Investment Timing, Agency and Information

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Abstract

This paper provides a model of investment timing by managers in a decentralized firm in the presence of agency conflicts and information asymmetries. When investment decisions are delegated to managers, contracts must be designed to provide incentives for managers to both extend effort and truthfully reveal private information. Using a real options approach, we show that an underlying option to invest can be decomposed into two components: a manager’s option and an owner’s option. The implied investment behavior differs significantly from that of the first-best no-agency solution. In particular, there will be greater inertia in investment, as the model predicts that the manager will have a more valuable option to wait than the owner.

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1 Introduction

One of the most important topics in corporate finance is the formulation of the optimal investment strategies of firms. There are two components of the investment decision: how much to invest and when to invest. The first is the capital allocation decision and the second is the investment timing decision. The standard textbook prescription for the capital allocation decision is that firms should only invest in projects if their net present values (NPV) are positive. Similarly, a standard framework for the investment timing decision is the real options approach. The real options approach posits that the opportunity to invest in a project is analogous to an American call option on the investment project, and the timing of investment is economically equivalent to the optimal exercise decision for an option. The real options approach is well summarized in Dixit and Pindyck (1994) and Trigeorgis (1996).¹

However, both the simple NPV rule and the standard real options approach fail to account for the presence of agency conflicts and information asymmetries. In most modern corporations, shareholders delegate the investment decision to managers, taking advantage of managers’ special skills and expertise. In such decentralized settings, there are likely to be both information asymmetries (e.g., managers are better informed than owners about projected cash flows) and agency issues (e.g., unobserved managerial effort, perquisite consumption, empire building). A number of papers in the corporate finance literature provide models of capital budgeting under asymmetric information and agency.² The focus of this literature is on the first element of the investment decision: the amount of capital allocated to managers for investment. Thus, this literature provides predictions on whether firms over- or under-invest relative to the first-best no-agency benchmark. The focus of this paper is on the second element of the investment decision: the timing of investment. We extend the real options framework to account for the issues of information and agency in a decentralized firm. Analogous to the notions of over- or under-investment, our paper provides results on hurried or delayed investment.


²See Stein (2001) for a useful summary.
In the standard real options paradigm, there are no agency conflicts as it is assumed that the option’s owner makes the exercise decision. However, in this paper, an owner delegates the option exercise decision to a manager. Thus, the timing of investment is determined by the manager. The owner’s problem is to design an optimal contract under both hidden action and hidden information. The true quality of the underlying project can be high or low. The hidden action problem is that the manager can influence the likelihood that the quality of the project is high. An optimal contract will have the property that the manager will be induced to provide costly (but unverifiable) effort. The hidden information problem is that the underlying project’s future value contains a component that is only privately observed by the manager. Absent any mechanism that induces the manager to reveal his private information voluntarily, the manager may have an incentive to lie about the true quality of the project and divert value for his private interests. For example, the manager could divert privately observed project value by consuming excessive perquisites, building empires, or by working less hard. An optimal contract will induce the manager to deliver to the owner the true value of the privately observed component of project value, and thus no actual value diversion will take place in equilibrium.

Importantly, we show that the underlying option can be decomposed into two components: a “manager’s option” and an “owner’s option.” The manager’s option has a payout upon exercise that is a function of the contingent compensation contract. Based on this contractual payout, the manager determines the exercise time. The owner’s option has a payout, received at the manager’s chosen exercise time, equal to the payoff from the underlying option minus the manager’s compensation. The model provides the solution for the optimal contract that comes as close as possible to the first-best no-agency solution.

The model implies investment behavior that differs substantially from that of the standard real options approach with no agency problems. In general, managers will display greater inertia in their investment behavior, in that they will invest later than implied by the first-best solution. In essence, this is a result of the manager (even in an optimal contract) not having a full ownership stake in the option payoff. This less than full ownership interest implies that the manager has a more valuable “option to wait” than the owner.

While our paper focuses on the agency issues that arise from the divergence of interests between owners and shareholders, similar issues exist between stockholders and bondholders. Mello and Parsons (1992), Mauer and Triantis (1994), Leland (1998), Mauer and Ott (2000), and Morellec (2001, 2003) examine the impact of agency conflicts on firm value using the real options approach.
An important aspect of the model is the interaction of hidden action and hidden information. We find that the nature of the optimal contract depends explicitly on the relative importance of these two forces. While we focus on the economically most interesting case in which both forces play a role in the optimal contract, it is instructive to consider two extremes. If the cost benefit ratio of inducing effort (a measure of the strength of the hidden effort component) is very low, then the hidden action component disappears from the optimal contract terms. Thus, if the nature of the underlying option is such that inducing effort is sufficiently inexpensive, then we are left with a simple problem of hidden information and the contract will simply reward the manager with information rents. This is similar to the setting of Maeland (2002), which considers a real options problem with only hidden information about the exercise cost.\footnote{Bjerksund and Stensland (2000) provide a similar model to Maeland (2002), where a principal delegates an investment decision to an agent who holds private information about the investment’s cost. Brennan (1990) considers a setting in which managers attempt to signal the true quality of “latent” assets to investors through converting them into observable assets (e.g., exercising real options).} Conversely, as the cost benefit ratio of inducing effort becomes very high, then the hidden action component dominates the optimal contract. The cost of inducing effort is so high as to no longer necessitate the payment of information rents. When the cost benefit ratio of inducing effort is in the intermediate range, both forces are in effect, and the optimal contract must induce both effort and truthful revelation of private information. Interestingly, the interplay between hidden information and hidden action may actually reduce the inefficiency in investment timing, compared with the setting in which hidden information is the only friction. This is because the manager’s additional option to exert effort makes his incentives more closely aligned with those of the owner.

We further generalize the model to allow managers to display greater impatience than owners. There are several potential justifications for such an assumption. First, there are various models of managerial myopia that attempt to explain managers’ preferences for choosing projects with quicker pay-backs, even in the face of eschewing more valuable long-term opportunities.\footnote{See Narayanan (1985), Stein (1989) and Bebchuk and Stole (1993).} Such models are based on information asymmetries and agency problems. Second, in our “investment timing” setting, greater impatience can represent the manager’s preference for empire building or greater perquisite consumption and reputation that comes from running a larger company sooner rather than later. Third, managers may simply have shorter horizons (due to job loss, alternative job offers, death, etc.). Phrased in real options...
terms, managerial impatience decreases the value of the manager’s option to wait. While the base case model predicts that investment will never occur sooner than the first-best case, in this generalized setting investment can occur either earlier or later than the first-best case.

The setting of our paper is most similar to that of Bernardo et al. (2001). In a decentralized firm under asymmetric information and moral hazard, they examine the capital allocation decision, while we examine the investment timing decision. In their model, the firm’s headquarters delegates the investment decision to a manager, who possesses private information about project quality. The manager can improve project quality through the exertion of effort, which is costly to the manager but unverifiable by headquarters. These two assumptions mirror our framework. In addition, managers have preferences for “empire building” in that they derive utility from overseeing large investment projects. This assumption is addressed in the generalized version of our model that appears in Section 5. Absent any explicit incentive mechanism, managers will always claim that all projects are of high quality and worthy of funding, and then provide the minimal amount of effort. As in our paper, they use an optimal contracting approach to jointly derive the optimal investment and compensation policies. An incentive contract is derived that induces truth-telling and minimizes agency costs. In equilibrium, they find that there will be under-investment in all states of the world. Our model provides an intertemporal analogy to their equilibrium: in our base case model, we find that in equilibrium there will be delayed investment due to the information asymmetries and agency costs.\(^6\)

While our paper derives an optimal contract that best aligns the incentives of owners and managers, other papers in the corporate finance literature analyze the capital budgeting problem under information asymmetry and agency using other control mechanisms. Harris et al. (1982) consider the case of capital allocation in a decentralized firm with multiple division managers. Managers have private information about project values. In addition, managers have private interests in overstating investment requirements, and then diverting

\(^6\)In a different setting, Holmstrom and Ricart i Costa (1986) provide a model that combines an optimal wage contract with capital rationing. In their model, the manager and the market learn about managerial talent over time by observing investment outcomes. A conflict of interest arises because the manager wants to choose investment to maximize the value of his human capital while the shareholders want to maximize firm value. The optimal wage contract has the option feature that insures the manager against the possibility that an investment reveals his ability to be of low quality, but allows the manager to captures the gains if he is revealed to be of high quality. This option feature of the wage contract encourages the manager to take on excessive risks. Rationing capital mitigates the manager’s incentive to overinvest. As a result, in equilibrium both under- and over-investment are possible.
the excess cash flows in order to minimize effort or to consume greater perquisites. They focus on the role of transfer prices in allocating capital. Firms offer managers a menu of allocation/transfer price combinations. In equilibrium, truth-telling is achieved, and there can be both under- and over-investment.\footnote{Antle and Eppen (1985) provide a model that is very similar to that of Harris et al (1982).} Harris and Raviv (1996) use a very similar framework, but focus on a random auditing technology. By combining probabilistic auditing with a capital restriction, headquarters is able to learn the true project quality from the manager. In equilibrium there will be both regions of under- and over-investment. Stulz (1990) considers a decentralized investment framework in which the manager has private information about investment quality and a preference for empire building. Absent any controls, the manager would always overstate the investment opportunities and invest all available cash. The owners of the firm use debt as a mechanism to align the interests of managers and shareholders. By increasing the required debt payment, managers have less free cash flow to spend on investment projects. The optimal level of debt is chosen to trade off the benefits of preventing managers from investing in negative NPV projects when investment opportunities are poor with the costs of rationing managers away from taking positive NPV projects when investment opportunities are good. Again, in equilibrium there will be both under- and over-investment.

The remainder of the paper is organized as follows. Section 2 describes the setup of the model. Section 3 simplifies the optimization program and solves for the optimal contracts. In Section 4, we analyze the implications of the model in terms of the stock price’s reaction to investment, equilibrium investment lags, and the erosion of the option value due to the agency problem. Section 5 generalizes the model to allow for managers to display greater impatience than owners. Section 6 concludes. Appendices contain the solution details of the optimal contracts.

2 Model

In this section, we begin with a description of the model. We then, as a useful benchmark, provide the solution to the first-best no-agency investment problem. Finally, we present the full principal-agent optimization problem faced by the owner.
2.1 Setup

The principal owns an option to invest in a single project. We assume that the principal (owner) delegates the exercise decision to an agent (manager). Once investment takes place, the project generates two sources of value. One portion is observable and contractible to both the owner and the manager, while the other portion is privately observed only by the manager. Let $P(t)$ represent the observable component of the project’s value, and $\theta$ the value of the privately observed component. Thus, the total value of the project is $P(t) + \theta$.\(^8\)

In a standard call option setting, exercise yields the difference between the observable value $P(t)$ of the underlying asset and the exercise price, $K$. Thus, the payoff from exercise is typically $P(t) - K$. However, in the present model, the payoff from exercise also includes a privately observed random variable, $\theta$, whose realization directly impacts the option payoff. Thus, in this model the net payoff from exercise is $P(t) + \theta - K$. Note that the problem could be equivalently formulated as one in which the total value of the project is $P(t)$ and the effective cost of exercising the option is $K - \theta$.

Let the value $P(t)$ of the observable component of the underlying project evolve as a geometric Brownian motion:

$$dP(t) = \alpha P(t) \, dt + \sigma P(t) \, dz(t),$$  

(1)

where $\alpha$ is the instantaneous conditional expected percentage change in $P(t)$ per unit time, $\sigma$ is the instantaneous conditional standard deviation per unit time, and $dz$ is the increment of a standard Wiener process. Let $P_0$ equal the value of the project at time zero, in that $P_0 = P(0)$. Both the owner and the manager are risk neutral, with the risk-free rate of interest denoted by $r$.\(^9\) For convergence, we assume that $r > \alpha$.

The assumption that a portion of project value is only observed by the manager and not verifiable by the owner is quite common in the capital budgeting literature. This information asymmetry invites a host of agency issues. Harris et al. (1982) posit that managers have

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\(^8\)For ease of presentation, we model the process $P(t)$ for the present value of observable cash flows. We could back up a step, and begin with an underlying process for observable cash flows. However, if observable cash flows follow a geometric Brownian motion, then the present value of expected future observable cash flows will also follow a geometric Brownian motion as above. Similarly, rather than modeling $\theta$ as the present value of unobservable cash flows, we could begin with an underlying process for the unobservable cash flows themselves.

\(^9\)We rule out the time-zero selling-the-firm contract between the owner and the manager. This may be justified, for example, if the manager is liquidity constrained and cannot obtain financing.
incentives to understate project payoffs and to divert the free cash flow to themselves. In their model, such value diversion takes the form of managers reducing their level of effort. Stulz (1990), Harris and Raviv (1996) and Bernardo et al. (2001) model managers as having preferences for perquisite consumption or empire building. In these models, managers have incentives to divert free cash flows to inefficient investments or to excessive perquisites. In all of these models, mechanisms are used by firms (i.e., incentive contracts, auditing, required debt payments) to mitigate such value diversion.

The private component of value, $\theta$, may take on two possible values: $\theta_1$ or $\theta_2$, with $\theta_1 > \theta_2$. We denote $\Delta \theta = \theta_1 - \theta_2 > 0$. One may interpret a draw of $\theta_1$ as a “higher quality” project and a draw of $\theta_2$ as a “lower quality” project. Although the owner cannot observe the true value of $\theta$, the owner does observe the amount handed over by the manager upon exercise. While in theory the manager could attempt to hand over $\theta_2$ when the true value is $\theta_1$, it will be seen in equilibrium that the amount transferred to the owner at exercise will always be the true value.$^{11}$

The effort of the manager plays an important role in determining the likelihood of obtaining a higher quality project. The manager may affect the likelihood of drawing $\theta_1$ by exerting a one-time effort, at time zero. If the manager exerts no effort,$^{12}$ the probability of drawing a higher quality project $\theta_1$ equals $q_L$. However, if the manager exerts effort, he incurs a cost $\xi > 0$ at time zero, but increases the likelihood of drawing a higher quality project $\theta_1$ from $q_L$ to $q_H$. Immediately after his exerting effort at time zero, the manager observes the private component of project quality. In order to ensure a positive “net” exercise price, we restrict $\theta_1 < K$.

Although the owner cannot contract on the private component of value, $\theta$, he can contract on the observable component of value, $P(t)$. Contingent on the level of $P(t)$ at exercise, the manager is paid a wage.$^{13}$ The manager has limited liability and is always free to walk

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$^{10}$In Section 3.3 we generalize the model to allow $\theta$ to have continuous distributions.

$^{11}$Off the equilibrium path, the manager could attempt to hand over $\theta_2$ when the true value is $\theta_1$. As will be discussed below, if the transferred value is less than $\theta_1$ at the trigger $P_1$, a non-pecuniary penalty is imposed on the manager. This penalty will ensure that it will never be in the manager’s interest not to hand over the true value of the project upon exercise.

$^{12}$Without loss of generality, we may normalize the manager’s lower effort level to zero.

$^{13}$Note that what we refer to as “wages” are payments contingent on the project’s quality. They are analogous to a payment scheme in which a fixed wage is paid to the manager for exercising, plus a “bonus” for delivering a higher quality project.
In summary, the owner faces a problem with both hidden information (the owner does not observe the true realization of \( \theta \)) and hidden action (the owner cannot verify the manager’s effort level). The owner needs to provide compensation incentive both (i) to induce the manager exert effort at time zero and (ii) to have the manager reveal his type voluntarily and truthfully, by choosing the equilibrium exercise strategy and supplying the corresponding unobservable component of firm value. Before analyzing the optimal contract, we first briefly review the first-best no-agency solution that is used as the benchmark.

### 2.2 First-Best Benchmark (The Standard Real Options Case)

As a benchmark, we consider the case in which there is no delegation of the exercise decision and the owner observes the true value of \( \theta \). Equivalently, this first-best solution can be achieved in a principal-agent setting, provided that \( \theta \) is both publicly observable and contractible. Let \( W(P; \theta) \) denote the value of the owner’s option, in a world where \( \theta \) is a known parameter and \( P \) is the current level of \( P(t) \). Using standard arguments [i.e., Dixit and Pindyck (1994)], \( W(P; \theta) \) must solve the following differential equation:

\[
0 = \frac{1}{2} \sigma^2 P^2 W_{PP} + \alpha PW_P - rW. \tag{2}
\]

Differential equation (2) must be solved subject to appropriate boundary conditions. These boundary conditions serve to ensure that an optimal exercise strategy is chosen:

\[
W(P^*(\theta), \theta) = P^*(\theta) + \theta - K, \tag{3}
\]

\[
W_P(P^*(\theta), \theta) = 1, \tag{4}
\]

\[
W(0, \theta) = 0. \tag{5}
\]

Here, \( P^*(\theta) \) is the value of \( P(t) \) that triggers entry. The first boundary condition is the value-matching condition. It simply states that at the moment the option is exercised, the payoff is \( P^*(\theta) + \theta - K \). The second boundary condition is the smooth-pasting or high-contact condition.\(^{15}\) This condition ensures that the exercise trigger is chosen so as to maximize the

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\(^{14}\) The limited-liability condition is essential in delivering the investment inefficiency result in this context. Otherwise, with risk-neutrality assumptions for both the owner and the manager, and no limited liability, the first-best optimal investment timing may be achieved even in the presence of hidden information and hidden action. For a related discussion of limited liability, see Innes (1990). An alternative mechanism of generating investment inefficiency in an agency context is to assume managerial risk aversion.

\(^{15}\) See Merton (1973) for a discussion of the high-contact condition.
value of the option. The third boundary condition reflects the fact that zero is an absorbing barrier for \( P(t) \).

The owner’s option value at time zero, \( W(P_0; \theta) \), and the exercise trigger \( P^*(\theta) \) are:

\[
W(P_0; \theta) = \begin{cases} 
\left( \frac{P_0}{P^*(\theta)} \right)^\beta (P^*(\theta) + \theta - K), & \text{for } P_0 < P^*(\theta), \\
                 P_0 + \theta - K,        & \text{for } P_0 \geq P^*(\theta), 
\end{cases}
\]

(6)

and

\[
P^*(\theta) = \frac{\beta}{\beta - 1} (K - \theta),
\]

(7)

where

\[
\beta = \frac{1}{\sigma^2} \left[ -\left( \alpha - \frac{\sigma^2}{2} \right) + \sqrt{\left( \alpha - \frac{\sigma^2}{2} \right)^2 + 2r\sigma^2} \right] > 1.
\]

(8)

Since the realized value of \( \theta \) can be either \( \theta_1 \) or \( \theta_2 \), we denote \( P^*(\theta_1) = P_1^* \) and \( P^*(\theta_2) = P_2^* \). We shall always assume that the initial value of the project is less than the lower trigger, \( P_0 < P_1^* \), to ensure that there is always some positive option value inherent in the project.

The ex-ante value of the owner’s option in the first-best no agency setting is \( q_H W(P_0; \theta_1) + (1 - q_H) W(P_0; \theta_2) \). We can therefore write this first-best option value, \( V^*(P_0) \), as:

\[
V^*(P_0) = q_H \left( \frac{P_0}{P_1^*} \right)^\beta (P_1^* + \theta_1 - K) + (1 - q_H) \left( \frac{P_0}{P_2^*} \right)^\beta (P_2^* + \theta_2 - K).
\]

(9)

It will prove useful in future calculations to define the present value of one dollar received at the first moment that a specified trigger \( \hat{P} \) is reached. Denote this present value operator by the discount function \( D(P_0; \hat{P}) \). This is simply the solution to differential equation (2) subject to the boundary conditions that \( D(\hat{P}; \hat{P}) = 1 \), and \( D(0; \hat{P}) = 0 \). The solution can be written as:

\[
D(P_0; \hat{P}) = \left( \frac{P_0}{\hat{P}} \right)^\beta, \quad P_0 \leq \hat{P}.
\]

(10)

2.3 A Principal-Agent Setting

The owner offers the manager a contract at time zero that commits the owner to pay the manager at the time of exercise.\(^{16}\) The payment can be made contingent on the observable component of the project’s value at the time of exercise. Thus, in principle, for any realized

\(^{16}\)Renegotiation is not allowed. While commitment leads to inefficiency in investment timing ex-post, it increases the value of the project ex-ante.
value of $P(t)$ obtained at the time of exercise, $\hat{P}$; a contracted wage $w(\hat{P})$ can be specified, provided that $w(\hat{P}) > 0$. The contract will endogenously provide incentives to ensure that the manager exercises the option in accordance with the owner’s rational expectations and delivers the true value of the project to the owner.

The principal-agent setting leads to a decomposition of the underlying option into two options: an owner’s option and a manager’s option. The owner’s option has a payoff function of $\hat{P} + \theta - K - w(\hat{P})$, and the manager’s option has a payoff function of $w(\hat{P})$. Upon exercise, the owner receives the value of the underlying project ($\hat{P} + \theta$), after paying the exercise price ($K$) and the manager’s wage ($w(\hat{P})$). The manager’s payoff is the value of the contingent wage, $w(\hat{P})$. Obviously, the sum of these payoff functions equals the payoff of the underlying option. The manager’s option is a traditional American call option, since the manager chooses the exercise time to maximize the value of his option. However, in this optimal contracting setting, it is the owner who sets the contract parameters that induce the manager to follow an exercise policy that maximizes the value of the owner’s option. In addition, the manager also possesses a compound option, since the manager has the option to exert effort at time zero to increase the total expected surplus. The properties of the manager’s option thus are contingent upon this initial effort choice.

Since there are only two possible values of $\theta$, for any $w(\hat{P})$ schedule, there can be at most two wage/exercise trigger pairs that will be chosen by the manager.\textsuperscript{17} Thus, the contract need only include two wage/exercise trigger pairs from which the manager can choose: one that will be chosen by a manager when he observes $\theta_1$, and one chosen by a manager when he observes $\theta_2$. Therefore, the owner will offer a contract that promises a wage of $w_1$ if the option is exercised at $P_1$ and a wage of $w_2$ if the option is exercised at $P_2$. The revelation principle will ensure that a manager who privately observes $\theta_1$ will exercise at the $P_1$ trigger, and a manager who privately observes $\theta_2$ will exercise at the $P_2$ trigger.

The owner’s option has a payout function of $P_1 + \theta_1 - K - w_1$, if $\theta = \theta_1$, and $P_2 + \theta_2 - K - w_2$, if $\theta = \theta_2$. Thus, using the discounting function $D(\cdot; \cdot)$ derived in (10), conditional on the manager exerting effort, the value of the owner’s option, $\pi^o(P_0; w_1, w_2, P_1, P_2)$, can be written

\textsuperscript{17}We allow for the possibility of a pooling equilibrium in which only one wage/exercise trigger pair is offered. However, this pooling equilibrium always will be dominated by a separating equilibrium with two wage/exercise trigger pairs.
as:

\[ \pi^o(P_0; w_1, w_2, P_1, P_2) = q_H D(P_0; P_1)(P_1 + \theta_1 - K - w_1) + (1 - q_H) D(P_0; P_2)(P_2 + \theta_2 - K - w_2). \]  

(11)

The manager’s option has a payout function of \( w_1 \) if \( \theta = \theta_1 \) and \( w_2 \) if \( \theta = \theta_2 \). Conditional on the manager exerting effort, the value of the manager’s option, \( \pi^m(P_0; w_1, w_2, P_1, P_2) \), can be written as:

\[ \pi^m(P_0; w_1, w_2, P_1, P_2) = q_H D(P_0; P_1)w_1 + (1 - q_H) D(P_0; P_2)w_2. \]  

(12)

For notational simplicity, we sometimes will drop the parameter arguments and simply write the owner’s and manager’s option values as \( \pi^o(P_0) \), and \( \pi^m(P_0) \), respectively.

The owner’s objective is to maximize its option value through its choice of the contract terms \( w_1, w_2, P_1, \) and \( P_2 \). Thus, the owner solves the following optimization problem:

\[
\max_{w_1, w_2, P_1, P_2} \quad \begin{align*}
q_H \left( \frac{P_0}{P_1} \right)^\beta (P_1 + \theta_1 - K - w_1) + (1 - q_H) \left( \frac{P_0}{P_2} \right)^\beta (P_2 + \theta_2 - K - w_2) \quad &- \xi \geq q_L \left( \frac{P_0}{P_1} \right)^\beta (P_1 + \theta_1 - K - w_1) + (1 - q_L) \left( \frac{P_0}{P_2} \right)^\beta (P_2 + \theta_2 - K - w_2).
\end{align*}
\]

(13)

This optimization is subject to a variety of constraints induced by the hidden information and hidden action of the manager. The contract must induce the manager to exert effort, exercise at the trigger \( P_1 \) and provide the owner with a project value of \( P_1 + \theta_1 \) if \( \theta = \theta_1 \), and exercise at the trigger \( P_2 \) and provide the owner with a project value of \( P_2 + \theta_2 \) if \( \theta = \theta_2 \). It is to the specification of these constraints that we now turn.

There are both ex-ante and ex-post constraints. The ex-ante constraints ensure that the manager exerts effort and that the contract is accepted. These are the standard constraints as in a static moral hazard/asymmetric information setting.

- **ex-ante** incentive constraint:

\[
q_H \left( \frac{P_0}{P_1} \right)^\beta w_1 + (1 - q_H) \left( \frac{P_0}{P_2} \right)^\beta w_2 - \xi \geq q_L \left( \frac{P_0}{P_1} \right)^\beta w_1 + (1 - q_L) \left( \frac{P_0}{P_2} \right)^\beta w_2.
\]

(14)

The left side of this inequality is the value of the manager’s option if effort is exerted minus the cost of effort. The right side is the value of the manager’s option if no effort is exerted. This constraint ensures that the manager will exert effort. Re-arranging the ex-ante incentive constraint (14) gives

\[
\left( \frac{P_0}{P_1} \right)^\beta w_1 - \left( \frac{P_0}{P_2} \right)^\beta w_2 \geq \frac{\xi}{\Delta q},
\]

(15)

where \( \Delta q = q_H - q_L > 0 \).
• *ex-ante* participation constraint:

\[ q_H \left( \frac{P_0}{P_1} \right)^\beta w_1 + (1 - q_H) \left( \frac{P_0}{P_2} \right)^\beta w_2 - \xi \geq 0. \]  

(16)

This constraint ensures that the total value to the manager of accepting the contract is non-negative.

The *ex-post* incentive constraints ensure that managers will exercise in accordance with the owner’s expectations. Specifically, managers will exercise \( \theta_1 \)-type projects at the \( P_1 \) trigger and will exercise \( \theta_2 \)-type projects at the \( P_2 \) trigger. To provide such a timing incentive, managers must not have any incentive to divert value. As discussed at the beginning of Section 2.1, managers with private information have an incentive to misrepresent cash flows and divert free cash flows to themselves. For example, the manager may have an incentive to lie and claim that a higher quality project is a lower quality project, and then divert the difference in values. This could be done by diverting cash for private benefits such as perquisites and empire building [as in Stulz (1990), Harris and Raviv (1996) and Bernardo et al. (2001)]. Importantly, these incentive compatibility conditions ensure that this value diversion does not occur; such deception only occurs off the equilibrium path.

• *ex-post* incentive constraints:

\[
\left( \frac{P_0}{P_1} \right)^\beta (w_1 - \Delta \theta) \leq \left( \frac{P_0}{P_2} \right)^\beta w_2. 
\]  

(18)

The second constraint will be shown not to bind, so only constraint (17) is relevant to our discussion. The first inequality ensures that a manager of a higher quality project will choose to exercise at \( P_1 \). By truthfully revealing the private quality \( \theta_1 \) through exercising at \( P_1 \), the manager receives the wage \( w_1 \). This inequality requires the payoff from truthful revelation to be greater than or equal to the present value of the payoff from misrepresenting the private quality by waiting until the trigger \( P_2 \). The payoff from misrepresenting \( \theta_1 \) as \( \theta_2 \) is equal to the wage \( w_2 \), plus the value of diverting the private component of value \( \Delta \theta \). These constraints are common in the literature on moral hazard and asymmetric information. For example, entirely analogous conditions appear in Bolton and Scharfstein (1990) and Harris et al. (1982).
While the two \textit{ex-post} incentive constraints ensure that the manager exercises in accordance with the owner’s rational expectations, we also need to ensure that a manager of a $\theta_1$-type project will indeed hand over $P_1 + \theta_1$ in value and not divert the unobservable amount $\Delta \theta$ of the project’s value.\footnote{Note that there is no need to worry about the opposite problem of a manager of a $\theta_2$-type project exercising at $P_2$ and handing over $P_2 + \theta_1$, since that would never be in the manager’s interest.} We assume that a non-pecuniary penalty of $\kappa$ can be imposed on a manager who fails to deliver $P_1 + \theta_1$ at the trigger $P_1$.\footnote{For non-pecuniary penalties and optimal contracting, see the seminal contribution of Diamond (1984).} Specifically, we assume that the penalty, $\kappa$, is large enough to satisfy the condition $\kappa \geq \Delta \theta - w_1$. Thus, when the manager with a high quality project exercises at $P_1$, it is in their interest to deliver a value of $P_1 + \theta_1$ and receive $w_1$ rather than deliver only $P_1 + \theta_2$ and receive the penalty $\kappa$.\footnote{A manager could never transfer a value of $\theta < \theta_2$, since it is common knowledge that $\theta_2$ is the lower bound of the distribution of $\theta$. See Bolton and Scharfstein (1990) for similar assumptions and justifications.} Such a penalty could be envisioned as a reputational penalty (i.e., managers who fail to deliver what they promise are given poor recommendations) or a job search cost (i.e., such managers are terminated and forced to seek new employment).\footnote{An alternative mechanism for ensuring \textit{ex-post} enforceability of the manager’s claim is through a costly state verification mechanism as in Townsend (1979) and Gale and Hellwig (1985). Specifically, the owner may possess a monitoring technology that permits, at a cost, the determination of the true value of $\theta$ after investment is undertaken. Provided that the cost is not too high, it can be easily shown that the owner would always choose to pay the monitoring cost for managers who signal high quality projects and only hand over $\theta_2$ in value.} Importantly, without such a penalty, any kind of contracting solution would likely break down since the manager would not have to live up to his claims.

- \textit{ex-post} limited-liability constraints:

\begin{equation}
    w_i \geq 0, \quad i = 1, 2.
\end{equation}

Note that non-negative $w_1$ and $w_2$ are necessary to provide an incentive for the manager to implement the exercise of the project. For example, if $w_2 < 0$, then upon learning that $\theta = \theta_2$, the manager would rather walk away from the contract then by sticking around and receiving a negative wage at $P_2$.\footnote{Even if the manager decided to try to “fool” the owner by exercising at $P_1$, the net payout to the manager would be $w_1 - \Delta \theta < 0$, where this inequality is displayed in Proposition 4.} It is assumed that if the manager walks away, the investment opportunity is lost, and thus the owner will ensure that the manager has an incentive to invest \textit{ex-post}.

Therefore, the owner’s problem can be summarized as the solution to the objective function in (13), subject to a total of six inequality constraints: the \textit{ex-ante} incentive and par-
ticipation constraints, and each of the two ex-post incentive and limited-liability constraints. Fortunately, we will see in the next section that the problem can be substantially simplified in that we can reduce the number of constraints to two.

3 Model Solution: Optimal Contracts

In this section, we provide the solution to the optimal contracting problem described in the previous section: maximizing (13) subject to the six inequality constraints (15) - (19). We find that the nature of the solution depends on the parameter values. In particular, the solution depends explicitly on the magnitude of the cost benefit ratio of inducing the manager’s effort. Depending on this magnitude, the optimal contract can take on three possible types: a “pure hidden information” type, a “joint hidden information/hidden action” type, and a “pure hidden action” type.

3.1 A Simplified Statement of the Principal-Agent Problem

Although the owner’s optimization problem is subject to six inequality constraints, the solution can be found through only considering two of the constraints. Appendix A proves four propositions, Proposition 1 through Proposition 4, that provide the underpinnings for this simplification.

Proposition 1 shows that the limited liability for the manager of a $\theta_1$-type project in constraint (19) does not bind, while Proposition 2 shows that the ex-ante participation constraint (16) does not bind. Proposition 3 demonstrates that the limited liability for the manager of a $\theta_2$-type project binds, and thus we can simply substitute $w_2 = 0$ into the problem. Proposition 4 implies that the ex-post incentive constraint for the manager of a $\theta_2$-type project does not bind.

These four propositions jointly simplify the owner’s optimization problem as follows:

$$\max_{w_1, P_1, P_2} \quad q_H \left( \frac{P_0}{P_1} \right)^\beta (P_1 + \theta_1 - K) - q_H \left( \frac{P_0}{P_1} \right)^\beta w_1 + (1 - q_H) \left( \frac{P_0}{P_2} \right)^\beta (P_2 + \theta_2 - K),$$

subject to

$$\left( \frac{P_0}{P_1} \right)^\beta w_1 \geq \left( \frac{P_0}{P_2} \right)^\beta \Delta \theta,$$

$$\left( \frac{P_0}{P_1} \right)^\beta w_1 \geq \frac{\xi}{\Delta q}.$$
In summary, we now have a simplified optimization problem for the owner. Equation (20) is the owner’s option value. Constraint (21) is the simplified ex-post incentive constraint for the manager of the $\theta_1$-type project. Constraint (22) ensures that it is in the manager’s interest to extend his effort at time zero.

Proposition 5, proved in Appendix A, demonstrates that at least one of the two constraints must bind. Note that the two constraints can be written more succinctly as

$$\left(\frac{P_0}{P_1}\right)^\beta w_1 \geq \max \left[ \left(\frac{P_0}{P_2}\right)^\beta \Delta \theta, \frac{\xi}{\Delta q}\right].$$

(23)

### 3.2 General Properties of the Solution

Before we provide the explicit solutions for the three contract regions, we discuss some general properties of contracts that hold for all regions.

The first property of the solution is that the manager of the higher quality project will exercise at the first-best level. Intuitively, for any manager’s option value that satisfies constraint (23), the owner will always prefer to choose the first-best timing trigger $P_1^*$, and vary wage $w_1$ to achieve the same level of compensation. On the margin, it is cheaper for the owner to increase the wage for the manager of a higher quality project than to have that manager deviate away from the first-best optimal timing strategy.

**Property 1.** The optimal contracts have $P_1 = P_1^*$, for all admissible parameter regions.

*Proof.* Consider any candidate optimal contract $(\tilde{w}_1, \tilde{P}_1, \tilde{P}_2)$ with $\tilde{P}_1 \neq P_1^*$. The owner may improve his surplus by proposing an alternative contract $(\hat{w}_1, P_1^*, \tilde{P}_2)$, in which $\hat{w}_1$ is chosen such that the manager’s option has the same value as the first contract, in that $(P_0/P_1^*)^\beta \hat{w}_1 = (P_0/\tilde{P}_1)^\beta \tilde{w}_1$. The newly proposed contract is clearly feasible, as it will also satisfy constraints (21) and (22). For all such constant levels of the manager’s option value, the owner’s objective function (20) is maximized by choosing $P_1 = P_1^* = \arg \max_x (P_0/x)^\beta (x + \theta_1 - K)$.

As we shall now see, it is less costly for the owner to distort $P_2$ away from the first-best level than to distort $P_1$ away from the first-best level in order to provide the appropriate incentives to the manager. The next property of the solutions is that delay (beyond first-best) for the lower quality project is needed in order to create enough incentives for the manager of a higher-quality project not to imitate the one of a lower-quality project.
Property 2. For all admissible parameter regions, the investment trigger for a manager of a \(\theta_2\)-type project is (weakly) later than the first-best, in that \(P_2 \geq P^*_2\).

Proof. Suppose \(P_2 < P^*_2\). It is simple to show that this contract is dominated by the contract with \(P_2 = P^*_2\). We can always increase \(P_2\) without violating constraint (21). Moreover, the objective function (20) is increasing in \(P_2\), for \(P_2 < P^*_2\), irrespective of which constraint binds. Thus, any contract with \(P_2 < P^*_2\) is dominated by one with \(P_2 = P^*_2\).

Intuitively, the necessity of ensuring that the manager of a higher-quality project not imitate one of a lower-quality project leads the manager of a lower-quality project to display a greater “option to wait” than the first-best solution. In order to dissuade the manager of a higher-quality project from exercising at the trigger \(P_2\), the contract must sufficiently increase \(P_2\) above \(P^*_2\) to make such “lying” unprofitable.

We shall see that the extent to which \(P_2\) exceeds \(P^*_2\) depends explicitly on the relative strengths of the forces of hidden information and hidden action. The amount of suboptimal delay will vary across the three regions, and will be discussed in greater detail below.

3.3 Optimal Contracts

We first define the three regions that serve to determine the nature of the optimal contract. As a result of Proposition 5, the solution will depend on which of the two constraints (21) and (22) bind. The key to the contract is the cost benefit ratio of inducing the manager’s effort, defined by \(\xi/\Delta q\). The numerator is the direct cost of extending effort, and the denominator is the change in the likelihood of drawing a higher quality project \(\theta_1\) due to effort. The regions are then defined by where this cost benefit ratio falls relative to the present value of receiving a payment of \(\Delta \theta\) at three particular trigger values: \(P^*_1 = P^* (\theta_1)\), \(P^*_2 = P^* (\theta_2)\), and \(P^*_3 = P^* (\theta_3)\), where

\[
\theta_3 = \theta_2 - \frac{q_H}{1 - q_H} \Delta \theta < \theta_2.
\]

These present values are ordered by \((P_0 / P^*_3)^\beta \Delta \theta < (P_0 / P^*_2)^\beta \Delta \theta < (P_0 / P^*_1)^\beta \Delta \theta\). Note that another potential region in which \(\xi/\Delta q > (P_0/P^*_1)^\beta \Delta \theta\) exists, however in this range the costs of effort are so high as to no longer justify the exertion of effort in equilibrium. Thus, we do not consider this region.\(^{23}\)

\(^{23}\)A proof of this result is available from the authors by request.
Because optimal contracts specify \( P_1 = P_1^* \) and \( w_2 = 0 \) across all three regions, we may focus on \( P_2 \) and \( w_1 \) when we describe the optimal contracts in each of the three regions. The proofs detailing the solution are provided in Appendix A.

- **Hidden Information Only Region:** \( \xi / \Delta q < (P_0 / P_3^*)^\beta \Delta \theta \)

In this region, we have

\[
P_2 = P_3^* = P^*(\theta_3) > P_2^*, \quad (25)
\]

\[
w_1 = \left( \frac{P_1^*}{P_3^*} \right)^\beta \Delta \theta, \quad (26)
\]

where \( \theta_3 \) is given in (24).

The net costs of inducing effort are low enough so that there is no need for the firm to compensate the manager for extending effort. In this range, the *ex-ante* incentive constraint does not bind, and therefore the cost of effort does not find its way into the optimal contract.\textsuperscript{24}

The payments that the manager of the \( \theta_1 \)-type project receives are purely information rents that induce the manager to exercise at the first-best trigger \( P_1^* \), in accordance with the revelation principle. Since \( w_1 \) is relatively low in this region, the \( P_2 \) trigger needs to be high (relative to the first-best trigger \( P_2^* \)) in order to dissuade the manager of the \( \theta_1 \)-type project from deviating from the equilibrium first-best trigger \( P_1^* \).

We can use these contract terms to place a value on the owner’s and manager’s option values. The owner’s and manager’s option values, \( \pi^o(P_0) \) and \( \pi^m(P_0) \), respectively, can be written as:

\[
\pi^o(P_0) = q_H \left( \frac{P_0}{P_1^*} \right)^\beta (P_1^* + \theta_1 - K) + (1 - q_H) \left( \frac{P_0}{P_3^*} \right)^\beta (P_3^* + \theta_3 - K), \quad (27)
\]

\[
\pi^m(P_0) = q_H \left( \frac{P_0}{P_3^*} \right)^\beta \Delta \theta. \quad (28)
\]

It is interesting to note that the solution for the owner’s option value is observationally equivalent to the first-best solution in which one substitutes \( \theta_3 \) for the lower project quality \( \theta_2 \). In such a setting, the owner will choose to exercise at \( P_1^* \) if \( \theta = \theta_1 \) and at \( P_3^* \) if \( \theta = \theta_3 \). Thus, the impact of the costs of hidden information is fully embodied by a reduction of project quality in the low state.

\textsuperscript{24}In a different setting where the hidden information is the cost of exercising, Maeland (2002) shows a similar result.
In this region, we have

\[ P_2 = P_J = P_0 \left( \frac{\Delta q \Delta \theta}{\xi} \right)^{1/\beta} > P_2^*, \]  

(29)

\[ w_1 = \left( \frac{P_1^*}{P_J} \right)^\beta \Delta \theta = \frac{\xi}{\Delta q} \left( \frac{P_1^*}{P_0} \right)^\beta. \]  

(30)

Here, both the \textit{ex-ante} and \textit{ex-post} constraints bind. Since now the manager must be induced into providing effort, \( w_1 \) must be high enough to provide enough compensation for the \textit{ex-ante} incentive constraint \( \text{(22)} \) to bind. This reflects the hidden action component of the contract. In addition, the exercise trigger \( P_2 \) must be high enough to dissuade the manager of the \( \theta_1 \)-type project from deviating from the equilibrium first-best trigger \( P_1^* \). Thus, in this region, \( P_2 \) is set so that the \textit{ex-post} incentive constraint \( \text{(21)} \) binds. This requires that \( P_2 \) be above the full-information trigger \( P_2^* \). This deviation from the full-information trigger reflects the hidden information component of the contract.

Importantly, \( P_2 \) is lower in this region than in the hidden information only region. This is due to the fact that in this joint region \( w_1 \) is now higher in order to induce effort. This higher wage makes it easier to satisfy the \textit{ex-post} incentive constraint, and the deviation from \( P_2^* \) required to prevent managers of the \( \theta_1 \)-type project from pretending to have a \( \theta_2 \)-type project becomes smaller. Surprisingly, moral hazard serves to increase investment timing efficiency since the increased share of the firm that must go to compensate the manager leads the manager to more fully internalize the benefits of relatively more efficient investment timing.

The owner’s and manager’s option values, \( \pi^o(P_0) \) and \( \pi^m(P_0) \), respectively, can be written as:

\[ \pi^o(P_0) = q_H \left( \frac{P_0}{P_1^*} \right)^\beta (P_1^* + \theta_1 - K) + \left(1 - q_H\right) \left( \frac{P_0}{P_J} \right)^\beta (P_J + \theta_3 - K), \]  

(31)

\[ \pi^m(P_0) = q_H \frac{\xi}{\Delta q}. \]  

(32)

The owner’s option value deviates from the first-best value, \( V^*(P_0) \) in \( \text{(9)} \) in two ways. First, the hidden information rents effectively make the manager mark down his privately observed component of project value from \( \theta_2 \) to \( \theta_3 \), similar to that in the pure hidden information region. Second, the exercise trigger for a manager of a \( \theta_2 \)-type project is equal to \( P_J \), which is larger than \( P_2^* \). Note that the only difference between \( \pi^o(P_0) \) in this region and in the
pure hidden-information region is the different terms for the exercise trigger: $P_J$ versus $P_3^*$. Here, the trigger is lower due to the hidden action component.

- **Hidden Action Only Region:** $(P_0/P_2^*)^\beta \Delta \theta < \xi/\Delta q < (P_0/P_1^*)^\beta \Delta \theta$

In this parameter range, we have

\[ P_2 = P_2^* , \quad w_1 = \frac{\xi}{\Delta q} \left( \frac{P_1^*}{P_0} \right)^\beta . \]

The equilibrium triggers equal those of the first-best outcomes. The moral hazard costs are so high that rents needed for motivating effort (via the *ex-ante* incentive constraint) is sufficiently large so that the *ex-post* incentive constraints do not demand additional rents. That is, the wage needed to motivate the manager to extend effort ends up being high enough so that the manager of the $\theta_1$-type project no longer needs $P_2$ to exceed $P_2^*$ in order to dissuade him from deviating from the equilibrium trigger $P_1^*$. Thus, the contract is entirely driven by the need to motivate *ex-ante* effort, as the *ex-post* incentive constraint that reflects hidden information does not bind.

The owner’s and manager’s option values, $\pi^o(P_0)$ and $\pi^m(P_0)$, respectively, can be written as:

\[ \pi^o(P_0) = V^*(P_0) - q_H \frac{\xi}{\Delta q} , \quad \pi^m(P_0) = q_H \frac{\xi}{\Delta q} . \]

The owner’s option value is equal to the first-best solution $V^*(P_0)$ characterized in (9), minus the present value of the rent paid to the manager in order to induce effort.

Figure 1 summarizes the details of the optimal contracts through the three regions. The upper and lower graphs plot the equilibrium trigger strategy $P_2$ and wage payment $w_1$ in terms of effort cost $\xi$, respectively. The upper graph shows that the trigger strategy for the manager of the $\theta_2$-type project is flat and equal to $P_3^*$ for $\xi$ in the pure hidden information region; is decreasing and convex in $\xi$ for the joint hidden action/hidden information region; and is flat and equal to the first-best trigger level $P_2^*$ for $\xi$ in the pure hidden action. The equilibrium trigger $P_2$ is closer to the first-best level, for higher level of $\xi$, *ceteris paribus*. The lower graph plots corresponding wage contracts for a manager of the $\theta_1$-type project. For
low levels of $\xi$ (pure hidden information region), he only needs to be compensated with pure information rents. As a result, wage is insensitive to effort cost $\xi$ and is flat in this region. In both the joint hidden information/hidden action region and the pure hidden action region, $w_1$ increases linearly in $\xi$.

### 3.4 An Extension to Cases with Continuous Distributions of $\theta$

For ease of presentation, our basic model uses a simple two-point distribution for $\theta$. In order to check the robustness of our results, we generalize our model to allow for admissible continuous distributions of $\theta$ on $[\underline{\theta}, \bar{\theta}]$ in Appendix B. In this setting, the principal designs the contract such that the manager will find it optimal to exert effort at time zero and then reveal his $\theta$ truthfully by choosing the recommended equilibrium strategy $P(\theta)$ and $w(\theta)$. As in the basic setting, we also suppose that the owner may impose a non-pecuniary penalty $\kappa$ on the manager if the manager fails to live up to his signaled (and true) value of the unobservable component $\theta$.\(^{25}\) The manager is protected by ex-post limited liability in that $w(\theta) \geq 0$ for all $\theta$. Also, the manager’s participation is voluntary at time zero. We show that the following key results remain valid:

1. Agency problems (hidden information and hidden action) lead to a delayed investment timing decision, compared with first-best trigger levels;

2. Introducing hidden action into the model at time zero lowers investment timing distortions, because the manager has an option to align his incentives better with the owner by exerting effort at time zero. This leads to an investment timing trigger closer to the first-best level.

In addition, the model predicts that the manager with the lowest privately observed project value $\underline{\theta}$ receives no rents, in that $w(\underline{\theta}) = 0$ as in our basic setting.\(^{26}\) The ex-ante participation constraint does not bind, because the limited liability condition for the manager and ex-ante incentive constraint together provide enough incentive for the manager with any ex-post realized $\theta$ to participate, as in our basic setting. For technical convenience, we have

\(^{25}\)A sufficient condition to deter the manager from diverting the unobservable incremental part of value $\theta - \underline{\theta}$ is to require that the non-pecuniary cost $\kappa$ is large enough to deter the manager with the highest-type $\bar{\theta}$, in that, $\kappa \geq \bar{\theta} - \underline{\theta} - w(\bar{\theta})$.

\(^{26}\)Recall that the manager with $\theta_2$ receives no rents.
assumed that the distribution of θ under effort first-order stochastically dominates that under no effort. Intuitively, the manager is more likely to draw a “better” distribution of θ after exerting effort than not exerting effort. Under those conditions,\textsuperscript{27} managers of higher quality projects will exercise at lower equilibrium trigger strategies and receive higher equilibrium wages.

We may further generalize our model by allowing for multiple discrete choices of effort levels. One can solve this problem by following a similar two-step procedure: (i) first solving for the optimal contract for each given level of effort; and (ii) then choosing the “optimal” level of effort for the owner by searching for the maximum among owner’s option value across all effort levels. Subtle technical issues arise when we allow for effort choice to be continuous.\textsuperscript{28} However, the basic approach and intuition remain valid.

4 Model Implications

In this section, we analyze several of the more important implications of the model. First, Section 4.1 examines the stock price reaction to investment (or failure to invest). We shall see that the stock price will move by a discrete jump due to the information released at the trigger $P^*_1$. Investment at $P^*_1$ signals good news about project quality and the stock price jumps upward; failure to invest at $P^*_1$ signals bad news about project quality and the stock price falls downward. Second, a clear prediction of our model is that the principal-agent problem will introduce inertia into a firm’s investment behavior, in that investment will on average be delayed beyond first-best. Section 4.2 considers the factors that influence the expected lag in investment. Third, specifically because the timing of investment differs from that of the first-best outcome, the principal-agent problem results in a social loss and reduction in the owner’s option value. Section 4.3 analyzes the comparative statics of the social loss and owner’s option value with respect to the key parameters of the model.

In this section, we focus our analysis on the contract that prevails in the joint hidden information/hidden action region. It is in this region that the incentive problems are the richest and most meaningful. Therefore, it should be noted that in this section when we refer

\textsuperscript{27}See Appendix B for other technical conditions.

\textsuperscript{28}We need to verify the validity of first-order approach, which refers to the practice of replacing an infinite number of global incentive constraints imposed by ex-ante incentive to exert effort, with simple local incentive constraints as captured by first-order condition associated with the global incentive constraints. See Rogerson (1985) and Jewitt (1988) for more on the first-order approach.
to contracting variables such as \( w_1 \) and \( P_J \), we are referring to the values of those variables that hold in this joint hidden information/hidden action region. The terms of the contract and resulting option values in this region are displayed in equations (29)–(32).

### 4.1 Stock Price Reaction to Investment

In this section, we analyze the stock price reaction to the information released via the manager’s investment decision.\(^{29}\) The manager’s investment decision will signal to the market the true value of \( \theta \), and the stock price will reflect this information revelation. Importantly, this will allow for the manager’s compensation contract to be contingent on the firm’s stock price. That is, while in the model we have made the wages in the incentive contract contingent on the manager’s investment decision, the wages can also be made contingent on the stock price.

The equity value of the firm is equal to the value of the owner’s option value given in (31). Prior to the point at which \( P(t) \) reaches the threshold \( P_1^* \), the market does not know the true value of \( \theta \): the market believes that \( \theta = \theta_1 \) with probability \( q_H \) and \( \theta = \theta_2 \) with probability \( 1 - q_H \).

Once the process \( P(t) \) hits the threshold \( P_1^* \), the manager’s unobserved component of project value is fully revealed. The manager’s investment behavior signals to the market the true value of \( \theta \). If the manager exercises the option at \( P_1^* \), then the manager reveals to the market that the privately observed component of project value is high. Therefore, the firm’s value instantly jumps to \( S_u \), given by

\[
S_u = P_1^* + \theta_1 - K - w_1 = P_1^* + \theta_1 - K - \left( \frac{P_1^*}{P_J} \right)^\beta \Delta \theta .
\]

If the manager does not exercise his option at \( P_2^* \), then the market infers that the manager’s privately observed component of project value is low. Then, the firm’s value instantly drops to \( S_d \), given by

\[
S_d = \left( \frac{P_1^*}{P_J} \right)^\beta (P_J + \theta_2 - K).
\]

Figure 2 plots the stock price \( S \) as a function of \( P \), the current value of the process \( P(t) \).

For all \( P < P_1^* \), \( S(P) = \pi^o(P) \), where \( \pi^o \) is given in (31). For \( P = P_1^* \), \( S(P) = S_u \) if investment is undertaken, and \( S(P) = S_d \) if investment is not undertaken. The jump in

\(^{29}\)We thank the referee for suggesting this discussion.
the stock price at \( P_1^* \) is a result of the information revealed by the manager’s investment decisions.

This result is consistent with the empirical findings in McConnell and Muscarella (1985). They find that announcements of unexpected increases in investment spending lead to increases in stock prices, and vice versa for unexpected decreases.

Since the stock price movement at the trigger \( P_1^* \) reveals the true value of \( \theta \), the manager’s incentive contract can be made contingent on the stock price. For example, the manager could be paid a bonus \( w_1 \) if the stock price jumps upward to \( S_u \). Since \( w_2 = 0 \), no bonus is paid if the stock price falls to \( S_d \). Similarly, such a contingent payoff could be implemented through a properly parameterized stock option grant.

### 4.2 Agency Problems and Investment Lags

In the standard real options setting, investment is triggered at the value maximizing triggers, \( P_1^* \) and \( P_2^* \), for the higher and lower project quality outcomes, respectively. However, in our setting, while the trigger for investment in the higher quality state remains at \( P_1^* \), investment in the lower quality state may be triggered at \( P_J \), which is higher than the first-best benchmark level \( P_2^* \).

Let \( T \) and \( T^* \) be the stopping times at which the option is exercised, in our model and the first-best setting, respectively. We denote \( \Gamma = E(T - T^*) \) as the expected time lag due to the principal-agent problem. A solution for such an expectation can be derived using Harrison (1985, Chapter 3). The expected lag is given by

\[
\Gamma = \left( \frac{1 - q_H}{\alpha - \sigma^2/2} \right) \ln \left( \frac{P_J}{P_2^*} \right) \quad \text{(39)}
\]

\[
= \left( \frac{1 - q_H}{\alpha - \sigma^2/2} \right) \left[ \ln \left( \frac{P_0}{K - \theta_2} \right) + \frac{1}{\beta} \ln \left( \frac{\Delta q \Delta \theta}{\xi} \right) - \ln \beta + \ln (\beta - 1) \right], \quad \text{(40)}
\]

where we assume that \( \alpha > \sigma^2/2 \) in order for this expectation to exist.

An important insight from Section 3 is that increases in the cost benefit ratio of inducing effort lead to less distortion in investment timing. That is, as the ratio \( \xi/\Delta q \) increases, the equilibrium trigger \( P_J \) becomes closer to the first-best trigger \( P_2^* \). This is confirmed by the following comparative static:

\[
\frac{\partial \Gamma}{\partial (\xi/\Delta q)} = - \left( \frac{1 - q_H}{\alpha - \sigma^2/2} \right) \frac{\Delta q}{\beta \xi} < 0. \quad \text{(41)}
\]
An increase in the volatility of the underlying project, $\sigma$, has an ambiguous effect on the expected time lag $\Gamma$. This can be seen from the following comparative static:

$$\frac{\partial \Gamma}{\partial \sigma} = -\left(\frac{1 - q_H}{\alpha - \sigma^2/2}\right) \frac{1}{\beta^2} \left[ \ln \left( \frac{\Delta q \Delta \theta}{\xi} \right) - \frac{\beta}{\beta - 1} \right] \frac{\partial \beta}{\partial \sigma} + \frac{(1 - q_H) \sigma}{(\alpha - \sigma^2/2)^2} \ln \left( \frac{P_J}{P_2^*} \right),$$

where $\partial \beta / \partial \sigma < 0$. An increase in $\sigma$ raises the option value and makes waiting more worthwhile, implying that both $P_2^*$ and $P_J$ are larger, ceteris paribus. However, if the cost benefit ratio for exerting effort is relatively high, in that

$$\ln \left( \frac{\xi}{\Delta q} \right) > \frac{\beta - 1}{\beta} \ln(\Delta \theta),$$

then the change of $P_J$ relative to the change in $P_2^*$ is larger. Therefore, under such conditions the expected time lag increases in volatility $\sigma$.

An increase in the expected growth rate of the project, $\alpha$, also has an ambiguous effect on the expected time lag $\Gamma$. This can be seen from the following comparative static:

$$\frac{\partial \Gamma}{\partial \alpha} = -\frac{1 - q_H}{(\alpha - \sigma^2/2)^2} \left[ \ln \left( \frac{P_J}{P_2^*} \right) - \frac{1}{\beta} \left( \ln \left( \frac{\Delta q \Delta \theta}{\xi} \right) - \frac{\beta}{\beta - 1} \right) \frac{\alpha - \sigma^2/2}{\sqrt{(\alpha - \sigma^2/2)^2 + 2r\sigma^2}} \right].$$

However, if (43) holds, then expected time lag decreases with drift $\alpha$.

### 4.3 Social Loss and Option Values

Although the owner chooses the value-maximizing contract to provide an incentive for the manager to extend effort, the agency problem ultimately still proves costly. In an owner-managed firm, the manager will extend effort and will exercise the option at the first-best stopping time. However, in firms with delegated management, there will be a social loss due to the firm’s suboptimal exercise strategy.

By a social loss, we are referring to the difference between the values of the first-best option value, $V^*(P_0)$ in (9), and the sum of the owner and manager options, $\pi^o(P_0)$ and $\pi^m(P_0)$ in (31) and (32). Thus, define the social loss due to agency issues as $L$, where $L = V^*(P_0) - [\pi^o(P_0) + \pi^m(P_0)]$. Simplifying, we have:

$$L = (1 - q_H) \left( P_0 \left( \frac{P_J}{P_2} \right)^\beta (P_2^* - K + \theta_2) - \left( \frac{P_0}{P_J} \right)^\beta (P_J - K + \theta_2) \right).$$

This social loss is likely to have economic ramifications on the structure of firms. For firms in industries with potentially large social losses due to agency costs, there will be powerful
forces that will push them to be privately held, or to be organized in a manner that provides the closest alignment between owners and managers.

There are two effects of a later-than-first-best exercising trigger ($P_J > P^*_2$) on the social loss $L$: (i) a larger payout ($P_J + \theta_2 - K$) reduces social loss, *ceteris paribus*, and (ii) a lower discount factor $[(P_0/P_J)^\beta < (P_0/P^*_2)^\beta]$ increases the social loss. The latter dominates the former, because $P_J > P^*_2$ and $P^*_2 = \arg\max (P_0/x)^\beta (P_0 + \theta_2 - K)$. Equation (45) suggests that social loss is driven by the distance of the equilibrium trigger $P_J$ from $P^*_2$. As previously discussed, the firm’s exercise timing becomes less distorted as the net cost benefit ratio of inducing effort increases. That is, as the ratio $\xi/\Delta q$ increases, the equilibrium trigger $P_J$ gets closer to the first-best trigger $P^*_2$, and thus:

$$\frac{\partial L}{\partial (\xi/\Delta q)} < 0.$$  \hspace{1cm} (46)

Note that with or without an agency problem, the owner’s value decreases as the cost of effort $\xi$ increases, in that $d\pi^o(P_0)/d\xi < 0$. Without an agency problem (e.g., the firm’s owner also manages the investment decisions), the owner’s value falls one for one with an increase in effort cost; the owner simply must increase his effort outlay. In the case of delegated management with agency costs, the owner’s value $\pi^o(P_0)$ also falls as the cost of effort increases. A question that we ask below is whether or not $\pi^o(P_0)$ falls by more or less than the first-best value does when the cost of effort increases.

In terms of the owner’s option value, the incentive problem represents a trade-off between timing efficiency and the surplus that must be paid to the manager to extend effort. One can obtain better intuition on the forces at work in the agency problem through the following decomposition. In the first-best solution, the owner pays the cost of effort $\xi$ and obtains the first-best option value $V^*(P_0)$. In the agency equilibrium, the owner delegates the cost of effort to the manager, but then holds the sub-optimal option value $\pi^o(P_0)$. The loss in the owner’s option value due to the incentive problem is therefore given by

$$\Delta \pi^o(P_0) \equiv V^*(P_0) - \xi - \pi^o(P_0) = L + V^m,$$  \hspace{1cm} (47)

where $L$ is the total social loss given in (45), and $V^m$ is the *ex-ante* expected surplus paid to the manager to exert effort, and is given by:

$$V^m = \pi^m(P_0) - \xi = q_H \frac{\xi}{\Delta q} - \xi = q_L \frac{\xi}{\Delta q},$$  \hspace{1cm} (48)
Decomposing the loss in the owner’s option value given in (47) into the sum of the timing component $(L)$ and the compensation component $(V^m)$ is useful for providing intuition. When the owner delegates the option exercise decision to the manager, the owner’s option value is lowered for two reasons: (i) the exercising inefficiency induced by agency and information asymmetry; and (ii) the surplus needed to pay the manager to induce him to extend effort and reveal his private information. The impact of a higher effort cost $\xi$ represents a trade-off in terms of the timing and compensation components. As shown in (46), a higher effort cost results in more efficient investment timing. This must be traded-off against the increased compensation that must be paid to provide appropriate incentives to the manager, as seen in (48). Therefore, the total effect on the loss of owner’s option value due to an increase in $\xi$ depends on whether the “timing effect” or “compensation effect” is larger, in that

$$
\frac{\partial}{\partial \xi} \Delta \pi_o(P_0) = - (1 - q_H) (\beta - 1) \left( \frac{P_0}{P_J} \right)^\beta \left( 1 - \frac{P^*_2}{P_J} \right) \frac{P_J}{\beta \xi} + \frac{q_L}{\Delta q} \quad (49)
$$

$$
= \frac{\beta - 1}{\beta} \frac{1}{\Delta q \Delta \theta} \left[ - (1 - q_H) (P_J - P^*_2) + q_L (P^*_2 - P^*_1) \right]. \quad (50)
$$

If the investment trigger $P_J$ is significantly larger than $P^*_2$, in that

$$(1 - q_H) (P_J - P^*_2) > q_L (P^*_2 - P^*_1), \quad (51)$$

then an increase in $\xi$ leads to a smaller loss in the owner’s option value, as the gain in timing efficiency overshadows the loss due to the manager’s increased compensation. That is, while the owner’s option value under agency falls as $\xi$ increases, it may fall by less than the full amount of the increase in $\xi$ due to the gain in timing efficiency.

5 Impatient Managers and Early Investment

So far, we have assumed that both owners and managers value payoffs identically. However, it may be the case that managers are more impatient than owners. There are several potential justifications for such an assumption. First, there are various models of managerial myopia

\[ \text{Note that the above condition is non-empty. This can be seen as follows. Condition (51) is equivalent to} \]

$$
P_J > \frac{1}{1 - q_H} [(1 - q_H) P^*_2 + q_L (P^*_2 - P^*_1)] = P^*_3 - \frac{\Delta q}{1 - q_H} (P^*_2 - P^*_1). \]

The joint hidden action/hidden information region is characterized by $P^*_2 \leq P_J \leq P^*_3$. Therefore, the above condition is met for some $P_J$.}

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that attempt to explain a manager’s preference for choosing projects with quicker pay-backs, even in the face of eschewing more valuable long-term opportunities. For example, Narayanan (1985) and Stein (1989) argue that concerns about either the firm’s short-term performance or labor-market reputation may give the manager an incentive to take actions that pay off in the near term at the expense of the long term. Second, in our “investment timing” setting, greater impatience can represent the manager’s preference for empire building or greater perquisite consumption and reputation that comes from running a larger company sooner rather than later. Third, managerial short-termism could be the result of the manager facing stochastic termination.\footnote{We assume that the owner can costlessly replace the manager in the event of separation.} This termination, for example, could be due to the manager leaving for a better job elsewhere or being fired. We can model such stochastic termination by supposing that the manager faces an exogenous termination driven by a Poisson process with intensity \( \zeta \). The addition of stochastic termination transforms the manager’s option to one in which his discount rate \( r \) is elevated to \( r + \zeta \) to reflect the stochastic termination.\footnote{We suppose that the manager receives his reservation value (normalized to zero), when the termination occurs. See Yaari (1965), Merton (1971) and Richard (1975) for analogous results on stochastic horizon.}

Phrased in real options terms, managerial impatience decreases the value of the manager’s option to wait. Thus, this generalization leads to very different predictions about investment timing. While the basic model predicts that investment will never occur earlier than the first-best case, in this generalized setting investment can occur earlier or later than the first-best case. This is similar to the result found in Stulz (1990) where there is which generates? both over- and under-investment in the capital allocation decision, as shareholders use debt to constrain managerial empire-building preferences.

Recall that the owner discounts future cash flows by the discount function \( D(P_0; \hat{P}) = \left( \frac{P_0}{\hat{P}} \right)^\beta \) for \( P_0 < \hat{P} \). We can therefore represent greater managerial impatience by defining a managerial discount function \( D^m(P_0; \hat{P}) = \left( \frac{P_0}{\hat{P}} \right)^\gamma \), where \( \gamma > \beta \) ensures that \( D^m(P_0; \hat{P}) < D(P_0; \hat{P}) \). That is, a dollar received at the stopping time described by the trigger strategy \( \hat{P} \) is worth less to the manager than to the owner.\footnote{Note that this is also consistent with the interpretation that the manager has a higher discount rate than the owner. Since \( \partial \beta / \partial r > 0 \), the manager’s higher discount rate is embodied by the condition \( \gamma > \beta \).}

This generalized problem is quite similar to that of Section 2, with the exception that the constraints all use \( \gamma \) rather than \( \beta \). Much of the solution methodology is the same. For example, Propositions 1 and 2 apply as before, using the same proof. In addition, Proposition

\footnote{\textit{we assume that the owner can costlessly replace the manager in the event of separation.}}
3 and 4 remain valid, and are demonstrated in Appendix C. Thus, the optimal contracting problem in the generalized setting can be written as:

$$\max_{w_1, P_1, P_2} q_H \left( \frac{P_0}{P_1} \right)^\beta (P_1 + \theta_1 - K) - q_H \left( \frac{P_0}{P_1} \right)^\beta w_1 + (1 - q_H) \left( \frac{P_0}{P_2} \right)^\beta (P_2 + \theta_2 - K),$$  \hspace{1cm} (52)

subject to

$$\left( \frac{P_0}{P_1} \right)^\gamma w_1 \geq \left( \frac{P_0}{P_2} \right)^\gamma \Delta \theta, \hspace{1cm} (53)$$

$$\left( \frac{P_0}{P_1} \right)^\gamma w_1 \geq \frac{\xi}{\Delta q}. \hspace{1cm} (54)$$

Similar to Proposition 5, at least one of (53) and (54) binds. Otherwise, the owner may strictly increases his payoff by lowering the wage payment $w_1$ without violating any constraints.

Just as in Section 3, there are three contracting regions: a hidden information region, a joint hidden information/hidden action region, and a hidden action region, depending on the level of cost/benefit ratio $\xi/\Delta q$. In this section, we focus on the joint hidden information/hidden action region.\(^{34}\)

The joint hidden information/hidden action region is defined by: $(P_0/\hat{P}_3^*)^\gamma \Delta \theta < \xi/\Delta q < (P_0/P_2^*)^\gamma \Delta \theta$, where $\hat{P}_3^*$ is defined in (C.11), and shown to be greater than the trigger $P_2^*$. In this region the optimal contract can be written as:

$$P_1 = \hat{P}_1, \hspace{1cm} (55)$$

$$P_2 = \hat{P}_J = P_0 \left( \frac{\Delta q \Delta \theta}{\xi} \right)^{1/\gamma}, \hspace{1cm} (56)$$

$$w_1 = \left( \frac{\hat{P}_1}{P_1} \right)^\gamma \Delta \theta < \Delta \theta, \hspace{1cm} (57)$$

$$w_2 = 0. \hspace{1cm} (58)$$

where $\hat{P}_1$ is the root of $H(x) = 0$, defined by

$$H(x) = \frac{\beta}{\beta - 1} \left[ K - \theta_1 + \left( 1 - \frac{\gamma}{\beta} \right) x \right] - x. \hspace{1cm} (59)$$

Unlike the results of the basic model, we now have the possibility of investment occurring before the first-best trigger is reached, in that $P_1 = \hat{P}_1 < P_1^*$. To see this, note that $H(0) = P_1^*$ and

$$H(P_1^*) = \frac{\beta}{\beta - 1} \left( 1 - \frac{\gamma}{\beta} \right) \left( \frac{P_1^*}{P_0} \right)^\gamma \frac{\xi}{\Delta q} < 0. \hspace{1cm} (60)$$

\(^{34}\)The derivations for the optimal contracts in the other regions are shown in Appendix C.
The derivative of $H(\cdot)$ is

$$H'(x) = \frac{\beta}{\beta - 1} \gamma \left(1 - \frac{\gamma}{\beta}\right) \left(\frac{x}{P_0}\right)^{\gamma-1} \frac{1}{P_0 \Delta q} - 1 < 0.$$ \hspace{1cm} (61)

Therefore, there exists a unique solution $P_1 = \hat{P}_1 < P_1^*$. As in the basic model, the trigger strategy for the manager of a $\theta_2$-type project is greater than the first-best trigger, $P_2^*$. Recall that $\hat{P}_J > P_2^*$ in the region $(P_0/\hat{P}_3^*)^\gamma \Delta \theta < \xi/\Delta q < (P_0/P_2^*)^\gamma \Delta \theta$, where $P_3^*$ is given in (C.11). However, for $\gamma > \beta$, the trigger is closer to the first-best trigger than for the standard case in which $\gamma = \beta$. This is true, since for $\gamma > \beta$,

$$\hat{P}_J = P_0 \left(\frac{\Delta q \Delta \theta}{\xi}\right)^{1/\gamma} < P_0 \left(\frac{\Delta q \Delta \theta}{\xi}\right)^{1/\beta} = P_J.$$ \hspace{1cm} (62)

Thus, when the manager is more impatient than the owner, equilibrium investment occurs sooner than it does in the standard principal-agent model described earlier in the paper. In particular, investment actually occurs prior to the first-best trigger is reached for the $\theta_1$-type project. The greater impatience on the part of the manager implies that it is in the owner’s interest to offer a contract that motivates earlier exercise. This results in both costs and benefits to the owner. By motivating investment for the $\theta_2$-type project earlier than the standard principal-agent model, investment timing trigger moves closer to the first-best one. Since the manager receives no surplus for the $\theta_2$-type project, the owner is the sole beneficiary of this timing efficiency. However, investment for the $\theta_1$-type project occurs earlier than that in the model of Section 2, which is the first-best outcome. Therefore, the owner is worse off with respect to the $\theta_1$-type projects for two reasons: investment occurs too early, and the wage paid to the manager in this state must be higher (than in the standard model) in order to motivate earlier investment. The net effect on ex-ante owner’s option value is ambiguous and is driven by the relative parameter values.

6 Conclusion

This paper extends the real options framework to account for the agency and information issues that are prevalent in many real-world applications. When investment decisions are delegated to managers, contracts must be designed to provide incentives for managers to both extend effort and to truthfully reveal their private information. This paper provides a model of optimal contracting in a continuous-time principal-agent setting in which there is
both moral hazard and adverse selection. The implied investment behavior differs significantly from that of the first-best no-agency solution. In particular, there will be greater inertia in investment, as the model predicts that the manager will have a more valuable option to wait than the owner. The interplay between the twin forces of hidden information and hidden action leads to markedly different investment outcomes than when only one of the two forces is at work. Allowing the manager to have an effort choice that affects the likelihood of getting a high quality project mitigates the investment inefficiency due to information asymmetry. When the model is generalized to include differing degrees of impatience between owners and managers, we find that investment may occur either earlier or later than optimal.

Some extensions of the model would prove interesting. First, the model could allow for repeated investment decisions. This richer setting would permit owners to update their beliefs over time, and for managers to establish reputations. Second, the model could also be generalized to include competition in both the labor and product markets. As shown by Grenadier (2002), the forces of competition greatly alter the investment behavior implied by standard real options models.
Appendices

A Solution to the Optimal Contracting Problem

This appendix provides a derivation of the optimal contracts detailed in Section 3.

First, we simplify the optimal contracting problem by presenting and proving the following four propositions. Proposition 1 shows that the limited liability for the manager of a $\theta_1$-type project in constraint (19) does not bind, while Proposition 2 shows that the *ex-ante* participation constraint (16) does not bind.

**Proposition 1.** The limited-liability condition for a manager of a $\theta_1$-type project does not bind. That is, $w_1 > 0$.

*Proof.*

\[
w_1 \geq \left( \frac{P_1}{P_2} \right) \beta (w_2 + \Delta \theta) \geq \left( \frac{P_1}{P_2} \right) \beta \Delta \theta > 0,
\]

The first and second inequalities follow from (17) and (19), respectively.

In order to motivate the manager to exert effort, we need to reward the manager with an option value larger than zero, which is the manager’s reservation value. This leads to the following result.

**Proposition 2.** The *ex-ante* participation constraint (16) does not bind.

*Proof.*

\[
\left( \frac{P_0}{P_1} \right)^\beta w_1 + \frac{1 - q_H}{q_H} \left( \frac{P_0}{P_2} \right)^\beta w_2 - \frac{\xi}{q_H} \geq \frac{\xi}{\Delta q} - \frac{\xi}{q_H} > 0,
\]

where the first inequality follows from the *ex-ante* incentive constraint (15) and the limited liability condition for the $\theta_2$ project.

Propositions 1 and 2 allow us to express the owner’s objective as maximizing the value of his option, given in (13), subject to (15), (17), (18) and $w_2 \geq 0$. Using the method of
Kuhn-Tucker, we form the Lagrangian as follows:

\[ L = \left( \frac{P_0}{P_1} \right)^\beta (P_1 + \theta_1 - K - w_1) + \frac{1 - q_H}{q_H} \left( \frac{P_0}{P_2} \right)^\beta (P_2 + \theta_2 - K - w_2) \]

\[ + \lambda_1 \left[ \left( \frac{P_0}{P_1} \right)^\beta w_1 - \left( \frac{P_0}{P_2} \right)^\beta (w_2 + \Delta \theta) \right] + \lambda_2 \left[ \left( \frac{P_0}{P_2} \right)^\beta w_2 - \left( \frac{P_0}{P_1} \right)^\beta (w_1 - \Delta \theta) \right] \]

\[ + \lambda_3 \left[ \left( \frac{P_0}{P_1} \right)^\beta w_1 - \left( \frac{P_0}{P_2} \right)^\beta w_2 - \frac{\xi}{\Delta q} \right] + \lambda_4 w_2, \quad (A.1) \]

with corresponding complementary slackness conditions for the four constraints.

The first-order condition with respect to \( w_1 \) gives

\[ \lambda_1 - \lambda_2 + \lambda_3 = 1. \quad (A.2) \]

The first-order condition with respect to \( w_2 \) implies

\[ \left( -\lambda_1 + \lambda_2 - \lambda_3 - \frac{1 - q_H}{q_H} \right) \left( \frac{P_0}{P_2} \right)^\beta + \lambda_4 = 0. \quad (A.3) \]

Simplifying (A.3) gives \( \lambda_4 = (P_0/P_2)^\beta / q_H > 0 \). Therefore, the complementary slackness condition \( \lambda_4 w_2 = 0 \) implies that \( w_2 = 0 \). This is summarized in Proposition 3.

**Proposition 3.** The limited liability for a manager of a \( \theta_2 \)-type project binds, in that \( w_2 = 0 \).

The intuition is straightforward. Giving the manager of a \( \theta_2 \)-type project any positive rents implies higher rents for managers of \( \theta_1 \)-type projects in order to meet the *ex-post* incentive constraint of the manager of a \( \theta_1 \)-type project. In order to minimize the rents subject to the manager’s participation and incentive constraints, the owner shall give the manager of a \( \theta_2 \)-type project no *ex-post* rents.

The first-order conditions with respect to \( P_1 \) and \( P_2 \) imply:

\[ P_1 = \frac{\beta}{\beta - 1} (K - \theta_1 - \lambda_2 \Delta \theta), \quad (A.4) \]

\[ P_2 = \frac{\beta}{\beta - 1} \left( K - \theta_2 + \frac{q_H}{1 - q_H} \lambda_1 \Delta \theta \right). \quad (A.5) \]

The following proposition states that the *ex-post* incentive constraint for the manager of a \( \theta_2 \)-type project, (18), does not bind, in that \( \lambda_2 = 0 \). We verify this conjecture, formalized in Proposition 4 for each region. Proposition 4 allows us to ignore (18) in the optimization problem.
Proposition 4. Optimal contracts imply \( w_1 \leq \Delta \theta \).

Intuitively, if \( w_1 > \Delta \theta \), then the manager of a \( \theta_2 \)-type project would never accept the equilibrium contract with \( w_2 = 0 \). This would clearly be inconsistent with Proposition 3.

Propositions 1–4 jointly simplify the owner’s optimization problem as the solution to the objective function (20), subject to constraints (21) and (22).

The following proposition demonstrates that at least one of the two constraints binds.

Proposition 5. At least one of (21) and (22) binds.

The argument is immediate. If Proposition 5 did not hold, then reducing \( w_1 \) will increase the owner’s value strictly without violating any of the constraints. With \( \lambda_2 = 0 \), then (A.2) may be written as \( \lambda_1 + \lambda_3 = 1 \). Therefore, it must be the case that at least one of (21) and (22) binds.

The problem summarized in equations (20)-(22) is now solved below.

A.1 The Hidden Information Only Region

Suppose that the \textit{ex-ante} incentive constraint (22) does not bind. Since (21) must hold as an equality and \( \lambda_1 = 1 \), (A.5) implies \( P_2 = P_3^* \) where \( P_3^* \) is given in (25) and \( w_1 \) is given in (26). The inequality \( P_1^* < P_3^* \) implies (18) does not bind, consistent with Proposition 4. Finally, in order to be consistent with the assumption that (22) does not bind, we require that \( \xi/\Delta q < (P_0/P_3^*)^\beta \Delta \theta \), the parameter range defining this hidden information only region.

A.2 The Joint Hidden Information/Hidden Action Region

We derive the optimal contract in this region by conjecturing that both (21) and (22) bind. Solving these two equality constraints gives (29) and (30). The inequality \( P_J > P_1^* \) confirms that (18) does not bind, consistent with Proposition 4. The solution for \( P_2 \) implies that \( \lambda_1 \) can be written as:

\[
\lambda_1 = \frac{\beta - 1}{\beta} \left( P_J - P_2^* \right) \frac{1 - q_H}{q_H \Delta \theta} .
\]  

(A.6)

The only possible region under which both constraints may bind\(^\text{35}\) is characterized by

\[
\left( \frac{P_0}{P_3^*} \right) = \frac{\Delta \theta}{\Delta q} \quad \text{and} \quad \left( \frac{P_0}{P_2^*} \right) = \frac{\xi}{\Delta q} .
\]  

(A.7)

\(^{33}\)If \((P_0/P_3^*)^\beta \Delta \theta > \xi/\Delta q > (P_0/P_2^*)^\beta \Delta \theta\), then only the third constraint binds. If \((P_0/P_3^*)^\beta \Delta \theta > \xi/\Delta q\), then only the first constraint binds. If \(\xi/\Delta q > (P_0/P_3^*)^\beta \Delta \theta\), then supporting high effort is no longer in the owner’s interest.

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We now show that indeed both (21) and (22) bind throughout this entire region. The region characterized by (A.7) can be equivalently expressed as \( P_2^* < P_J < P_3^* \). Because (A.6) implies that \( \lambda_1 \) is monotonically increasing in \( P_J \), therefore, \( 0 < \lambda_1 < 1 \) in this region. Since \( \lambda_3 = 1 - \lambda_1 \), we also have \( 0 < \lambda_3 < 1 \). By complementary slackness conditions, both (21) and (22) bind in this joint region, confirming the result that (A.7) is the whole region, with both constraints binding.\(^{36}\)

### A.3 The Hidden Action Only Region

Suppose that (21) does not bind and (22) binds, then \( \lambda_1 = 0 \) by complementary slackness, and \( \lambda_3 = 1 \). Therefore, \( P_2 = P_2^* \) given in (33) and \( w_1 \) is given in (34). We need to verify that (21) and (18) do not bind. The constraint (18) is non-binding (consistent with Proposition 4) if and only if \( P_1^* < P_J \). The constraint (21) is non-binding if and only if \( P_J < P_2^* \). Thus, together these imply that \( P_1^* < P_J < P_2^* \), which is identical to the condition \( (P_0/P_1^*)^\beta \Delta \theta < \xi/\Delta q < (P_0/P_2^*)^\beta \Delta \theta \), that defines this region.

If the parameters do not fall in any of the three regions, namely, \( \xi/\Delta q > (P_0/P_1^*)^\beta \Delta \theta \) then it can be shown that the owner will not choose to motivate the manager to exert effort. The cost of effort is so high as to overwhelm any potential benefits of motivating effort. A proof of this result is available from the authors upon request.

### B Optimal Contracting with a Continuous Distribution of \( \theta \)

This appendix contains the derivation of the optimal contracts when the distribution of the project’s unobserved component \( \theta \) of value is continuous.

Denote the manager’s time-zero expected utility as \( u(\hat{\theta}, \theta) \), if he reports that his privately observed component of project value is \( \hat{\theta} \), and the true level of his privately observed value is \( \theta \). His time-zero expected utility is then given by

\[
u(\hat{\theta}, \theta) = \left( \frac{P_0}{P(\theta)} \right)^\beta \left( w(\hat{\theta}) + \theta - \hat{\theta} \right). \tag{B.1}\]

We denote \( U(\theta) \) as the value function of the manager whose privately observed component

\(^{36}\)We also need additional technical conditions to ensure that inducing the manager to extend high effort is in the interest of the owner.
of project value is \( \theta \). That is,

\[
U(\theta) = u(\theta, \theta) = \left( \frac{P_0}{P(\theta)} \right)^\beta w(\theta).
\]  

(B.2)

As in Section 2, we denote \( \xi \) as the cost of extending effort at time zero. Let \( F_H(\theta) \) and \( F_L(\theta) \) be the cumulative distribution functions of \( \theta \) drawn if the manager extends effort and if he does not extend effort, respectively. Using the revelation principle, we may write the principal’s optimization problem as follows:

\[
\max_{P(\cdot), w(\cdot)} \int_\theta^\bar{\theta} \left( \frac{P_0}{P(\theta)} \right)^\beta (P(\theta) + \theta - K - w(\theta)) \, dF_H(\theta),
\]  

subject to:

1. **ex-post** incentive-compatibility condition:

   \[
   U(\theta) \geq u(\hat{\theta}, \theta), \quad \text{for any } \hat{\theta} \text{ and } \theta; \quad \text{(B.4)}
   \]

2. limited-liability condition:

   \[
   w(\theta) \geq 0, \quad \text{for any } \theta; \quad \text{(B.5)}
   \]

3. **ex-ante** incentive compatibility condition:

   \[
   \int_\theta^\bar{\theta} U(\theta) \, dF_H(\theta) - \xi \geq \int_\theta^\bar{\theta} U(\theta) \, dF_L(\theta); \quad \text{(B.6)}
   \]

4. **ex-ante** participation constraint:

   \[
   \int_\theta^\bar{\theta} U(\theta) \, dF_H(\theta) - \xi \geq 0. \quad \text{(B.7)}
   \]

**Proposition 6.** The **ex-ante** participation constraint (B.7) does not bind.

**Proof.** The **ex-ante** incentive constraint (B.6) and the limited-liability condition (B.5) together imply that the **ex-ante** participation constraint (B.7) does not bind.

First, we simplify the **ex-ante** incentive constraint (B.6) using integration by parts. This gives

\[
\int_\theta^\bar{\theta} M(\theta) \, dU(\theta) \geq \xi, \quad \text{(B.8)}
\]

where \( M(\theta) = -(F_H(\theta) - F_L(\theta)) \).
Next, we simplify the ex-post incentive-compatibility condition (B.4) by totally differentiating $U(\theta)$, the value function for a manager of a type-$\theta$ project, with respect to $\theta$. This gives

$$\frac{dU(\theta)}{d\theta} = u_1 \frac{d\hat{\theta}}{d\theta} + u_2,$$  \hspace{1cm} (B.9)

where

$$u_1 = \frac{\partial u(\hat{\theta}, \theta)}{\partial \hat{\theta}} \bigg|_{\hat{\theta}=\theta}, \quad \text{and} \quad u_2 = \frac{\partial u(\hat{\theta}, \theta)}{\partial \theta} \bigg|_{\hat{\theta}=\theta}.$$  \hspace{1cm} (B.10)

Since the manager optimally reveals his project quality by choosing recommended equilibrium strategy, we have $u_1(\theta, \theta) = 0$. Therefore, we have $U'(\theta) = u_2$. Integration gives

$$U(\theta) = U(\theta) + \int_{\theta}^{\theta} u_2(s, s) \, ds = U(\theta) + \int_{\theta}^{\theta} \left( \frac{P_0}{P(s)} \right)^{\beta} \, ds.$$  \hspace{1cm} (B.11)

We note that the Spence-Mirrlees condition is satisfied.\(^{37}\)

A standard result in the contracting literature with asymmetric information states that the limited-liability condition for a manager of a $\theta$-type project binds, in that $U(\theta) = w(\theta) = 0$. Therefore, the information rents $U(\theta)$ that accrues to the manager of a $\theta$-type project is given by

$$U(\theta) = \int_{\theta}^{\theta} \left( \frac{P_0}{P(s)} \right)^{\beta} \, ds.$$  \hspace{1cm} (B.12)

The relationship between $U(\theta)$ and the equilibrium wage implies that

$$w(\theta) = \left( \frac{P(\theta)}{P_0} \right)^{\beta} U(\theta) = \int_{\theta}^{\theta} \left( \frac{P(\theta)}{P(s)} \right)^{\beta} \, ds.$$  \hspace{1cm} (B.13)

Using (B.13), we simplify the present value of expected wage payment as follows:

$$\int_{\theta}^{\theta} \left( \frac{P_0}{P(\theta)} \right)^{\beta} w(\theta) \, dF_H(\theta) = \int_{\theta}^{\theta} \left[ \int_{\theta}^{\theta} \left( \frac{P_0}{P(s)} \right)^{\beta} \, ds \right] dF_H(\theta),$$

$$= \left[ \int_{\theta}^{\theta} \left( \frac{P_0}{P(s)} \right)^{\beta} \, ds \right] F_H(\theta) \bigg|_{\theta}^{\theta} - \int_{\theta}^{\theta} F_H(\theta) \left( \frac{P_0}{P(\theta)} \right)^{\beta} \, d\theta,$$

$$= \int_{\theta}^{\theta} \lambda_H(\theta) \left( \frac{P_0}{P(\theta)} \right)^{\beta} \, dF_H(\theta),$$.  \hspace{1cm} (B.14)

where

$$\lambda_H(\theta) = \frac{1 - F_H(\theta)}{f_H(\theta)}$$  \hspace{1cm} (B.15)

is the inverse of the hazard rate under $F_H(\cdot)$.

\(^{37}\)Details are available upon request.
Using (B.14) allows us to simplify the principal’s optimization problem as follows:

$$\max_{P(\cdot)} \int_{\theta}^{\tilde{\theta}} \left( \frac{P_0}{P(\theta)} \right)^\beta (P(\theta) + \theta - K - \lambda_H(\theta)) \ dF_H(\theta),$$  \hspace{1cm} (B.16)  

subject to the *ex-ante* incentive constraint (B.8) and *ex-post* limited-liability condition (B.5). The equilibrium wage is then obtained by using (B.13).

Similar to the model with discrete values for \( \theta \), optimal contracts, characterized by the pair of trigger strategy and wage payment functions, depend on the region in which effort cost \( \xi \) lies. Subsection B.1 solves for the optimal contracts in the region under which the *ex-ante* incentive constraint does not bind. Subsection B.2 solves for the optimal contracts in the region under which both the *ex-ante* incentive constraint and the *ex-post* incentive constraint bind.

### B.1 The Hidden Information Only Region

If effort cost \( \xi \) is low enough, then no additional rent is needed to induce the manager to extend effort. The following condition ensures that the *ex-ante* incentive constraint (B.8) does not bind.

**Condition 1.**

$$\int_{\bar{\theta}}^{\tilde{\theta}} M(\theta) \left( \frac{P_0}{P_3^*(\theta)} \right)^\beta d\theta \geq \xi,$$  \hspace{1cm} (B.17)  

where

$$P_3^*(\theta) = \frac{\beta}{\beta - 1} \left[ K - \theta + \lambda_H(\theta) \right].$$  \hspace{1cm} (B.18)  

Maximizing (B.16) may be done point by point. This gives the candidate optimal trigger level \( P(\theta) = P_3^*(\theta) \), where \( P_3^*(\theta) \) is given in (B.18). Because \( \lambda_H(\theta) > 0 \), the exercise trigger is larger than the first-best level, confirming the intuition delivered in Section 3 using the two-point distribution of \( \theta \). A verification easily confirms that (B.8) does not bind under Condition 1.

The following condition ensures that the candidate trigger strategy is positive for any \( \theta \).

**Condition 2.** For all \( \theta \) on the support, \( \theta - \lambda_H(\theta) < K \).

Finally, we ensure that the candidate trigger strategy decreases in \( \theta \) by requiring the following condition:
Condition 3. \( d\lambda H(\theta)/d\theta < 1 \).

It is straightforward to note that Conditions 2 and 3 also imply that wage is positive and increases in the project quality \( \theta \), as seen from (B.13).

### B.2 The Joint Hidden Information/Hidden Action Region

When the effort cost is higher, both the \textit{ex-ante} incentive constraint (B.6) and the \textit{ex-post} incentive constraint (B.4) bind. The condition governing the parameters for this region when \( \theta \) is drawn from a continuous distribution is given as follows.

**Condition 4.**

\[
\int_{\theta}^{\tilde{\theta}} M(\theta) \left( \frac{P_0}{P_3^*(\theta)} \right)^\beta d\theta \leq \xi \leq \int_{\theta}^{\tilde{\theta}} M(\theta) \left( \frac{P_0}{P_2^*(\theta)} \right)^\beta d\theta ,
\]

where \( P_3^*(\theta) \) is given in (B.18) and

\[
P_2^*(\theta) = \frac{\beta}{\beta - 1} \left( K - \theta + \frac{1 - F_L(\theta)}{f_H(\theta)} \right).
\]

Denote \( l \) as the Lagrangian multiplier for (B.8). Then, the candidate equilibrium trigger strategy is given by

\[
P(\theta) = P_J(\theta) = \frac{\beta}{\beta - 1} \left( K - \theta + \lambda H(\theta) - l \frac{M(\theta)}{f_H(\theta)} \right).
\]

The Lagrangian multiplier \( l \) is positive under Condition 4. Therefore, the optimal trigger with the exception of the one for the manager of the lowest quality project \( \theta \) is larger than the first-best \( P^*(\theta) = \beta (K - \theta)/(\beta - 1) \). Because (B.8) holds with strict equality, we may combine (B.8), (B.12) and (B.21) in order to obtain

\[
\xi = \int_{\theta}^{\tilde{\theta}} M(\theta) \left( \frac{P_0}{P_J(\theta)} \right)^\beta d\theta .
\]

Solving the above equation gives the Lagrangian multiplier \( l \). It is straightforward to show that the Lagrangian multiplier \( l \) increases in effort cost \( \xi \), in that

\[
\frac{dl}{d\xi} = \frac{P_0}{\beta - 1} \left[ \int_{\theta}^{\tilde{\theta}} M^2(\theta) \left( \frac{\beta}{\beta - 1} \right)^2 \left( \frac{P_0}{P_J(\theta)} \right)^{\beta + 1} d\theta \right]^{-1}.
\]

Therefore, as effort cost \( \xi \) increases, the optimal trigger \( P_J(\theta) \) decreases, as shown below:

\[
\frac{dP_J(\theta)}{d\xi} = -\frac{\beta}{\beta - 1} \frac{M(\theta)}{f_H(\theta)} \frac{dl}{d\xi} < 0.
\]
This is consistent with our intuition and results in Section 3 that a higher effort cost mitigates investment inefficiency by pushing the exercise trigger towards the first-best level.

The following two conditions ensure that the conjectured candidate solutions $P_J(\theta)$ is positive and decreasing in $\theta$.

**Condition 5.**
\[
\frac{d}{d\theta} \left( \theta - \lambda_H(\theta) + l \frac{M(\theta)}{f_H(\theta)} \right) > 0, \quad (B.25)
\]
for $0 \leq l \leq 1$.

**Condition 6.** The distribution $F_H(\cdot)$ first-order stochastically dominates $F_L(\cdot)$, in that
\[
F_H(\theta) \leq F_L(\theta), \quad \text{for all } \theta. \quad (B.26)
\]
This implies $M(\theta) \geq 0$, for all $\theta$.

We note that, under Conditions 5 and 6, wage is positive and increasing in $\theta$, in that
\[
\begin{align*}
w(\theta_2) &= \int_{\theta_2}^{\theta} \left( \frac{P(\theta_2)}{P(s)} \right)^\beta ds > \int_{\theta_2}^{\theta_1} \left( \frac{P(\theta_1)}{P(s)} \right)^\beta ds > \int_{\theta_2}^{\theta_1} \left( \frac{P(\theta_1)}{P(s)} \right)^\beta ds = w(\theta_1), \quad (B.27)
\end{align*}
\]
for $\theta_2 > \theta_1$. The first inequality follows from the monotonicity of $P(\theta)$.

## C Derivations of Optimal Contracts in Section 5

This appendix provides a derivation of the optimal contracts for the generalized model of Section 5. Propositions 1 and 2 apply as in Appendix A. Using the method of Kuhn-Tucker, we form the Lagrangian as follows:

\[
\mathcal{L} = \left( \frac{P_0}{P_1} \right)^\beta (P_1 + \theta_1 - K - w_1) + \frac{1 - q_H}{q_H} \left( \frac{P_0}{P_2} \right)^\beta (P_2 + \theta_2 - K - w_2) \\
+ \lambda_1 \left[ \left( \frac{P_0}{P_1} \right)^\gamma w_1 - \left( \frac{P_0}{P_2} \right)^\gamma (w_2 + \Delta \theta) \right] + \lambda_2 \left[ \left( \frac{P_0}{P_2} \right)^\gamma w_2 - \left( \frac{P_0}{P_1} \right)^\gamma (w_1 - \Delta \theta) \right] \\
+ \lambda_3 \left[ \left( \frac{P_0}{P_1} \right)^\gamma w_1 - \left( \frac{P_0}{P_2} \right)^\gamma w_2 - \frac{\xi}{\Delta q} \right] + \lambda_4 w_2, \quad (C.1)
\]

with corresponding complementary slackness conditions for the four constraints. As in Appendix A, we also conjecture that the ex-post incentive constraint does not bind, in that the Lagrangian multiplier $\lambda_2$ associated with the constraint below is zero:

\[
\left( \frac{P_0}{P_2} \right)^\gamma w_2 \geq \left( \frac{P_0}{P_1} \right)^\gamma (w_1 - \Delta \theta). \quad (C.2)
\]
We will verify this conjecture for each region.

The first-order conditions with respect to $w_1$ and $w_2$ imply

$$0 = (\lambda_1 + \lambda_3) \left( \frac{P_0}{P_1} \right)^{\gamma} - \left( \frac{P_0}{P_1} \right)^{\beta}, \quad (C.3)$$

$$0 = - (\lambda_1 + \lambda_3) \left( \frac{P_0}{P_2} \right)^{\gamma} - 1 - q_H \left( \frac{P_0}{P_2} \right)^{\beta} + \lambda_4. \quad (C.4)$$

Using (C.3) to simplify (C.4) gives $\lambda_4 > 0$. The complementary slackness condition implies that $w_2 = 0$. The first-order conditions with respect to $P_1$ and $P_2$ are given by

$$P_1 = \frac{\beta}{\beta - 1} \left[ K - \theta_1 + \left( 1 - \frac{\gamma}{\beta} \right) w_1 \right], \quad (C.5)$$

$$P_2 = \frac{\beta}{\beta - 1} \left[ K - \theta_2 + \lambda_1 q_H \frac{\gamma}{1 - q_H \beta} \left( \frac{P_0}{P_2} \right)^{\gamma - \beta} \Delta \theta \right]. \quad (C.6)$$

Therefore, it must be the case that at least one of (53) and (54) binds (Similar to Proposition 5 of Appendix A). Depending on the cost benefit ratio $\xi/\Delta q$, we have three disjoint regions to be analyzed below, similar to the analyses in Section 3.

### C.1 The Hidden Information Only Region

Suppose that the constraint (54) does not bind and thus $\lambda_3 = 0$. Then, $\lambda_1 = (P_0/P_1)^{\beta - \gamma} > 1$. A binding ex-ante incentive constraint (53) implies that the wage payment is $w_1 = (P_1/P_2)^{\gamma} \Delta \theta$. The first-order conditions (C.5) and (C.6) give the following coupled equations:

$$P_1 = \frac{\beta}{\beta - 1} \left[ K - \theta_1 + \left( 1 - \frac{\gamma}{\beta} \right) \left( \frac{P_1}{P_2} \right)^{\gamma} \Delta \theta \right], \quad (C.7)$$

$$P_2 = \frac{\beta}{\beta - 1} \left[ K - \theta_2 + \lambda_1 q_H \frac{\gamma}{1 - q_H \beta} \left( \frac{P_0}{P_2} \right)^{\gamma - \beta} \Delta \theta \right]. \quad (C.8)$$

Note that with $\gamma > \beta$, we immediately have $P_1 < P_1^*$ and $P_2 > P_2^*$. Therefore, $w_1 < \Delta \theta$, as conjectured, confirming that (C.2) does not bind and $\lambda_2 = 0$.

Define the ratio $x = P_1/P_2$. The coupled equations (C.7) and (C.8) allow us to first solve for ratio $x^*$, in that

$$G(x^*) = 0, \quad (C.9)$$

where

$$G(x) = x \left[ K - \theta_2 + \frac{\gamma}{\beta} \left( \frac{q_H}{1 - q_H} \right) x^{\gamma - \beta} \Delta \theta \right] - \left[ K - \theta_1 + \left( 1 - \frac{\gamma}{\beta} \right) x^{\gamma} \Delta \theta \right] = 0. \quad (C.10)$$
First, note that $G(0) = -(K - \theta_1) < 0$ and $G(1) = \gamma \Delta \theta / (\beta (1-q_H)) > 0$. Second,

$$G'(x) = K - \theta_2 + \frac{\gamma + (\gamma - \beta) q_H}{\beta} \frac{\gamma - \beta}{1 - q_H} x^\gamma \Delta \theta + \gamma x^\gamma \Delta \theta > 0,$$

for $\gamma > \beta$. Therefore, there exist a unique $x^* \in (0, 1)$ solving (C.9).

Therefore, for the region defined by $\xi/\Delta q < (P/\hat{P}_3^*)^\gamma \Delta \theta$, where

$$\hat{P}_3^* = \frac{\beta}{\beta - 1} \left[ K - \theta_2 + \frac{q_H}{1 - q_H} \beta (x^*)^{\gamma - \beta} \Delta \theta \right],$$

the optimal contract can be written as:

$$P_1 = \frac{\beta}{\beta - 1} \left[ K - \theta_1 + \left( 1 - \frac{\gamma}{\beta} \right) (x^*)^\gamma \Delta \theta \right],$$

$$P_2 = \hat{P}_3^*,$$

$$w_1 = \left( \frac{P_1}{P_2} \right)^\gamma \Delta \theta,$$

$$w_2 = 0.$$

Finally, if $\xi/\Delta q < (P_0/\hat{P}_3^*)^\gamma \Delta \theta$, constraint (54) is indeed not binding, consistent with our conjecture.

### C.2 The Joint Hidden Information/Hidden Action Region

We derive the optimal contract in this region by conjecturing that both (53) and (54) bind. Solving these two equality constraints gives (56) and (57). Plugging (56) and (57) into the first-order condition (C.5) gives

$$P_1 = \frac{\beta}{\beta - 1} \left[ K - \theta_1 + \left( 1 - \frac{\gamma}{\beta} \right) \left( \frac{P_1}{P_2} \right)^\gamma \Delta \theta \right].$$

The solution for $P_1$ is $\hat{P}_1$, the same solution for $P_1$ as in the hidden action region. In Section 5 we proved that a unique $\hat{P}_1$ exists, where $\hat{P}_1 \in (0, P_1^*)$. Naturally, we have $w_1 = \left( P_1/\hat{P}_J \right)^\gamma \Delta \theta$. As before, we have verified that (C.2) does not bind in this region, because $\hat{P}_1 < \hat{P}_J$ implies that $w_1 < \Delta \theta$.

We know that the only possible regions in which both (53) and (54) bind is $(P_0/\hat{P}_3^*)^\gamma \Delta \theta \leq \xi/\Delta q < (P_0/P_2^*)^\gamma \Delta \theta$, since we have already shown that in the other regions only one of these constraints binds.\(^38\) Equivalently stated in terms of $\hat{P}_J$, this region is characterized by

\(^{38}\)Note that in the region $\xi/\Delta q > (P_0/\hat{P}_1)^\gamma \Delta \theta$, it can be shown that effort cannot be induced. This result is available upon request.
\[ P_2^* < \hat{P}_J < \hat{P}_3^* \]. We now verify that the above solutions are indeed optimal for this entire region. Recall that \( \lambda_1 + \lambda_3 = \left( \frac{P_0}{\hat{P}_1} \right)^{\beta - \gamma} \). Therefore, if we show that \( \lambda_1 \) lies within the range defined by

\[
0 < \lambda_1 < \left( \frac{P_0}{\hat{P}_1} \right)^{\beta - \gamma}, \tag{C.17}
\]

then we have shown both (53) and (54) bind \((\lambda_1, \lambda_3 \neq 0)\).

The first-order condition with respect to \( P_2 \) implies that

\[
\lambda_1 = \left[ \frac{\beta}{\beta - 1} - \frac{q_H}{\beta} \gamma \left( \frac{P_0}{\hat{P}_1} \right)^{\gamma - \beta} J_{\theta} \right]^{-1} \left( \hat{P}_J - P^*_2 \right). \tag{C.18}
\]

Since \( \hat{P}_J > P^*_2 \), we have confirmed that \( \lambda_1 > 0 \). We next prove that \( \lambda_1 < \left( \frac{P_0}{\hat{P}_1} \right)^{\beta - \gamma} \).

Expressing \( \lambda_1 \) as a function of \( \hat{P}_J \), we can rewrite (C.18) as:

\[
\lambda_1(\hat{P}_J) = \left( \frac{P_0}{\hat{P}_1(\hat{P}_J)} \right)^{\beta - \gamma} \left( \frac{\hat{P}_1(\hat{P}_J)}{\hat{P}_J} \right)^{\beta - \gamma} \left[ \frac{q_H}{1 - q_H} \frac{\gamma}{\beta} (P^*_2 - P^*_1) \right]^{-1} \left( \hat{P}_J - P^*_2 \right). \tag{C.19}
\]

Note that from (C.16), \( \hat{P}_1 \) is a function of \( \hat{P}_J \); we make this functional dependence explicit in the above equation. Proving that \( \lambda_1 < \left( \frac{P_0}{\hat{P}_1} \right)^{\beta - \gamma} \) over the region \( \hat{P}_J \in (P^*_2, \hat{P}^*_3) \) is equivalent to showing that

\[ N(x) > 0, \quad \text{for} \quad x \in (P^*_2, \hat{P}^*_3), \]

where \( N(x) \) is defined by

\[
N(x) = P^*_2 + \left( \frac{\hat{P}_1(x)}{x} \right)^{\gamma - \beta} \frac{q_H}{1 - q_H} \frac{\gamma}{\beta} (P^*_2 - P^*_1) - x. \tag{C.20}
\]

Using implicit differentiation in (C.16), we can write:

\[
\frac{d\hat{P}_1(x)}{dx} = \frac{\hat{P}_1(x)}{x} \frac{\gamma(\hat{P}_1(x) - P^*_1)}{\gamma(\hat{P}_1(x) - P^*_1) - \hat{P}_1(x)} > 0, \tag{C.21}
\]

because \( \hat{P}_1(x) < P^*_1 \) in this region. Therefore,

\[
\frac{dL(x)}{dx} = (\gamma - \beta) \frac{q_H}{1 - q_H} \frac{\gamma}{\beta} (P^*_2 - P^*_1) \left( \frac{\hat{P}_1(x)}{x} \right)^{\gamma - \beta - 1} \frac{1}{x^2} \left( x \frac{d\hat{P}_1(x)}{dx} - \hat{P}_1(x) \right) - 1. \tag{C.22}
\]

From (C.21),

\[
x \frac{d\hat{P}_1(x)}{dx} - \hat{P}_1(x) = \frac{\left( \frac{\hat{P}_1(x)}{x} \right)^2}{\gamma(\hat{P}_1(x) - P^*_1) - \hat{P}_1(x)} < 0, \tag{C.23}
\]

because \( \hat{P}_1(x) < P^*_1 \) in this region. Therefore, \( N'(x) < 0 \), for \( x \in (P^*_2, \hat{P}^*_3) \). Since \( N(\hat{P}^*_3) = 0 \), we thus have \( N(x) > 0 \) for \( x \in (P^*_2, \hat{P}^*_3) \). This confirms that \( \lambda_1, \lambda_3 > 0 \) in this entire region, and therefore both (53) and (54) bind.
C.3 The Hidden Action Only Region

Suppose that (54) binds, while (53) does not. Thus, $\lambda_1 = 0$ and $\lambda_3 = (P_0/P_1)^{\beta-\gamma} > 1$. With $\lambda_1 = 0$, equation (C.6) implies that $P_2 = P_2^*$. A binding (54) implies that the wage payment is

$$w_1 = \left( \frac{P_1}{P_0} \right)^\gamma \frac{\xi}{\Delta q} = \left( \frac{P_1}{\hat{P}_J} \right)^\gamma \Delta \theta,$$

where $\hat{P}_J$ is given in (56). Substituting (C.24) into (C.5) gives $P_1 = \hat{P}_1$, the root of the expression given in (59). Section 5 proves that a unique $\hat{P}_1$ exists, where $\hat{P}_1 \in (0, P_1^*)$.

To ensure that our conjecture that (C.2) is not binding, (C.24) implies that we need to check if $\hat{P}_1 < \hat{P}_J$ holds. This inequality can be written as $\xi/\Delta q < (P_0/\hat{P}_1)^\gamma \Delta \theta$, which is assured to hold in this region. In order to be consistent with the fact that (53) does not bind, we need $(P_0/P_2^*)^\gamma \Delta \theta > \xi/\Delta q$, which again holds in this region.
References


Figure 1: Optimal incentive contracts across the three parameter regions. The upper and lower graphs plot the equilibrium trigger strategy $P_2$ and wage payment $w_1$ in terms of effort cost $\xi$, respectively. As the cost of effort increases, the hidden action problem becomes more pronounced. The upper graph demonstrates that as the cost of effort increases, the equilibrium trigger strategy $P_2$ decreases, as it approaches the first best trigger $P_2^*$. The lower graph demonstrates that as the cost of effort increases, the wage payment must increase in order to induce effort from the manager. In summary, as the cost of inducing hidden effort increases, the timing of investment becomes more efficient while the value of the compensation package increases.
Figure 2: Stock price reaction to investment. This graph plots the stock price as a function of $P$, the present value of the observed component of cash flows. Whenever the level of $P$ is below the lower investment trigger $P_1^*$, the market does not know the true value of $\theta$, the present value of the unobserved component of cash flows. Thus, for all $P$ below $P_1^*$, the stock price equals the value of the owner’s option given in (31). At the moment the process $P$ hits the trigger $P_1^*$, the true value of $\theta$ is revealed through the manager’s action: if the manager invests, then the value of $\theta$ is the higher value $\theta_1$; if the manager does not invest, then the value of $\theta$ is the lower value $\theta_2$. Thus, the stock price is discontinuous at $P_1^*$. Investment signals good news and the stock price jumps to $S_u$, while failure to invest signals bad news and the stock price drops to $S_d$, where $S_u$ and $S_d$ are given in (37) and (38), respectively.