FM LASER OSCILLATION - THEORY AND EXPERIMENT

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Summary

We consider a type of laser oscillation wherein the laser modes oscillate with approximately FM phases and nearly Bessel function amplitudes, and thereby comprise the sidebands of a frequency-modulated light signal. This type of FM oscillation is induced by an intra-cavity phase perturbation which is driven at a frequency which is approximately, but not exactly, that of the axial mode spacing. The resulting laser oscillation frequency may, in effect, be swept over the entire Doppler linewidth. A first order theory, some experimental results, and an interesting application are considered.

Introduction

A typical present day gas laser oscillates in a large number of relatively independent axial modes. These modes are driven by spontaneous emission; they saturate essentially independently; and are, to a good approximation, uncoupled. The output of such a laser is not nearly as coherent as might be desired for many communication and spectroscopic applications.

Recently, Harris and Targ have demonstrated FM oscillation of a He-Ne laser.[1] Such oscillation is achieved by the utilization of an intra-cavity phase perturbation; and is a parametric oscillation wherein the previously uncoupled laser modes become the spectral components or sidebands of a frequency modulated light signal. The intra-cavity phase perturbation is driven at a frequency which is approximately, but not exactly, the frequency of the axial mode spacing (i.e., $c/2L$ cycles per second, where L is the length of the optical resonator). The resulting laser oscillation consists of a set of modes which have nearly Bessel function amplitudes and FM phases; and which, in effect, is swept over the entire Doppler linewidth at a sweep frequency which is approximately that of the axial mode spacing.

As opposed to normal multi-mode laser oscillation, FM laser oscillation is truly coherent. Both the relative amplitude and relative phase of its spectral components are completely specified. In fact, Massey, Oshman, and Targ[2] have demonstrated that the output from an FM laser may be converted to a single and essentially monochromatic optical signal containing all of the power previously distributed between the spectral components of the FM signal.

In the following sections we consider a linear theory of FM laser oscillation, describe the initial experiments and experimental techniques, and discuss a particularly intersting application of the FM laser. Before proceeding, it should be noted that an alternate method of establishing laser coherence has been proposed and demonstrated earlier by Hargrove, et al.,[3] who utilized an intra-cavity time-varying loss to achieve an AM type phase locking wherein the laser modes oscillated with nearly equal amplitudes and AM phases; with the result that the time domain output consisted of a periodic set of extremely narrow pulses. Such AM type phase locking has been analyzed by DiDomenico[4] and Yariv.[5] It should also be noted that the effect of phase perturbations on a passive resonant optical cavity was first considered by Gordon and Rigden.[6] The "on-frequency" case of a time-varying phase perturbation in an active cavity was first considered by Yariv.[5,7] The "off-frequency" case, i.e., that of steady-state FM laser oscillation, was first considered by Harris and McDuff,[8] and will be described in the following section.

First Order Theory

A first order theory of FM laser oscillation may be obtained by utilizing a set of equations derived by Lamb,[9] and termed as "self-consistency" equations. These equations describe the effect of an arbitrary optical polarization on an optical cavity, and are as follows:

$$(\nu_n + \dot{\varphi}_n - \Omega_n)\, E_n = -\frac{1}{2}\left(\frac{\nu}{\epsilon_0}\right) C_n(t) \quad (1a)$$

and

$$\dot{E}_n + \frac{1}{2}\left(\frac{\nu}{Q_n}\right) E_n = -\frac{1}{2}\left(\frac{\nu}{\epsilon_0}\right) S_n(t) \quad . \text{(1b)}$$

In the above equations, E_n , ν_n , and φ_n are the amplitude, frequency,† and phase, respectively, of the nth cavity mode; and $C_n(t)$ and $S_n(t)$ are the in-phase and quadrature components of its driving polarization. That is, the total cavity electromagnetic field is given

$$E(z,t) = \sum_n E_n(t) \cos\,[\nu_n t + \varphi_n(t)]\; U_n(z) \quad , \text{(2a)}$$

†Following Lamb's notation, we adopt the convention that all symbols for frequencies shall denote circular frequencies.

where $U_n(Z) = \sin n \pi Z/L$; and the polarization driving the n^{th} mode has the form

$$P_n(t) = C_n(t) \cos [\nu_n t + \varphi_n(t)] + S_n(t) \sin [\nu_n t + \varphi_n(t)] \quad . \tag{2b}$$

Other symbols are defined as follows: Ω_n = frequency of the n^{th} mode in the absence of a driving polarization; $\Delta\Omega$ = frequency interval between axial modes $(\Delta\Omega = \pi c/L)$; $Q_n = Q$ of the n^{th} mode; ν = average optical frequency. Once the cavity polarization, $P_n(t)$, and therefore $C_n(t)$ and $S_n(t)$ are known in terms of $E_n(t)$; then Eqs. (1a) and (1b) completely determine the amplitude, frequency, and phase of the optical frequency oscillations.

The cavity polarization $P_n(t)$ consists of a parametric contribution resulting from the intracavity time-varying dielectric perturbation, and of an atomic contribution resulting from the presence of the inverted atomic media. The dielectric or phase perturbation might be accomplished by means of an electro-optic crystal situated at one end of the laser cavity, or perhaps by some type of acoustic mirror vibration. We assume the phase perturbation to have an instantaneous phase retardation of $\delta \cos \nu_m t$ radians, where δ is the peak phase retardation which an optical signal collects on a single one-way pass through the perturbing element, and ν_m is the driving frequency. We also assume that the perturbation is situated at one end of the laser cavity, and that it occupies a length which is only a small fraction of the total cavity length. If the cavity Q is sufficiently high that only the contributions of immediately adjacent modes need be retained, then it may be shown[8] that the parametric polarization driving the n^{th} cavity mode is given by

$$P_n(t) = \frac{E_0 \, \delta \, c}{\nu L} [E_{n+1} \cos (\nu_n t + \varphi_{n+1}) + E_{n-1} \cos (\nu_n t + \varphi_{n-1})] \quad . \tag{3}$$

Thus a particular cavity mode is driven by only the immediately adjacent cavity modes. It might be noted again that Eq. (3) implies a phase perturbation which occupies only a small fraction of the total cavity length. If a longer perturbing element is used, its spatial variation is of particular importance. For instance, it may readily be shown that a perturbation which uniformly fills the entire laser cavity will produce no driving polarization at adjacent modes.

We introduce the atomic contribution to the polarization by means of macroscopic quadrature and in-phase components of susceptibility, denoted by χ_n'' and χ_n' , respectively. In an exact theory,[9] χ_n'' and χ_n' depend upon E_n and thereby include the effects of atomic saturation, power dependent mode pulling and pushing, and non-linear coupling effects. We next resolve $P_n(t)$ of Eq. (3) into in-phase and quadrature components of the form of Eq. (2b). We then add the atomic polarizability terms, and substitute the resulting $C_n(t)$ and $S_n(t)$ into Eqs. (1a) and (1b), respectively. The oscillation frequency of the n^{th} mode, i.e., ν_n , is assumed to be that of some central mode whose oscillation frequency is Ω_0 , plus $n\nu_m$. Thus $\Omega_0 + n\nu_m - \Omega_n = n\Delta\nu$ where $\Delta\nu$ is the frequency difference between the modulation frequency and the axial mode spacing frequency. Equations (1a) and (1b) then become

$$\left[\dot{\varphi}_n + n\Delta\nu + \frac{1}{2} \nu\chi_n'\right] E_n = -\frac{\delta c}{2L} [E_{n+1} \cos (\varphi_{n+1} - \varphi_n) + E_{n-1} \cos(\varphi_n - \varphi_{n-1})] \tag{4a}$$

$$\dot{E}_n + \frac{\nu}{2}\left[\frac{1}{Q_n} + \chi_n''\right] E_n = -\frac{\delta c}{2L} [-E_{n+1} \sin (\varphi_{n+1} - \varphi_n) + E_{n-1} \sin (\varphi_n - \varphi_{n-1})] . \tag{4b}$$

We are interested in the possible steady-state solutions of Eqs. (4a) and (4b) and thus set $\dot{E}_n = \dot{\varphi}_n = 0$. We also define

$$\Gamma = \frac{c}{L\Delta\nu} \delta = \frac{1}{\pi} \frac{\Delta\Omega}{\Delta\nu} \delta \tag{5a}$$

and

$$\rho_n = \frac{2c\delta}{L\nu\left[\frac{1}{Q_n} + \chi''\right]} \tag{5b}$$

$$\theta_n = \varphi_{n+1} - \varphi_n \quad . \tag{5c}$$

If we drop the term $1/2 \; \nu\chi_n'$ from Eq. (4a), and thereby neglect mode pulling effects, then Eqs. (4a) and (4b) become

$$\frac{2n}{\Gamma} E_n = -[E_{n+1} \cos \theta_{n+1} + E_{n-1} \cos \theta_n] \tag{6a}$$

$$\frac{2}{\rho_n} E_n = -[-E_{n+1} \sin \theta_{n+1} + E_{n-1} \sin \theta_n] \quad . \tag{6b}$$

The form of the solution of Eqs. (6) depends on the relative magnitude of Γ and ρ_n . In order to obtain a perfectly pure FM signal, it is necessary to assume that $\rho_n = \infty$ for all modes from $n = -\infty$ to $n = +\infty$. Then, by noting the Bessel identity

$$\frac{2n}{Z} J_n(Z) = J_{n-1}(Z) + J_{n+1}(Z) \quad , \qquad (7)$$

it is seen that Eqs. (6) have the solution $E_n = J_n(\Gamma)$ and

$$\theta_n = \theta_{n+1} = \pi \quad . \qquad (8)$$

The E_n and θ_n may be substituted into Eq. (2a) to yield a cavity electromagnetic given by

$$E(z,t) = \sum_{n=-\infty}^{+\infty} J_n(\Gamma) \left[\cos\,[(\Omega_0 + n\nu_m)t + n\pi]\right] \sin \frac{(N_0 + n)\,\pi z}{L} \quad , \qquad (9)$$

where a change of variable in the mode number has been made such that N_0 is now the central or carrier mode of the FM signal.

The quantity ρ_n is inversely proportional to the net or excess laser gain in the presence of the parametric oscillation; i.e., χ''_n and therefore ρ_n depend on E_n . For a free-running laser (with no parametric perturbation), all modes would saturate at gain = loss and therefore ρ_n would be infinite for all oscillating modes. However, in the presence of a perturbation which itself effects the mode amplitudes, this could no longer be the case. It is thus expected that some or perhaps a great deal of distortion of the ideal solution should be present in an actual laser. The exact evaluation of this distortion must await the solution of the non-linear problem.

It is of interest to note that by the use of standard trigometric and Bessel identities, Eq. (9) may be put into the closed form:

$$E(z,t) = \frac{1}{2} \sin \left[\Omega_0 t + \frac{N_0 \pi z}{L} + \Gamma \sin(\nu_m t + \frac{\pi z}{L})\right] - \frac{1}{2} \sin \left[\Omega_0 t - \frac{N_0 \pi z}{L} + \Gamma \sin(\nu_m t - \frac{\pi z}{L})\right] \quad , \qquad (10)$$

which consists of two FM traveling waves, moving in opposite directions. Equation (10) may be further reduced to the standing wave form

$$E(z,t) = \cos\,[\Omega_0 t + \Gamma \sin \nu_m t \cos \frac{\pi z}{L}] \times \sin \left[\frac{N_0 \pi z}{L} + \Gamma \cos \nu_m t \sin \frac{\pi z}{L}\right] \quad . \qquad (11)$$

It may be noted that at any particular point of space within the cavity, the total electromagnetic field is not frequency modulated.

Experimental

In this section we will review the results of the FM laser experiments reported by Harris and Targ,[1] and discuss some of the techniques which may be used to study FM laser oscillation.

In these experiments, the phase perturbation was obtained by utilizing the electro-optic effect in a 1 cm long crystal of KH_2PO_4 (KDP) situated in a 100 Mc tuned circuit. The crystal was anti-reflection coated and placed near one mirror of the laser cavity. Its orientation was such that the light traveled along its optic axis, and such that one of its electrically induced principal axes was along the direction of the laser polarization. In this orientation the crystal should ideally introduce a pure phase perturbation into the laser cavity. An rf power input of 2 watts was sufficient to produce a single-pass phase retardation δ of about 0.05 radians at the optical frequency. The laser used in these experiments was a Spectra-Physics He-Ne Model 116 , operated at 6328Å. The axial mode spacing was 100.5 Mc. Due to the large insertion loss caused by the KDP crystal, it was necessary to operate the laser with nearly opaque mirrors. An output power of about 0.1 mw was obtained by specular reflection from the Brewster angle windows.

Three techniques were useful in studying FM laser oscillation. These were: (1) Observation of the laser beat notes on an rf spectrum analyzer, (2) Demodulation of the FM signal by means of a Michelson interferometer and a birefringent discriminator, and (3) The examination of the laser mode amplitude by means of a scanning Fabry-Perot interferometer. A schematic of the experimental arrangement is shown in Fig. 1.

In all of the experiments, it was consistently observed that FM laser oscillation could not be obtained unless the modulation frequency was sufficiently detuned from the axial mode spacing frequency. The necessary detuning was approximately 250 kc, and was somewhat dependent on the single pass phase retardation δ .

If the modulation frequency was detuned from the axial mode interval by 250 kc, and δ were then increased, then at a δ of about .04, a quenching of the original laser beat notes was observed. After quenching of the original axial beat notes, a small amount of rf beat power was observed at harmonics of the modulation frequency. At the second and third harmonics, this power level was 25 dB below that of the original beat amplitude. At the fundamental and fourth harmonics, this level was at least 12 dB below that of the original signal. However, measurements at the latter two frequencies were limited by pickup and poor photomultiplier sensitivity, respectively.

Direct demodulation of the FM signal was accomplished by using both a Michelson interferometer[10] and a birefringent discriminator.[11,12] Both instruments function by separating the incident optical signal into two components which travel different path lengths and then interfere. The resulting intensity transmission characteristic is that of Fig. 2. The periodicity of the characteristic is determined by gross differences in path length traveled by the two components of the optical signal. Its exact position on the frequency scale has been termed as the bias of the interferometer, and is determined by fine variations of optical path difference. It may be shown[11,12] that if an FM light signal with a modulation depth Γ is incident on either a properly biased Michelson Interferometer or birefringent discriminator, the per cent amplitude modulation at the fundamental frequency of the transmitted light signal is given by

$$m = \frac{I_{ac}}{I_{dc}} = 2\, J_1\left(2\Gamma \sin\frac{\pi}{2}\frac{f_m}{f_{0m}}\right), \qquad (12)$$

where f_m is the modulation frequency, and f_{0m} is that frequency at which the relative time delay between the two optical path lengths is one-half wavelength. For very low Γ's it is thus advantageous to choose the relative time delay equal to one-half wavelength at the modulation frequency. For higher Γ's this relative time-delay must be correspondingly shortened.

By varying the path length of the interferometer, and observing the value at which the modulation index m was maximized, it was possible to determine Γ. In our experiments Γ ranged between about 2 and 6. To within a factor of two, its value was found to be correctly predicted by Eq. (5a). It may be noted that at a modulation frequency of 100 Mc, a Γ of 6 corresponds to a peak-to-peak frequency swing of 1200 Mc.

The most striking results of our experiments were obtained by utilization of a Spectra-Physics scanning interferometer, thus allowing the direct display of laser mode amplitude versus frequency. The results are shown in Fig. 3. In the absence of modulation, the laser modes appear as in Fig. 3a. As the modulation depth is increased, the central mode amplitude falls, and the first pair of sidebands increase. At still larger modulation depths, the second and third pair of sidebands achieve significant amplitude, and there is a general diffusion of power toward the wings of the Doppler line. Fig. 3 is captioned both in terms of the internal drive strength δ, and in terms of the Γ to which the resulting mode intensity amplitudes $[J_n^2(\Gamma)]$ appear to correspond.

Application

One extremely interesting application of the FM laser has been suggested by Massey, and demonstrated by Massey, Oshman, and Targ.[2] In their experiments, they took the output light signal from the FM laser and passed it through an external phase modulator which was driven at a Γ exactly equal to that at which the FM laser was running. By properly adjusting the phase of the external modulator with respect to that of the internal modulator, the resulting light signal could be made to have a resultant frequency deviation Γ_R anywhere between 0 and 2Γ. In particular, when Γ_R was adjusted to zero, then all of the energy that was previously distributed between the various sidebands of the FM laser signal appeared in a single "super-mode" -- as it has been termed by Targ. In other words, the function of the second modulator is to convert the output of the FM laser into a light signal which is no longer frequency modulated, but which contains all of the power of the original FM signal. A schematic of their experimental arrangement is shown in Fig. 4.

It should however be noted that the effect of saturation on the behavior of the FM laser has not at this time been properly considered. Thus it is not certain whether the FM laser will run at high power levels, and the functional utility of the "super-mode" technique remains to be determined.

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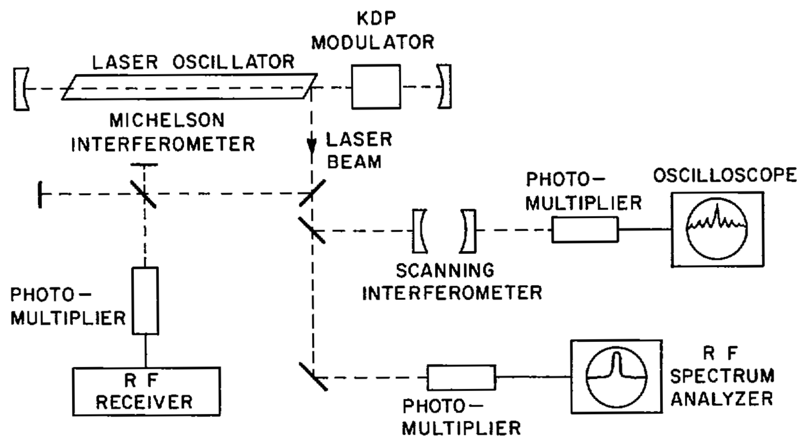

Fig. 1—Schematic of experimental arrangement.

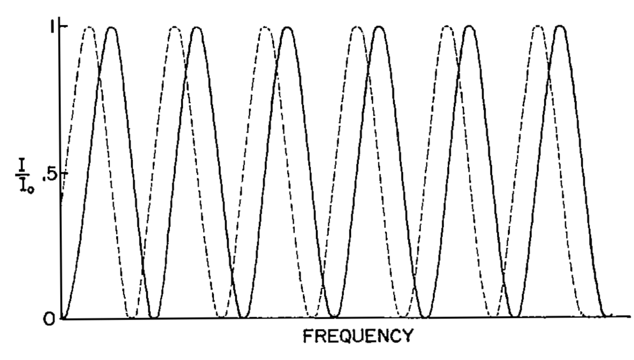

Fig. 2—Transmission versus frequency for the two different biases of a birefringent discriminator or a Michelson interferometer.

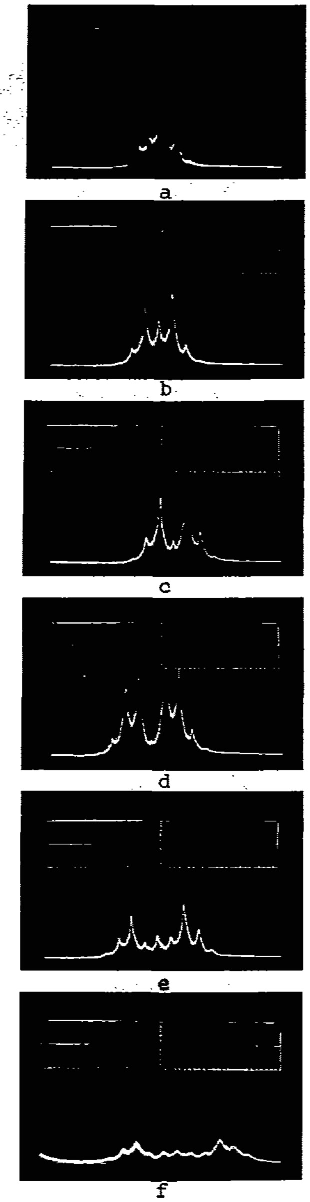

Fig. 3—Laser mode amplitude versus optical frequency for variable modulation depth.

(a) free running laser
(b) δ = 0.045 $\Gamma \sim 2$
(c) δ = 0.063 $\Gamma \sim 2.2$
(d) δ = 0.069 $\Gamma \sim 2.4$
(e) δ = 0.072 $\Gamma \sim 3$
(f) δ = 0.088 $\Gamma \sim 4.5$

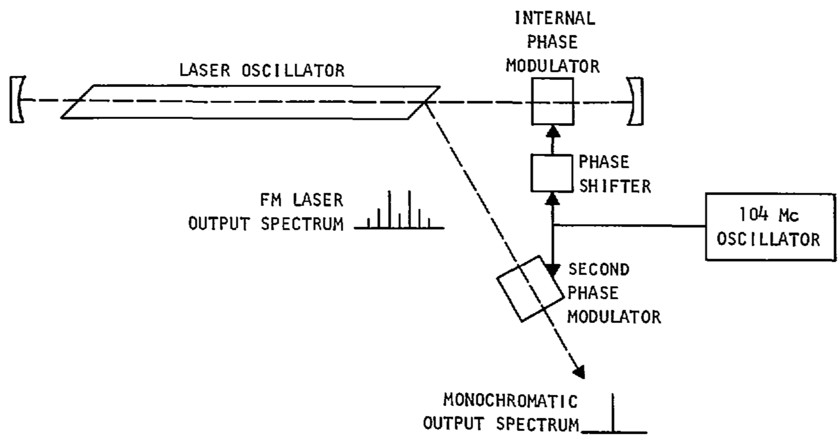

Fig. 4—Conversion of FM light to single frequency light.