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Control of Feshbach resonances by quantum interference

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We describe how a quasibound state that is in Feshbach resonance with an incoming atom wave and is coupled to a nondecaying bound state by an electromagnetic field will exhibit a quantum interference that controls the collision dynamics. By varying the frequency of a coupling field, the elastic cross section may be modulated. By varying the amplitude of a coupling field, the bound diatomic population may be increased by several orders of magnitude.

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It is known from the work of Fano that quasibound (autoionizing) states whose energies lie above the first ionization potential of an atom may exhibit a quantum interference with the continuum or with other discrete states, which will preclude absorption of radiation at certain frequencies in the vacuum ultraviolet [1]. A general technique for creating an interference where one does not exist is to apply an electromagnetic field that couples the quasibound state to a bound nondecaying state. The equivalent (dressed) system then consists of two closely spaced states that decay to the same continuum and interfere. An incident optical beam tuned to what was previously the line center will have near-zero absorption [2–6].

It is the aim of this paper to apply similar ideas to the control of the collision cross sections of cold atoms. A free atom whose asymptotic total energy is the same as the energy of a quasibound diatomic molecular state is said to be in Feshbach resonance and its collision cross section may be orders of magnitude larger than the size of the atom [7–10]. The essential idea for obtaining a quantum interference is to apply an electromagnetic field that couples the quasibound molecular state to a nondecaying molecular state. In the spirit of the preceding paragraph, but with the optical probe replaced by an incoming atom wave, it is expected that the cross section for elastic collision will have an interference profile with a zero at the (previous) Feshbach maximum. Interesting dynamic effects become possible: by varying the frequency of the coupling laser the cross section may be modulated; by varying its amplitude the bound molecular population may be increased, as compared to its equilibrium value, by several orders of magnitude.

A schematic of the system to be studied is shown in Fig. 1. A quasibound state $|1\rangle$ of a diatomic molecule is magnetically tuned to near Feshbach resonance with an incoming plane wave of particles J_0. State $|1\rangle$ dissociates at a rate Γ_1 to a continuum of unbound states $|\mu_q\rangle$. An electromagnetic field couples state $|1\rangle$ to state $|2\rangle$ of the molecule. If state $|2\rangle$ does not dissociate there will be a complete interference in the cross section for elastic scattering. If state $|2\rangle$ does dissociate (for example, if it lies above state $|1\rangle$) the incoming particle beam will have a cross section for inelastic collision where it gains kinetic energy equal to the $|1\rangle$-$|2\rangle$ transition energy. This inelastic cross section maximizes at the point of destructive interference of the elastic cross section.

There has been considerable previous work, both theoretical and experimental, which demonstrates control of collisions of both hot [11,12] and cold atoms [13,14] by quantum interference and laser coupling. There are also connections between this work and the field of photoassociation spectroscopy [15,16]. In photoassociation spectroscopy a laser is used to cause transitions of unbound atoms to bound molecular states of an upper electronic manifold [17]. These states could be used as intermediate states to Raman couple states $|1\rangle$ and $|2\rangle$ of Fig. 1. Of particular pertinence, Courteille *et al.* [9] have found a strong increase in photoassociation when tuned to Feshbach resonance, and Heinzen *et al.* describe the Raman coupling of a quasibound and bound state and its use to affect the dynamics of coupled condensates [18]. In other work, Fedichev *et al.* discuss laser manipulation of scattering lengths [19], Fatemi *et al.* have observed an optically induced Feshbach resonance in cold sodium vapor [20], and Kokkelmans *et al.* have shown how by sweeping through a Feshbach resonance, one may enhance the Raman transition probability for conversion of an atomic condensate to a molecular condensate [21]. A recently submitted paper describes the use of stimulated Raman adiabatic passage (STIRAP) to attain the same objective [22].

We analyze the problem by a method that keeps track of the molecular population, gives the *S*-matrix elements, and allows numerical simulation of the temporal dynamics. We

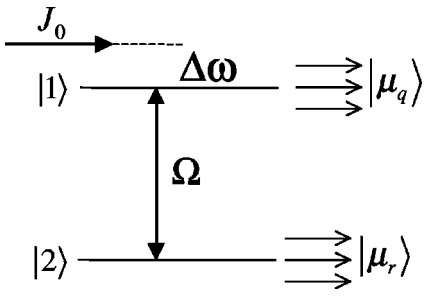

FIG. 1. (a) A quasibound state $|1\rangle$ of a diatomic molecule is tuned to near Feshbach resonance with an incoming plane wave of particles J_0. State $|1\rangle$ dissociates at a rate Γ_1 to a continuum of unbound states $|\mu_q\rangle$. An electromagnetic field with Rabi frequency Ω couples this state to state $|2\rangle$ which, in general, decays to a different continuum $|\mu_r\rangle$. In the ideal case when state $|2\rangle$ does not decay, there will be a perfect quantum interference in the elastic cross section of the incoming wave.

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assume that the energy of the atom beam is sufficiently low that only s waves are pertinent and take the colliding particles to be nonidentical. Noting Fig. 1, we begin by expanding the wave function as

$$|\psi(r,t)\rangle = c_1 \exp(-i\omega_1 t)|\mu_1\rangle + c_2 \exp(-i\omega_2 t)|\mu_2\rangle + \int c_q \exp(-i\omega_q t)|\mu_q\rangle d\omega_q + \int c_r \exp(-i\omega_r t)|\mu_r\rangle d\omega_r . \quad (1)$$

In Eq. (1) the quantities $|\mu_1\rangle$ and $|\mu_2\rangle$ are bound eigenstates of the diatomic molecule with probability amplitudes $c_1(t)$ and $c_2(t)$. For example, the number of molecules per unit volume in state $|2\rangle$ is $|c_2|^2$ multiplied by the number of atoms per unit volume. The $|\mu_q\rangle$ are dense s-wave unbound states that have energies near state $|1\rangle$ and have asymptotic form $c_q(t)n_q \sin(k_q r+\delta_1)$. The quantity δ_1 is the background phase shift and n_q is the normalization constant. The continuum eigenstates are energy normalized with energy in units of $\hbar\omega$. With the particle velocity denoted by v_q, the normalization constant is $n_q=\sqrt{2/\pi v_q}$, the density of states $\rho(\omega)=1$, and $c_q(t)$ is the probability amplitude per square root of angular frequency space. Similar definitions apply to states $|r\rangle$ that are near state $|2\rangle$ and have phase shift δ_2.

We work in the interaction picture and write the coupled equations for the probability amplitudes. The matrix element between the continuum and bound states is expressed in terms of the decay rate Γ and resonant shift $\delta\omega$ of these states. For state $|1\rangle$, $\Gamma_1=2\pi|\langle\mu_1|H'|\mu_q\rangle|^2$ and $\delta\omega_1=\mathcal{P}\int[|\langle\mu_1|H'|\mu_q\rangle|^2/(\omega_1-\omega_q)]d\omega_q$, with similar definitions for state $|2\rangle$. We eliminate the resonant shifts by setting the frequency of the coupling field equal to the difference of the shifted resonances. With the rotating wave approximation, the coupled equations are then

$$\frac{\partial c_1}{\partial t}+\frac{\Gamma_1}{2}c_1=-i\left(\frac{\Gamma_1}{2\pi}\right)^{1/2}\xi_0 \exp(-i\Delta\omega t)+i\frac{\Omega(t)}{2}c_2, \quad (2a)$$

$$\frac{\partial c_2}{\partial t}+\frac{\Gamma_2}{2}c_2=i\frac{\Omega^*(t)}{2}c_1, \quad (2b)$$

$$c_q=-i\left(\frac{\Gamma_1}{2\pi}\right)^{1/2}\int_{-\infty}^{t} c_1(\tau)\exp[i(\omega_q-\omega_1)\tau]d\tau, \quad (2c)$$

$$c_r=-i\left(\frac{\Gamma_2}{2\pi}\right)^{1/2}\int_{-\infty}^{t} c_2(\tau)\exp[i(\omega_r-\omega_2)\tau]d\tau. \quad (2d)$$

The probability amplitude of state $|1\rangle$ is driven by a monochromatic plane wave of particles J_0 with an energy $\Delta\omega$ above state $|1\rangle$. ξ_0 is the s-wave portion of this driving wave. The value of ξ_0 is obtained by matching the asymptotic form of the normalized continuum wave function with phase shift δ_1 with the asymptotic form of the incoming plane wave and is $\xi_0=\sqrt{(\pi/2k^2)|J_0|}\exp(i\delta_1)$. Because it is chosen on line center, the frequency of the coupling field does not appear in Eqs. (2). To vary this frequency by $\delta\omega$, take $\Omega(t)=\Omega_0\exp(-i\delta\omega t)$ in Eq. (2a) and $\Omega^*(t)=\Omega_0^*\exp(i\delta\omega t)$ in Eq. (2b). If state $|2\rangle$ does not decay via a Feshbach resonance, the quantity Γ_2 on the left-hand side of Eq. (2b) may be retained to account for other losses.

The time-varying populations of the molecular states $|1\rangle$ and $|2\rangle$ are determined by Eqs. (2a) and (2b). These populations, in turn, generate scattered waves, as per Eqs. (2c) and (2d). To obtain the energy spectrum of the scattered waves we extend the upper limit of the integrals in Eqs. (2c) and (2d) to infinity. For a coupling pulse of finite duration, the spectrum is the Fourier transform of the coupling field. For a monochromatic coupling field, and a monochromatic incoming plane wave, then in steady state, the probability amplitudes $c_1(t)$ and $c_2(t)$ are also monochromatic and vary as $\exp(-i\Delta\omega t)$. We change to time-independent variables b_1 and b_2 by $c_1(t)=b_1\exp(-i\Delta\omega t)$ and $c_2(t)=b_2\exp(-i\Delta\omega t)$. Solving for b_1 and b_2, the monochromatic scattered waves are $c_q'=\int c_q(\omega)d\omega=-i\sqrt{2\pi\Gamma_1}b_1$ and $c_r'=\int c_r(\omega)d\omega=-i\sqrt{2\pi\Gamma_2}b_2$. Because we do not keep track of the depletion of the incident atom beam the treatment is perturbative, and it is required that c_q' and c_r' must be small as compared to ξ_0.

We use Eqs. (2) to obtain the scattering matrix elements that in turn determine the collision cross sections and scattering length. Equating the asymptotic forms of the continuum functions and the s-wave portion of the incident and outgoing waves, we obtain the relation between the S-matrix elements and the quantities c_q', c_r', and ξ_0. With background phase shifts δ_1 and δ_2 for states $|1\rangle$ and $|2\rangle$, respectively, and Ω real, the necessary relations are $S_{11}=(1+c_q'/\xi_0)\exp(i2\delta_1)$ and $S_{21}=(c_r'/\xi_0)\exp[i(\delta_1+\delta_2)]$. S_{11} and S_{21} are then

$$S_{11}=e^{i2\delta_1}\left[1-\frac{2\Gamma_1(\Gamma_2-2i\Delta\omega)}{(\Gamma_1-2i\Delta\omega)(\Gamma_2-2i\Delta\omega)+\Omega^2}\right],$$

$$S_{21}=e^{i(\delta_1+\delta_2)}\left[\frac{-2i\sqrt{\Gamma_1\Gamma_2}\Omega}{(\Gamma_1-2i\Delta\omega)(\Gamma_2-2i\Delta\omega)+\Omega^2}\right]. \quad (3)$$

The matrix element S_{22} is obtained from S_{11} by replacing Γ_1, Γ_2, and δ_1 by Γ_2, Γ_1, and δ_2, respectively. $S_{12}=S_{21}$ and the S matrix is unitary. We note that the scattering matrix elements of Eq. (3) may alternatively be obtained by using the coupled-channel formalism of Bohn and Julienne [13]. [Noting Bohn and Julienne Eqs. (4.8) through (4.11), the necessary variable changes to obtain the scattering matrix elements of Eq. (3) are the replacement of the Feshbach decay rate of this work with the stimulated rate of their first laser, and the interchange of our c_r channel with their artificial channel.]

Cross sections for nonidentical particles are related to the S-matrix elements by $\sigma_{\text{elastic}}=(\pi/k^2)|1-S_{11}|^2$ and $\sigma_{\text{inelastic}}=(\pi/k^2)|S_{21}|^2$. Figure 2 shows the cross section for elastic collision vs detuning from resonance for the normal Breit-Wigner profile ($\Omega=0$) and for the ideal ($\Gamma_2=0$) interference profile of this work. The elastic collision cross section

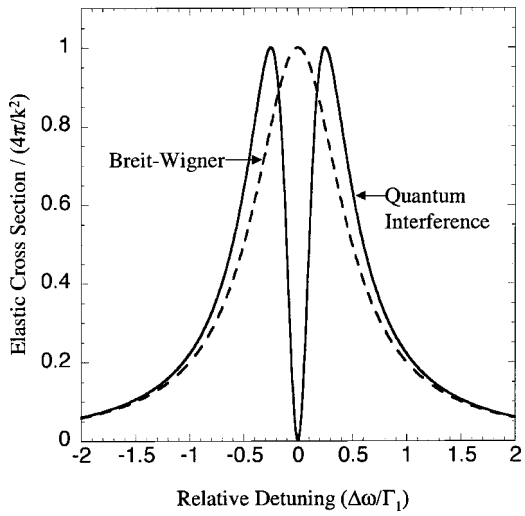

FIG. 2. Elastic collision cross section vs detuning for a Feshbach resonance; Breit-Wigner profile (dashed curve) and quantum interference profile (solid curve) caused by a coupling field with $\Omega=0.5$ and $\Gamma_2=0$. For both curves the Feshbach decay rate $\Gamma_1=1$ and the background cross sections $\delta_1=\delta_2=0$.

is double peaked and is zero at its Breit-Wigner maximum. By choosing Ω, the interference profile may be either narrower or wider than the Breit-Wigner profile. The latter case may be useful to reduce the sensitivity to variation of the magnetic field.

We use S_{11} to obtain the expression for the zero-energy scattering length. For near-zero energies we require the dependence of Γ_1 on the collision energy. Following Moerdijk *et al.* [23], we take Γ_1 to vary as the square root of energy and write $\Gamma_1(k)=(k/k_0)\Gamma_1^{(0)}$, where $\Gamma_1^{(0)}$ is the measured value of Γ_1 at wavelength k_0. With a_0 as the background scattering length and $S_{11}\equiv\exp(-2ika)$, the complex zero energy scattering length a is

$$a=a_0+\left(\frac{\Gamma_1^{(0)}}{k_0}\right)\frac{i(\Gamma_2-2i\Delta\omega)}{(\Gamma_2-2i\Delta\omega)(\Gamma_a-2i\Delta\omega)+\Omega^2}. \quad (4)$$

In Eq. (4), we have allowed for an additional decay Γ_a from state $|1\rangle$. This artificial decay is used to represent any additional loss for atoms in state $|1\rangle$ and causes the scattering matrix to be no longer unitary.

With the coupling field off, Eq. (4) reduces to the complex Breit-Wigner scattering length [23]. With the coupling field on and with $\Omega\gg\Gamma_2,\Gamma_a,\Delta\omega$, the lowest-order terms in the series expansion of the scattering length are

$$a=a_0+\left(\frac{2\Gamma_1^{(0)}}{k_0}\right)\frac{1}{\Omega^2}\left(\Delta\omega+i\frac{\Gamma_2}{2}\right)+\cdots. \quad (5)$$

In the ideal case with $\Gamma_2=0$, as in electromagnetically induced transparency with optical waves, the scattering length is real and has a steep dispersive profile. This variation of the scattering length might be used to control either the group velocity or the nonlinearity of a Bose condensate. Also, as in electromagnetically induced transparency, the additional decay from state $|1\rangle$, to this order, is absent from the scattering length and does not cause depletion of an incident *s* wave.

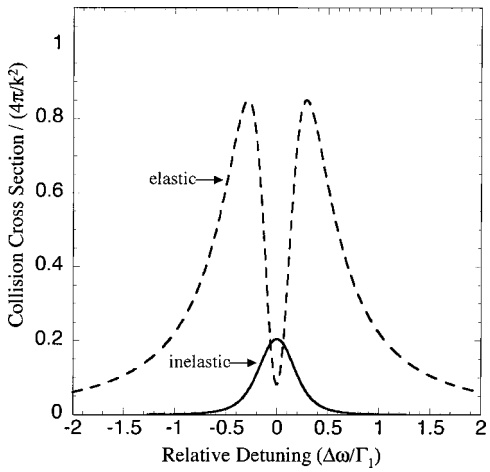

FIG. 3. Cross section for elastic and inelastic collisions vs detuning. The interference is destructive in the elastic profile and constructive in the inelastic profile. The parameters are $\Omega=0.5$, $\Gamma_1=1$, $\Gamma_2=0.1$, and $\delta_1=\delta_2=0$.

An *s* wave of atoms, which is incident on state $|1\rangle$ may either be elastically scattered or inelastically translated in energy by the frequency of the coupling field and scattered as an *s* wave from state $|2\rangle$. Neglecting the background phase shift the cross sections at the point of interference are

$$\sigma_{\text{elastic}}=\frac{\pi}{k^2}\left(\frac{2\Gamma_1\Gamma_2}{|\Omega|^2+\Gamma_1\Gamma_2}\right)^2,$$

$$\sigma_{\text{inelastic}}=\frac{\pi}{k^2}\left[\frac{\Gamma_1\Gamma_2|\Omega^2|}{(|\Omega|^2+\Gamma_1\Gamma_2)^2}\right]. \quad (6)$$

Figure 3 shows the frequency dependence of these cross sections. The inelastic collision profile peaks at the frequency where the elastic profile is zero; the interference is therefore destructive for elastic collision and constructive for inelastic collision.

We next consider the populations of the diatomic molecule. Returning to Eqs. (2), with the atom density denoted by N_0 so that the incident plane-wave particle current is v_0N_0, and again noting the relation between the driving current J_0 and its *s* wave portion ξ_0, i.e., $|\xi_0|^2=\pi/(2k)^2J_0$, the magnitudes of the molecular populations are

$$N_1=|c_1|^2N_0=\left(\frac{v_0}{2k^2}\right)\frac{\Gamma_1\Gamma_2^2}{(|\Omega|^2+\Gamma_1\Gamma_2)^2}N_0^2,$$

$$N_2=|c_2|^2N_0=\left(\frac{v_0}{2k^2}\right)\frac{\Gamma_1\Omega^2}{(|\Omega|^2+\Gamma_1\Gamma_2)^2}N_0^2. \quad (7)$$

With the coupling field off, $N_1=v_0N_0^2/(2k^2\Gamma_1)$. For example, for $N_0=10^{14}$ atoms of ^{87}Rb per cubic centimeter with a velocity of 1 cm/s and a dissociation rate of $\Gamma_1=2\pi\times10^6$, the ratio of state $|1\rangle$ molecules to atoms is 4.2

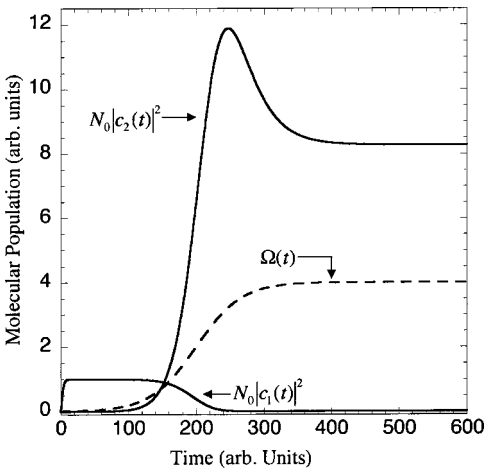

FIG. 4. Molecular population as a function of time. As the coupling field (dashed curve) is applied, the populations of states $|1\rangle$ and $|2\rangle$ move toward their new equilibrium values. The parameters are $\Gamma_1=1$, $\Gamma_2=0.01$, $\xi_0=\sqrt{\pi/2}$, and $\delta_1=\delta_2=0$. The Rabi frequency of the coupling field is $\Omega(t)=\sqrt{0.025}\{1+\tanh[0.015(t-200)]\}$.

$\times 10^{-4}$. With the coupling field on, for small Γ_2, this ratio is increased by Γ_1^2/Ω^2. If we assume that the Rabi frequency of the coupling field may be reduced to the atom beam energy of $1\,\mu$K, then an increase of almost three orders of magnitude may be possible.

Figure 4 shows the populations of states $|1\rangle$ and $|2\rangle$ of the diatomic molecule when the coupling field increases from zero to a constant value. Equations (2a) and (2b) are solved numerically with the initial condition $c_1(0)=c_2(0)=0$. With the magnitude of $\Omega(t)$ still near zero, the population of state $|1\rangle$ rises rapidly from zero to its equilibrium value of one unit. As the coupling field increases, the population of state $|1\rangle$ falls. The population of state $|2\rangle$ first rises and then approaches its new equilibrium value of 8.27 units, in agreement with Eqs. (7).

There are several possibilities for coupling states $|1\rangle$ and $|2\rangle$. At low temperatures microwave magnetic fields with power densities of about $1\,\mathrm{W/cm^2}$ might be used. First experiments are more likely to use a Raman system [18], where coupling is attained through an intermediate electronic state $|3\rangle$. We have numerically simulated such a system by adiabatically eliminating state $|3\rangle$ to obtain expressions for effective values of the prototype quantities $\Gamma 1$, $\Gamma 2$, and Ω. Taking the radiative lifetime of state $|3\rangle$ equal to the dissociative time of state $|1\rangle$, with both equal to one unit, detuning from state $|3\rangle$ by ten units, and choosing the Raman-Rabi frequencies $\Omega_{13}=\Omega_{23}$ equal to two units, we obtain near-perfect contrast in the interference profile with an increase in the molecular population of a factor of 20. Detuning further, the effective Rabi frequency and contrast are reduced, but the population increase may exceed a factor of several hundreds. Based on the power broadening measurements of Courteille *et al.* [9], near resonant Raman coupling should require laser power densities of about 1 $\mathrm{W/cm^2}$. For heteronuclear molecules direct dipole coupling of states $|1\rangle$ and $|2\rangle$ by an incident microwave field will be possible.

This paper has shown how one may produce a quantum interference in a system of coupled Feshbach resonances. In the ideal case, at resonance, both the inelastic and elastic cross sections are zero and the molecular population is nondecaying and stable. Because the interference width may be less than the natural Feshbach width, applications to subnatural enhancement of nonlinear matter wave interactions are expected.

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[1] U. Fano, Phys. Rev. **124**, 1866 (1961).
[2] K.-J. Boller, A. Imamoglu, and S.E. Harris, Phys. Rev. Lett. **66**, 2593 (1991); S.E. Harris, *ibid.* **62**, 1033 (1989); A. Imamoglu and S.E. Harris, Opt. Lett. **14**, 1344 (1989).
[3] E. Arimondo, in *Progress in Optics*, edited by E. Wolf (Elsevier Science, Amsterdam, 1996), p. 257.
[4] M.O. Scully and M.S. Zubairy, *Quantum Optics* (Cambridge University Press, Cambridge, England, 1997).
[5] O. Kocharovskaya and P. Mandel, Quantum Opt. **6**, 217 (1994).
[6] P.L. Knight *et al.*, Phys. Rep. **190**, 1 (1990).
[7] S. Inouye *et al.*, Nature (London) **392**, 151 (1998).
[8] J.L. Roberts *et al.*, Phys. Rev. Lett. **81**, 5109 (1998).
[9] Ph. Courteille *et al.*, Phys. Rev. Lett. **81**, 69 (1998).
[10] V. Vuletić *et al.*, Phys. Rev. Lett. **82**, 1406 (1999); C. Chin *et al.*, *ibid.* **85**, 2717 (2000).
[11] M. Shapiro and P. Brumer, Int. Rev. Phys. Chem. **13**, 187 (1994).
[12] A. Shnitman *et al.*, Phys. Rev. Lett. **76**, 2886 (1996).
[13] J.L. Bohn and P.S. Julienne, Phys. Rev. A **60**, 414 (1999).
[14] A.J. Moerdijk, B.J. Vehaar, and T.M. Nagtegaal, Phys. Rev. A **53**, 4343 (1996).
[15] J. Weiner *et al.*, Rev. Mod. Phys. **71**, 1 (1999).
[16] H.R. Sadeghpour *et al.*, J. Phys. B **33**, R93 (2000).
[17] R. Côté *et al.*, Phys. Rev. Lett. **74**, 3581 (1995).
[18] D.J. Heinzen *et al.*, Phys. Rev. Lett. **84**, 5029 (2000).
[19] P.O. Fedichev *et al.*, Phys. Rev. Lett. **77**, 2913 (1996).
[20] F.K. Fatemi, K.M. Jones, and P.D. Lett, Phys. Rev. Lett. **85**, 4462 (2000).
[21] S.J.J.M.F. Kokkelmans *et al.*, Phys. Rev. A **63**, 031601 (2001).
[22] P.D. Drummond, K.V. Kheruntsyan, D.J. Heinzen, and R.H. Wynar (unpublished).
[23] A.J. Moerdijk, B.J. Verhaar, and A. Axelsson, Phys. Rev. A **51**, 4852 (1995).