Pondermotive Forces with Slow Light

S. E. Harris
Edward L. Ginzton Laboratory, Stanford University, Stanford, California 94305
(Received 21 July 2000)

This work describes atomic processes which result from the greatly enhanced longitudinal gradient force which is inherent to the propagation of slow light. These processes are (1) ballistic atom motion and atom surfing, and (2) a type of local pondermotive nonlinearity or scattering which results from free-particle sinusoidal motion and the density variation caused by this motion.

PACS numbers: 32.80.Lg, 32.80.Qk, 42.50.Vk

By combining electromagnetically induced transparency (EIT) with cold atom technology, one may produce a transmission window in an otherwise optically thick medium. Because this window is much narrower than the natural linewidth (Fig. 1) of an isolated atom, the dispersion of the medium is very steep and results in a group velocity V_g that can be less than 10^{-7} c [1]. As a result of this slow group velocity, an optical pulse that enters the EIT medium is spatially compressed [2], as compared to its length in free space, in the ratio of c/V_g. An atom in this medium will experience a longitudinally directed gradient force that is enhanced in this same ratio.

This Letter describes atomic processes which result from this greatly enhanced gradient force. These are (1) ballistic atom motion and atom surfing, and (2) a type of local optical nonlinearity or light scattering which results from free-particle sinusoidal motion and the density variation caused by this motion.

Before proceeding, we note that the use of slow light to phase match and enhance acousto-optical interactions in an ion-doped fiber has recently been described by Matsko *et al.* [3]. This work departs from their work in that, here, the described effects are based on independent (nonpropagating) particle motion. We are primarily interested in conditions which are characteristic of (noncondensed) trapped, cold atoms where the atom density is too low and the mean free path for collision is too long to support a propagating acoustic wave.

This work should be considered in the context of a considerable ongoing effort to use the unique absorptive and dispersive properties of EIT to enhance nonlinear optical processes of many types [4]. Representative publications which describe all-optical nonlinearities (no motional coordinate) are given in Ref. [5]; publications which discuss the enhancement of Raman processes are given in Ref. [6]; and, as noted above, acousto-optic interactions are described in Ref. [3].

Throughout this work we will assume dilute, two-state test atoms with a transition energy $\omega_b - \omega_a$ which are immersed in an EIT medium with a group velocity of V_g. By using test atoms at a density which is small as compared to the EIT atoms we insure that the motion of the test atoms does not interact with the light-slowing EIT process. In an early experiment, the test atoms might be a different isotope of the same species which is used to produce the slow light.

In general, the pondermotive force is the force on atoms in a dielectric medium, or on electrons or ions in a plasma, which results from the spatial variation of the optical power density [7]. For a monochromatic plane wave, the optical power density is independent of distance and the longitudinal component of the pondermotive force is zero. For an optical pulse in a dispersive medium, the spatial extent of the traveling pulse varies as the optical group velocity V_g and at slow group velocities, even at modest pulse energies, will transfer substantial momentum to the test atoms with which it collides.

We write the slowly moving electromagnetic pulse which will impart momentum to the test atoms as

$$\Omega(t,z) = \mathrm{Re}\left[\Omega\left(t - \frac{z}{V_g}\right)\exp j(\omega_0 t - kz)\right]. \quad (1)$$

The carrier frequency ω_0 is tuned to the center of the EIT transparency (Fig. 1) and is detuned by $\Delta\omega = \omega_0 - (\omega_b - \omega_a)$ from the transition of the test atom. This test atom transition is assumed to be well outside the EIT profile. Because the test atoms are dilute, and the refractive index of the EIT medium is unity, the k vector is k_{vac}. The Fourier components of the pulse envelope $\Omega(t - z/V_g)$ are taken to lie sufficiently within the transparency window that, to within a good approximation, the pulse propagates without change of shape. We also assume that the spectral width of the envelope is small as compared to the detuning from the test atom transition $\Delta\omega$.

The longitudinal force on a test atom results from both a radiation pressure term and a gradient term and may be written as [8]

$$F_z(t,z) = \frac{\hbar}{2}\left\{-jk\Omega\left(t - \frac{z}{V_g}\right) + \frac{\partial}{\partial z}\left[\Omega\left(t - \frac{z}{V_g}\right)\right]\right\}\rho^*_{ab}(t,z) + \text{c.c.}. \quad (2)$$

We work in the interaction picture and define ρ_{ab} as the coherence of the two-state test atom. When on line center ρ_{ab} is in phase quadrature with $\Omega(t - z/V_g)$ and only the

 0031-9007/00/85(19)/4032(4)$15.00

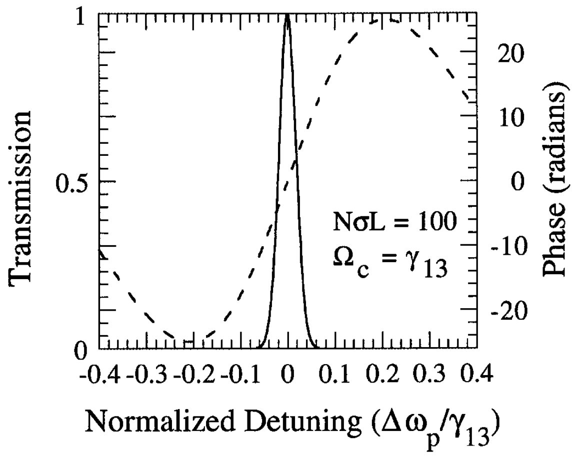

FIG. 1. Transmission and phase versus frequency for subnatural linewidth electromagnetically induced transparency. The quantity $\Delta\omega_p$ is the detuning from line center of an otherwise opaque transition with an Einstein A coefficient of $2\gamma_{13}$ and an optical depth of 100. The transition is made transparent by applying a time-independent coupling laser with a Rabi frequency $\Omega_c = \gamma_{13}$. The group velocity and spatial extent of a propagating pulse vary inversely as the atom density N. [Reprinted from Harris and Hau [5].]

radiation pressure term in Eq. (2) is nonzero. When this is the case, less than a single quantum of momentum may be transferred during a radiative lifetime. Here, we assume that the detuning $\Delta\omega$ is sufficiently large that ρ_{ab} is in phase with $\Omega(t - z/V_g)$. In this case, only the gradient term is nonzero and a large number of quanta may be transferred during the applied pulse. With these assumptions $\rho_{ab} = -\Omega/(2\Delta\omega)$ and the longitudinal gradient force is

$$\begin{aligned} F_z(t,z) &= -\frac{\hbar}{4\Delta\omega}\frac{\partial}{\partial z}\left|\Omega\left(t - \frac{z}{V_g}\right)\right|^2 \\ &= \hbar k\left(\frac{c}{V_g}\right)\left(\frac{1}{4\omega_0\Delta\omega}\right)\frac{\partial}{\partial t}\left|\Omega\left(t - \frac{z}{V_g}\right)\right|^2 . \end{aligned} \quad (3)$$

As a first example of the effect of this force we consider the one-dimensional interaction of a Gaussian shaped temporal pulse with a cold test atom which has an initial velocity V_0. Newton's law is $m(dV/dt) = F_z[t - z(t)/V_g]$. The right-hand side of this equation denotes the force seen by the moving test atom, i.e., $z(t)$ is the coordinate of the test atom as a function of time. In the absence of friction and assuming that the sign of the force is such as to push on the atom, two types of behavior are possible: For a sufficiently weak force, the atom is first accelerated and then decelerated so as to return to its initial velocity. For a sufficiently strong force, the atom acquires a final velocity of $2V_g - V_0$. The break point between these two types of behavior occurs when the peak pondermotive energy is less than or greater than the kinetic energy of the particle as viewed in the frame of the moving light pulse; that is, at that energy where

$$\frac{m(V_0 - V_g)^2}{2} = \frac{\hbar|\Omega(0)|^2}{4\Delta\omega} . \quad (4)$$

In the frame of the moving light pulse, the light pulse-particle encounter may be viewed as an elastic scattering. If the particle has sufficient initial energy, it moves through the pondermotive barrier with no change in its initial velocity. If the particle has insufficient initial energy to move through the barrier, its velocity in the moving frame is reversed and, in the stationary frame, becomes $2V_g - V_0$. Figure 2 shows, in the laboratory frame, the applied pulse shape, force, and atom velocity. At $t = 0$ the atom is stationary. In the first column the pondermotive energy is 5% below the reflection condition of Eq. (4). In the second column, the pondermotive energy is 5% greater and the atom acquires a terminal velocity of $2V_g$.

Slow light moving through test atoms with a given velocity distribution will alter that distribution. For example, a distribution centered at V_g will be inverted in velocity

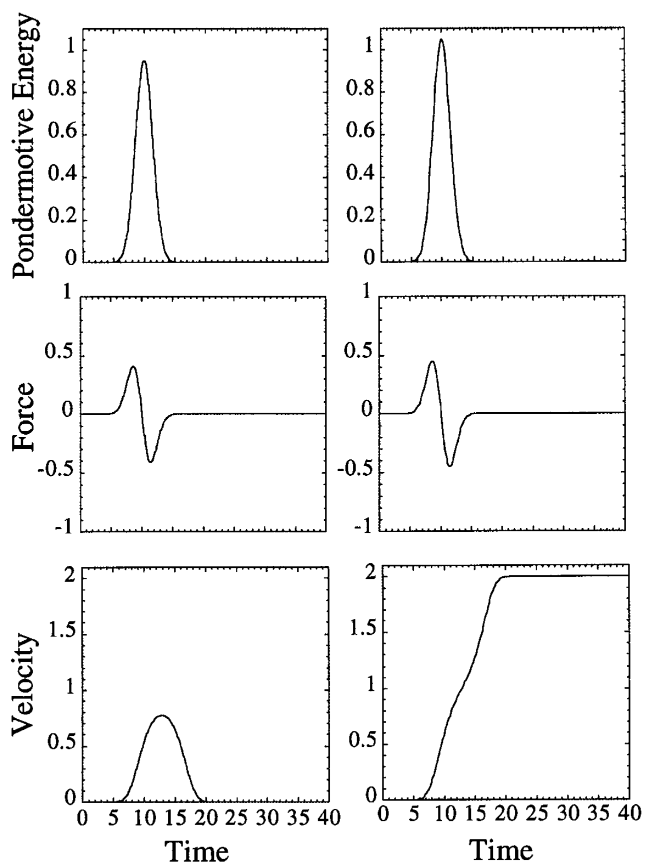

FIG. 2. Ballistic interaction of a previously stationary atom with a slowly moving light pulse. In the first column the peak pondermotive energy is 5% less than the critical energy of Eq. (4). Here the particle returns to zero velocity. In the second column the peak pondermotive energy is 5% greater than this energy and the particle acquires a terminal velocity of $2V_g$. The parameters for all parts of the figure are $V_0 = 0$, $V_g = 1$, and $m = 2$; the critical pondermotive energy is therefore one unit.

space; a distribution centered at $2V_g$ will be inverted and translated to zero velocity.

If a force proportional to and opposing the velocity (friction) is added to the problem, the ballistic solution, which is described above, is no longer obtained. Now, for a weak force, the atom velocity damps to zero. For a sufficiently strong force, the atom surfs on the front edge of the pulse with a velocity of V_g. This behavior is shown in Fig. 3.

The pulse energy necessary to observe these effects is modest. For a detuning of 100 MHz from a test atom transition with a matrix element of 1 a.u., a group velocity of 1 m/s, a pulse length of 1 μs, and a 1-mm beam diameter, the pulse energy necessary to satisfy Eq. (4) for a previously stationary atom is 0.35 μJ.

We continue with a second example of the effect of these unusually strong longitudinal light forces. This is a type of collinear light scattering which is in the spirit of stimulated Brillouin and Raman-Nath scattering, but differs in that it is based on local particle motion and does not involve a propagating acoustic wave. This type of scattering or nonlinearity is of importance when the atom density is too low and the mean free path for collision is too long to support a propagating acoustic wave.

In the experiment which is considered, two monochromatic laser beams separated by an acoustic frequency ω_a are applied to a sample of cold atoms which are embedded in a slow light medium. The time-varying pondermotive force drives z-directed oscillatory motion. In turn, this motion causes the atom density to vary sinusoidally with time and distance. As the beams propagate, the time-varying density generates a collinear comb of sidebands which are separated by ω_a.

Cumulative generation of a spectrum of sidebands requires that each of the generated sidebands propagate with a k vector that is equal to the k vector of its driving polarization. Because, at any z, the atom density depends only on the *electromagnetic fields at that z,* the electromagnetic nonlinearity is local in space, and k-vector matching is insured by the near-linear dispersive profile of EIT. Because the second derivative of k with respect to ω is zero in the EIT medium, to the extent that higher derivatives may be neglected [2], the frequencies ω_q and k vectors k_q of the generated sidebands are

$$\omega_q = \omega_0 + q\omega_a,$$
$$k_q = k_0 + \left(\frac{\partial k}{\partial \omega}\bigg|_{\omega_0}\right) q\omega_a,$$
$$= k_0 + q\frac{\omega_a}{V_g}. \quad (5)$$

Irrespective of the acoustic frequency, each sideband is phase matched. (For a propagating sound wave, the optical nonlinearity is not local and, for cumulative growth, there is the additional requirement that the optical group velocity equal the phase velocity of the sound wave [3].)

We continue with the formalism described above. To allow for the generated as well as the applied frequencies we write the envelope $\Omega(t - z/V_g)$ of Eq. (1) as

$$\Omega(t - z/V_g) = \sum_{-\infty}^{+\infty} \Omega_q \exp\left[-jq\omega_a\left(t - \frac{z}{V_g}\right)\right]. \quad (6)$$

The driving beams have Rabi frequencies Ω_0 and Ω_{-1}. We assume that all frequencies lie well within the transmission window of Fig. 1 and satisfy the linear dispersion condition of Eq. (6). The force on each atom is given by Eq. (3) and the atom velocity follows from Newton's law. With N as the number of atoms per volume, the normalized atom density $\rho(t,z)$ is defined as $N = N_0[1 + \rho(t,z)]$ and is obtained from the one-dimensional equation of continuity $\partial\rho/\partial t = -\partial v/\partial z$. In general, $\rho(t,z)$ has Fourier components at all multiples of ω_a and is

$$\rho(t,z) = \sum_{r\neq 0} \rho_r \exp\left[jr\omega_a\left(t - \frac{z}{V_g}\right)\right],$$
$$\rho_r = \frac{\hbar}{4\Delta\omega m V_g^2} \sum_s \Omega_{r+s}\Omega_s^*. \quad (7)$$

The slowly varying envelope equation for the propagating sidebands is

$$\frac{\partial \Omega_q}{\partial z} = -j\zeta\Omega_q - j\zeta \sum_{r\neq 0} \rho_r \Omega_{q-r}. \quad (8)$$

The quantity $\zeta = \omega_0(n - 1)/c$, where the refractive index n of the test atoms is the same for all sidebands and, to within the rotating wave approximations, is $n = 1 - (|\mu|^2 N_0)/(2\epsilon_0 \hbar \Delta\omega)$.

We eliminate the first term in Eq. (8) by the change of variable $\Omega_q = \tilde{\Omega}_q \exp(-j\zeta z)$. Following Harris and Sokolov [6], it may be verified that an exact solution of both the motional and slowly varying envelope equations is

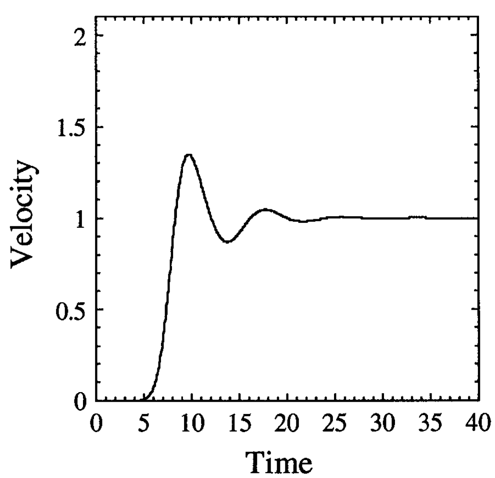

FIG. 3. Atom surfing on a slowly moving light pulse. Here, the parameters of Fig. 2 are modified by including a frictional force. The magnitude of this force is chosen so that the $1/e$ damping time for the particle is $\frac{1}{2}$ of the pulse length.

$$\tilde{\Omega}_q(z) = \Omega_0 \exp\left[j\left(\varphi(0) - \frac{\pi}{2}\right)q \right] J_q(\gamma z) + \Omega_{-1} \exp\left[j\left(\varphi(0) - \frac{\pi}{2}\right)(q + 1) \right] J_{q+1}(\gamma z),$$
$$\rho(t,z) = \frac{\hbar}{4\Delta\omega m V_g^2} \Omega_0 \Omega_{-1}^* \exp\left[j\omega_a\left(t - \frac{z}{V_g}\right)\right] + \text{c.c.}. \tag{9}$$

The quantities γ and $\varphi(0)$ are $\gamma = 2\zeta|\rho(0)|$ and $\varphi(0) = \arg[\rho(0)]$. Though the generated spectrum changes with distance, the magnitude and phase of the density wave $\rho(t,z)$ are independent of z, and equal to their values at $z = 0$. The reason for this is that the envelope of a frequency-modulated signal is time independent, and it is only the beat note between the two frequency-modulated input frequencies that drives the atoms. Mathematically, the solution follows from the identities $2\frac{\partial J_n(x)}{\partial x} = [J_{n-1}(x) = J_{n+1}(x)]$ and $\sum_n J_{n+q}(x) J_{n+p}(x) = \delta_{pq}$. In the more general case, where the dispersion is not linear and Bessel function amplitudes are not obtained, the acoustic density will vary not only as ω_a, but also at harmonics of ω_a. Each spectral sideband is coupled to all other sidebands and not only to its nearest neighbor.

If ρ is sufficiently small that only the first anti-Stokes sideband has significant amplitude, the ratio of the generated anti-Stokes electric field Ω_1 to the pump field Ω_0 is $|\Omega_{+1}/\Omega_0| = \frac{\omega_0}{c}(n - 1)|\rho(0)|L$. To observe the generated signal one might measure the photobeat at frequency $2\omega_a$. This beat varies as $(1/V_g)^2$ and, at slow group velocities and typical MOT conditions, should be readily observable. It may be shown that the condition for the pondermotive nonlinearity to exceed the $\chi^{(3)}$ nonlinearity of the test atom is $\hbar\Delta\omega > mV_g^2$. Therefore, this type of scattering is observable only with slow light.

It is of interest to compare the scattering amplitude of Eq. (9) to that which is obtained with an acoustic wave which has a phase velocity equal to the optical group velocity. The atom density now satisfies the driven acoustic wave equation, and the pondermotive nonlinearity is no longer local. Following Ref. [3], with the acoustic wavelength denoted by λ_a, and assuming that the group velocity of the optical field equals the phase velocity of the sound wave, the generated electric field is increased by a factor of $2\pi L/\lambda_a$. An example is provided by Bose condensation. As condensation occurs and the previously independent particles become coupled, the particle motion changes from local to wavelike. By varying the acoustic frequency within the transparency profile and thereby varying the acoustic wavelength from long to short as compared to the healing length of the condensate, it may be possible to move continuously between the local and wavelike regimes of the nonlinearity. Such experiments would be in the character of the recent work of Stamper-Kurn and colleagues [9].

This work has shown how the steep linear dispersive profile of EIT will increase the z-directed pondermotive force so as to allow the observation of effects such as ballistic z-directed atom pumping, atom surfing, and a type of local pondermotive light scattering which varies as $1/V_g^2$. Though we have assumed the use of test atoms in an EIT medium, in practice, it is likely that a fourth state of the EIT atom could replace the test atom.

This work was supported by the U.S. Air Force Office of Scientific Research, the U.S. Army Research Office, the U.S. Office of Naval Research, and the OSD Multidisciplinary University Research Initiative Program. The author thanks Phil Bucksbaum for suggesting the utility of the moving light frame and Lene Hau for many discussions on topics such as velocity synchronism and the possibility of atom surfing. Helpful discussions with Danielle Manuszak, Alexei Sokolov, Deniz Yavuz, and Guang-Yu Yin are gratefully acknowledged.

[1] L. V. Hau, S. E. Harris, Z. Dutton, and C. H. Behroozi, Nature (London) **397**, 594 (1999); M. M. Kash, V. A. Sautenkov, A. S. Zibrov, L. Hollberg, G. R. Welch, M. D. Lukin, Y. Rostovtsev, E. S. Fry, and M. O. Scully, Phys. Rev. Lett. **82**, 5229 (1999); D. Budker, D. F. Kimball, S. M. Rochester, and V. V. Yashchuk, Phys. Rev. Lett. **83**, 1767 (1999).
[2] S. E. Harris, J. E. Field, and A. Kasapi, Phys. Rev. A **46**, R29 (1992); A. Kasapi, M. Jain, G. Y. Yin, and S. E. Harris, Phys. Rev. Lett. **74**, 2447–2450 (1995).
[3] A. B. Matsko, Y. V. Rostovtsev, H. Z. Cummins, and M. O. Scully, Phys. Rev. Lett. **84**, 5752 (2000).
[4] E. Arimondo, in *Progress in Optics,* edited by E. Wolf (Elsevier Science, Amsterdam, 1996), p. 257; S. E. Harris, Phys. Today **50**, No. 7, 36 (1997); M. O. Scully and M. S. Zubairy, *Quantum Optics* (Cambridge University Press, Cambridge, England, 1997).
[5] H. Schmidt and A. Imamoglu, Opt. Lett. **21**, 1936 (1996); G. S. Agarwal and W. Harshawardhan, Phys. Rev. Lett. **77**, 1039 (1996); M. D. Lukin and A. Imamoglu, Phys. Rev. Lett. **84**, 1419 (2000); M. Fleischhauer and M. D. Lukin, Phys. Rev. Lett. **84**, 5094 (2000); B. S. Ham and P. R. Hemmer, Phys. Rev. Lett. **84**, 4080 (2000); S. E. Harris and L. V. Hau, Phys. Rev. Lett. **82**, 4611 (1999).
[6] K. Hakuta, M. Suzuki, M. Katsuragawa, and J. Z. Li, Phys. Rev. Lett. **79**, 209 (1997); S. E. Harris and A. V. Sokolov, Phys. Rev. Lett. **81**, 2894 (1998).
[7] J. P. Gordon, Phys. Rev. A **8**, 14 (1973).
[8] H. J. Metcalf and P. van der Straten, *Laser Cooling and Trapping* (Springer-Verlag, New York, 1999).
[9] D. M. Stamper-Kurn, A. P. Chikkatur, A. Gorlitz, S. Inouye, S. Gupta, D. E. Pritchard, and W. Ketterle, Phys. Rev. Lett. **83**, 2876 (1999).