deformation of crystals. Double intersection cross slip, where the segment that has been pulled into the cross-slip plane soon encounters another attractive intersection that pulls it back into a new layer of the primary glide plane, provides a reasonable mechanism for the growth of slip bands at low temperatures. The importance of intersecting dislocations for the growth of slip bands can be experimentally checked when it becomes possible to study their growth in otherwise dislocation free crystals. However, there is already some evidence that intersections are important. When the total number of intersecting dislocations met by a moving dislocation in traversing the entire cross section on the slip plane becomes small, slip-band growth should become more difficult. This indeed seems to be the case as relatively high stresses are required for propagation of slip in very small crystals.[5]

For the hcp structure the common glide plane for three dislocations of the type $\frac{1}{3}\langle 2\bar{1}\bar{1}0\rangle$ is the basal plane. Therefore, attractive intersections in which a junction dislocation is formed can assist "cross slip" from the prism plane into the basal plane but not vise versa. This may help to explain why continuous *thermally activated* cross slip of new loops into the prism plane seems to be necessary for prismatic slip. No dislocation near screw orientation should be able to move far on the prism plane before it is pulled into the basal plane by an attractive intersection. These intersections should be extremely frequent because of the relative ease with which dislocations move and multiply on the basal systems.

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PROPOSED FREQUENCY STABILIZATION OF THE FM LASER[1]

(KDP intracavity phase perturbation; T/E)

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We describe a novel and sensitive technique for stabilizing the center frequency (or "carrier") of an FM laser with respect to the center of the atomic gain profile. In this Letter we establish the existence of the frequency discriminant.

It has recently been demonstrated[2] that the modes of an FM laser have approximately the amplitudes and phases of an FM spectrum. Subsequent theoretical[3,4] and experimental[5] investigations have shown that the deviation of this signal from an ideal FM spectrum can be made very small. As a result, essentially all of the power associated with the previously uncoupled laser modes becomes available in the form of a single coherent oscillation.[6]

The stabilization technique described herein was proposed by Harris and Oshman and utilizes the small distortion which is always present in FM laser oscillation. Under proper (quenched[4,5]) conditions of FM laser oscillation, this distortion appears as small residual beats at harmonics of the driving frequency of the internal phase perturbation when the FM laser output is detected by a phototube. Of importance is the fact that the amplitudes of the odd harmonic beats are extremely sensitive to the position of the center frequency of the FM oscillation with respect to the center of the atomic line, while the amplitudes of the even harmonic beats are nearly independent of this position. In addition, the phase of the beat at odd harmonics changes abruptly as the center of the FM oscillation moves from one side of the atomic line center to the other.

If the output of the FM laser is incident on a square-law photodetector, then the amplitude of the q^{th} beat is given by

$$w_q(t) = \sum_{n=-\infty}^{\infty} E_n E_{n+q} \cos\left(q\nu_m t + \phi_{n+q} - \phi_n\right) \tag{1}$$

where ν_m is the driving frequency of the perturbation and E_n and ϕ_n are the amplitude and phase of the n^{th} laser mode. If all modes have the Bessel function amplitudes and the appropriate phases of an ideal FM signal, then for $q \neq 0$, all $w_q(t) = 0$. In practice, however, the mode amplitudes and phases will deviate slightly from these ideal values. To first order, the relative phase of the n^{th} mode may be shown to be given by[4]

$$\sin\left(\phi_n - \phi_{n-1}\right) = \frac{1}{J_n(\Gamma) J_{n-1}(\Gamma)} \frac{1}{\delta} \sum_{q=-\infty}^{n-1} \rho_q J_q^2(\Gamma) \tag{2}$$

where δ is the single-pass phase delay of the intracavity phase perturbation, ρ_q is the net saturated single-pass gain of the q^{th} mode in the presence of the FM oscillation, and Γ is the modulation depth of the FM signal and is given by[3,4]

$$\Gamma = \frac{\delta}{\pi} \frac{\text{axial-mode interval}}{\text{detuning frequency}}. \tag{3}$$

The detuning frequency is the difference between ν_m and the axial-mode interval. To first order, the amplitude of the n^{th} mode is $J_n(\Gamma)$. To second order, its amplitude may be shown to be[4]

$$E_n = J_n(\Gamma) + \frac{\Gamma}{2} \sum_{\substack{q \\ q\neq 0}} \sum_{m} \frac{1}{q} J_{n+q}(\Gamma) J_{m+q}(\Gamma) P_m \tag{4}$$

where

$$P_n = J_{n+1}(\Gamma)\left[1 - \cos\left(\phi_{n+1} - \phi_n\right)\right] + J_{n-1}(\Gamma)\left[1 - \cos\left(\phi_n - \phi_{n-1}\right)\right] + \frac{\psi_n}{\delta} J_n \tag{5}$$

and ψ_n is the additional round-trip phase retardation of the n^{th} mode resulting from the real part of the atomic susceptibility. Higher order amplitudes and phases may be obtained by an iterative procedure wherein the $J_n(\Gamma)$ of Eqs. (2) and (5) is replaced by the E_n of Eq. (4) calculated from the previous iteration.[4]

Equations (2) and (4) predict that if the center frequency of the FM oscillation is at the center of a symmetrical atomic line, all odd harmonic beats will be zero. Odd harmonic beats from sidebands which are above the center frequency of the FM oscillation are exactly cancelled by contributions from sidebands below the center frequency of the oscillation. As the center frequency of the FM oscillation moves away from the center of the atomic line, the cancellation of upper and lower contributions is not complete, and odd harmonic distortion rapidly increases. No such cancellation occurs for even harmonics.

FM laser oscillation was obtained experimentally using a KDP intracavity phase perturbation in a Spectra-Physics Model 116 He-Ne laser operating at 6328 Å with an axial-mode interval of 100 Mc. A piezoelectric crystal was attached to one mirror of the laser and a sawtooth voltage was applied which swept the center frequency of the FM oscillation by approximately an axial-mode interval. The various beat signals were monitored and displayed as a function of the position of the driven laser mirror. Experiments were performed under a number of FM laser conditions as both δ and Γ were varied.

Figure 1 is a typical result showing the ampli-

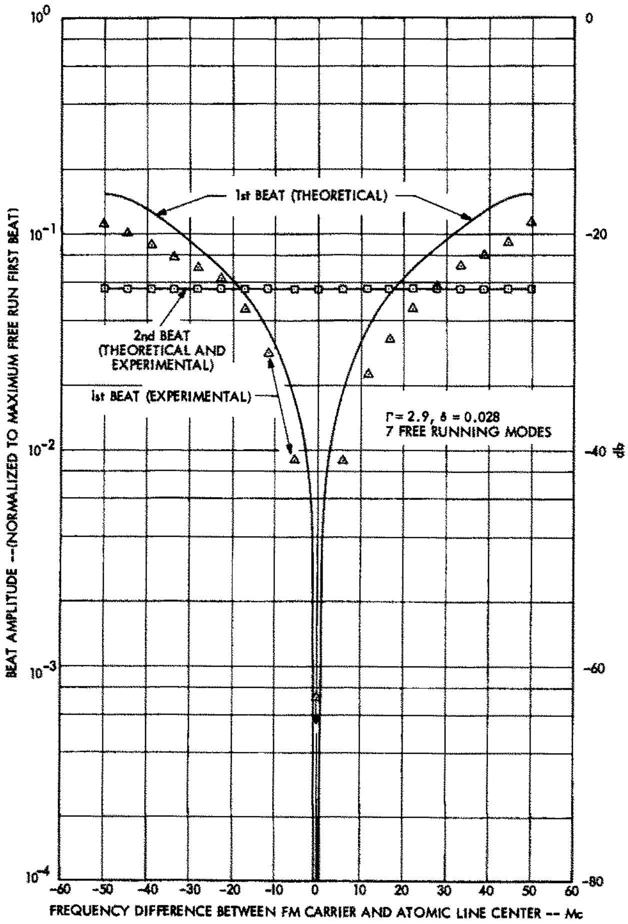

Fig. 1. Experimental and calculated values of the first and second beat amplitudes as a function of the frequency difference between the FM carrier and the atomic line center.

tude of the first beat plotted as a function of the relative position of the FM oscillation with respect to the center of the gain profile. The free-running laser (no applied phase perturbation) had seven modes above threshold. The ordinate of Fig. 1 is normalized so that 0 dB represents the maximum amplitude of the fundamental beat of the free-running laser. The point at line center was the limit of our detector sensitivity. The accuracy of the experimental points is approximately ±30% in amplitude.

The solid curves of Fig. 1 give the theoretical result obtained from five iterations of Eqs. (2), (4), and (5) under the conditions of the above experiment. A Gaussian atomic line shape and inhomogeneous saturation of the n^{th} mode of the form $(1 - \beta E_n^2)$ were assumed. The theoretical results were normalized to the experimental results by making the second-beat amplitudes equal for the two cases.

The theoretical result also showed that the phase of the first harmonic beat (with respect to the driving perturbation) was essentially constant on either side of line center, but changed abruptly by 180° at line center.

In a practical stability scheme, one might detect the ratio of the first- and second-beat amplitudes, thereby eliminating the possibility that spurious variations in laser output would be incorrectly interpreted. One limitation of the proposed technique might be the atomic line asymmetry resulting, for instance, from the mixture of isotopes which are normally present;[7,8] however in such cases stabilization could still be accomplished at the nonzero minimum of the fundamental beat amplitude.

The FM laser, when used in conjunction with the supermode technique[9] and the stabilization technique described herein, may make possible the concentration of large amounts of optical power into a stabilized and nearly single-frequency oscillation.

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PROPAGATION AND OBLIQUE REFLECTION OF SLANT-MODE HYPERSONIC WAVES IN QUARTZ

(microwave acoustics; anisotropy; E)

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The coherent detection of perpendicularly and obliquely reflected slant-mode acoustic pulses at frequencies of 2.975 and 9.375 Gc at temperatures of 4°K is reported. Such reflections demonstrate the noncolinearity of the Poynting vector and wave normal for slant modes and show for oblique reflections of these modes how to minimize coupling losses by maintaining the parallelism between acoustic phase fronts and crystal faces. Previously pulses of microwave frequency acoustic energy propagating in modes with parallel Poynting and wave normal vectors have been perpendicularly reflected between the parallel faces of crystals of many materials.[1] The coherent detection of oblique reflections at 400 Mc in quartz has been reported by Merkulov and Yakovlev.[2] Heat pulse detectors in quartz and sapphire have recorded disturbances due to a spectrum of slant modes propagating between parallel crystal faces.[3] Optical methods have been used by Bömmel and Dransfeld[4] to show noncolinearity of Poynting and wave normal vectors.

The characteristics of slant modes are developed