entrepreneurs must have the same exposure to idiosyncratic risk v_{cet} and the same k/ℓ ratios, and that workers and entrepreneurs follow their Euler equations and aggregate risk sharing conditions, which take r_t and π_t as given. This implies that prices appear in the incentive constraints for privately optimal contracts. This is the ultimate source of inefficiency. The planner realizes that a change in aggregate consumption changes equilibrium prices r_t and π_t that appear in the IC constraints of privately optimal contracts. So the planner only needs to respect constraint (67), which has no prices in it. For any path for consumption for entrepreneurs and workers that respects (67) there is a set of prices r_t and π_t that make it consistent with agents' private Euler and risk sharing equations.

In the case where the planner observes trades, we drop constraint (23) and treat η_t as a choice variable, or equivalently, choose c_{et} and c_{wt} separately. In this case the planner and privately optimal contracts would both want entrepreneurs to follow a (modified) inverse Euler equation. For an entrepreneur i,

$$\lambda_t e^{\rho_e t} c_{it} \times \frac{1}{1 + v_{cet}^2/\rho_e} \tag{83}$$

must be a martingale, where $\lambda_t = e^{-\rho_w t} c_{wt}^{-1}$ is the pricing kernel. As a comparison, the

[10]See Di Tella and Sannikov (2016) and Di Tella (2019).

inverse Euler equation says that $\lambda_t e^{\rho_e t} c_{it}$ is a martingale, and the Euler equation says that $\lambda_t^{-1} e^{-\rho_e t} c_{it}^{-1}$ is a martingale. With log preferences, the difference between the Euler equation and the traditional inverse Euler equation is the precautionary motive, which arises because the marginal utility c_{it}^{-1} is a convex function of c_{it}. As a result, the inverse Euler equation wants to front-load consumption relative to the Euler equation. The planner would like to eliminate the precautionary motive, which played such an important role in the competitive equilibrium. The modified inverse Euler equation (83) also eliminates the precautionary motive, but it adds the factor $1/(1 + v_{cet}^2/\rho)$. This captures the fact that giving consumption to an entrepreneur not only gives utility, as in the classic Rogerson (1985) setting, but also relaxes the idiosyncratic risk sharing constraint (22).

We can aggregate (83) to obtain

$$e^{(\rho_e - \rho_w)t} \frac{c_{et}}{c_{wt}} \propto 1 + v_{cet}^2/\rho_e. \tag{84}$$

This expression has two implications. First, without hidden trade, the planner wants to give relatively more consumption to entrepreneurs when idiosyncratic risk is high, in order to relax idiosyncratic risk sharing, which depends on entrepreneurs' consumption, $v_{cet} = f(k_t, \ell_t)/c_{et} \times \rho_e v_t$. With hidden trade, in the competitive equilibrium, we have instead $c_{et} = \eta_t c_t$ and $\sigma_{\eta t} = 0$ because agents can privately share aggregate risk, so the ratio c_{et}/c_{wt} cannot react on impact to a risk shock.

In other words, what the planner would really like to do is to front-load entrepreneurs' consumption c_{et} when risk is high to relax idiosyncratic risk sharing, but the planner cannot do that when agents have access to hidden trade, because they trade in financial markets to share aggregate risk. So the planner must front-load *aggregate* consumption c_t to relax idiosyncratic risk sharing, at the cost of distorting workers' intertemporal consumption smoothing and labor supply. A subsidy to labor helps correct this distortion.

Second, without hidden trade, since entrepreneurs are more impatient than workers $\rho_e > \rho_w$, their consumption share vanishes in the long run, $\eta_t \to 0$. This reflects that the planner wants entrepreneurs to follow the inverse Euler equation, which doesn't have a precautionary saving motive. Once we add hidden trade, entrepreneurs use their Euler equation with a precautionary motive, and the planner is constrained to give them consumption even in the long-run, η_t does not limit to zero. But while the planner is forced to give some of the consumption to entrepreneurs, he is not forced to make workers supply labor, so he also uses a labor tax in the long run to correct the income effect on labor supply.

This long-run argument is why we can drop entrepreneur's utility from the planner's objective function in the long-run. But it does not depend on assuming entrepreneurs are more impatient. If instead we assume $\rho_e = \rho_w$, we get the same result. In that case, without hidden trade η_t would converge to some ergodic distribution, according to (84). With hidden trade entrepreneurs would account for all of consumption in the long-run,

$\eta_t \to 1$, because of their precautionary savings motive, according to (23). The marginal utility of entrepreneurs' consumption would then limit to zero relative to the marginal utility of workers' consumption, so we would be able to drop entrepreneurs utility from the objective function, just as in the case with $\rho_e > \rho_w$. In other words, the elimination of entrepreneurs' utility from the objective function does not depend on $\rho_e > \rho_w$, but rather on the fundamental disagreement between private entrepreneurs and the planner about how to evaluate consumption paths—private entrepreneurs want to follow an Euler equation with a precautionary motive that assigns a high marginal value to consumption in the future, while the planner wants them to follow an inverse Euler equation that does not.