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# Stabilization and Modulation of Laser Oscillators by Internal Time-Varying Perturbation

S. E. HARRIS

*Abstract*—**The paper reviews the application of internal time-varying perturbation to the problem of laser mode control and stabilization. The spectral characteristics and time domain behavior obtained by means of phase type and loss type perturbations are considered. Two techniques which allow the attainment of high single frequency output powers from normally multimode lasers are described. A method for the absolute frequency stabilization of an FM laser is considered, and a brief discussion of an efficient method of internal modulation, termed coupling modulation, is given.**

## I. Introduction

To a large extent the gain seen by the modes of an optical maser is a result of their independent interaction with essentially different atomic populations. In gas masers, this is primarily the result of Doppler broadening [1], while in solid-state masers it is often the result of what may be termed as spatial broadening [2]. If the spectrum of such a multimode laser is viewed with a scanning Fabry-Perot interferometer, it is typically observed to be unstable both in amplitude and frequency; and in certain cases of importance such as that of high gain argon, it is completely erratic, displaying violent amplitude fluctuations and random appearance of the different modes [3]. An oscillator spectrum of this type severely limits the application of the laser. For example, the distortion-free bandwidth which is available for communications is immediately restricted to the axial mode interval—which for many gas lasers is approximately 100 Mc/s.

The purpose of this paper is to consider the use of internal time-varying perturbation to establish coherence between and to achieve stabilization of the laser modes. By an internal time-varying perturbation is meant an element either whose optical path length, or whose loss, may be varied by an externally applied modulating signal. An element of the first type is termed a phase perturbation, while one of the second type is termed a loss perturbation. Both types of perturbations may be achieved by means of either the linear electrooptic effect in crystals such as KDP, or in certain cases by acoustic means.

Consider first a situation wherein an internal loss perturbation is weakly driven at a frequency which is approximately that of the axial mode interval. Its effect is to produce sidebands at all frequencies which are spaced by the modulation frequency from each of the original free-running axial modes. By displaying the beat spectrum of the laser on an RF spectrum analyzer we could observe that the modes of the free-running laser are slightly

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The author is with the Department of Electrical Engineering, Stanford University, Stanford, Calif.

pulled toward the parametrically generated sidebands of the perturbation. If the drive strength of the perturbation is increased, a point is reached such that one of the free-running modes is pulled into lock with the perturbing sideband from an adjacent mode. At a further increase of drive strength, the last free-running mode of the laser is pulled into synchronism, and a complete phase locking of the laser occurs. Phase locking of this type was first demonstrated by Hargrove, Fork, and Pollack [4] by acoustic modulation of the intracavity loss of a 6328 Å He-Ne laser. These authors noted a greatly increased stability of the optical spectrum and observed that the time domain output corresponding to such a phase locked spectrum is a repetitive series of narrow pulses having a repetition frequency equal to that of the perturbation.

Consider next the case of fairly strong internal phase perturbation driven at a frequency which is almost, but not exactly, that of the axial mode interval. The situation here is best described somewhat differently. The effect of the perturbation is again to associate a set of sidebands with each of the previously free-running laser modes. For a sufficiently strong perturbation these sidebands will have approximately Bessel function relative amplitudes and FM phases, with the modulation depth determined by the strength of the phase perturbation and by the difference in frequency between the driving frequency of phase perturbation and the axial mode interval. In effect, each of the free-running laser modes becomes the center frequency or carrier of an FM signal. The resulting FM oscillations, that is the resulting sets of sidebands, then compete for the atomic population in the same sense as did the previously free-running modes. However, while the free-running modes experienced their gain from somewhat independent atomic populations, the competing FM oscillations to a large extent see the same atomic population. For instance, the first upper sideband of an FM oscillation which is centered at the center of the atomic fluorescence line is in the same homogeneous linewidth, and therefore sees the same atomic population, as does the center frequency of an FM oscillation which is centered one mode above the center of the atomic fluorescence line. The competing FM oscillations are thus much more tightly coupled than were the previous free-running laser modes. In the cases of principal interest the strongest of the FM oscillations—usually the oscillation whose carrier is at the center of the atomic line—will be able to completely quench the competing weaker oscillations. The result is that shown in the lower part of Fig. 1 wherein the sidebands of a single coherent FM oscillation deplete most of the inverted population of the atomic line. Such FM laser oscillation was first demonstrated by Harris and Targ [5] in a 6328 Å He-Ne laser.

In the following sections of the paper, we will describe in detail the spectral characteristics which are obtained by means of internal phase or loss type perturbation. Consideration will be given to the time domain behavior which is characteristic of the different spectral outputs, and to the questions of distortion and power output. Two

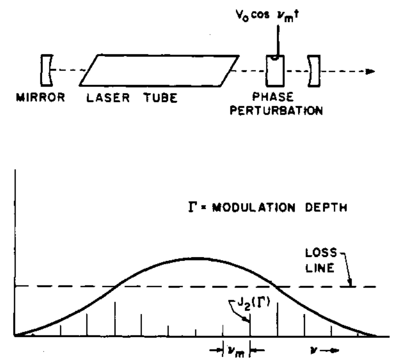


Fig. 1. Schematic of FM laser oscillation. Figure from Harris and McDuff [9].

techniques which allow the attainment of high single frequency output powers and have been termed as the super-mode technique, and as the method of frequency selective coupling, will be considered. A technique for absolute frequency stabilization of an FM laser will then be described. Finally a brief discussion of an efficient method of internal modulation, termed coupling modulation, will be given.

## II. Internal Phase Perturbation

As noted in the Introduction, the effect of an internal phase perturbation is to associate a set of sidebands with each of the previously free-running modes. The relative amplitudes of these sidebands depend in part on the strength of the internal phase perturbation, in part on the frequency difference between the driving frequency of the internal phase perturbation and the axial mode interval (hereafter referred to as the detuning), and in part on the atomic gains, cavity losses, and reactive mode pulling effects which are present in the system. Depending on the relative magnitude of these parameters, a number of different effects may be observed. For large perturbation strength and a certain range of detuning, FM laser oscillation may be obtained. Alternately, very small detunings and lower perturbation strengths yield a time domain pulsing behavior which is very similar to that obtained by means of internal loss perturbation.

### *A. Basic Equations and Discussion of Parameters*

The basic equations which describe the effect of an internal phase perturbation on a laser oscillator have been given by Gordon and Rigden [6], Yariv [7], and Harris and McDuff [8], [9]. The form of Harris and McDuff [9] is as follows:

$$[\dot{\phi}_n - n\Delta\nu + \tfrac{1}{2}\nu\chi_n']E_n = -\frac{\delta c}{2L}[E_{n+1}\cos(\phi_{n+1}-\phi_n) + E_{n-1}\cos(\phi_n-\phi_{n-1})] \quad (1a)$$

$$\dot{E}_n + \frac{\nu}{2}\left[\frac{1}{Q_n} + \chi_n''\right]E_n = \frac{\delta c}{2L}[E_{n+1}\sin(\phi_{n+1}-\phi_n) - E_{n-1}\sin(\phi_n-\phi_{n-1})]. \quad (1b)$$

In these equations $E_n$ and $\phi_n$ are the amplitude and phase, respectively, of the $n$th cavity mode, such that the total cavity electromagnetic field as a function of space and time is given by

$$E(z, t) = \sum_n E_n(t) \cos [\nu_n t + \phi_n(t)] U_n(z), \quad (2)$$

where $U_n(z) = \sin (n_0+n)\pi z/L$. The integer $n_0$ is the number of spatial variations of some central mode which we choose to be that mode whose frequency is closest to the center of the atomic fluorescence line and whose circular frequency is $\Omega_0 = n_0\pi c/L$. Built into these equations is the convention that if $\dot{\phi}_n = 0$, then the oscillation frequency of the $n$th mode $\nu_n$ is equal to $\Omega_0 + n\nu_m$ where $\nu_m$ is the driving frequency of the internal phase perturbation. Thus if all $\dot{\phi}_n$ are zero, the oscillating modes are spaced by $\nu_m$, and the frequency of the central ($n=0$) mode is $\Omega_0$. The appearance of a nonzero $\dot{\phi}_n$ in the solution of these equations denotes that the actual oscillation frequency of the $n$th mode is then $\Omega_0 + n\nu_m + \dot{\phi}_n$. The quantity $\Delta\nu$ in (1a) is the frequency difference between the axial mode interval and the driving frequency of the internal phase perturbation, i e., $\Delta\nu = \Delta\Omega - \nu_m$; where positive $\Delta\nu$ denotes a driving frequency less than the axial mode interval.

The quantity $\chi_n''$ is the quadrature component of the atomic susceptibility and for small saturation is related to the single pass power gain by the relation

$$\frac{\nu L}{c} \chi_n'' = - g_n \left(1 - \sum_m \beta_{nm} E_m^2\right), \quad (3)$$

where $g_n$ is the unsaturated single pass power gain of the $n$th mode and the $\beta_{nm}$ are saturation parameters which represent the effect of the $m$th mode on the gain of the $n$th mode [10]; $\chi_n'$ is the in-phase component of the atomic susceptibility and may be expressed as

$$\frac{\nu L \chi_n'}{c} = \sigma_n + \sum_m \tau_{nm} E_m^2, \quad (4)$$

where $\sigma_n$ is the additional round trip phase retardation which is seen by the $n$th mode as a result of power independent mode pulling, and the $\tau_{nm}$ represent power dependent pulling and pushing effects [10]. We also note that the $Q$ of the $n$th mode may be written as

$$\frac{\nu L}{c} \frac{1}{Q_n} = \alpha_n, \quad (5)$$

where $\alpha_n$ is the single pass power loss which is experienced by the $n$th mode as a result of nonzero output coupling, scattering, and diffraction; $L$ is the laser cavity length, and $\nu$ is the average optical frequency. We note that $Q_n$ is not meant to contain a contribution resulting from a parametric gain or loss in that such contributions are accounted for by the right side of (1b).

The quantity $\delta$ is the coupling coefficient between adjacent axial modes. In practice, the time-varying phase perturbation will often be achieved by means of a small perturbing element which has no significant spatial variation in the $z$ direction. If we let $\delta_m$ denote the peak single pass phase retardation of such an element of length $a$, then it may be shown [9] that the coupling coefficient $\delta$ is given by

$$\delta = \frac{L}{a} \frac{2}{\pi} \left(\sin \frac{a}{L} \frac{\pi}{2}\right) \left(\cos \frac{z_0 \pi}{L}\right) \delta_m, \quad (6)$$

where $a$ is the length of the perturbing element, and $z_0$ is its distance from an end mirror of the optical cavity. If $a/L$ is small, then we have $\delta \cong (\cos z_0\pi/L)\delta_m$. It is therefore desirable for such a perturbing element to be situated as closely as possible to the end of the optical cavity. In this case the coupling coefficient $\delta$ will then be very nearly the readily measurable peak single pass phase retardation of the perturbing element. It should be noted that if the perturbing element is not small then its spatial variation may be of importance. As an extreme case, if the perturbing element is spatially uniform and completely fills the optical cavity then $\delta$ would be zero.

### B. Solution of the Linear Approximation

Before proceeding to the results of a more exact solution of (1a) and (1b), it is instructive to examine their solution with nonlinearities and mode pulling neglected, and with the assumption of an infinity of laser modes all having a single pass gain exactly equal to their single pass loss.

We first look for steady-state solutions such that $\dot{E}_n = 0$ and $\dot{\phi}_n$ is constant and independent of $n$. Noting the Bessel function identity

$$\frac{2n}{\Gamma} J_n(\Gamma) = J_{n+1}(\Gamma) + J_{n-1}(\Gamma), \quad (7)$$

we see that (1a) and (1b) are satisfied by the set of $q$ solutions

$$\begin{aligned} \dot{E}_n &= 0 \\ \dot{\phi}_n &= q\Delta\nu \\ E_n &= J_{n-q}(\Gamma) \\ \phi_{n+1} - \phi_n &= 0, \end{aligned} \quad (8)$$

where $q$ is an integer, and $\Gamma$ is given by

$$\Gamma = \frac{c}{L} \frac{1}{\Delta\nu} \delta = \frac{1}{\pi} \frac{\Delta\Omega}{\Delta\nu} \delta = \frac{1}{\pi} \frac{\text{axial mode interval}}{\text{detuning frequency}} \delta. \quad (9)$$

These solutions correspond to FM oscillations having a modulation depth of $\Gamma$, and a center frequency at the $q$th mode [9]. A schematic of the $q=0$ and $q=1$ solutions is shown in Fig. 2.

We see that in a linear theory any of the previously free-running laser modes may become the carrier of an FM oscillation. These FM oscillations, all having a modulation depth $\Gamma$, are the normal modes of the lossless sys-

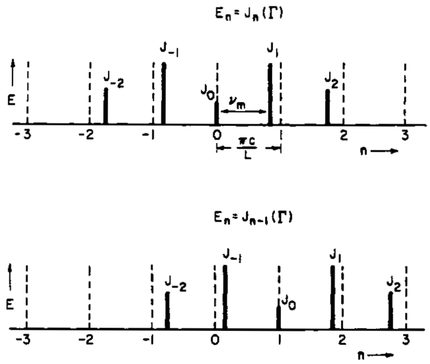


Fig. 2. Schematic of $q=0$ and $q=1$ solutions. Figure from Harris and McDuff [9].

tem in the sense that if a particular oscillation could be independently excited, it would be indefinitely sustained. In the linear approximation of this section any number of these FM oscillations may be independently excited, and may exist simultaneously with arbitrary amplitudes and phases [9]. However, in the presence of the nonlinear atomic gain, these oscillations compete for the atomic population with the result that in many cases of interest, a single steady-state FM oscillation may be attained.

We next consider the solution of (1) for the case where the detuning $\Delta\nu=0$. In this case $\Gamma$ as defined by (9) is infinite and the solutions considered thus far are indeterminate. Equations (1a) and (1b) have the solution

$$\begin{aligned} E_n &= E_{n+1} \\ \phi_{n+1} - \phi_n &= p\pi \\ \dot{\phi}_n &= (-1)^{p+1}\frac{\delta c}{L} = (-1)^{p+1}\frac{\delta}{\pi}\Delta\Omega, \end{aligned} \qquad (10)$$

where $p$ is an integer. In the time domain this solution corresponds to a behavior which consists of a repetitive series of pulses, and is thus similar to that obtained by means of internal loss perturbation. Of interest is the presence of the constant $\dot{\phi}_n$, which denotes a uniform comb-type shift of all modes from their free-running frequencies.

### *C. Details of the FM Steady-State Solution*

In the approximation of Section II-B, nonlinearities and mode-pulling were neglected, and an infinity of laser modes all with gain equal to loss were assumed. It was found that any of the FM oscillations had exactly Bessel function relative amplitudes and zero relative phases and thus were completely distortion free. When finite atomic line width, mode pulling, and atomic saturation are included, this is no longer the case. We define the quantities $\Psi_n$ and $\rho_n$ as follows:

$$\begin{aligned} \Psi_n &= \frac{\nu L \chi_n'}{c} \\ \rho_n &= \frac{\nu L}{c}\left[\frac{1}{Q_n} + \chi_n''\right]. \end{aligned} \qquad (11)$$

The quantity $\Psi_n$ is the additional round trip phase retardation which is seen by the $n$th mode as a result of the real part of the atomic susceptibility and is described in (4). The quantity $\rho_n$ described in (3) and (5) is the net saturated single pass power gain of the $n$th mode. In a free-running laser, all modes oscillate at a power level such that single pass gain equals single pass loss, and therefore $\rho_n=0$ for all oscillating modes. In the presence of the parametric phase perturbation, this is no longer the case. For those modes which oscillate at a level which is greater than their free-running counterpart, $\rho_n$ is positive; while for those modes which oscillate at a level which is less than their free-running counterparts, $\rho_n$ is negative.

For typical experimental situations, the nonzero $\rho_n$ result in more distortion than do the $\Psi_n$. Harris and McDuff [9] have shown that, for such situations, the first-order, relative mode amplitudes are still $J_n(\Gamma)$, and that the relative phase angles are given by

$$\sin(\phi_n - \phi_{n-1}) = \frac{1}{J_n(\Gamma)J_{n-1}(\Gamma)}\,\frac{1}{\delta}\sum_{q=-\infty}^{n-1}\rho_q J_q^2(\Gamma). \qquad (12)$$

Thus to first order, the nonzero net saturated atomic gains do not affect the relative mode amplitudes, but do affect the relative phase. The nonzero relative phase angles result from accumulation of nonzero net saturated gains. In general an FM laser operated at a $\Gamma$ such that the relative mode amplitudes, to at least some extent, approximate those of a free-running laser will have less phase distortion at a given $\delta$ than an FM laser whose mode amplitudes depart sharply from those of the free-running case. In particular, distortion will be increased when operating at a $\Gamma$ such that the amplitude of some mode is driven close to zero, or alternately when operating at a large $\Gamma$ such that a number of modes having appreciable amplitudes are outside of the spectral width of the free-running laser. A particularly important point is that phase distortion may be made arbitrarily small by making the perturbation strength $\delta$ increasingly large. It is therefore desirable to obtain a given $\Gamma$ by working with the largest available $\delta$ and thus from (9) with a correspondingly large detuning.

At smaller perturbation strengths, or at small detunings where the effect of mode pulling becomes more important, significant distortion of relative amplitudes may also result. To the next order, relative mode amplitudes are given by [9]:

$$E_n = J_n(\Gamma) + \frac{\Gamma}{2}\sum_{\substack{q \\ q\neq 0}}\sum_m \frac{1}{q}J_{n-q}(\Gamma)J_{m-q}(\Gamma)\mu_m \qquad (13)$$

where

$$\begin{aligned} \mu_n = {} & J_{n+1}(\Gamma)[1-\cos(\phi_{n+1}-\phi_n)] \\ & + J_{n+1}(\Gamma)[1-\cos(\phi_n-\phi_{n-1})] - J_n(\Gamma)\frac{\Psi_n}{\delta}. \end{aligned} \qquad (14)$$

At sufficiently large $\delta$ the cosine of the relative phase angles approach unity and their contributions to $\mu_n$ van-

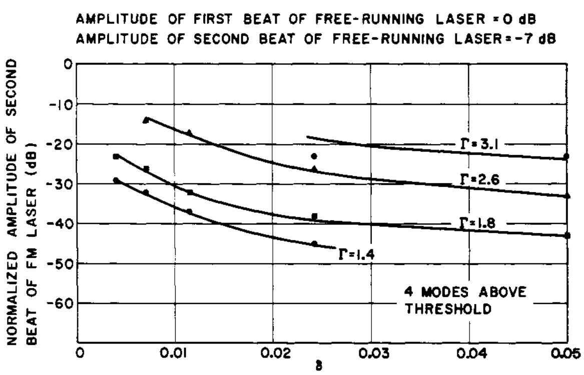


Fig. 3. Beat amplitude vs. $\delta$ at constant $\Gamma$. Figure from Ammann, McMurtry, and Oshman [11].

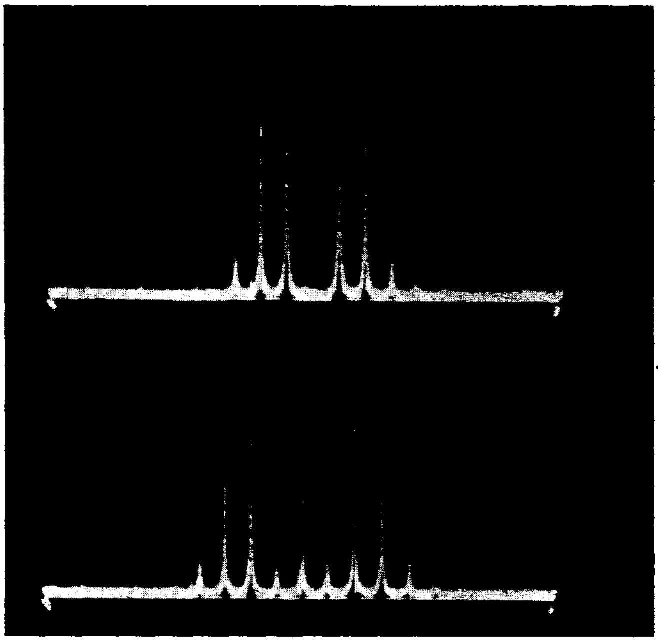

Fig. 4. Typical FM spectra. Figure from Ammann, McMurtry, and Oshman [11].

ish. The contribution to $\mu_n$ resulting from mode pulling also varies inversely as $\delta$, and thus at large $\delta$ the $\mu_n$ approach 0 and the $E_n$ approach $J_n(\Gamma)$.

One manifestation of distortion is the existence of beat notes when the output of an FM laser is incident on a photodetector. An ideal FM signal has no variation of amplitude with time and thus all beats are zero. In the presence of phase or amplitude distortion, such beats are present and according to the previous formulas should decrease if $\delta$ is increased and $\Delta\nu$ is adjusted to hold $\Gamma$ constant. Figure 3 shows results obtained by Ammann, McMurtry, and Oshman in their experiments with a He-Ne FM laser [11]. The gain of the laser in the experiment was such that four free-running modes were above threshold. The figure shows the amplitude of the second beat vs. $\delta$ at constant $\Gamma$. Beat amplitudes are normalized to the first beat of the free-running laser. These curves are in good qualitative agreement with curves based on (12)–(14) [9].

Some typical FM spectra obtained by means of a scanning interferometer by Ammann et al. [11] are shown in Fig. 4. These were obtained at $\delta \cong 0.05$ and show little distortion.

### D. Power Output

We return to (1b), multiply both sides through by $E_n$, and sum over $n$ from $-\infty$ to $+\infty$. Examination shows the right side to be zero and we thus obtain

$$\sum_{n=-\infty}^{+\infty} \left\{ E_n \dot{E}_n + \frac{\nu}{2}\left[\frac{1}{Q_n} + \chi_n''\right] E_n^2 \right\} = 0. \qquad (15)$$

Noting that $E_n\dot{E}_n = (d/dt)(E_n^2/2)$, it is seen that the foregoing equation is a statement of power conservation. That is, the rate of change of total stored energy plus the net power dissipated or absorbed in all modes is zero. Though in the absence of the parametric perturbation, power is approximately conserved by all modes individually, in its presence it is jointly conserved. In the steady state, all $\dot{E}_n = 0$, and thus if the relative mode amplitudes $E_n$ are known, then (15) is an equation in one unknown, and determines the level of oscillation. If $\delta$ is maintained sufficiently large that relative mode amplitudes are $J_n(\Gamma)$, and $\Gamma$ is increased by decreasing the detuning $\Delta\nu$, then the laser output power will decrease as modes with large relative amplitudes appear further from the center of the atomic line. From (15) we find that the condition for positive power output from an FM laser is that

$$\sum_n (g_n - \alpha_n) J_n^2(\Gamma) > 0. \qquad (16)$$

This condition is analogous to the condition $g_0 > \alpha_0$ for positive power output for a free-running single mode laser. Thus at sufficiently large $\delta$, an FM laser may be extinguished by a sufficient decrease of the detuning $\Delta\nu$. For optimum $\Gamma$'s, the maximum theoretically possible power output of an FM laser has been found to be approximately that of the equivalent free-running laser [9], [11].

One of the problems associated with FM laser oscillation is that of constructing an internal perturbing element which is sufficiently lossless that it does not significantly reduce the available output power. Early experiments were performed with antireflection coated z-cut KDP crystals [5], [11]. This method was not satisfactory and yielded maximum FM powers of about 1 mW where, in the absence of the internal element, free-running powers of about 15 mW could be obtained. The trouble with the above method was that of achieving a sufficient polish on the surfaces of the KDP crystal. Targ has overcome this difficulty by means of antireflection coated cover-glasses which are contacted to the crystal by means of index matching cement. Using this technique, Targ and McMurtry [12] have recently obtained FM power outputs of 53 mW from a Spectra-Physics Model 125 laser which furnishes 68 mW in the absence of the internal perturbation. Using an argon laser, Osterink has recently obtained an FM power output of about 100 mW from a laser capable of about 120 mW [22]. The construction of low loss internal modulation elements has also been considered by Uchida [13].

### *E. The Phase Locked Region*

In previous sections we have primarily discussed what may be termed the FM region of operation of an FM laser. However, if either $\delta$ or the detuning $\Delta\nu$ is sufficiently decreased, the behavior predicted by (1a) and (1b) becomes considerably more complex. In particular for small detuning there is a region where the modes have nearly equal amplitudes and is such that the behavior in the time domain consists of a series of spikes or pulses. This type of solution was indicated by (10), and has been termed a phase locked solution. In order to illustrate a number of interesting effects we consider the results of a numerical solution of (1a) and (1b) for a situation corresponding to five free-running modes in a He-Ne laser. It should be noted that the following numerical solutions assume atomic saturation of the form $(1-\beta E_n^2)$, and thus, especially at higher gains, should not be expected to agree exactly with experimental results [14], [15]. Further details of these solutions are given by Harris and McDuff [9].

In Fig. 5 we show laser mode intensities $(\frac{1}{2}E_n^2)$ vs. optical frequency—such as would be observed on a scanning Fabry-Perot interferometer. Figure 5(a) shows the intensities of the modes of the free-running laser, i.e., with $\delta=0$. In Fig. 5(b), $\delta$ is set equal to 0.015, and the frequency of the parametric drives adjusted such that it is exactly equal to the axial mode interval ($\Delta\nu=0$). A widening of the optical spectrum and some tendency toward equalization of the mode intensities is observed. Relative phases are found to have values between 0 and 50°; a uniform, angular frequency shift of all modes from their free-running positions of $\phi=0.94\delta(\Delta\Omega/\pi)$ is obtained. The direction of this shift is dependent on initial conditions and is too small to show on the scale of Fig. 5. We note that a shift of this type has recently been observed experimentally by McMurtry and Targ [11].

In Figs. 5(c)–5(f), $\delta$ is left constant at 0.015, and the detuning $\Delta\nu$ is increased in steps. At the small detuning of Fig. 5(c) ($\Delta\nu=0.00035\Delta\Omega$), we observe an interesting shift of the envelope of the modes of about $2\Delta\nu$. Associated with this gross envelope shift is a decrease in laser power as peak relative amplitudes are moved further from the center of the Doppler line. As the detuning is further increased, the envelope shift and decrease in power continues until, as shown in Fig. 5(d), the laser is extinguished. This interesting envelope shift has also been observed by Ammann, McMurtry, and Oshman [11] and is shown in Fig. 6.

Figure 5(d) might be considered as the beginning of the steady-state FM region wherein there is a single FM oscillation with its center frequency at the center of the atomic line and with a modulation depth $\Gamma$ approximately given by (9). For the detuning of Fig. 5(d), $\Gamma$ is approximately

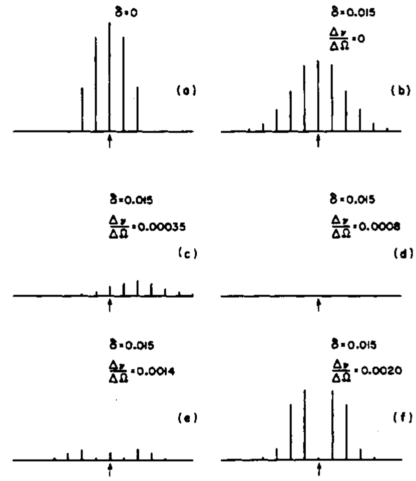


Fig. 5. Laser mode intensities at constant $\delta$ and variable detuning: $g_0=0.075$; $\alpha_n=0.070$. Five modes free running. Figure from Harris and McDuff [9].

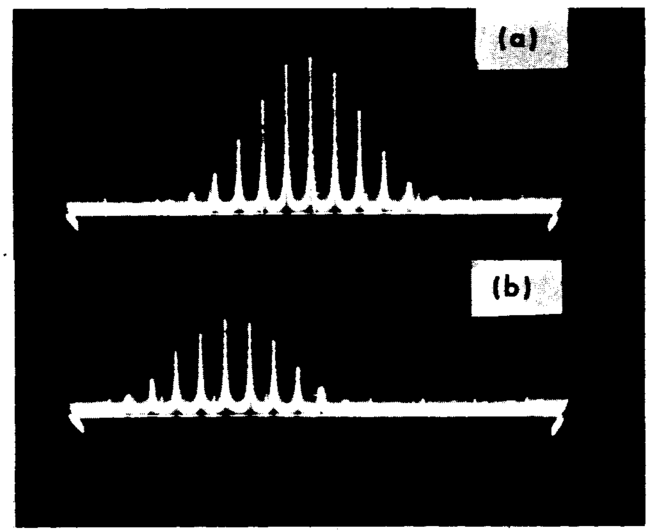


Fig. 6. Shift of envelope of laser modes in phase locked region. (a) Zero detuning. (b) Finite detuning. Figure from Ammann, McMurtry, and Oshman [11].

six, with the result that (16) is not satisfied and the oscillation is below threshold. In Figs. 5(e) and 5(f) the detuning is further increased with the result that $\Gamma$ decreases, relative amplitudes are concentrated closer to the center of the atomic line, and output power increases. If the detuning is increased past that of Fig. 5(f) we enter a region of multiple FM oscillation. In this region more than one mode acts as a carrier for an FM oscillation and a steady solution in the sense of zero $\dot{E}_n$ does not exist.

Figure 7 shows the time domain behavior which corresponds to the spectrums of Figs. 5(b) and 5(f). Output intensities are normalized to the total average intensity

$$\left(\frac{1}{2}\sum_n E_n^2\right)$$

of the free-running laser. We note that the pulsing of the phase locked solution is at the driving frequency of the internal phase perturbation and has peak intensities which

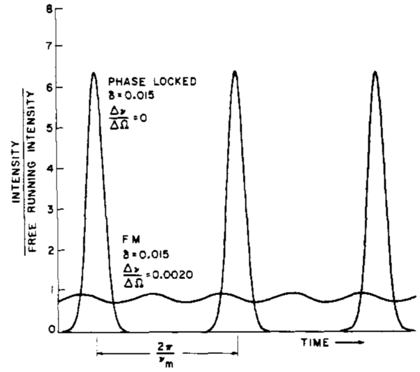


Fig. 7. Output intensity vs. time for phase locked and FM operation: $g_0=0.075$; $\alpha_n=0.070$. Five modes free running. Figure from Harris and McDuff [9].

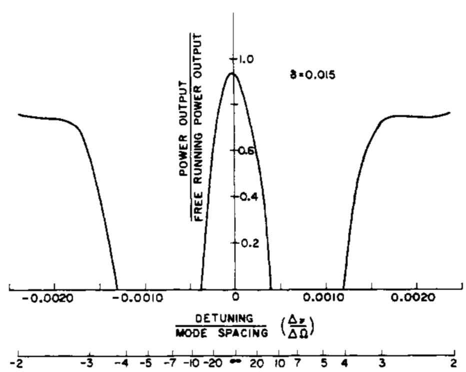


Fig. 8. Output power vs. detuning: $g_0=0.075$; $\alpha_n=0.070$. Five modes free running. Figure from Harris and McDuff [9].

are approximately six times the average intensity of the free-running laser. By contrast the envelope which corresponds to the FM spectrum of Fig. 5(f) is more nearly constant and independent of time. The ripple is entirely even harmonic and is a result of the distortion of amplitudes and phases from those of an ideal FM signal. As discussed in Section II-D, this ripple can be made arbitrarily small if $\delta$ is made sufficiently large. We might note that, as opposed to the periodic behavior of either the phase locked or FM solution, the time domain behavior of the envelope of a free-running laser consists of an erratic fluctuation with peak intensities almost as great as those obtained in the phase locked region.

In Fig. 8 we show average intensity

$$\left(\frac{1}{2}\sum_n E_n^2\right)$$

as a function of the normalized detuning. At zero detuning, i.e., in the phase locked region, the output power is about 0.95 that of the free-running laser. As detuning is increased the mode envelope shifts away from the center of the Doppler line and output power decreases to zero. The oscillation remains below threshold until a detuning corresponding to a $\Gamma$ of $\sim$4. For further increase of detuning, $\Gamma$ decreases and the output power rapidly rises. We note that for the case considered either phase locked or FM operation is obtained at nearly full power of the free running laser.

The behavior vs. detuning that has been seen in Figs. 5–8 has been somewhat simpler than it would have been had $\delta$ not been chosen sufficiently large. At low $\delta$ the region in the above figures where the laser was extinguished becomes instead an unquenched region where $\dot{E}_n$ is not equal to zero and a number of highly distorted FM oscillations are simultaneously above threshold. At still lower $\delta$, this unquenched region extends into what was previously the steady-state FM region. Finally for very low $\delta$ the FM solution entirely disappears. There remains a steady-state phase locked solution for very small $\Delta\upsilon$ and, for all other $\Delta\upsilon$, the situation is that of multiple oscillations. In such small $\delta$ cases, the final steady-state mode amplitudes of the phase locked solution are approximately those of the free-running laser and the principal effect of the perturbation is to cause a locking of the phases of these respective modes.

It might also be noted that if $\delta$ is very large, then the phase locked solution completely disappears, and the laser is extinguished in the region of detuning between the positive and negative (detuning) FM solutions [9], [11].

## III. Internal Loss Perturbation

The establishment of coherence between the modes of an oscillating laser may also be achieved by time-varying perturbation of the internal loss. Early experimental work in this area was performed by Hargrove, Fork, and Pollack [4] and by Gürs and Muller [16], [17]. Linear analyses and discussion have been given by DiDomenico [18], Yariv [7], and Crowell [19]; and a linearized transient analysis has been given by Pantell and Kohn [20]. An analysis of the nonlinear problem has been given by McDuff and Harris [21].

### *A. Equations and Discussions of Parameters*

The equations describing the effect of an internal time varying loss have been given by several authors [7], [18], [19], [20], [21]. The form of McDuff and Harris is as follows:

$$[\dot{\phi}_n - n\Delta\nu + \tfrac{1}{2}\nu\chi_n']E_n = -\frac{\alpha_c c}{2L}[E_{n+1}\sin(\phi_{n+1}-\phi_n) - E_{n-1}\sin(\phi_n-\phi_{n-1})] \quad (17a)$$

$$\dot{E}_n + \frac{\nu}{2}\left[\frac{1}{Q_n} + \chi_n''\right]E_n = -\frac{\alpha c}{2L}E_n - \frac{\alpha_c c}{2L}[E_{n+1}\cos(\phi_{n+1}-\phi_n) + E_{n-1}\cos(\phi_n-\phi_{n-1})]. \quad (17b)$$

These equations assume a time-varying loss of the form $\alpha(z)[1+\cos\nu_m t]$ to be present internally to the laser cavity. The quantity $\alpha_c$ is the coupling coefficient between adjacent axial modes and is given by

$$\alpha_c = \frac{1}{2}\int_0^L \alpha(z) \sin \frac{\pi z}{L}\, dz. \quad (18)$$

The quantity $\alpha$ is the average single pass power loss of the perturbing element,

$$\alpha = \int_0^L \alpha(z)\, dz. \quad (19)$$

If the perturbing element has a loss which is independent of $z$, is of length $a$, and is situated a distance $z_0$ from the end of the laser cavity, then $\alpha_c$ of (18) becomes

$$\alpha_c = \frac{L}{a}\,\frac{\alpha}{\pi} \sin\left(\frac{\pi a}{2L}\right) \cos \frac{z_0 \pi}{L}\,. \quad (20)$$

Thus if, as is often the case, $a/L$ is small and the perturbing element is situated sufficiently close to the end of the laser cavity, then we have $\alpha_c = \alpha/2$.

Other quantities in (17a) and (17b) are defined in Section II.

### B. Solution of the Linear Approximation

With the assumption that an infinity of laser modes saturate at gain equal to loss, and setting the detuning $\Delta\nu = 0$, DiDomenico [18] has given the solution of (17a) and (17b) as:

$$E_n = E_{n+1}$$
$$\phi_{n+1} - \phi_n = \pi$$
$$\dot{\phi}_n = 0. \quad (21)$$

This solution consists of an infinite number of laser modes having equal amplitudes and $\pi$ relative phases, and corresponds to a repetitive series of infinitely sharp pulses having a repetition frequency equal to the driving frequency of the loss perturbation.

For $\Delta\nu \neq 0$, Yariv [7] has given the solution

$$E_n = I_n(\xi)$$
$$\phi_{n+1} - \phi_n = \frac{\pi}{2},$$
$$\dot{\phi}_n = 0, \quad (22)$$

where the $I_n$ are modified Bessel functions and $\xi = (\alpha_c/\pi)(\Delta\Omega/\Delta\nu)$. However for a reason to be considered in the following section, this solution does not appear to be pertinent to the actual nonlinear problem.

### C. Power Level

Multiplying (17b) by $E_n$ and summing over $n$, we obtain the following conservation condition:

$$\sum_n E_n \dot{E}_n + \frac{\nu}{2}\left[\frac{1}{Q_n} + \chi_n''\right] E_n^2$$
$$= -\frac{\alpha c}{2L} \sum_n E_n^2 - \frac{\alpha_c c}{L} \sum_n E_n E_{n+1} \cos(\phi_{n+1} - \phi_n). \quad (23)$$

In the steady state all $\dot{E}_n = 0$, and thus if the relative amplitudes are known (23) gives the level of oscillation.

From (23), we see that in general the loss perturbation is an additional loss to the system and thus tends to reduce the overall oscillation level from that of the free-running laser. However, to the extent that DiDomenico's solution [18] is sufficiently exact, then $\phi_{n+1} - \phi_n = \pi$ and $E_n = E_{n+1}$, and thus if $\alpha_c = \alpha/2$, then the last term on the right-hand side of (23) is equal in magnitude and of opposite sign to the second term on the right-hand side; and thus the total loss of the internal perturbation is zero. This ideal situation has been recognized by Crowell [19], who noted that it can be explained in terms of an infinitely sharp light pulse which passes through the modulator at that instant of time when its attenuation is exactly zero.

On the other hand, Yariv's solution (22) has $\phi_{n+1} - \phi_n = \pi/2$ and thus the average loss of the modulator is not canceled. The result is a solution that would run at a considerably lower level than the true solution and is therefore not competitive.

### D. Effect of Perturbation Strength and Detuning

The amplitudes and phases of the various modes are in part determined by the perturbing element, and in part determined by the atomic gains and cavity losses. For a very small loss perturbation, the mode amplitudes are approximately those of the free-running laser, and the mode phases are given by [21]:

$$\sin(\phi_n - \phi_{n-1}) = \frac{1}{E_n E_{n-1}} \frac{1}{\alpha_c} \sum_{q=n}^{q=\infty} \left[\frac{-q\Delta\nu 2L}{c} + \psi_q\right] E_q^2. \quad (24)$$

At higher perturbation strengths the mode amplitudes depart considerably from those of the free-running laser and a broadening of the spectrum is observed. Figure 9(a) shows the spectrum and time domain behavior which correspond to a loss perturbation which is just barely sufficient to obtain mode locking [21]. The data for these figures were obtained by computer solution of (17a) and (17b), and approximately correspond to the situation of five free-running modes in a 6328 Å He-Ne laser. The poor quality time domain pulses are primarily a result of distortion of the relative phase angles from their ideal value of $\pi$. Figures 9(b) and 9(c) show the spectrum and time domain behavior, for the same atomic line at successively larger perturbation strengths.

The question of the optimum value for the internal loss perturbation has been studied by McDuff and Harris [21]. Predicted peak intensity, normalized to free-running intensity, as a function of the strength of the internal perturbation is shown in Fig. 10. It is seen that peak pulse intensities first rise and then fall. The initial rise is caused primarily by an improvement in the relative phases and amplitudes of the various modes, while the decrease is caused by the increased average value of the perturbation loss and by spectral energy extending beyond the fluorescence line. Figure 11 shows pulse width as a function

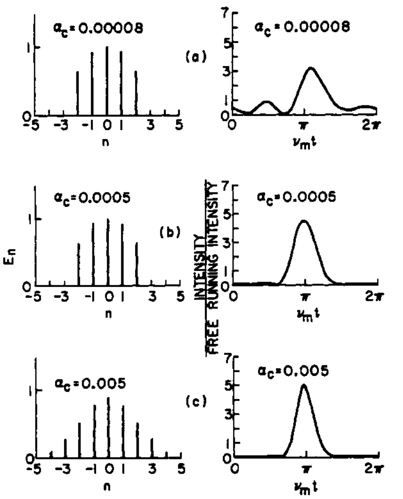


Fig. 9. Spectra and time domain behavior for loss perturbation. Figure from McDuff and Harris [21].

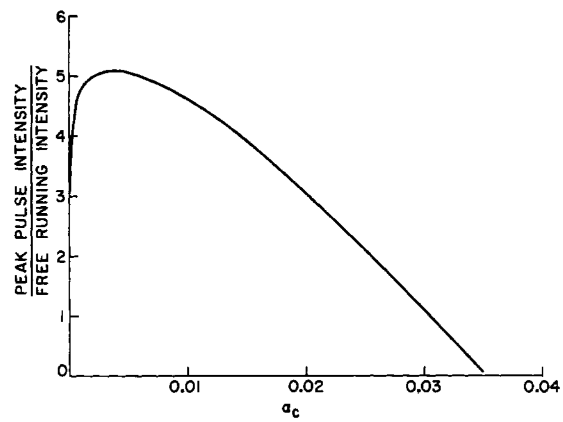


Fig. 10. Peak pulse intensity vs. $\alpha_c$. Five modes free running. Figure from McDuff and Harris [21].

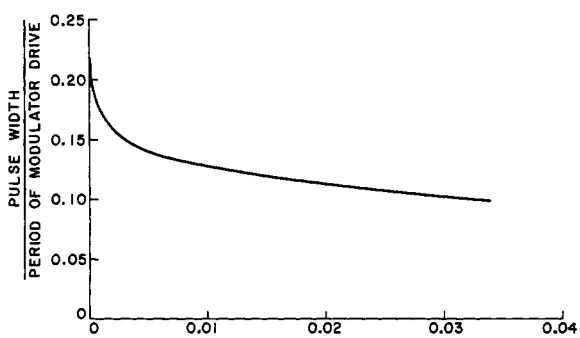


Fig. 11. Pulse width vs. $\alpha_c$. Five modes free running. Figure from McDuff and Harris [21].

of the strength of the internal loss perturbation. It is seen that the pulses continue to narrow as the perturbation strength is increased.

Crowell [19] has noted that the tuning of the perturbation frequency is fairly critical. As seen from (24), if the detuning $\Delta\nu$ is not zero, then relative phase angles will distort. This results in distortion of the pulse shape and in a lowering of the level of oscillation.

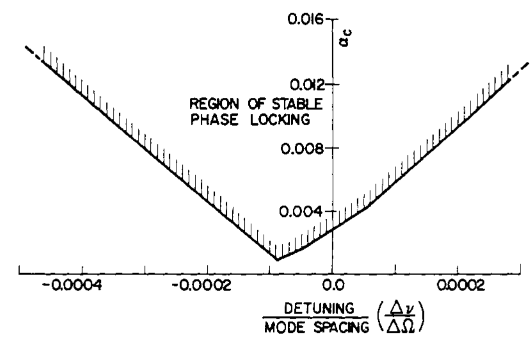


Fig. 12. $\alpha_c$(threshold) vs. detuning. Nine modes free running. Figure from McDuff and Harris [21].

### E. Threshold for Locking

As a result of the nonlinear portion of the atomic mode pulling, a certain minimum perturbation strength is required to achieve mode locking. If the cavity modes were equally spaced, then in the limit of perfect tuning of the modulation frequency, a vanishingly small perturbation would be required. In the presence of mode pulling, the free-running modes are unequally spaced and the perturbation must be sufficiently strong to pull these modes until their frequency spacing is equal to that of the modulator drive frequency.

A necessary condition on the minimum value of $\alpha_c$ may be obtained from (24) [21]. The condition is simply that the $|\sin(\phi_n - \phi_{n-1})| \leq 1$ for all $n$. Since at low perturbation strengths the mode amplitudes are approximately unaffected, free-running amplitudes may be used in the evaluation of (24). As $\alpha_c$ is reduced, a point will be reached where the magnitude of one of these sins will exceed unity, and it is this value of $\alpha_c$ which is the minimum perturbation required for locking.

A plot of the minimum $\alpha_c$ necessary to obtain locking vs. detuning for a case corresponding to nine free-running modes in a He-Ne laser is shown in Fig. 12. It is seen that the optimum drive frequency is slightly greater than the $c/2L$ frequency of the laser. This results in that the effect of the nonlinear portion of the power independent mode pulling is to push the modes further from the center of the atomic line than they would otherwise be, and thus to increase the average mode spacing.

## IV. Super-Mode and Frequency Selective Coupling

Two techniques have been proposed whereby it is possible to obtain a high power single frequency output from a normally multimode laser. The first of these is termed the super-mode technique, and was demonstrated by Massey, Oshman, and Targ [22]. In this technique, shown in Fig. 13, the output signal from an FM laser is passed through an external phase modulator which is operated with a single pass phase retardation which is exactly equal to the $\Gamma$ at which the FM laser is running. By properly adjusting the phase of the external modulator with respect to that of the internal phase perturbation, the resultant light signal can be made to have a modulation depth anywhere between 0 and $2\Gamma$. In particular, when

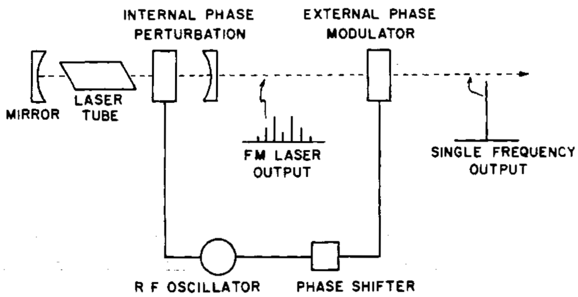


Fig. 13. Schematic of super-mode techniques. Figure from Harris and McDuff [9].

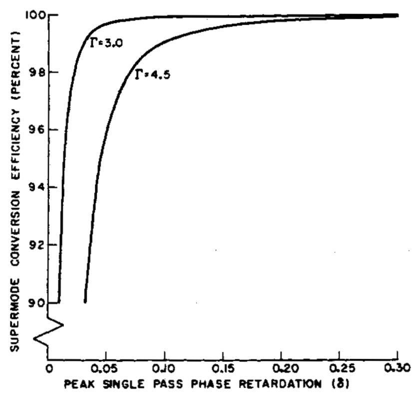


Fig. 14. Super-mode conversion efficiency vs. $\delta: g_0=0.085$; $\alpha_n=0.070$. Nine modes free running. Figure from Harris and McDuff [9].

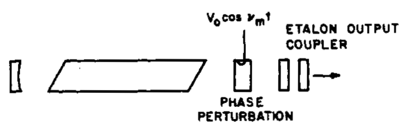


Fig. 15. Method of frequency selective coupling.

the resultant modulation depth is adjusted to 0, then in principle all of the energy which was previously distributed between all of the sidebands of the FM signal should appear as a single monochromatic optical signal.

If the FM signal emerging from the FM laser were completely free of distortion then, neglecting losses in the external modulator, the conversion process would be 100 percent efficient. However, as noted in Section II, a certain amount of distortion of the FM signal must always exist. It is therefore of interest to consider a super-mode conversion efficiency which we define as the ratio of power in the super-mode to the total power of the incident FM signal. Figure 14 shows the super-mode conversion efficiency as a function of $\delta$ at constant $\Gamma$, for a case corresponding to nine free-running modes in a He-Ne laser [9]. It is seen that, as $\delta$ becomes increasingly large, the distortion of the FM signal is reduced, and the super-mode conversion efficiency approaches 100 percent. It is to be noted that these curves assume that the external modulator is lossless and is operating at the correct and appropriately phased $\Gamma$.

The second technique for obtaining a high power output from a normally multimode laser has been termed the method of frequency selective coupling and has been demonstrated by Harris and McMurtry [23]. This technique shown schematically in Fig. 15 makes use of the joint saturation which is present in FM laser oscillation. As was discussed in Section II, if the intracavity phase perturbation $\delta$ of an FM laser is made sufficiently large as compared to the atomic gains and cavity losses of the various laser modes, then these modes will have approximately Bessel function relative amplitudes and will saturate as an entity. If the gain or loss of any of the modes is changed, relative amplitudes will still be very nearly maintained, and the oscillation level will adjust so that the net average power absorbed and dissipated by all modes remains zero. The selective coupling is obtained by replacing one mirror of the FM laser with a Fabry-Perot etalon, and adjusting the etalon so that only the desired mode has a nonzero output coupling.

Osterink et al. [24], have recently shown that successful operation of the selective coupling method is also possible in the phase locked region of operation of an FM laser discussed in Section II-E. In fact, at higher laser gains, operation in the phase locked region appears to be essential. Osterink et al., find that when coupling is attempted to some particular mode of an FM oscillation, the entire FM oscillation often shifts its carrier frequency by a mode and thereby runs in a condition where it sees a lower loss. In the phase locked region, all modes have more nearly the same amplitude, and this difficulty is better avoided.

It should be noted that at the present time there is considerable practical difficulty associated with either the super-mode method or the method of frequency selective coupling. Both methods require a very low loss internal perturbing element. The super-mode method requires an external modulator having a single pass phase retardation which is larger than is at present conveniently obtainable. The method of frequency selective coupling suffers from problems of mirror alignment and stability.

## V. Absolute Frequency Stabilization

As a result of fluctuations in the length of the laser cavity, all modes of an FM or phase locked laser will drift with respect to the center of the atomic fluorescence line. In the case of a free-running or phase locked laser, the relative mode amplitudes are primarily determined by the atomic gain profile. Thus as the modes drift they continuously change their amplitudes so that their envelope remains approximately unchanged.

In an FM laser the situation is considerably different. In this case relative mode amplitudes are determined by the internal phase perturbation, and the primary question is that of the competition between the different potential FM oscillations. It is thus expected that if the drift from the center of the atomic line is greater than about one half of an axial mode interval, one of the adjacent FM oscillations will become dominant and a discrete jump in all mode amplitudes will occur. This type of behavior has been observed experimentally and is discussed by Am-

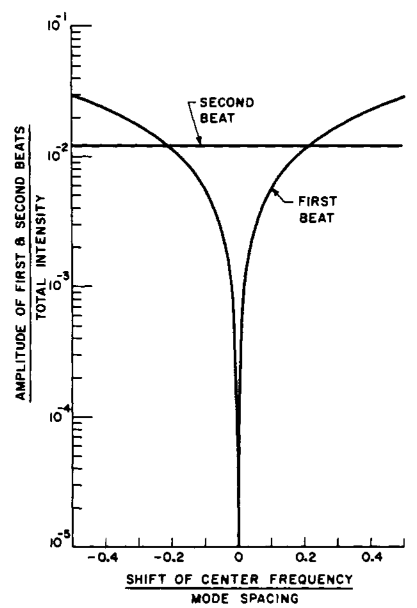


Fig. 16. Amplitude of first and second beat vs. position of center frequency: $g_0=0.085$; $\alpha_n=0.070$. Nine modes free running. Figure from Harris and McDuff [9].

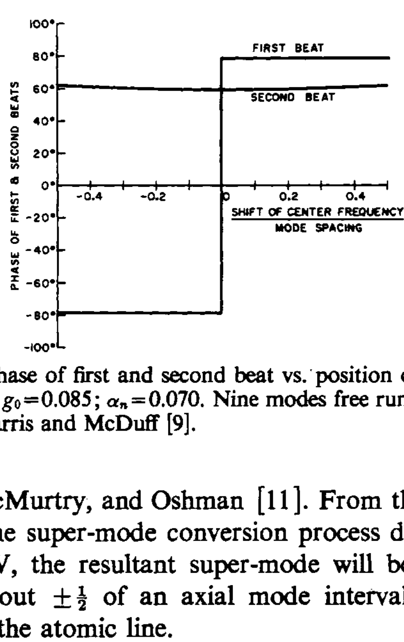


Fig. 17. Phase of first and second beat vs. position of center frequency: $g_0=0.085$; $\alpha_n=0.070$. Nine modes free running. Figure from Harris and McDuff [9].

mann, McMurtry, and Oshman [11]. From the point of view of the super-mode conversion process discussed in Section IV, the resultant super-mode will be stable to within about $\pm\frac{1}{2}$ of an axial mode interval from the center of the atomic line.

If the center frequency of an FM oscillation is exactly at the center of a symmetrical atomic line, then all odd harmonic beat notes will be identically zero [9], [11]. This results because the contributions to the odd harmonic beats from sidebands which are above the center frequency of the FM oscillation are exactly canceled by contributions from sidebands below the center frequency of the oscillation. As the center frequency moves off line center, this is no longer the case. Cancellation of upper and lower contributions is no longer complete, and odd harmonic distortion rapidly increases.

From another point of view, the FM signal can be thought of as a swinging frequency with its amplitude distorted by the atomic lineshape; thus for a symmetrical line, this AM distortion will be even harmonic.

These ideas provide the basis for a technique for the absolute frequency stabilization of an FM laser [25]. Figures 16 and 17 show the amplitude and phase, respectively, of the first and second beats as a function of the position of the center frequency of an FM oscillation with respect to the center of the atomic line. The data for these figures correspond to a case of nine free-running modes in a He-Ne laser, with $\delta=0.15$, and $\Gamma=3.0$. It is seen that the amplitude of the first beat is extremely sensitive to the position of the center frequency of the FM oscillation, while the amplitude of the second beat is nearly independent of this position. In addition, the phase of the first beat (with respect to the phase of the driving perturbation) changes abruptly as the center of the FM oscillation moves from one side of the atomic line to the other.

In a practical stability scheme, one might detect the ratio of the first and second beat amplitudes, and thereby eliminate the possibility that spurious variations in output power would be incorrectly interpreted. By homodyning with the driving source, the phase information could provide the sign of the necessary correction in mirror position. In cases where the atomic fluorescence line is asymmetrical, stabilization could still be accomplished with respect to the nonzero minimum of the fundamental beat amplitude.

## VI. Coupling Modulation

In previous sections we have considered the application of internal time-varying perturbation to the problem of spectral control of multimode laser oscillators. Another application of internal time-varying elements is that of coupling modulation. By coupling modulation is meant a technique wherein the internal perturbation itself, or in conjunction with another element, becomes the output coupler of the laser oscillator. The optimum coupling loss for most lasers is on the order of a few percent, and is typically obtained by means of the finite transmission of an output mirror. If instead, the mirrors of the laser are opaque, and if this entire coupling loss is obtained via the internal element, then the maximum available power of the laser may be obtained as a modulated signal.

### *A. Amplitude Modulation*

The technique of coupling modulation, as applied to amplitude modulation, was first proposed and demonstrated by Gürs and Muller, and is shown schematically in Fig. 18 [16], [17]. The internal coupling element consists of a KDP crystal followed by a reflective type polarizing element such as a Rochon prism. The KDP crystal is oriented with its optic axis in the longitudinal direction and with its electrically induced principal axes at 45° to the polarization of the laser. When in this orientation, the KDP crystal acts to create AM type sidebands which are orthogonally polarized to the main laser beam. The polarizing element couples these sidebands directly from the laser, while allowing the main polarization to pass through unchanged. For components as described above, the resulting modulation will be entirely at the second harmonic of the driving frequency of the KDP crystal. If, as is usual, it is desired to obtain modulation at the fundamental frequency, then a bias equal to the peak value of the ac signal must be applied to the crystal. This bias may be obtained either electrically, by means of a retardation plate, or by deliberately misorienting the KDP crystal so that there is a small angle between its optic axis and the direction of light propagation. To obtain 100 percent modulation of the maximum laser output, both the bias loss and the peak ac loss should be adjusted to equal the optimum coupling loss of the laser. Kaminow has noted that even in the presence of a properly adjusted bias a certain amount of higher harmonic distortion will always be present [26].

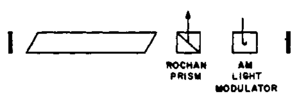

Fig. 18. AM type coupling modulation.

In that coupling modulation does not require modulation of the average internal stored energy of the laser, it is not restricted in frequency by either the laser cavity bandwidth or the atomic fluorescence linewidth. It is however restricted by distortion considerations. The problem is, that associated with the desired AM modulation of the orthogonal polarization, there is also a simultaneous and unavoidable AM modulation of the primary laser polarization [18], [26]. If the modulation frequency is far from a $c/2L$ frequency, then the AM sidebands do not appreciably affect the behavior of other modes. However, as the modulation frequency approaches an axial mode frequency or multiple thereof, the internal stored energy of the laser will be appreciably affected, and the internal mode amplitudes will behave as described in Section III. The situation will be particularly bad if the modulation frequency is such as to produce a sideband near a frequency where there is already a free-running laser mode, but serious distortion will also result if a cold Fabry-Perot mode is approached. The question of distortion in coupling modulation has been considered analytically by DiDomenico [18], Kaminow [26], and Uchida [13], and has been studied experimentally by Uchida [13].

A number of methods have been suggested for increasing the distortion free bandwidth of coupling modulation. All of these are based on keeping the internal AM perturbation coefficient $\alpha_c$ of Section III at as low a level as possible. Gürs and Muller have suggested a push-pull coupling element wherein two modulators are fed 180° out of phase, so that the total internal energy remains constant [16], [17], [26]. Another technique suggested by Fox, is to place the coupling elements as close as possible to the center of the laser cavity and to precede it by a second polarizing element [26]. The transit time of the light between successive passes through the modulator will then be very nearly one-half period at the modulation frequency, and the internal modulation experienced on successive passes will approximately cancel.

### *B. Frequency Translation*

Coupling modulation may also be used to accomplish efficient optical frequency translation (alternately termed as single-sideband-suppressed carrier modulation). The first such technique was proposed and demonstrated by Siegman et al. [27], and is shown in Fig. 19. This technique utilizes the Brillouin scattering from a traveling acoustic wave to achieve the frequency translated output from the laser cavity. An optical beam which is incident on a traveling acoustic wave at approximately the Bragg angle will be partially reflected from the acoustic wavefronts and translated in frequency by the acoustic frequency. The direction of this frequency shift is upward or downward, respectively, depending on whether a component of the incident optical velocity is in the direction opposite to or in the same direction as the velocity of the acoustic wave [28], [29]. When attempted externally to a laser cavity the practicality of the Brillouin scattering technique is presently limited by the large acoustic drive strengths needed to achieve efficient conversion. When placed internally to the laser cavity, the acoustic element scatters the same fraction of the very much larger light intensity which exists inside the laser cavity; and as noted above, if the fraction which is acoustically coupled out is the same as the fraction ordinarily transmitted by the laser mirrors, then the intensity of the frequency translated beams will be comparable to that of the free-running laser.

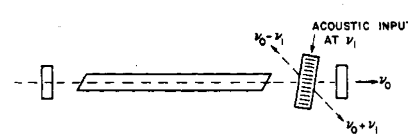

Fig. 19. Acoustic frequency translation.

Since the internal energy of the laser is not affected, the efficiency of this technique is not influenced by the relation of the driving acoustic frequency to the $c/2L$ frequencies of the laser. For the configuration of Fig. 19, it is noted that two output beams are obtained—one shifted up and the other down in frequency. A single beam output could be obtained by operation in a ring type resonator.

Methods of optical frequency translation which utilize the electrooptic effect have been proposed and demonstrated by Targ et al. [30], and also by Peterson and Yariv [31]. The configuration for Targ's method is similar to that for AM type coupling modulation which was described above and shown in Fig. 18. The differences are: first, the path length between the center of the KDP crystal and the 100 percent reflective right-hand end mirror is set equal to an odd number of quarter wavelengths at the modulation frequency; and second, the crystal is misoriented in order to achieve a quarter wavelength of necessary distributed birefringence. (Alternately, a retardation plate of lumped $\lambda/8$ birefringence may be employed.) The principle of operation of this coupling element is similar to that of a single sideband modulator which was proposed earlier by Buhrer [30]. A linearly polarized optical signal propagating from left to right through the electrooptic crystal generates a pair of orthogonally polarized sidebands. The signal then traverses the air path and returns to the electrooptic crystal with sideband phases such that on its second path through the crystal the magnitude of one sideband is enhanced while the other sideband is eliminated. The remaining sideband is coupled out of the laser cavity via the polarizing element. By changing the sign of the bias, either the upper or lower sideband may be obtained.

Peterson and Yariv's method [31] utilizes a KDP crystal oriented as a phase modulator, followed by an etalon output coupler. The length of the etalon is adjusted so that (ideally) it is 100 percent reflective at the frequency of the laser oscillation, and completely transparent at the desired translated frequency. Their experiments were performed with a 6328 Å He-Ne laser, and a translation frequency of 8.9 Gc/s. Either the upper or lower sideband could be obtained by tuning of the etalon.

## Acknowledgment

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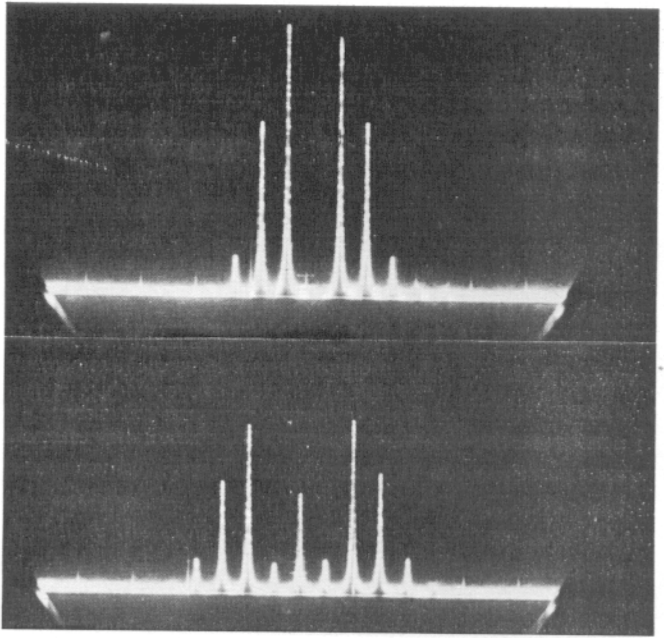

Fig. 4. Typical FM spectra. Figure from Ammann, McMurtry, and Oshman [11].

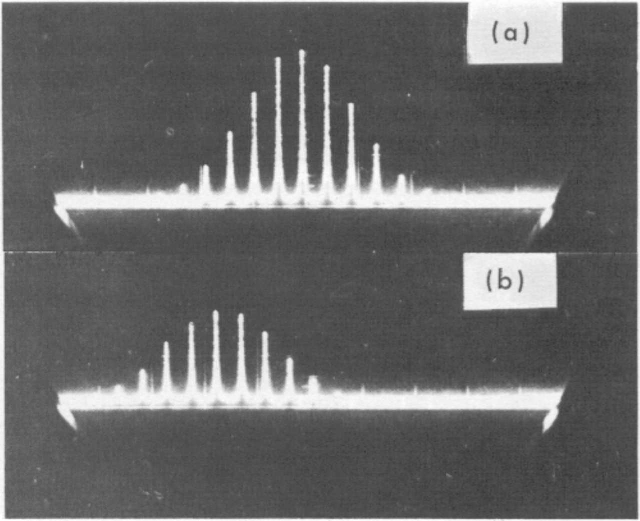

Fig. 6. Shift of envelope of laser modes in phase locked region. (a) Zero detuning. (b) Finite detuning. Figure from Ammann, McMurtry, and Oshman [11].