Designing Efficient Mortgage Foreclosure Sale

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Abstract

Lenders with delinquent mortgages recover their lending by foreclosure, which is a legal process to sell the mortgage property via public auction. In the U.S., mortgage lenders are allowed to bid in such foreclosure auctions, and they win in such auctions very frequently. I study the question of why mortgage lenders win in most of those auctions. I develop a theoretical model of ascending auctions with private values. I find that the lender’s optimal bidding strategy is the same as the optimal reserve price of an auction seller, if it is below the debt balance. In other words, the lender exercises monopoly power as would an auction seller, up to the remaining debt. I next derive a mechanism that achieves efficient allocation of the foreclosed property. Finally, I match the model with data for 15,860 foreclosure auctions in Miami-Date County, Florida. The lender’s monopoly power partially explains the reason why the lender wins in 88% of the auctions. I also find evidence that mortgage lenders may have a systematically higher valuation of the mortgage property compared to other bidders.

1 Introduction

Especially after the financial crisis, mortgage foreclosure has drawn much attention in the public and the academic world. Indeed, foreclosures have a profound effect on the economy. Mian et al. (2015) analyze the negative effect of foreclosures on housing market and its impact on real economy: between 2007 and 2009, foreclosures account for 33% of the decline in house prices, 20% of the decline in residential investment, and 20% of the decline in auto sales in the United States. As such, designing an efficient foreclosure system is highly important to the economy.

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Yet, sale of foreclosed properties exhibits characteristics that cast serious doubts on its efficiency. Mortgage lenders\(^1\) win a vast majority of foreclosure auctions, despite that their ultimate objective is to resell the foreclosed property and gain cash. In Miami-Dade County of Florida that is the focus of this paper, mortgage lenders won 88% of the foreclosure auctions conducted between January and July, 2010. This is a widely observed phenomenon across time and states; see, for instance, Wechsler (1985); Stark (1997); Lambie-Hanson et al. (2015). Wechsler (1985) condemns that foreclosure by sale “often functions as a meaningless ceremony whereby the mortgagee exchanges the property for the debt” (p. 884).

This paper presents a simple model that explains why lenders tend to win foreclosure auctions and a mechanism that achieves efficient allocation of foreclosed property. First, I model the currently dominant foreclosure auction mechanism as an ascending auction of independent private values, and derive that the lender bids higher than her value of the foreclosed property while other bidders bid their values. The lender’s optimal bidding strategy is exactly the optimal reserve price of an auction seller, if it is below the debt balance. Intuitively, the lender behaves as seller up to the debt balance and as buyer beyond it. The monopoly power conferred to the lender implies that foreclosed properties are not efficiently allocated under the current system. Second, I derive a mechanism that achieves efficient allocation of the foreclosed property. There are multiple possibilities of efficient mechanisms. Among them, I focus on the mechanism that yields the highest payoff to the lender. This lender-optimal mechanism first releases the mortgage debt in full, then runs a second price auction among the bidders including the lender, and distributes the proceeds between the lender and the borrower. The borrower may receive some amount even if the lender does not collect the debt fully.

I then test whether the model prediction above is consistent with data. I use data for 15,860 foreclosure auctions in Miami-Dade County, Florida that were conducted between January and July, 2010 and finished with a sale. I first observe that the lenders’ bidding behavior is not consistent with model prediction for the small number of cases with debt balance less than 60,000 USD. In these low-debt cases, the lenders submit the debt balance as maximum bids. This is puzzling because property values tend to be higher than the debt balance. In cases with more than 60,000 USD of debt, the lenders’ bidding seems consistent with model prediction. However, lenders’ monopoly power in setting reserve prices may not be a sufficient explanation for the extremely high chance that lenders win or the lenders’ maximum bids that seems to be several times higher than the actual property value. To test this, I run a simulation assuming that the lender and the bidders face potential buyers at resale that have the same lognormal value distribution. The simulation results

\(^{1}\)More accurately, the owner of the mortgage property is called “mortgagor,” while the creditor of the debt for whom the property is mortgaged is called “mortgagee.” In this paper, I use the terms “(mortgage) borrower” and “(mortgage) lender” that the readers are probably more familiar with, even though the mortgage debt does not necessarily come from a loan.
can be consistent with data only if we assume a substantially skewed value distribution of resale buyers. This implies that the lenders may have a systematically higher valuation compared to other bidders.

Legal studies have pointed out the possible inefficiencies of foreclosure sale in the United States. Remarkably, some legal scholars have collected data to analyze efficiencies of foreclosure sales. Wechsler (1985) is the first of those studies; he analyzed all mortgage foreclosure actions that begun in Onondaga county, New York, during 1979 and ended in completion of foreclosure sale. Stark (1997) followed, with data on all foreclosure actions commenced in Cook County Chancery Court of Illinois in July 1993 and in July 1994. Though these studies offer valuable insights, they offer only descriptive evidence and are limited in their depth of economic analysis and in the data size (118 in Wechsler (1985) and 276 in Stark (1997)). Surprisingly, there have been very few economic studies on foreclosure auction process in general.

To the best of my knowledge, Niedermayer et al. (2018) is the only economic analysis of foreclosure sale process, and this work does not analyze efficiency of foreclosure auctions. This is in stark contrast to the volume of economic literature that analyzes the consequences of foreclosure on real property market and the economy in general, taking the current situation of foreclosure sale as given. Niedermayer et al. (2018) develops a model of foreclosure auctions and test their theoretical predictions with data of foreclosure auctions in Palm Beach County, Florida. They model the auctions as second-price auctions, allowing for the possibility of common values between the seller and the bidders. They find that the data are consistent with (i) the model predictions of bunching of lender’s bids at the judgment amount and (ii) information asymmetry between the lender and the bidders. My work extends their framework to ascending auctions and demonstrate that the same intuition — the lender behaving as auction seller up to the debt balance — holds, while abstracting away from informational asymmetry explained by a common value model. I also address the question of why the lenders win very frequently, which is not the focus of Niedermayer et al. (2018).

The paper proceeds as follows. Section 2 gives an overview of foreclosure sale system in the United States in general and Miami-Dade County, FL in particular. Section 3 develops a model of foreclosure auction. After setting up the model framework in Subsection 3.1, Subsection 3.2 analyzes ascending auction, the current foreclosure sale system. Subsection 3.3 solves for efficient mechanism of foreclosure sale. Section 4 analyzes the data for foreclosure auctions in Miami-Dade County, FL. Section 5 discusses the results and possible directions for future research.
2 Overview of Foreclosure Sale in the United States

2.1 Brief History and Purpose of Foreclosure Sale

Mortgage is a collateral of debt payment based on the value of land, and mortgage foreclosure is a legally mandated procedure to enforce mortgage lender’s rights to collect debt out of the land value. To understand the objective of mortgage foreclosure sale, let us briefly review the history of mortgage foreclosure (see Wechsler (1985), Section I).

In its beginning under English law, mortgage was transfer of land to the lender with the condition that the borrower regained title if he repaid the debt fully on time. If the borrower failed to repay the debt by the due date, the borrower immediately lost any right to the land. Because of the harshness of this treatment, equity courts started to protect mortgage borrowers even if they failed to pay on time. By the end of the seventeenth century in England, equity courts routinely granted delinquent mortgage borrowers to redeem (i.e., regain by payment) their land within a reasonable period after the due date. Now the pendulum had swung to borrower-friendly extreme, because mortgage lenders could not be certain when their right to mortgage property would be secure. As a result of pressures from lenders, courts started to grant “foreclosure,” which meant terminating the borrower’s right to redeem the mortgage property.

In its original form, the foreclosure action left the mortgage lender with full title to the property and resulted, in effect, in the mortgagee exchanging debt for land. This procedure is now called “strict foreclosure.” Although American laws on foreclosure initially followed this form of English law, most states quickly switched to “foreclosure by sale” by the early 1800s, which is still the dominant regime. Under foreclosure by sale, the mortgage property is sold by public sale (auction), and the proceeds will be allocated to debt payment. If the proceeds exceed the debt amount, the remainder will be paid to the borrower. If, on the contrary, the debt exceeds the sale proceeds, then the lender is allowed to collect the remaining balance by obtaining a judgment (called deficiency judgment).

Foreclosure by sale gained popularity in legal practice because it avoided the harshness of strict foreclosure that the mortgage borrower loses property that may be far more valuable than the mortgage debt. It was also premised on the assumption that public sales achieve “fair prices” by producing the best possible price from potential buyers. This assumption seems to have been vindicated in the early days of foreclosure by sale, because property values were rising rapidly and foreclosure sale often resulted in surpluses above debt balance. However, during the economic depression of the 1820s, it became apparent that the public was only willing to pay bargain prices at foreclosure sales, and mortgage lenders usually won at foreclosure sales, paying low prices. There were legislative responses to such phenomenon (like strengthening the borrower’s right to redeem), but the auction mechanism of foreclosure sale has remained basically the same despite
the fact that the auction usually fails to sell to third party bidders.

Although the law seems to expect that the foreclosure auction maximizes sale price, it seems silent about what the objective of foreclosure sale is in the majority case when the lender wins the auction. As such, this paper focuses on the efficiency of allocating the mortgage property as the principal objective of foreclosure sale.

2.2 Current Foreclosure Sale System

Mortgage foreclosure process in the United States is governed by state laws, and is therefore different from state to state. With the exception of a few states that adopts strict foreclosure, mortgage property is foreclosed in either of two forms: judicial foreclosure sale and nonjudicial foreclosure sale (also called power of sale). The lender must file a court action to foreclose on the property in the former, while the latter requires procedures with minimal or no court involvement. Judicial foreclosure is available in every state, while nonjudicial foreclosure is available only in some states. Since judicial foreclosure sale requires more time and cost, lenders usually resort to nonjudicial foreclosure if available. Hence, it is customary to call states that allow nonjudicial foreclosure sale as “nonjudicial foreclosure states” and call the rest “judicial foreclosure states.” (See, for instance, Wechsler (1985) Section I.C. and Stark (1997) Section I.A. for the above.)

In both judicial and nonjudicial foreclosure, foreclosed properties are sold through public sale, which takes the form of English auction (ascending auction) (Nelson and Whitman (2004)), but the precise form may differ from state to state and from county to county. Although the auctions have traditionally been held orally in the courtroom, sheriff’s office or other places, some states / counties have moved to online auctions partially or entirely.

The sale proceeds are paid first to the lender up to the debt balance (which is determined in a judgment before the sale in the case of judicial foreclosure) and then to the borrower if there is excess. If the lender is the winning bidder, the debt balance is deducted from the lender’s payments, so that the lender ends up paying to the borrower only if the price is above the debt balance.

In most cases, foreclosure sale proceeds cannot satisfy the debt balance; in our dataset, the debt balance exceeds the sale proceeds in 95.6% of the samples. As explained, if the sale price is below the debt balance and the lender wants to recover the remaining amount, the lender must bring a lawsuit to obtain a deficiency judgment.

When the lender wins the auction, the lender tries to resell the property. Anecdotally, they aim to resell quickly (Shilling et al. (1990)) by retaining a real estate agent (Stark (1997)) and do not actively maintain the properties acquired through foreclosure (Mallach (2010)). The third party bidders also seem to try to resell the property (or rent it, if the rental market is active). In our dataset, 1,376 out of 1,679 successful third party bidders are corporate entities and thus are
unlikely to be ultimate users of the properties.

2.3 Foreclosure Auction in Miami-Dade County, FL

Florida is a judicial foreclosure state, and every foreclosure sale requires a court order that authorizes sale and is administered by the court pursuant to that order. The court governing Miami-Dade County, FL, started to run foreclosure auctions online in January, 2010, instead of the traditional oral auctions. The county had the seventh-highest number of foreclosed homes in the United States, and online auctions helped clear up the backlog of foreclosure cases after the financial crisis. Going online was also a convenient way to enhance transparency and fairness of the auctions, which used to suffer from dirty tactics by repeat participants. The relevant court document explains the motivation for moving online more fully (Eleventh Judicial Circuit of Florida (2010)):

“(1) the elimination of police officers, security guards, and video cameras at on-site sales to ensure the safety of auction participants; (2) the elimination of impropriety and collusion among bidders by enhancing the fairness and security of the process; (3) increased ability of the Clerk to schedule sales from three to five times weekly and an increased amount of properties that can be sold at each auction; (4) accessibility to interested parties to view, research, bid and manage cases twenty-four hours, seven days a week; ...

Here is a brief explanation of the auction process. Before the auction, potential bidders may access the auction website\(^2\) and browse a list of scheduled auctions for each day. Bidders can (i) see relevant information and (ii) submit maximum bid for proxy bidding, which can be canceled any time before the auction (see below for proxy bidding). Panel (a) of Figure 1 shows an example of auction listing and a popup window for entering maximum bid before the auction starts. Here, potential bidders may see relevant information, including: case number, final judgment amount (debt balance confirmed by the court in its order to conduct foreclosure sale), property address, Parcel ID (assessor’s parcel number (APN) that uniquely identifies the mortgage property to be auctioned, assigned by the Office of the Property Appraiser), assessed value (value of the property assessed by the Office of the Property Appraiser for the purpose of calculating property tax) and plaintiff max bid (the mortgage lender’s maximum bid for proxy bidding; see below for bidding). The plaintiff max bid may be hidden from bidders at the choice of the mortgage lender (in such case, bidders see the word “Hidden” in the auction listing). The list also contains a link to a page detailing the name of the parties, which has links to a copy of the final judgment (court order authorizing sale of the property), websites of the Office of the Property Appraiser providing

\(^2\)https://www.miamidade.realfclose.com/
Figure 1: Examples of auction listing and a popup for submitting bids (Panel (a): before the auction, Panel (b): during the auction). The popup for bidding on the right appears by clicking “Place Bid” button in the auction listing on the left. Final judgment amount refers to the debt balance confirmed by the court in its order to conduct foreclosure sale. Parcel ID is assessor’s parcel number (APN) that uniquely identifies the mortgage property to be auctioned, assigned by the Office of the Property Appraiser. Assessed Value is the value of the property assessed by the Office of the Property Appraiser for the purpose of calculating property tax. Plaintiff max bid is the mortgage lender’s maximum bid for proxy bidding.

On each day, the auctions proceed sequentially, with one auction starting after another closes. The format is an ascending auction, starting from $0 and receiving bids in $100 increments. The auction lasts for 1 minute, and will be extended for 1 minute every time a new high bid is submitted with less than 30 seconds remaining. Panel (b) of Figure 1 shows how the auction listing and bidding popup window look like when the auction is active. The bidders see the current high bid at each moment, but do not observe to whom the high bid belongs (except when it is obvious from the lender’s maximum bid) or the identity or number of other participating bidders.

The bidders submit maximum bid for proxy bidding, not the bids themselves. This works as follows: each bidder specifies a “maximum bid,” and the auction platform submits a bid on behalf of the bidder so that the new bid exceeds the current high bid by $100 if it is at or below the specified maximum bid. The maximum bid can be freely modified or canceled before the auction starts; it can only be modified upwards after the auction starts. If the plaintiff’s maximum bid is
disclosed, bidders are discouraged from submitting bids below the plaintiff’s maximum bid since
they have no chance of winning by submitting those bids.\textsuperscript{34}

3 A Model of Foreclosure Sale

3.1 Setup

Consider the following setting of $I + 2$ players: the mortgage lender (numbered 0), third party
bidders $1, \ldots, I$ and the mortgage borrower whose property is being foreclosed. The borrower is
a passive player who does not take any action. Each player $i \in \{0, \ldots, I\}$ other than the borrower
values the foreclosed property at $\theta_i$. Let $\theta_0, \ldots, \theta_I$ be independently drawn from distributions with
CDFs $\theta_0 \sim F_0$ and $\theta_i \sim F$ for $i \geq 1$, with support $[0, \theta_0]$ and $[0, \theta]$ respectively. I assume that (i)
$F$ is continuous, (ii) the corresponding PDF $f$ is positive and continuous in the interior of their
supports and (iii) $F$ is regular, i.e., $\text{MR}(\theta) = \theta - (1 - F(\theta))/f(\theta)$ is strictly increasing in $\theta$. Let
$\Theta = [0, \theta_0] \times [0, \theta]^I$ denote the type space.

We consider mechanisms through which each player $i \in \{0, \ldots, I\}$ other than the borrower
obtains the property with probability $x_i$ and pays $t_i$. The proceeds $\sum_{i=0}^I t_i$ are then paid to the lender
up to $D$, the debt balance at the time of foreclosure sale, and then to the borrower beyond $D$. In
other words, the lender obtains $\pi(\sum_{i=0}^I t_i)$ and the borrower $\max\{0, \sum_{i=0}^I t_i - D\}$ from the sale
proceeds, where $\pi(p) = \min\{p, D\}$.

I assume the players have quasi-linear utility in money. Hence, payoffs are $U_0 = \theta_0 x_0 - t_0 +$ $\pi(\sum_{i=0}^I t_i)$ for the lender, $U_i = \theta_i x_i - t_i$ for other bidders $(i \geq 1)$ and $U_{\text{borrower}} = \max\{0, \sum_{i=0}^I t_i - D\}$ for the borrower. (I ignore the possibility that the lender recovers via deficiency judgment.)

3.2 Analysis of Ascending Auction

3.2.1 Auction Mechanism

First, I solve for the equilibrium of ascending auction, with a twist. Based on the auction format in
Miami-Dade County described in Subsection 2.3, I analyze the following auction:

- Stage 1: The lender submits its maximum bid and chooses whether to disclose the maximum
  bid to other bidders or not.

\textsuperscript{3}As a penalty, bidders will be barred from attending future auctions for two consecutive auction dates if they submit bids below the lender’s maximal bid that is disclosed. Repeat offenders may be permanently banned from the auction process.

\textsuperscript{4}As a minor point, bidders must pay a nonrefundable deposit of 5% of their bids before the auction date.
• Stage 2: The auction opens, and all third party bidders submit bids that start from 0 and increase in infinitesimally small increments.

Here, I assume that the lender moves first and will not revise the maximum bid during the auction. In reality, the lender might revise its bids until the end of the auction as described before. But given the large volume of auctions conducted each day and the short duration of each auction, it is probably rare that lenders revise their maximum bids during the auction.

3.2.2 Solving for Equilibrium

In Stage 2, it is a weakly dominant strategy for all third party bidders to bid their values, regardless of the lender’s action in Stage 1.

In Stage 1, note first that the lender is indifferent between disclosing her bid or keeping it confidential. This is because her bid does not affect other bidders’ bids in Stage 2.

Let $\theta_{-0}^{(n)}$ be the $n$-th order statistic of $\theta_1, \ldots, \theta_I$, and $F^{(n)}, f^{(n)}$ be the corresponding CDF and PDF. To describe the behavior of $\mathbb{E} [U_0 | \theta_0, b_0]$ as a function of $b_0$, define $\theta_{r-0} = \theta_{r-0}(\theta_0)$ as a value that satisfies:

$$\theta_0 = \text{MR}(\theta_{r-0}),$$

with $\theta_{r-0}(\theta_0) = 0$ if such value does not exist (i.e., $\theta_0 > \theta$). We obtain the following result (all proofs are in the Appendix):

**Proposition 1.** In the ascending auction model of foreclosure sale, in Stage 1, the lender’s optimal bid function $b_0^*(\theta_0)$ is as follows:

$$b_0^*(\theta_0) = \begin{cases} 
\min \{ \theta_{r-0}(\theta_0), D, \theta \} & (\theta_0 \leq D) \\
\min \{ \theta_0, \theta \} & (\theta_0 > D). 
\end{cases}$$

The lender is indifferent between disclosing her bid or keeping it confidential. In Stage 2, it is a weakly dominant strategy for all third party bidders to bid their values, regardless of the lender’s action in stage 1.

See figure 2 for an example of optimal bid function.

3.2.3 Discussion

Proposition 1 indicates that the lender’s problem of deciding maximum bid is equivalent to auction seller’s problem of setting the reserve price if it is below the debt balance (Myerson (1981)), and to buyer’s problem of setting drop-out price if it is above the debt balance. The intuition is as follows. Consider first that $\theta_0$ is very low, and that the lender considers raising the maximum
Figure 2: Lender’s optimal bid function when $F = \text{Uniform}[0, 1]$ and $D = 0.65$. The bid function is depicted in solid line. The brown line corresponds to $\theta_0^*(\theta_0)$, and the green line corresponds to 45 degree line.

The lender faces the following tradeoff: (1) raising the maximum bid raises the revenue from sale if a single third party bidder bids above the maximum bid, while (2) reducing the chance that the property will be sold to a third party bidder and increasing the chance that the lender is confined to $\theta_0$, the reservation utility. This is exactly the tradeoff faced by an auction seller in setting the reserve price. As $\theta_0$ increases, the optimal maximum bid reaches $D$, in which case the effect (1) disappears since the lender will not collect revenue above $D$. As such, the optimal maximum bid forms a plateau at $D$. If $\theta_0$ is above $D$, bidding above or below $\theta_0$ is a weakly dominated strategy, which is a variant of the familiar argument for plain vanilla ascending auction.\(^5\) In short, the lender behaves as seller up to the debt amount and as buyer beyond that (Niedermayer et al. (2018)), which is induced by the payoff structure that the lender obtains the sale proceeds up to the debt amount.

This result reveals that the lender bids higher than her value of the property up to the debt balance. In other words, the lender’s monopoly power in setting “reserve price” generates allocative inefficiencies. Since most foreclosed properties sell below the debt amount, it is a plausible explanation to why the lender wins most of the foreclosure auctions.

\(^5\)A minor difference is that the lender obtains up to $D$ if she loses the auction. But this does not change the argument since it is smaller than $\theta_0$. 

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3.3 Designing Efficient Mechanism

3.3.1 Constraints in Designing Mechanism

Next, I design a mechanism that achieves efficient allocation of the foreclosed property under the setup described in Subsection 3.1. I consider mechanisms that are dominant strategy incentive compatible.

By virtue of Revelation Principle, we may limit our attention to direct revelation mechanisms \((x_0, \ldots, x_I, t_0, \ldots, t_I) : \Theta \rightarrow [0, 1]^{I+1} \times \mathbb{R}^{I+1}\), with \(\sum_{i=0}^I x_i(\theta) \leq 1\) for any \(\theta \in \Theta\). As before, \(x_i(\theta)\) is the probability that player \(i\) obtains the foreclosed property and \(t_i(\theta)\) is player \(i\)'s payment, conditional on the reported types being \(\theta\).

Players’ payoffs can be described as follows:

\[
U_0(\theta) = \theta_0 x_0(\theta) - t_0(\theta) + \pi \left( \sum_{i=0}^I t_i(\theta) \right), \quad \text{(Def}_0) \]

\[
U_i(\theta) = \theta_i x_i(\theta) - t_i(\theta), \quad i \in \{1, \ldots, I\}, \quad \text{(Def}_i) \]

\[
U_{\text{borrower}}(\theta) = \max \left\{ 0, \sum_{i=0}^I t_i(\theta) - D \right\}. \quad \text{(O)}
\]

I further limit attention to mechanisms that are ex post individual rational and ex post budget balancing, i.e.,

\[
U_i(\theta) \geq 0 \quad \forall i \in \{0, \ldots, I\}, \forall \theta \in \Theta, \quad \text{(EXPIR)}
\]

\[
\sum_{i=0}^I t_i(\theta) \geq 0, \quad \forall \theta \in \Theta. \quad \text{(BB)}
\]

Moreover, we need to disallow the lender to collect sale proceeds outside of loan collection so that the borrower will be paid if there is excess in sale proceeds:

\[
t_0(\theta) \geq 0, \quad \forall \theta \in \Theta. \quad \text{(O)}
\]

Dominant-strategy incentive compatibility pins down the payoffs a great deal. Adapting the proof of Proposition 23.D.2 of Mas-Colell et al. (1995), we obtain that dominant-strategy incentive compatibility is equivalent to the following, for each of \(i \in \{0, \ldots, I\}\):

\[
U_i(\theta_i, \theta_{-i}) = U_i(0, \theta_{-i}) + \int_0^{\theta_i} x_i(\hat{\theta}_i, \theta_{-i}) d\hat{\theta}_i, \quad \forall \theta_{-i}, \quad \text{(DICFOC}_i) \]

\[
x_i(\theta_i, \theta_{-i}) : \text{nondecreasing in } \theta_i. \quad \text{(M)}
\]
3.3.2 Efficient Mechanisms

Efficient allocation is achieved if (with some arbitrary tie-breaking rule)

\[ x_i(\theta) = 1(\theta_i > \theta_{-i}). \]  

(1)

Our task is to find \( t_0(\cdot), \ldots, t_I(\cdot) \) that satisfy suitable conditions. Note that this allocation satisfies (M), which will be omitted from the discussion below. Under this allocation, \( \text{DICFOC}_i \) is equivalent to

\[ U_i(\theta_i, \theta_{-i}) = (\theta_i - \theta_{1-i})x_i(\theta_i, \theta_{-i}) + U_i(0, \theta_{-i}), \]

where \( \theta_{(n)} \) is the \( n \)-th order statistic of \( \theta_j \)'s for \( j \in \{0, \ldots, I\} \setminus \{i\} \). By (Def\( _0 \)) and (Def\( _i \)),

\[ t_0(\theta) = \theta_{1-0}x_0(\theta) - U_0(0, \theta_{-0}) + \pi \left( \sum_{i=0}^{I} t_i(\theta) \right), \quad (\text{Def'}_0) \]

\[ t_i(\theta) = \theta_{1-i}x_i(\theta) - U_i(0, \theta_{-i}). \quad (\text{Def'}_i) \]

Also, by (DICFOC\( _i \)), (EXPIR) can be reduced to

\[ U_i(0, \theta_{-i}) \geq 0 \quad \forall i \in \{0, \ldots, I\}, \forall \theta_{-i}. \] \hspace{1cm} (EXPIR')

Thus, the problem reduces to choosing a payment mechanism \( t_0(\cdot), \ldots, t_I(\cdot) \) (and lowest-level utility \( U_0(0, \theta_{-0}), \ldots, U_I(0, \theta_{-i}) \)) that satisfies (Def\( _0' \)), (Def\( _i' \)), (EXPIR'), (BB) and (O).

One easy solution is:

\[ t_0(\theta) = D + \theta_{1-0}x_0(\theta), \]

\[ t_i(\theta) = \theta_{1-i}x_i(\theta), \]

\[ U_0(0, \theta_{-0}) = 0, \]

\[ U_i(0, \theta_{-i}) = 0 \quad (i = 1, \ldots, I). \]

In essence, this mechanism (i) releases the debt balance \( D \) in full and then (ii) runs a second price / English auction among the lender and the bidders, the proceeds of which will be paid to the borrower. However, this solution frustrates the main purpose of mortgage, which is to satisfy the loan by selling mortgaged property when the borrower defaults. Unless the lender wins the auction, the lender will get nothing (zero payoff).

3.3.3 Lender-Optimal Efficient Mechanism

There can be many possible payment rules \( \{t_i(\cdot)\} \) that achieve efficient allocation, corresponding to various values of \( \{U_i(0, \theta_{-i})\} \). To achieve the purpose of mortgage system, I solve for lender-optimal efficient mechanism, i.e., the mechanism that maximizes the lender’s payoff among those
that achieve efficient allocation. The lender-optimal efficient mechanism maximizes the lender’s payoff for all profiles of \( \theta \in \Theta \), not just an expected payoff. In other words, it maximizes \( U_0(0, \theta_0) \) for all \( \theta_0 \).

**Proposition 2.** Among efficient mechanisms (i.e., those that achieve (1)) that are dominant strategy incentive compatible, ex post budget balancing and ex post individual rational, the following is a lender-optimal efficient mechanism:

\[
t_0(\theta) = \theta^{(1)}_0 x_0(\theta) + D - \min\left\{ \theta^{(2)}_0, D \right\}, \quad t_i(\theta) = \theta^{(1)}_i x_i(\theta).
\]

Then, the utilities are as follows:

\[
U_0(\theta) = (\theta_0 - \theta^{(1)}_0)x_0(\theta) + \min\left\{ \theta^{(2)}_0, D \right\},
\]

\[
U_i(\theta) = (\theta_i - \theta^{(1)}_i)x_i(\theta) \quad (i \geq 1),
\]

\[
U_{\text{borrower}}(\theta) = \theta^{(2)}_0 - \min\left\{ \theta^{(2)}_0, D \right\}.
\]

(If \( I = 1 \), substitute 0 for \( \theta^{(2)}_0 \).)

This lender-optimal efficient mechanism (i) releases the debt balance \( D \) in full and then (ii) runs a second price / English auction among the lender and the bidders. Out of the proceeds, \( \theta^{(2)}_0 \) is paid to the lender so long as it does not exceed \( D \); the rest is paid to the borrower. It may be informative to compare this mechanism with the “easy solution” described above: of the proceeds that are to be paid under the “easy solution,” \( \theta^{(2)}_0 \) is paid to the lender (assuming it is below \( D \)), which is the maximum amount not affected by the lender’s reporting of \( \theta_0 \) so that the lender will still tell the truth. As a result, the difference \( \theta^{(2)}_0 - \theta^{(2)}_0 \) cannot be distributed among the bidders and will belong to the borrower, even if the debt is not fully satisfied. Note also that this mechanism is detail-free, i.e., it does not require knowledge of distribution of \( \theta_i \).

4 Empirical Analysis of Foreclosure Auctions in Miami-Dade, FL

4.1 Data Description, Summary Statistics

I collect data for all mortgage foreclosure auctions from January to July, 2010 from the website of Miami-Dade Clerk of the Court. The data include: name of the winner, each bid submitted by the platform on behalf of bidders (paired with a bidder ID number assigned by the platform), in addi-
tion to the basic information observed by bidders (see Figure 1 and the description in Subsection 2.3).

Table 1 shows how the scheduled auctions were disposed. Among the 26,276 scheduled auctions, 6,15,860 of them resulted in completion of the auction by sale (either to the mortgage lender or to a third party bidder). 2,237 of them (8.5%) were canceled because the borrower filed for bankruptcy. Most of the remaining cases were canceled before the auction for various reasons, such as that the lender and borrower reached an agreement on rescheduled payments.

Table 2 gives a list of summary statistics. There were 116.1 auctions on average daily, each of which lasted for 201.5 seconds on average. There is a good amount of bidder participation: there were on average 4.97 bidders participating in the auction, and there were only 1,035 auctions (6.5%) that attracted no bids other than from the lender.

For auctions finished with sale, I matched the auction data with the CoreLogic Tax and Deed data to obtain resale price. 8,936 auctions were matched with resale transactions, and resale prices were available in 6,131 of them.

The summary statistics of dollar amounts reveal the features of a typical foreclosure auction. The auction sale price (mean $78,351) is well below the tax assessment value of the property (mean $191,273) and even further below the debt balance (judgment amount; mean $319,184).\(^7\) The lender’s maximum bid, if disclosed (mean $233,813), is on average in between the tax assessment value (and much higher than resale price) and the debt balance.

<table>
<thead>
<tr>
<th>Case Disposition</th>
<th>Obs.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sold</td>
<td>15,860</td>
</tr>
<tr>
<td>Bidder walked away</td>
<td>389</td>
</tr>
<tr>
<td>Canceled per bankruptcy</td>
<td>2,237</td>
</tr>
<tr>
<td>Canceled for other reasons</td>
<td>6,688</td>
</tr>
<tr>
<td>Rescheduled</td>
<td>466</td>
</tr>
<tr>
<td>Other</td>
<td>636</td>
</tr>
<tr>
<td>Total</td>
<td>26,276</td>
</tr>
</tbody>
</table>

Table 1: Case disposition of foreclosure auctions in Miami-Dade County, FL, scheduled between January and July, 2010.

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\(^6\)The Court website includes a small number of auctions other than mortgage foreclosure sale of real property ownership, such as auctions of timeshares and liquor licenses. These auctions are excluded from the dataset.

\(^7\)As explained, bidders are discouraged from submitting bids below the lender’s maximum bid that is disclosed. Thus, auction sale price is biased downward if the lender’s maximum bid is disclosed. However, the general observation in the text holds even if I consider only cases where the lenders’ maximum bids are hidden (mean sale price for such cases is $83,487).
Table 2: Summary statistics of foreclosure auctions in the dataset. The number of bidders includes the mortgage lender but exclude bidders who did not pay enough deposits are excluded. See the notes to Figure 1 for the definition of judgment amount, assessment value and plaintiff max bid.

Lenders win the vast majority of foreclosure auctions; they win 13,942 auctions out of 15,860 (88%) in the dataset. This may be partly due to the lender’s monopoly power in effectively setting reserve prices. In this subsection, I provide evidence as to whether the lenders set their maximum bids in consistent with the bidding strategies in Proposition 1.

4.2.1 Puzzle with Small Debt Cases (Judgment Amount < 60,000 USD)

Panel (a) of Figure 3 plots the relation between debt balance (judgment amount) and auction sale price. The left plot shows auctions where lenders won, and the right plot shows auctions where third party bidders won. The axes are log-scaled, and the blue line represents 45 degree line (all of this apply to panels (b) and (c) as well). We observe one striking feature if debt balance is below 60,000 USD (to the left of the vertical broken lines): the sale price is almost always below the debt balance when the lender wins, while it is most often above the debt balance if a third party bidder wins.

This pattern is explained by the lenders’ bidding behavior. Panel (b) of Figure 3 shows the relation between debt balance and lenders’ maximum bids when such bids are disclosed to bidders. When the debt amount is less than 60,000 USD, the plots are on the 45 degree line, regardless of who wins the auction. This means that the lenders submit maximum bids that are equal to the corresponding debt amounts for cases with less than 60,000 USD of debt. Even though Panel (b) does not show plots for auctions when the lender’s maximum bids are hidden, Panel (a) strongly implies that the lenders’ maximum bids show the same pattern regardless of whether the lender discloses the maximum bids or not, if the debt amount is less than 60,000 USD.

This bidding pattern does not reflect the property values. Panel (c) of Figure 3 shows the
relation between debt balance and resale price, when resale price is available. When the debt balance is less than 60,000 USD, the resale price tends to be higher than the debt balance (which, as explained, is almost equal to lender’s maximum bid) and is not highly correlated with the debt balance.

The bidding pattern observed here seems to be at odds with the model prediction. The model predicts that, if the value of the property is above the debt amount, the lender would submit maximum bid equal to the lender’s valuation of the property. In any case, the model does not anticipate that the lender would submit maximum bids below the valuation. But the data seem to suggest that, for cases with less than 60,000 USD of debt, lenders submit maximum bids that are well below the expected resale value of the property.

This pattern implies the lender may have a different incentive than is modeled above. For instance, lenders may need to incur costs (whether actual costs or some internal, administrative costs) to assess property value, and they do not think the cost is worth if the debt amount (their total investments) is low and just aim to recover the debt. In any case, it still remains a puzzle why lenders win the majority of the auctions in these low-debt cases (567 out of 909 auctions, or 62%).

4.2.2 Large Debt Cases (Judgment Amount \( \geq 60,000 \) USD)

When the debt amount exceeds 60,000 USD, lenders’ bidding behavior is, at first sight, consistent with the model prediction. Panels (b) and (c) of Figure 3 imply that both the lender’s maximum bid and resale price are at or below the debt balance in most cases. Figure 4 shows an empirical PDF of the ratio of lender’s maximum bid (when that is disclosed) to resale price, for cases with judgment amount above 60,000 USD and when lenders won. The blue line shows 45 degree line If we regard the lender’s resale price as a proxy for the lender’s valuation of the property, we expect this ratio to be above 1, since the lender’s maximum bid is essentially a reserve price according to Proposition 1. The empirical PDF is somewhat consistent with this prediction, having most of the mass above 1. There are about 23% of the cases where the ratio is less than 1; this can be explained by the noisy nature of the resale price, which cannot be perfectly foreseen at the time of the auction.

On a closer look, however, the ratio seems to be very fat-tailed to the right, presumably generating the incredibly high chance of lenders’ winning (88%). This raises doubt that the lenders face a systematically higher valuation of the property than other bidders. Figure 5 seems to suggest that

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8In about 32% of auctions where the lender’s maximum bid is disclosed, such bid is higher than the judgment amount. But the ratio of lender’s maximum bid to judgment amount is close to 1 even if it exceeds 1 (90% quantile of the ratio is 1.09). This implies that these maximum bids are often at or below the debt balance, which is the sum of the judgment amount (debt balance as of the judgment date) and the interests from the judgment date. There are at least a few months between the judgment and auction, but unfortunately I do not have this time interval or interest rates in the dataset to compute the debt balance at the time of the auction.
this is the case. This is a scatter plot of auction sale price (horizontal axis) and the resale price (vertical axis) for cases with judgment amount above 60,000 USD and lenders’ maximum bids hidden, again with log scales on both axes and 45 degree line in blue. The left plot shows auctions where lenders won. This plot indicates that, when lenders won, the highest valuation among the third party bidders (auction sale price) is generally lower than the lender’s valuation, proxied by the resale price.

To test if it is likely that the lenders systematically face a higher valuation than third party bidders, I run a simulation of the lender’s winning probability and the ratio of the lender’s maximum bid to resale price. Assume that there are \( n = 4 \) third party bidders (to match the data, where the average number of bidders is 4.97 including the lender), who draw their values \( \theta_i \) from \( F \) independently of each other and of the lender’s value. Let \( F \) be a lognormal distribution with parameters \((-\sigma^2/2, \sigma^2)\). Figure 6 plots the PDF and marginal revenue of \( \theta_i \), i.e., \( f(\theta) \) and \( \text{MR}(\theta) = \theta - (1 - F(\theta))/f(\theta) \). Assume that the lender faces a pool of potential buyers with the same value distribution as that of potential buyers from third party bidders, except that the lender may face a larger number of potential buyers. More specifically, the lender’s proceeds from resale (if the lender wins the auction) is \( \theta^{(1:N)} \) (denoted as \( \theta_0 \)), the first order statistic of \( N \) independent draws from \( F \) that are also independent of the bidders’ valuations (\( N = 1 \) corresponds to the case where the lender faces the exactly the same number of potential buyers as third party bidders do). Assuming that the debt amount is infinitely large, the lender’s optimal maximum bid from Proposition 1 is

\[
\theta^*(\mathbb{E}[\theta_0]) = \text{MR}^{-1}\left(\mathbb{E}\left[\theta^{(1:N)}\right]\right).
\]

The lender wins with probability \( \Pr\left(\theta^{(1:n)} < \theta^*(\mathbb{E}[\theta_0])\right) = F\left(\theta^*(\mathbb{E}[\theta_0])\right)^n \), where \( \theta^{(1:n)} \) is the first order statistic of the valuation of third party bidders. Figure 7 shows the probability of lenders’ winning for different values of \( \sigma \) (horizontal axis) and \( N \) (corresponding to each curve). To get a number close to the actual rate of lender’s winning (88%), we need a high number of \( N \), say \( N = 5 \). If \( N = 2 \), this high win rate may be achieved only if we assume a very high \( \sigma \) (i.e., extremely fat tailed distribution of \( F \)).

Figure 8 shows the PDF of \( \theta^*/\theta_0 \), the ratio of the lender’s maximum bid to the realized resale price. When \( N = 1 \), \( \sigma = 0.6 \) may be a good fit to the empirical PDF in Figure 4, but the lenders’ win rate is too low as mentioned before. When \( N = 5 \), the PDFs shift leftward, probably because \( \theta_0 \) moves upward and away from 0. In this case, \( \sigma = 1 \) seems a good fit, and the lenders’ simulated win rate is close to the observed rate of 88%. However, \( \sigma = 1 \) still implies a substantially skewed value distribution.

\[9\text{The first parameter } \mu \text{ of lognormal distribution merely scales the value of } \theta_i \text{ by a scalar multiplication. Here I picked } \mu = -\sigma^2/2 \text{ so that the mean of } \theta_i \text{ is 1. The variance is exp}(\sigma^2) - 1.\]
These simple analyses show that it may be difficult to achieve the extremely high chance of lender’s winning plus a high maximum bid by lenders, if we assume that the potential resale buyers from the lender and other bidders have the same value distributions. The high rate itself might be explained by a larger number of potential buyers by the lender (though why the lender has a larger pool is not very clear given that lenders retain real estate agents), but assuming a larger pool for the lender would make it difficult to explain the high ratio of lender’s max bid to resale price (i.e., it requires a very skewed value distribution).

5 Discussion

As seen, the current mechanism of foreclosure sale, ascending auction, confers the lender monopoly power and induces her to submit a bid higher than her value of the foreclosed property. This explains, at least partially, why the lender wins the foreclosure auction in most cases. Even if the lenders’ objective is to resell the property, they are better off bidding higher than their expected resale price than lowering bids and attracting sale at the foreclosure auction.

However, this does not seem to be the entire story. First, the lenders set the debt balance as the maximum bid in cases with debt balance less than 60,000 USD, despite the expected resale price that is higher than the maximum bid. In cases with more than 60,000 USD of debt, the lenders’ bidding seems, at first glance, consistent with model prediction that the lender’s maximum bid is effectively a reserve price. But the simulation results imply that the lenders may face a systematically higher valuation at resale compared to third party bidders.

I propose the following potential explanation. The lenders believe the mortgage property will sell at a much higher price at resale than at the foreclosure auction. As such, they often submit maximal bids that are much higher than property value (even higher than optimal reserve price) and so close to the debt balance, just to satisfy the formality of foreclosure and obtain title to the mortgage property. The situation might improve if auctions attract higher bids, but they do not because of informational asymmetry with the lender or the disadvantage of smaller buyer pool compared to negotiated sale at the resale stage (Mayer (1998)). Some facts suggest existence of informational asymmetry: Unlike sale of nondistressed properties, the buyers cannot inspect foreclosed property to mitigate informational asymmetry if the property is occupied. Additionally, there may be title defects (such as senior mortgages) and tax delinquencies. In other words, the market of foreclosure auction has mostly unravelled because of information asymmetry. This story seems consistent with the observation from Panels (a) and (b) of Figure 3: when the third party wins (right hand of the plots), both the sale price and the lender’s maximum bids seem to be

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10This kind of issue will most likely be resolved if the buyers conduct due diligence such as inspecting title documents in county recorder’s office, but the information acquisition costs could be high.
substantially below the debt amount (45 degree line).

A next step would be to find ways to test this story, such as the information asymmetry among bidders analyzed in Hendricks and Porter (1988). However, I note one difference from Hendricks and Porter (1988). They observe that, in a common value auction with asymmetric information, uninformed bidders earn zero profits on average, both theoretically and empirically. But Figure 5 suggests that the third party bidders who won may have consistently made profits (though I do not observe their costs, particularly of investments). This suggests that, in foreclosure auctions, third party bidders’ private information and their information rents may also need to be considered.

Another, subtler reason is private mortgage insurance. Private mortgage insurance insures the mortgage lender for the borrower’s failure to repay the loan, and is issued when the loan amount is above 80% of the value of the mortgage property at the time of mortgage underwriting. If the insurance disbursement amount is calculated based on the lender’s bid at foreclosure auctions, the lender may be incentivized to bid high, regardless of its true valuation of the property.
Appendix

A Proof of Proposition 1

Here, I assume that \( I \geq 2 \) (if \( I = 1 \), some terms in the following equations drop out and the final results are the same). First, I compute the lender’s expected payoff \( \mathbb{E}[U_0 \mid b_0, \theta_0] \) as a function of her drop-out price \( b_0 \) given type \( \theta_0 \). If \( b_0 \leq D \), the lender’s payoff \( U_0 \) after the auction is as follows:

\[
U_0 = \begin{cases} 
\theta_0 & \text{if } \theta_0^{(1)} < b_0 \\
 b_0 & \text{if } \theta_0^{(2)} < b_0 \leq \theta_0^{(1)} \\
 \theta_0^{(2)} & \text{if } b_0 \leq \theta_0^{(2)} \leq D \\
 D & \text{if } \theta_0^{(2)} > D 
\end{cases}
\]

(with probability \( F^{(1)}(b_0) \))

(with probability \( F^{(2)}(b_0) - F^{(1)}(b_0) \))

(with probability \( F^{(2)}(D) - F^{(2)}(b_0) \))

(with probability \( 1 - F^{(2)}(D) \)).

If \( b_0 > D \),

\[
U_0 = \begin{cases} 
\theta_0 & \text{if } \theta_0^{(1)} \leq D \\
 \theta_0 - \theta_0^{(1)} + D & \text{if } D < \theta_0^{(1)} < b_0 \\
 D & \text{if } \theta_0^{(1)} \geq b_0 
\end{cases}
\]

(with probability \( F^{(1)}(D) \))

(with probability \( F^{(1)}(b_0) - F^{(1)}(D) \))

(with probability \( 1 - F^{(1)}(b_0) \)).

Thus, the lender’s payoff from bidding \( b_0 \) expected at Stage 1 is:

\[
\mathbb{E}[U_0 \mid \theta_0, b_0] = 
\begin{cases} 
\theta_0 F^{(1)}(b_0) + b_0 \left( F^{(2)}(b_0) - F^{(1)}(b_0) \right) + \int_{b_0}^{D} \theta_0^{(2)} dF^{(2)}(\theta_0^{(2)}) + D \left( 1 - F^{(2)}(D) \right) & (b_0 \leq D) \\
\theta_0 F^{(1)}(D) + \int_{D}^{b_0} \left( \theta_0 - \theta_0^{(1)} + D \right) dF^{(1)}(\theta_0^{(1)}) + D \left( 1 - F^{(1)}(b_0) \right) & (b_0 > D) 
\end{cases}
\]

The derivative with respect to \( b_0 \) for the first case is:

\[
\theta_0 f^{(1)}(b_0) + \left( F^{(2)}(b_0) - F^{(1)}(b_0) \right) + b_0 \left( f^{(2)}(b_0) - f^{(1)}(b_0) \right) - b_0 f^{(2)}(b_0).
\]

Noting \( F^{(1)}(b_0) = F(b_0)^I \) and \( F^{(2)}(b_0) = IF(b_0)^{I-1} - (I-1)F(b_0)^I \),

\[
\frac{\partial}{\partial b_0} \mathbb{E}[U_0 \mid \theta_0, b_0] = 
\begin{cases} 
f^{(1)}(b_0) \left[ \theta_0 - \text{MR}(b_0) \right] & (b_0 \leq D) \\
\left( \theta_0 - b_0 \right) f^{(1)}(b_0) & (b_0 > D) 
\end{cases}
\]

Then, expected payoff of the lender behaves as follows:

- If \( b_0 \leq D \): For \( b_0 \leq \theta \), if \( \theta^{r_0}(\theta_0) \in [0, \theta] \), then \( \mathbb{E}[U_0 \mid \theta_0, b_0] \) is strictly increasing in \( b_0 \).
for \( b_0 < \theta^*_0(\theta_0) \) and strictly decreasing in \( b_0 \) for \( b_0 > \theta^*_0(\theta_0) \). If \( \theta^*_0(\theta_0) \not\in [0, \overline{\theta}] \), then \( \mathbb{E}[U_0 | \theta_0, b_0] \) is strictly increasing in \( b_0 \). For \( b_0 > \overline{\theta} \), \( \mathbb{E}[U_0 | \theta_0, b_0] \) is constant in \( b_0 \).

- If \( b_0 > D \): For \( b_0 \leq \overline{\theta} \), \( \mathbb{E}[U_0 | \theta_0, b_0] \) is strictly increasing in \( b_0 \) for \( b_0 < \theta_0 \) and strictly decreasing in \( b_0 \) for \( b_0 > \theta_0 \). For \( b_0 > \overline{\theta} \), \( \mathbb{E}[U_0 | \theta_0, b_0] \) is constant in \( b_0 \).

The proposition follows from the fact that (i) \( \theta^*_0(\theta_0) \) is strictly increasing in \( \theta_0 \), and (ii) \( \theta^*_0(\theta_0) \geq \theta_0 \), with strict inequality for \( \theta_0 \in [0, \overline{\theta}] \).

### B Proof of Proposition 2

First, I show that an efficient mechanism must satisfy \( U_0(0, \theta_{-0}) \leq \min \left\{ \theta^{(2)}_{-0}, D \right\} \) for all \( \theta_{-0} \). For each \( \theta \in \Theta \), there are two possibilities:

- \( \sum_{i=0}^{I} t_i(\theta) \geq D \). In this case, (Def\(_0\)) implies that \( t_0(\theta) = \theta^{(1)}_{-0} x_0(\theta) - U_0(0, \theta_{-0}) + D \). Using (Def\(_I\)), \( \sum_{i=0}^{I} t_i(\theta) \geq D \) requires that

\[
D \leq \left( \theta^{(1)}_{-0} x_0(\theta) - U_0(0, \theta_{-0}) + D \right) + \sum_{i=1}^{I} \left( \theta^{(1)}_{-i} x_i(\theta) - U_i(0, \theta_{-i}) \right)
\]

\[
= \sum_{i=0}^{I} \left( \theta^{(1)}_{-i} x_i(\theta) - U_i(0, \theta_{-i}) \right) + D.
\]

Since \( \sum_{i=0}^{I} \theta^{(1)}_{-i} x_i(\theta) = \theta^{(2)} \), \( \sum_{i=0}^{I} U_i(0, \theta_{-i}) \leq \theta^{(2)} \). Additionally, (O) implies that \( U_0(0, \theta_{-0}) \leq \theta^{(1)}_{-0} x_0(\theta) + D \).

- \( \sum_{i=0}^{I} t_i(\theta) < D \). In this case, (Def\(_0\)) implies \( t_0(\theta) = \theta^{(1)}_{-0} x_0(\theta) - U_0(0, \theta_{-0}) + \sum_{i=0}^{I} t_i(\theta) \), so \( 0 = \theta^{(1)}_{-0} x_0(\theta) - U_0(0, \theta_{-0}) + \sum_{i=1}^{I} t_i(\theta) \). Using (Def\(_I\)), \( 0 = \theta^{(1)}_{-0} x_0(\theta) - U_0(0, \theta_{-0}) + \sum_{i=1}^{I} \left( \theta^{(1)}_{-i} x_i(\theta) - U_i(0, \theta_{-i}) \right) \), so we obtain \( \sum_{i=0}^{I} U_i(0, \theta_{-i}) = \sum_{i=0}^{I} \theta^{(1)}_{-i} x_i(\theta) = \theta^{(2)} \). Additionally, (O) and \( \sum_{i=0}^{I} t_i(\theta) < D \) implies that \( D > \sum_{i=1}^{I} t_i(\theta) = U_0(0, \theta_{-0}) - \theta^{(1)}_{-0} x_0(\theta) \), so \( U_0(0, \theta_{-0}) < \theta^{(1)}_{-0} x_0(\theta) + D \).

In both cases, it is necessary that \( \sum_{i=0}^{I} U_i(0, \theta_{-i}) \leq \theta^{(2)} \) and \( U_0(0, \theta_{-0}) \leq \theta^{(1)}_{-0} x_0(\theta) + D \) for all \( \theta \in \Theta \). Taking \( \theta_0 = 0 \) and noting (EXPR'), we obtain the necessary condition that \( U_0(0, \theta_{-0}) \leq \theta^{(2)}_{-0} \) and \( U_0(0, \theta_{-0}) \leq D \).

Conversely, it is easy to show that the payment mechanism \( t_0(\theta) = \theta^{(1)}_{-0} x_0(\theta) + D - \min \left\{ \theta^{(2)}_{-0}, D \right\} \) and \( t_i(\theta) = \theta^{(1)}_{-i} x_i(\theta) \), satisfies \( U_0(0, \theta_{-0}) = \min \left\{ \theta^{(2)}_{-0}, D \right\} \), \( U_i(0, \theta_{-i}) = 0 \) for \( i \geq 1 \) and (Def\(_0\)), (Def\(_I\)), (EXPR'), (BB) and (O) (note that \( \sum_{i=0}^{I} t_i(\theta) \) is always \( D \) or greater).
References


Figure 3: Scatter plots of the relation between the debt balance (judgment amount) on the horizontal axis and (a) auction sale price, (b) lenders’ maximum bids and (c) resale price on the vertical axis. For each panel, the left plot shows auctions where lenders won, and the right plot shows auctions where third party bidders won. The axes are log-scaled, and the blue line represents 45 degree line. The vertical broken line represents debt balance of 60,000 USD.
Figure 4: The empirical distribution (PDF) of the ratio of lender’s maximum bid to resale price, for cases with judgment amount above 60,000 USD and when lenders won. It uses smoothing by Epanechnikov kernel with the rule-of-thumb bandwidth. The blue vertical line has intercept 1. The mean of the ratio is 2.04.

Figure 5: Scatter plots of the relation between auction sale price on the horizontal axis and resale price on the vertical axis, for cases with judgment amount above 60,000 USD and lenders’ maximum bids being hidden. The left plot shows auctions where lenders won, and the right plot shows auctions where third party bidders won. The axes are log-scaled, and the blue line represents 45 degree line.
Figure 6: Left panel: PDF of lognormal distributions with parameter $(-\sigma^2/2, \sigma^2)$ for different values of $\sigma$. Right panel: MR function. The horizontal line has intercept of zero.

Figure 7: Simulated probability that the lender wins the foreclosure auction (assuming lender submits a maximum bid of $\theta^* (E[\theta_0]))$, for different values of $N$ and $\sigma$. The number of bidders $n$ is set to be 5.
Figure 8: Simulated empirical distribution of $\theta' / \theta$ for different values of $N$ and $\sigma$ (assuming lender submits a maximum bid of $\theta' (E[\theta_0])$).