shows a typical spectrum analyzer picture of a 7.5 MHz beat frequency similar to that of Fig. 1. However, the frequency dispersion has been narrowed to 30 kHz/cm and the IF bandwidth to 1 kHz. The combination of a 1 second exposure time and the 3×10^{-3} second/cm sweep rate of Fig. 2(a) allowed the integration of approximately 33 consecutive traces on the photograph. Thus Fig. 2(a) indicated an approximately 20 kHz wide envelope for the combined frequency drift and jitter contributed by both lasers. For an exposure time of 3×10^{-2} second, an approximately 9 kHz envelope width was typical. Single-scan pictures at 3×10^{-3} second/cm sweep rates seldom yielded anything different from the frequency spike caused by a Hewlett-Packard 608 generator which is shown in Fig. 2(b). At this point, it should be emphasized that the effective resolution of a spectrum analyzer is not necessarily given by the IF bandwidth B, but rather by the combination of B (in hertz), sweep time T (in seconds), and dispersion Δf (in hertz). For most conventional spectrum analyzers with a Gaussian IF response, the effective resolution R may be shown to be

$$R = B\left[1 + 0.195\left(\frac{\Delta f}{TB^2}\right)^2\right]^{1/2}. \tag{2}$$

Thus, in spite of a 1 kHz IF bandwidth, the effective resolution was approximately 4.5 kHz for Fig. 2(a) and (b), both of which were photographed with identical analyzer and camera settings.

The stability thus far observed is considered poor because of relatively bad environmental conditions. On one occasion at least, the beat frequency fluctuation stayed well within the approximately 500 Hz resolution limit of a Panoramic spectrum analyzer for several seconds and the slow drift did not exceed a few kilohertz on a 1 kHz/cm display for several minutes. This observation shows that the conservative but more typical stability figures given previously can be improved with proper care and equipment.

The measurements described here are considered only preliminary in order to test some of the design features of the lasers and to facilitate the choice among the different approaches pertaining to more refined frequency stability measurements as partially outlined by Siegman et al.[3] and in the PROCEEDINGS OF THE IEEE.[1] We hope that future experiments with improved lasers and experimental setup will result in better stability figures.

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REFERENCES

[1] "Special Issue on Frequency Stability," *Proc. IEEE*, vol. 54, February 1966.
[2] T. S. Jaseja, A. Javan, and C. H. Townes, "Frequency stability of the He-Ne masers and measurements of length," *Phys. Rev. Lett.*, vol. 10, pp. 165–167, March 1, 1963.
[3] A. E. Siegman, B. Daino, and K. R. Manes, "Preliminary measurements of laser short-term frequency fluctuations," this issue, page 180.
[4] R. A. Brandewie, W. T. Haswell, III, and R. H. Harada, "Heterodyne detection and linewidth measurement with high power CO_2 lasers," *IEEE J. of Quantum Electronics*, vol. QE-2, pp. 756–757, November 1966.
[5] M. Scully and W. E. Lamb, Jr., "Quantum theory of an optical maser," *Phys. Rev. Lett.*, vol. 16, pp. 853–855, May 9, 1966.
[6] H. J. Pauwels, "Phase and amplitude fluctuations of the laser oscillator," *IEEE J.* of *Quantum Electronics*, vol. QE-2, pp. 54–62, March 1966.
[7] M. Lax, "Quantum noise V: Phase noise in a homogeneously broadened maser," in *Physics of Quantum Electronics*. New York: McGraw-Hill, 1966, pp. 735–747.
[8] V. Korenman, "Quantum theory of the laser," *ibid.*, pp. 748–758.
[9] C. Freed and H. A. Haus, "Photocurrent spectrum and photoelectron counts produced by a gas laser," *Phys. Rev.*, vol. 141, pp. 287–298, January 1966.
[10] T. Li, "Diffraction loss and selection of modes in maser resonators with circular mirrors," *Bell Sys. Tech. J.*, vol. 44, pp. 917–932, May 1965.
[11] R. Carbone, private communication.

[1] Operated with support from the U. S. Air Force.

Threshold of Phase-Locked Parametric Oscillators

In a recent correspondence,[1] the author considered the threshold for a multimode parametric system consisting of a set of idler modes with the same $c/2L$ frequency spacing as the laser pump source, all coupled through a nonlinear element to a *single* signal mode. It was shown that the threshold for such a system is determined by the total average pump power, and that the distribution and phasing of this pump power among the various pump modes is irrelevant to threshold.

As a result of the high peak powers which are now available from phase-locked lasers,[2–4] it is of interest to consider a parametric system whose threshold is determined by the peak as opposed to the average power of the laser pulse train. We show below that such a system is simply that of an idler and signal cavity which *each has a c/2L frequency spacing equal to that of the phase-locked pumping laser*. No time-varying or phase-locking elements are necessary in either the signal or idler cavities. Before proceeding, we note that other authors have considered the use of phase-locked pulses to enhance nonlinear optical processes such as harmonic generation.[5]

The equations which determine threshold of the foregoing system may be shown to be as follows:

$$\begin{aligned}\frac{dE_{sq}}{dt} &= -\alpha_s E_{sq} + \nu_s \kappa \sum_t E_{pt} E_{i(t-q)} \\ \frac{dE_{iq}}{dt} &= -\alpha_i E_{iq} + \nu_i \kappa \sum_t E_{pt} E_{s(t-q)}\end{aligned} \tag{1}$$

where E_{sq}, E_{iq}, and E_{pq} are the amplitudes of the q signal, idler, and pump modes, respectively; α_s is the single-pass power loss of all the signal modes, and α_i is the single-pass power loss of all idler modes. ν_s and ν_i are the mean frequencies of the signal and idler modes, and κ is the parametric coupling constant. The above equations have assumed a set of pump modes which have zero relative phase angles, such that they synthesize a pulse train in the time domain having a peak power of $[\sum_q E_{pq}]^2$. It is also assumed that the $\bar{k}$ matching condition is exactly satisfied by all interacting modes, and that therefore relative parametric phases adjust to the optimum value.

To find the threshold for this system, we set time derivatives equal to zero, and thus have

$$E_{sq} = \frac{\nu_s \kappa}{\alpha_s} \sum_t E_{pt} E_{i(t-q)} \tag{2a}$$

$$E_{iq} = \frac{\nu_i \kappa}{\alpha_i} \sum_t E_{pt} E_{s(t-q)}. \tag{2b}$$

Substituting (2b) into (2a) we have

$$E_{sq} = \frac{\nu_s \nu_i \kappa^2}{\alpha_s \alpha_i} \sum_t \sum_n E_{pt} E_{pn} E_{s(n-t+q)}. \tag{3}$$

We now sum both sides of (3) over q and note that

$$\sum_q E_{sq} = \sum_q E_{s(n-t+q)}$$

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[1] S. E. Harris, "Threshold of multimode parametric oscillators," *IEEE J. of Quantum Electronics (Correspondence)*, vol. QE-2, pp. 701–702, October 1966.
[2] S. E. Harris, "Stabilization and modulation of laser oscillators by internal time-varying perturbation," *Proc. IEEE*, vol. 54, pp. 1401–1413, October 1966.
[3] M. DiDomenico, Jr., J. E. Geusic, H. M. Marcos, and R. G. Smith, "Generation of ultrashort optical pulses by mode locking the YAIG:Nd laser," *Appl. Phys. Lett.*, vol. 8, p. 180, April 1, 1966.
[4] A. J. DeMaria, D. A. Stetser, and H. Heynau, "Self mode-locking of lasers with saturable absorbers," *Appl. Phys. Lett.*, vol. 8, p. 176, April 1, 1966.
[5] R. L. Kohn and R. H. Pantell, "Second-harmonic enhancement with an internally-modulated ruby laser," *Appl. Phys. Lett.*, vol. 9, p. 231, May 1, 1966.

(all sums are from $-\infty$ to $+\infty$). We then obtain

$$\sum_{l} \sum_{n} E_{pl} E_{pn} = \frac{\alpha_s \alpha_i}{\nu_s \nu_i K^2}$$

or (4)

$$[\sum_{q} E_{pq}]^2 = \frac{\alpha_s \alpha_i}{\nu_s \nu_i K^2}$$

which is the desired result. We thus have found that it is peak rather than average pump power which determines threshold for this type of parametric system. Physically, the picture is that of parametric signal and idler pulses which build up with relative mode phases such that they pass through the nonlinear element synchronously with the pulses of the pumping laser.

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A Note on "Excitation Cross Section of Some of the States of NeII, ArII, KrII by Electron Collision"

In the above paper,[1] excitation cross sections were calculated using the sudden perturbation approximation. There it was assumed that the ground state after losing an electron was either 100 percent in a $^2P_{3/2}$ state or else 100 percent in a $^2P_{1/2}$ state. However, it may be possible to approximately determine in what J-value state the resulting ion will be left after losing an electron, if we change the order of coupling of the ground-state atom from LS to JJ coupling before the loss of the electron by collision; i.e.,

$$|l^6 LSJM\rangle = \sum_{\bar{J}, j, \bar{\psi}} (\psi\{|\bar{\psi})[\bar{J}]^{1/2}[j]^{1/2}[L]^{1/2}[S]^{1/2} \cdot \begin{Bmatrix} \bar{L} & l & L \\ \bar{S} & s & S \\ \bar{J} & j & J \end{Bmatrix} |l^5 \bar{L}\bar{S}\bar{J}, lsj, JM\rangle, \quad (1)$$

where the atomic ground state $\psi = |LSJM\rangle$ has first been expressed in terms of its parent states and the order of coupling has been changed. The summation is overall possible l^5 core states $\bar{\psi} = |\bar{L}\bar{S}\rangle$ and all possible j and $\bar{J}$ values where $\bar{J} = \bar{L} + \bar{S}$. The quantities $(\psi\{|\bar{\psi})$ are the coefficients of fractional parentage[2] which for the P^6 configuration of the noble gas atoms is equal to one since there is only one possible state $\bar{\psi} = |^2P\rangle$. The quantity in curly brackets is the usual 9-j symbol[3] and the notation $[x] = 2x + 1$.

The probability that the atom will be left in any $\bar{J}$ value state after the loss of any one of its core electrons will be α_J, where

$$\alpha_J = \sum_{j} [\bar{J}][j][L][S] \begin{Bmatrix} \bar{L} & l & L \\ \bar{S} & s & S \\ \bar{J} & j & J \end{Bmatrix}^2. \quad (2)$$

For noble gas atoms, J is either 1/2 or 3/2, $L = S = J = 0$, $\bar{L} = 1$, and $\bar{S} = s = 1/2$. It is seen after carrying out the algebra that

$$\alpha_{3/2} = 2/3 \quad \text{and} \quad \alpha_{1/2} = 1/3.$$

From this calculation, it is seen that the probability of production of states with total $J = 3/2$ is favored two to one as compared to states with $J = 1/2$.

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[1] S. H. Koozekanani, *J. of Quantum Electronics*, vol. QE-2, pp. 770–773, December 1966.

[2] B. R. Judd, *Operator Techniques in Atomic Spectroscopy*. New York: McGraw-Hill, 1963. Also C. W. Nielson and G. F. Koster, *Spectroscopic Coefficients for p^n, d^n and f^n Configurations*. Cambridge, Mass.: M.I.T. Press, 1963.

[3] A. R. Edmonds, *Angular Momentum in Quantum Mechanics*. Princeton, N. J.: Princeton University Press, 1957.

[4] Formerly the Antenna Laboratory.