Non-Binding Secret Reserve Prices: The Case of Wholesale Used-Car Auctions

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Abstract

I study secret reserve prices in auctions that are non-binding in the sense that the sellers can accept bids below them. Such a reserve price has a bite only when the winning bid exceeds it, in which case the winning bid is accepted without seller's action. This work investigates the motivation for this puzzling practice that many real-world auctions take, such as wholesale used-car auctions. I estimate a structural model of ascending auctions using the auction data in the wholesale used-car market. To microfound seller's decision of the secret reserve price, I posit that the seller has uncertainty as to the value of the item when she sets the reserve price and that this uncertainty is resolved after she observes the auction price. I compare the status quo with two counterfactual auction formats: (i) no reserve prices and the seller gets to accept or reject every winning bid, and (ii) the seller commits to the secret reserve price. I observe very little difference among them in terms of probability of trade, seller's payoff and revenue. I discuss how the current format may be rationalized as reducing transaction costs for asking sellers' confirmation of all winning bids and avoiding sellers' cognitive cost of committing to a reserve price.

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1 Introduction

Auctions are one of the fundamental tools to sell items and are used widely. A vast theoretical literature analyzes auctions; however, auctions in the real world often differ from those in theoretical studies in subtle but important ways. One such difference is the practice of reserve prices that are (i) secret and (ii) non-binding. In other words, (i) the bidders do not observe the reserve prices when they are bidding, and (ii) the seller may choose to accept the winning bid and sell the item at that price even if the winning bid is below the reserve price. This is puzzling in that the only case when the reserve price takes effect is when the auction price (winning bid) exceeds the reserve price: in such case, the item sells at the winning bid without seller's action. On the other hand, the seller has a last look if the auction price is below the reserve price: the seller has the choice to either sell at the auction price, or reject the auction price and withdraw the item.

This practice stands in stark contrast to the reserve prices in classical auction literature, such as Myerson (1981) and Riley and Samuelson (1981), which are public and binding. When reserve prices are public, bidders' strategies depend on the reserve price, sometimes increasing the winning bid (e.g., when only one bidder values the auction item above the reserve price in an ascending auction). With binding reserve prices, bids below them are ignored. None of these features appear under secret non-binding reserve prices.

Several past studies have offered explanations for secret reserve prices, but the literature has yet to converge on a standard model. For instance, Elyakime et al. (1994) explain that secret reserve prices may yield higher revenue than public reserve prices, even though the former would lead to lower seller payoff. Here, the difference between revenue and seller payoff is that the former counts 0, and the latter counts the seller's value of the item, when the item does not sell. Though an attractive explanation, the explanation by Elyakime et al. (1994) does not address secret reserve prices that are non-binding. More recently, Andreyanov and Caoui (2020) study secret non-binding reserve prices using data on the sale of timber tracts in France by first-price auctions. They explain sellers' motivation in setting high secret reserve prices as follows: the sellers do not observe the value of the item perfectly when setting the reserve price, so they set high reserve prices to wait until after the auction and learn the value of the item. However, Andreyanov and

Caoui (2020) do not address the auctioneer's motivations separately from sellers', as the seller is the auction designer in their application.

The U.S. wholesale used-car market is an ideal setting to study non-binding secret reserve prices. It is a huge market that supplies much of the inventory of U.S. car dealers, where cars totaling to 100 billion dollars were sold in 2020. The market operates by auctioning used cars and has embraced the practice of non-binding secret reserve prices widely.

I use data on auctions in six U.S. auction houses in January 2007 to March 2010, first used by Larsen (2021), to study the motivations for the practice of non-binding secret reserve prices. In this dataset, the auction proceeds in three steps: first, the seller sets the secret reserve price. Second, the auctioneer solicits bids in an ascending manner. The third step (which I call "after-auction process") has two cases: If the highest bid (auction price) exceeds the reserve price, the item is sold at the auction price without any action by the seller or the buyer. If, on the other hand, the auction price does not meet the reserve price, the seller makes a decision between selling the item at the auction price, or rejecting the auction price and withdrawing the item.[1] I compare this practice with two other natural formats: (A) infinite non-binding secret reserve price, which is equivalent to repealing reserve prices and letting the seller decide whether to accept the auction price or not for any value of the auction price, and (B) binding secret reserve price. I simulate these counterfactual cases to compare the impact on seller's payoff and revenue.

As a prerequisite to the counterfactual simulations, I estimate a structural model of ascending auctions. I model that the valuation of the item by the seller and the bidders are additively separable into three components: (i) idiosyncratic, private component, (ii) a component that explains game-level heterogeneity, which is observed by and common knowledge to the seller and the bidders, and (iii) a component that indicates *hotness* of the market on the auction day, which is observed by the bidders but unobserved by the seller when setting the reserve price, and observed by the seller in the after-auction process. The component (ii) is introduced to explain the correlation between auction price and secret reserve price that reflects both characteristics that are observable to the econometrician (e.g., make, model, odometer reading) and those that are unobservable (certain aspects of item

[1] Another possible outcome, which I do not focus on in this study, is post-auction bargaining between the seller and high bidder.

quality not recorded in the dataset).

The component (iii) is motivated by the following institutional details. The seller sets the reserve price a few days in advance of the auction day, so the seller does not observe how strong the demand in the market would be on the auction day. I refer to this day-specific demand component as the hotness of the market On the auction day, the auction house conducts several hundred auctions live. I model the setting the bidders knowing this market hotness on the day of the auction. The seller, on the other hand, learns the market hotness after she sets the reserve price but before the after-auction process, either by watching the auction live herself or by receiving soft information from the auctioneer. This component is necessary to justify, in the spirit of Andreyanov and Caoui (2020), the empirical observation that there are a significant number of cases where the auction price is accepted even though it is below the secret reserve price. To identify and estimate the model, I assume that this component is observed by the econometrician and equals the average price of all auction prices on each day in auction house after controlling for game-level heterogeneity observed to the econometrician.

I estimate the distributions of the components above as follows. First, I remove game-level heterogeneity observed to the econometrician by a pooled regression of secret reserve price and auction price on a rich set of item characteristics observed in the data. I next decompose the joint distribution of secret reserve price and auction price into the distributions of (a) the private component of seller value, (b) the sum of private component of auction price and market hotness, and (c) the game-level heterogeneity component unobserved to the econometrician, using maximum likelihood under a flexible parametrization of the distributions. I then deconvolve the distribution of the market hotness from the distribution of (b). Finally, I obtain the distribution of seller's private value component.

Some of these identification and estimation steps are straightforward applications of methods used in the literature, including Larsen (2021). One contribution of my paper to the literature is to show how the model primitives — the CDF of private value component of sellers in particular — can be estimated in an environment of ascending auctions with non-binding secret reserve prices. I show that the probability that the seller accepts an auction price in the after-auction process is a convolution of the distribution of game-level heterogeneity (Comonent (ii)), the distribution of market hotness (Component (iii)), and the CDF of private value component of sellers minus the CDF of homogenized secret reserve prices. This allows for an estimation of the CDF of private value component by a flexible parametrization and matching the convolution with the probability of sellers' acceptance in the after-auction process.

I then simulate two counterfactuals A and B above: infinite non-binding reserve prices and binding secret reserve prices. I find that the results are very similar to each other in the probability of trade, seller payoff, and revenue. This suggests that the auctioneer has chosen the status quo of non-binding secret reserve prices out of reasons that are more subtle than payoffs and revenue modeled here. I conjecture that the auctioneer prefers the status quo to infinite non-binding reserve prices as they find it costly to contact the seller for approval of auction price in all cases. They, on the other hand, may not be able to implement binding secret reserve prices as sellers may be reluctant to commit to reserve prices in advance. The status quo may thus be the result of this compromise between the auctioneer's willingness to save transaction costs and sellers' aversion to the cognitive cost of committing to a reserve price.

Related Literature. This work contributes to the literature of secret reserve prices in auctions. Elyakime et al. (1994) find that, under first-price auctions with independent private values, the seller's profit is lower with secret reserve prices than with public ones. There have been several explanations for the common use of secret reserve prices nonetheless. Elyakime et al. (1994) show that the secret reserve prices may result in higher expected revenue ("sales" in their word) under first price auctions. In the same spirit, Eklöf and Lunander (2003) show, by simulation, that secret reserve prices under ascending auctions result in higher probability of trade (though lower seller profits) than public ones. Ashenfelter (1989) and Ashenfelter and Graddy (2003) suggest that auctioneers may keep the reserve price secret to mitigate concerns of bidder collusion. Vincent (1995) proves that, in a common value model of second-price auctions, keeping the reserve price may encourage bidder participation and information revelation by bidders, resulting in higher seller profit by the linkage principle (Milgrom and Weber, 1982). Bajari and Hortaçsu (2003) support and expand this argument with the observation that sellers in eBay ascending auctions use secret reserve prices, especially when the item is expensive. Li and Tan (2017) and Brisset and Naegelen (2006) show that bidder risk aversion could generate higher seller payoff with secret reserve prices rather than public ones under first-price auctions and ascending auctions with jump bidding (but not under button auctions). Coey et al. (2021) indicate that, in a setting of repeated auctions, a seller could potentially benefit from secret reserve prices because they observe more bids and learn more about future auctions. Some studies, such as Jehiel and Lamy (2015), motivate secret reserve prices assuming deviations from rational agents.

The literature seems to have been less conscious about the role of sellers' commitment to reserve price, with some notable exceptions. Horstmann and LaCasse (1997) indicate that the seller may signal item quality and raise profits by rejecting a bid above public reserve prices and reauctioning. Katkar and Reiley (2006) note that, on eBay, the seller may set a high secret reserve price and email the high bidder later, indicating her willingness to sell at the auction price, so she doesn't have to pay commissions to eBay. Grant et al. (2006) compare, theoretically, auctions with secret non-binding reserve prices and public binding reserve prices, when the seller has the option to reauction. Andreyanov and Caoui (2020) study, using data for timber tract auctions in France, seller's motivation in setting non-binding secret reserve prices: the seller sets a high reserve price to wait and see the auction price, and learn the value of the item before she makes the final decision. My work applies this idea in a similar setting but focuses on the motivation of auction design by a third-party auctioneer.

Larsen (2021) studies the same setting of wholesale used-car auctions, and my work builds on his work and dataset. Our focuses are different: He studies the efficiency of bargaining that takes place after the auctions. I focus on the reserve prices, while simplifying the bargaining process as a take-it-or-leave-it offer by the winning bidder.[2] This simplification is justified by the empirical observation that there are few cases in which the seller makes a counteroffer to the winning bid and then agree with the buyer to trade — a fact consistent with Larsen (2021)'s observation that the bargaining outcome is much less efficient than the theoretical efficient frontier.

This paper is organized as follows. Section 2 explains the institutional setting,

[2] In the dataset, in addition to accepting the auction price or rejecting the sale of the item, the seller also has the option to make a counteroffer. If the seller chooses so, the buyer can also accept it, make a counteroffer, or walk away from the bargaining. I simplify this bargaining process by looking at whether the seller accepts the auction price, or rejects the auction price (which includes the case when the seller counteroffers). See Section 2 for details.

including the current auction mechanism, in more detail. Section 3 develops the empirical model. Section 4 discusses the identification of the empirical model and develops the estimation strategy. Section 5 shows model estimates. Section 6 simulates counterfactuals and discusses possible motivations of the current auction mechanism. Section 7 concludes.

2 Background and Data

The United States has a large wholesale market of used cars. In 2020, 7.9 million cars were sold at a total value of 100 billion dollars (National Auto Auction Association, 2021). In this market, car dealers, as buyers, procure their inventory through auctions. There are two types of sellers: car dealers and other types of institutions. Car dealers (as sellers) can sell vehicles that they obtained by trade-ins or can just try to sell off their excess inventory. Other types of institutions include rental car companies and fleet companies that try to divest vehicles they no longer lease, and financial institutions that try to sell vehicles they obtained through foreclosures. I follow Larsen (2021) and label the former type of sellers as "dealers" and the latter type of sellers as "fleet/lease sellers."

The auction house specializes in the sale of used cars. They receive consignments of vehicles from sellers and sell hundreds to thousands of cars by running live auctions, typically on a fixed day, once a week. The auction process is quick, which on average takes about 60 seconds per vehicle. In these auctions, the seller can set a reserve price ("floor price" or "the low"). However, even if the winning bid is below the reserve price, the auctioneer allows trade if the seller, after the auction concludes, agrees to accept the winning bid; such bids are called "if bids" in industry jargon. [3]

I use data from six auction houses in the United States between January 2007 and March 2010, first used by Larsen (2021). These auction houses organized the auctions in the following manner. A seller brings a used car to the auction house and sets a secret reserve price several days in advance. Then, on the auction day, potential buyers bid for the car in an ascending auction. If the winning auction price exceeds the secret reserve price, the car is sold to the winning bidder at the

[3] https://www.naaa.com/about_us/all_about_auctions/all_about_auctions.html. Accessed on 2021-09-29.

auction price. If not, the seller is contacted by the auction house and can accept or reject the high bid. The seller may also give a counteroffer, but, as shown below, this is less common.

In the dataset, each observation is an auction, and it records the auction price, the secret reserve price set by the seller, whether parties agree or not, the final price if agreed, and rich car characteristics such as make, model, year, item condition, and a blue book estimate provided by the auction house.

	Mean	SD
1(auction price ≥ reserve)	0.094	
1(buyer walks away after auction)	0.012	
1(seller accepts auction price)	0.577	
1(seller quits after auction)	0.170	
1(seller counters and trade)	0.034	
1(seller counters and no trade)	0.113	
Reserve price ($)	7,405	5,196
Auction price ($)	6,253	4,881
Auction price if agreed ($)	6,057	4,673
Final price if agreed ($)	6,075	4,681
# of bidders (lower bound)	2.92	0.36
Seller fee (if trade, $)	146	54
Buyer fee (if trade, $)	162	36
Blue book price ($)	6,820	4,828
Odometer reading (mi)	97,938	46,445
# obs	133,523	

Table 1: Summary statistics for dealer samples. The unit of observation is an auction. The number of bidders is the lower bound as recorded in the data. Blue book price is an estimate of the market value of the car, provided by the auction house.

Table 1 shows summary statistics for the dealer samples, which is the focus of this paper. There are 133,523 auctions recorded. Out of those auctions, only 9.4% resulted in trade because the auction price exceeded the reserve price, while 57.7% resulted in trade because the seller agreed to sell the item at the auction price despite being below the reserve price. This is a central feature of the dataset that any empirical model needs to explain.

My analysis focuses on whether (i) the auction price exceeds the reserve price, (ii) the seller accepts an auction price that is below reserve price, or (iii) the seller rejects an auction price. That is a simplification of the post-auction process modeled in Larsen (2021). The detailed breakdown of the auction results, in chlonorogical order, is as follows (the first six rows of Table 1): (1) 9.4% of auctions end because the auction price exceeded the reserve price. (2) In 1.2% of auctions, the winning bidder finds out that the auction price is below the reserve price and then withdraws his bid. If the winning bidder does not withdraw his bid, the seller takes one of the following three actions. (3) In 57.7% of auctions, the seller accepts the winning bid (agrees to sell the item at the auction price). (4) In 17.0% of auctions, the seller walks away from negotiation with the winning bidder. (5) In 14.7% of auctions, the seller makes a counteroffer. When the seller does so, 3.4% result in trade, while the remaining 11.3% result in no trade. In my analysis, I treat cases (2), (4), and (5) as the seller rejecting an auction price that is below the reserve price (note that both of cases (4) and (5) imply that the seller rejected the auction price). Doing so is an abstraction of the detailed bargaining process, but will simplify my analysis greatly. I adopt this simplification because my focus is on producing a model with the feature that sellers accept auction prices below their reserve prices, a feature that Larsen (2021) abstracts away from. Another justification for this simplification is that the average auction price, conditional on trade, is very close to the average final price ($6,057 vs. $6,075).

Another notable feature that can be seen in Table 1 is that the secret reserve price is often much higher than the auction price: on average, the secret reserve price is $7,405, while the average auction price is $6,253. This is the reason why the auction price exceeds the secret reserve price in only 9.4% of auctions.

3 Model

The aim of this paper is to simulate two counterfactuals (infinite non-binding secret reserve prices and binding secret reserve prices) and compare these to the mechanism used in practice. To this end, I need a structural model of auctions and an estimate of underlying value distributions. Section (3.1) explains the setup of the structural model, and Section (3.2) solves the model. The structural model is agnostic about how exactly the seller sets the non-binding secret reserve prices, and can acocommodate different microfoundations about the reserve prices. In Section (3.3), I offer one simple microfoundation to explain a number of features observed in the data and/or used in the identification and estimation of the structural model.

3.1 Setup of Structural Model

I first explain the setup of the structural model. The players of the game are the seller (whom I refer to as "she/her") and bidders (indexed by i, each of whom I refer to as "he/him"). The seller sells a single item by an ascending auction.

Stages.

The model consists of three stages:

1. The seller sets a secret reserve price R.
2. The auctioneer runs an ascending auction for the item, resulting in auction price P^A.
3. After-auction process:
   (a) If $P^A \geq R$, the item is immediately sold to the winning bidder.
   (b) If $P^A < R$, the sellerdecides whether to accept P^A or not.

If P^A is accepted, the seller receives utility P^A, the winning bidder (call him bidder i) receives utility $B_i - P^A$, and the other bidders receive zero utility. If P^A is rejected, the seller receives utility S and the bidders receive zero utility.

Assumptions.

(A1) There are $N(\geq 2)$ bidders, where N is exogenously determined.

(A2) The seller is risk neutral and values the item at $S = S_0 + Y + Z$.

(A3) Each bidder i is risk neutral and values the item at $B_i = B_{0i} + Y + Z$.

(A4) S_0, B_{0i}, Y, Z are drawn independently of each other and iid across games.

(A5) S_0 is observed only by the seller, and B_{0i} is observed only by bidder i. The econometrician does not observe either S_0 or B_{0i}.

(A6) Y is observed by the players. Y is decomposed as $Y = \boldsymbol{x}'\boldsymbol{\beta} + \widetilde{Y}$, where $\boldsymbol{x}$ is the characteristic of the game observable to the econometrician but $\boldsymbol{\beta}$ and $\widetilde{Y}$ are not. The variables $\boldsymbol{x}$ and $\widetilde{Y}$ are independent of each other.

(A7) Z is observed by the bidders. The seller only knows the distribution of Z at Stage 1 (when the seller sets R). The seller perfectly observes Z at Stage 3(b) in the after-auction process.

(A8) The econometrician observes R, P^A, seller's decision in Stage 3(b), and $\boldsymbol{x}$. Additionally, Z is the average of $P^A - \boldsymbol{x}'\boldsymbol{\beta}$ on each auction day in each auction house, demeaned so that $\mathbb{E}[Z] = 0$.

Discussions of Model Setup

My model is a static ascending auction model, with S_0 and B_{0i} being private components of the seller and bidder i. In reality, in the event that the seller fails to sell an item, the seller may try to sell the item by bringing the same item to the same auction house and rerunning an auction, or by finding another sale channel (such as another auction house). This means that the seller's value S represents, in a reduced-form manner, the seller's option value from waiting and reselling.

Y represents the quality of the item that is observed by both the seller and the bidders. I incorporate this term to explain the correlation between the auction price and the secret reserve price observed in the data, This term includes factors such as car make, model, trim, year, and mileage (observable to the econometrician) and odors and dents (unobservable to the econometrician). Assumption (A6) assumes that Y is additively separable into a linear combination of characteristics observable to the econometrician ($\boldsymbol{x}$) and a component unobserved by the econometrician ($\widetilde{Y}$).

I incorporate the component Z to explain that the seller often accepts an auction price below the secret reserve price in the after-auction process. Assume, to the contrary, that there is no such component (in other words, assume Z is degenerate at zero). I show that, with this assumption and under a microfoundation of seller's choice of reserve price discussed below, the seller's optimal reserve price is below her value of the item S (Proposition 3, Part 4). If, then, the auction price is below the reserve price, it is below S. This implies that there should not be cases where the seller accepts an auction price below the secret reserve price, contrary to what I observe in the data. By introducing a component Z of seller's value that the seller learns after the auction, I can explain this data pattern: the seller will set a high secret reserve price to wait and see the auction price and the realized value of Z

(Andreyanov and Caoui, 2020).

The exact modeling choice of Z is motivated by the following institutional setting. Z stands for hotness of the market on each day in each auction house, which equals the average price (net of item heterogeneity explained by characteristics observed by the econometrician). In wholesale used-car auctions, the seller sets the secret reserve price several days in advance of the auction day. As such, the seller does not observe Z when she sets the reserve price (Stage 1). On the auction day, the auction house runs live auctions for hundreds of items. The bidders watch the first few auctions of the day and get a sense of the market hotness before they place bids in the auction (Stage 2). The seller learns the market hotness before the after-auction process (Stage 3) in one of two channels: If the seller physically watches the auctions, she has the same information about market hotness as the bidders. If, on the other hand, the seller is not present at the auction house, the auctioneer may convey soft information about the market hotness. This occurs when the auctioneer calls up the seller to ask her willingness to sell the item at the auction price that is below the reserve price.[4] The model simplifies this learning process of market hotness by assuming that the seller perfectly observes Z in the after-auction process.[5]

Assumption (A8) on the observability of Z to the econometrician is admittedly a strong assumption. This assumption is necessary to separate Z from the distribution of $P^A - Y$. This is a different assumption than that in Andreyanov and Caoui (2020). In their work, the econometrician observes sellers' appraisal of the item, independent of sellers' private value, so they treat it as conveying a noisy signal of the component of seller value learned by the seller after the auction. I do not have such an appraisal in the dataset, so I need additional assumption on the observability of Z.

[4] According to those familiar with the industry, the phone conversation between the seller and the auctioneer representative may include the representative saying something like: "The auction price is below your reserve price, but it was a pretty good price for today's market. Demand was just low today."

[5] An alternative, arguably more realistic modeling assumption would be to posit that the seller updates her belief on Z by Bayes' rule after observing the auction price. The current assumption that the seller observes Z perfectly is a simplification that still captures the seller's learning.

3.2 Solving the Model

I solve the model by backward induction.

3.2.1 Stage 3(b): Seller's Decision to Accept or Reject P^A

At Stage 3(b), the seller perfectly observes her value of the item S. The seller compares selling the item at the auction price P^A and rejecting the sale to enjoy her reservation utility S. As such, I have the following result. All proofs are found in Appendix A.

Proposition 1. *In Stage 3(b), the seller's optimal action is to accept P^A if and only if $P^A \geq S$.*

3.2.2 Stage 2: Bidders' Bidding Strategy

Each bidder observes his value of the item B_i when he is bidding. This implies that Stage 2 is an ascending auction with private values by bidders. Despite the existence of Stage 3, I can prove the following in the same manner as in standard models of ascending auctions:

Proposition 2. *For each bidder i, it is a weakly dominant strategy for him to stay in the ascending auction if and only if the price is below his value of the item B_i.*

I assume that the bidders follow this weakly dominant strategy. Then, the auction price P^A is the second order statistic of B_i: $P^A = B_i^{(N-1:N)}$. It is the sum of Y, Z, and the second order statistic of B_{0i}, which I denote P_0^A and call the "private component" of the auction price:

$$P^A = P_0^A + Y + Z.$$

3.2.3 Stage 1: Secret Reserve Price

My empirical model here is agnostic about how the seller sets R. I do, however, assume the following:

Assumption. (A9) There exists a strictly increasing, finite function $R_0(\cdot)$ such that $R = R_0(S_0) + Y$.

The next section, Section (3.3), offers one simple microfoundation of the reserve price. I prove that, under that microfoundation, Assumption (A9) holds by imposing some additional mild assumptions.

3.3 Microfounding Seller's Choice of Non-Binding Secret Reserve Price

In this section, I microfound the seller's decision of setting a secret reserve price to explain a number of features in the data and derive properties that I use for identification and estimation.

The main component of the microfoundation is that the seller incurs some transaction cost when she decides, in the after-auction process, whether she accepts or rejects the auction price. Without some kind of cost on the seller, the seller's only motivation in setting the reserve price is to "wait and see" the auction price, and the seller can just choose infinite reserve prices. By assuming such a transaction cost, the seller trades off setting a high reserve price to "wait and see" and setting a low reserve price to reduce the chance she has to pay the transaction cost. This tradeoff makes it optimal for the seller to set a finite reserve price R.

This transaction cost introduced here reflects the following institutional feature: The auction house does not bother to contact the seller after the auction if the auctioneer perceives the reserve price to be excessively high compared to the reserve price. Another example of such transaction cost is that the seller finds it costly to take up a phone call from the auction house if the auction price is below the reserve price.

3.3.1 Microfoundation: Model Setting

The model slightly modifies the setting of Section 3.1. The timeline is as follows:

1. The seller observes S_0 and Y, and sets the secret reserve price R.
2. Auction price $P^A = P_0^A + Y + Z$ is drawn. (Second order statistic of bidder values)
3. After-auction process:

(a) If $P^A \geq R$, then P^A is automatically accepted. The seller enjoys utility P^A.

(b) If $P^A < R$, then the seller observes Z and decides to accept or reject P^A. If the seller accepts, she enjoys utility $P^A - c$. If the seller rejects, she enjoys utility $S - c$. (The cost c is necessary for the seller to learn the auction price and Z.)

3.3.2 Optimal Non-Binding Secret Reserve Price under the Microfoundation

The seller's ex ante expected payoff from setting a reserve price R is

$$\pi^S(R \mid S_0, Y) \equiv \mathbb{E}_{Z, P_0^A}\left[\mathbf{1}(P^A \geq R)P^A + \mathbf{1}(P^A < R)\big(\max\{P^A, S\} - c\big) \,\Big|\, Y, S_0\right].$$

For simplicity, assume that P_0^A has a continuous distribution with support on $(-\infty, \infty)$ and that Z has a bounded support.

Proposition 3. *Under the setting in this section, the optimal reserve price R satisfies the following:*

1. *There exists a function R_0 that does not depend on S_0 such that $R = R_0(S_0) + Y$.*

2. *Assume that Z has a continuous distribution with positive probability density on $(\underline{z}, \overline{z})$. If $R_0(S_0) - S_0 \in (\underline{z}, \overline{z})$, then R_0 is strictly increasing at S_0.*

3. *If $c = 0$, then setting $R = \infty$ is optimal for the seller. If $c > 0$, then the optimal R is finite.*

4. *If Z is a degenerate random variable with value 0, then $R = S_0 + Y - c$.*

Part 1 means that R is additively separable with respect to Y, which is a natural consequence of the additive separability of all players' values. Part 2 means that a seller with a higher value of the item sets a higher reserve price. Part 3 means that, if the seller does not incur any cost for observing Z and deciding to accept or reject auction offer that is below the auction price, then the seller is best served by setting infinite reserve price and "wait and see" what the auction brings about. If, on the other hand, the seller has to incur a transaction cost c when the auction price is below the reserve price, the seller trades off between a desire to set a high reserve price to "wait and see" and a motivation to save the transaction cost by lowering the reserve price.

Part 4 means that if the seller perfectly observes her value of the item when setting the reserve price, the seller sets her reserve price to be below, but close to (when $c \approx 0$), her value of the item. Notably, if the auction price $P^A = P_0^A + Y$ is below the reserve price R, then it means $P^A < S_0 + Y = S$, so the seller always rejects such an auction price. This cannot explain our empirical observation that the seller often accepts an auction price that is below the reserve price.

4 Identification and Estimation

I observe (1) secret reserve price set by the seller R, (2) the number of bidders N, (3) the auction price P^A, and (4) the seller's decision of accepting/rejecting P^A as sale price if P^A is below R. Additionally, Z is also observable by virtue of Assumption (A8).

4.1 Removing Observed Heterogeneity

I first "homogenize" the secret reserve prices and auction prices as in Haile et al. (2003). I jointly regress the auction price P^A and reserve price R on a rich set of observed characteristics of the auction $\boldsymbol{x}$:

$$\begin{cases} R = \boldsymbol{x}'\boldsymbol{\beta} + \widetilde{R}, \\ P^A = \boldsymbol{x}'\boldsymbol{\beta} + \widetilde{P}^A. \end{cases}$$

This way, I obtain estimates for $\widetilde{P}^A \equiv P^A - \boldsymbol{x}'\boldsymbol{\beta} = P_0^A + \widetilde{Y} + Z$ and $\widetilde{R} \equiv R - \boldsymbol{x}'\boldsymbol{\beta} = R_0(S_0) + \widetilde{Y}$. The observable characteristics $\boldsymbol{x}$ include:

1. Fifth-order polynomial terms in the auction houses' blue-book estimate and the odometer reading.

2. The number of previous attempts to sell the car; the number of pictures displayed on the auction houses' websites; a dummy for whethe or not the odometer reading is considered accurate, and an interaction of this dummy with the odometer reading; the interaction of the odometer reading with car-make dummies.

3. Dummies for each make-model-year-trim-age combination (age is the number of years between the car model year and the auction day); dummies for condition report grade; dummies for the year-month combination; dummies for auction house location interacted with hour of sale; dummies for 32 different vehicle damage categories recorded by the auction house; dummies for each seller who appears in at least 500 observations.

4. Dummies for discrete odometer bins: four equally sized bins for mileage in [0, 20000); eight equally sized bins for mileage in [20000, 100000); four equally sized bins for mileage in [100000, 200000); one bin for mileage in [200000, 250000); and one bin for mileage greater than 250000.

5. Several measures of the thickness of the market during a given sale, computed as follows: for a given car on a given sale date at a given auction house, I compute the number of remaining vehicles still in queue to be sold at the same auction house on the same day lying in the same category as the car in consideration. The six categories I consider are make, make-by-model, make-by-age, make-by-model-by-age, age, or seller identity.

6. Controls for the run numbers, which represents the order in which cars are auctioned. I include fifth-order polynomials for both the run number within an auction-house-by-day combination, and the run number within an auction-house-by-day-by-lane combination.

See Online Appendix D.1.2 of Larsen (2021).

4.2 Accounting for Game-Level Unobserved Heterogeneity

I observe the secret reserve price (after homogenizing with respect to observed heterogeneity) $\widetilde{P}^A = (P_0^A + Z) + \widetilde{Y}$ and $\widetilde{R} = R_0 + \widetilde{Y}$. By a statistical result due to Kotlarski (1967), the joint distribution of $\widetilde{P}^A$ and $\widetilde{R}$ nonparametrically identifies the marginal distribution of (i) the auction-level unobserved heterogeneity $\widetilde{Y}$, (ii) the "homogenized" secret reserve price R_0, and (iii) $P_0^A + Z$. (This identification is up to a shift by a constant; it can be uniquely determined if I fix a parameter for parallel shift of $\widetilde{Y}$). This identification argument has been used in the literature to account for unobserved heterogeneity, such as Krasnokutskaya (2011).[6]

To actually estimate the distribution of $\widetilde{Y}$, R_0, and $P_0^A + Z$, I follow Step 2 of estimation by Larsen (2021) and use maximum likelihood, instead of the estimation method by Krasnokutskaya (2011) that closely follows the identification argument. Other works using maximum likelihood to control for unobserved heterogeneity include Athey et al. (2011) and Freyberger and Larsen (2017). I parametrize the probability densities of $W \in \{\widetilde{Y}, R_0, P_0^A + Z\}$ as

$$f_W(w) = \frac{1}{\sigma_W}\left(\sum_{k=1}^{5} \theta_k^W H_k\left(\frac{w-\mu_W}{\sigma_W}\right)\right)\phi\left(\frac{x-\mu_W}{\sigma_W}\right), \tag{1}$$

using the first five Hermite polynomials $H_1, \ldots, H_5$ and standard normal pdf ϕ ($\mu_W, \sigma_W, \theta_1^W, \ldots, \theta_5^W$ are the parameters). I set $\mu_{\widetilde{Y}} = 0$, which is a parameter for parallel shift of $\widetilde{Y}$. I then maximize the following likelihood:

$$\mathcal{L}(f_{\widetilde{Y}}, f_{R_0}, f_{P_0^A+Z}) = \prod_j \int f_{R_0}(\widetilde{r}_j - y) f_{P_0^A+Z}(\widetilde{p}_j^A - y) f_{\widetilde{Y}}(y)\, dy,$$

where j is an index for observations (auctions) and $(\widetilde{r}_j, \widetilde{p}_j^A)$ is the j-th observation for the secret reserve price $\widetilde{R}$ and the auction price $\widetilde{P}^A$ (after removing observed heterogeneity). See Online Appendix D.2 of Larsen (2021).

4.3 Estimating the Distribution of Private Component of Auction Price

The steps in Sections 4.1 and 4.2 follow the estimation process in Larsen (2021); the procedures this section and onward are new in this paper. As Z is observed, I can identify the distribution of P_0^A from the distribution of $P_0^A + Z$ using the following equation:

$$F_{P_0^A+Z}(p) = \int_{\mathbb{R}} F_{P_0^A}(p-z) f_Z(z)\, dz. \tag{2}$$

[6] Roberts (2013) allows for a more flexible functional form of the reserve price with respect to the unobserved heterogeneity. Importantly, however, his approach does not allow for seller's private value component. This is justified in his case because the auctioneer, not the seller, sets the reserve price, unlike this paper's setting.

One obtains identification of $F_{P_0^A}$ by taking the characteristic function (Fourier transform) of $F_{P_0^A+Z}$.

The main idea of the estimation is to approximate $F_{P_0^A}$ by a linear spline function with parameter vector θ^P, and then obtain an estimate $\widehat{F}_{P_0^A} = F_{P_0^A}(\cdot;\widehat{\theta}^P)$ by

$$\widehat{\theta}^P = \arg\min_{\theta^P} \left\| F_{P_0^A+Z}(p) - \int F_{P_0^A}(p-z;\theta^P)\, f_Z(z)\, dz \right\|_2 .$$

Here, the norm $\|\cdot\|_2$ is the L^2 norm of functions in p.

The details of the estimation procedure are as follows. We first need the distribution of Z, which I estimate as the *demeaned* average of $\widetilde{P}^A$ within each auction house-day pair (in other words, fixed effects when regressing $\widetilde{P}^A$ on auction-day fixed effects and no regressors). Each auction-day pair has one value of Z, and each of those observations has an equal weight, regardless of the number of auctions over which the prices are averaged.

The CDF $F_{P_0^A}$ is parametrized in the following manner. I take a sufficiently granular grid of knots $\underline{q} = q_0 < q_1 < \cdots < q_{K^P} < q_{K^P+1} = \overline{q}$ on the domain of $F_{P_0^A}$. In the estimation, I choose $\underline{q}$ to be the 0.1 percentile of $P_0^A + Z$ minus 99.9 percentile of Z, and $\overline{q}$ to be the 99.9 percentile of $P_0^A + Z$ minus 0.1 percentile of Z. I pick $K^P = 200$ and select the knots to be evenly spaced. I then define basis functions $B_1^P, \ldots, B_{K_P+1}^P$ by:

$$B_k^P(x) = \begin{cases} \dfrac{x-q_{k-1}}{q_k-q_{k-1}}\mathbf{1}(q_{k-1} \le x < q_k) + \dfrac{q_{k+1}-x}{q_{k+1}-q_k}\mathbf{1}(q_k \le x \le q_{k+1}) & \text{if } k = 1,\ldots,K^P \\ \dfrac{x-q_{K^P}}{q_{K^P+1}-q_{K^P}}\mathbf{1}(q_{K^P} \le x \le q_{K^P+1}) & \text{if } k = K_p+1, \end{cases}$$

and parametrize $F_{P_0^A}(p)$ as

$$F_{P_0^A}(p) = B^P(p)' \begin{bmatrix} \theta^P \\ 1 \end{bmatrix} + \mathbf{1}(p > \overline{q}),$$

where $B^P(p) = \begin{bmatrix} B_1^P(p) \\ \vdots \\ B_{K^P+1}^P(p) \end{bmatrix}$ and $\theta^P = \begin{bmatrix} \theta_1^P \\ \vdots \\ \theta_{K^P}^P \end{bmatrix}$. Note that this is equivalent to parametrizing $F_{P_0^A}$ such that $F_{P_0^A}(\underline{q}) = 0$, $F_{P_0^A}(q_k) = \theta_k^P$ for $k = 1,\ldots,K^P$, $F_{P_0^A}(\overline{q}) = 1$ and it is linear in between the knots.

Under this parametrization, the right hand side of equation (2) can be simplified greatly. For each value of p, it can be written as:

$$x(p)'\theta^P + y(p),$$

where $x(p) = \begin{bmatrix} \int B_1^P(p-z) f_Z(z)\, dz \\ \vdots \\ \int B_{K^P}^P(p-z) f_Z(z)\, dz \end{bmatrix}$, $y(p) = \int \left(B_{K^P+1}^P(p-z) + \mathbf{1}(p-z > \overline{q})\right) f_Z(z)\, dz.$

We have, by a straightforward computation using integration by parts,

$$\begin{aligned} x_k(p) &= \frac{1}{q_k - q_{k-1}} \int_{p-q_k}^{p-q_{k-1}} F_Z(z)\, dz - \frac{1}{q_{k+1} - q_k} \int_{p-q_{k+1}}^{p-q_k} F_Z(z)\, dz, \\ y(p) &= \frac{1}{\overline{q} - q_{K^P}} \int_{p-\overline{q}}^{p-q_{K^P}} F_Z(z)\, dz. \end{aligned}$$

For a fixed value of p, both $x(p)$ and $y(p)$ can be computed easily using the empirical CDF of Z for F_Z.

I then find the value of the parameter θ^P as follows. I take a set of equal-sized grid points $p_1, \ldots, p_{L^P}$ on the domain of $P_0^A + Z$. In the estimation, I pick p_1 as the 0.1 percentile of $P_0^A + Z$, p_{L^P} as the 99.9 percentile of $P_0^A + Z$, and $L^P = 300$. The parameter θ^P is estimated by:

$$\min_{\theta^P} \sum_{l=1}^{L^P} \left(p_l - x(p_i)'\theta^P - y(p_i)\right)^2$$
$$\text{subject to } 0 \le \theta_1^P \le \theta_2^P \le \cdots \le \theta_{K^P}^P \le 1.$$

4.4 Estimating Seller Value Distribution

Recovering the distribution of S_0, the private component of the seller's value, is the most challenging step in this work. The seller's decision in the after-auction process reveals her preference, but we need to deconvolve $\widetilde{Y}$ and Z. Additionally, we get to observe the seller's decision only when the auction price is below the reserve price, which in turn is a function of S_0.

To solve this issue, I first fix $p = \widetilde{P}^A$ and observe the ratio of buyer offers p that are below the secret reserve price but are accepted by the seller nonetheless:

$\Pr(\widetilde{S} \leq \widetilde{P}^A < \widetilde{R} \mid \widetilde{P}^A = p)$, where $\widetilde{S} = S - \boldsymbol{x}'\boldsymbol{\beta}$. This ratio identifies the distribution F_{S_0} of the seller's private component of her value as in the following proposition.

Proposition 4. *Under Assumptions (A1)–(A9),*

$$\begin{aligned}
&\Pr(\widetilde{S} \leq \widetilde{P}^A < \widetilde{R} \mid \widetilde{P}^A = p) \\
&= \iint_{\mathbb{R}^2} \max\{F_{S_0}(p-y-z) - F_{R_0}(p-y), 0\} f_{\widetilde{Y}}(y) f_Z(z) f_{P_0^A}(p-y-z)\, dy\, dz \Big/ M(p),
\end{aligned} \tag{3}$$

where $M(p) = \iint_{\mathbb{R}^2} f_{\widetilde{Y}}(y) f_Z(z) f_{P_0}(p-y-z)\, dy\, dz.$

The following is a sketch of the proof. When the values of $\widetilde{P}^A$, $\widetilde{Y}$, and Z are fixed at p, y, and z, then the event $\widetilde{S} \leq \widetilde{P}^A < \widetilde{R}$ is equivalent to the intersection of $S_0 \leq p - y - z$ and $R_0(S_0) > p - y$, or $R_0^{-1}(p-y) < S_0 \leq p - y - z$ (I need using Assumption (A9) to take an inverse of R_0). The probability of such event conditional on p, y, z is $\max\{F_{S_0}(p-y-z) - F_{R_0}(p-y), 0\}$, using the fact that $R_0 = F_{R_0^{-1}} \circ F_{S_0}$. I take the maximum with zero to account for the case when $R_0^{-1}(p-y) > p - y - z$. To express the left-hand side of equation 3 using this conditional probability, I need to integrate this conditional probability using the distribution of $\widetilde{Y}$ and Z conditional on $\widetilde{P}^A = p$. This conditional distribution is given by $f_{\widetilde{Y}}(y) f_Z(z) f_{P_0^A}(p-y-z)/M(p)$, where $f_{\widetilde{Y}}$ and f_Z are PDFs of $\widetilde{Y}$ and Z.

Using this proposition, F_{S_0} is estimated in the following manner. I estimate $\Pr(\widetilde{S} \leq \widetilde{P}^A < \widetilde{R} \mid \widetilde{P}^A = p)$ by a local linear regression of $\mathbf{1}(\widetilde{S} \leq \widetilde{P}^A < \widetilde{R})$ on $\widetilde{P}^A$ on a grid of values for $\widetilde{P}^A$. I take a sufficiently granular grid of knots on the support of F_{S_0}, and approximate F_{S_0} by a linear spline function. The distribution $f_{\widetilde{Y}}$ is estimated in Section 4.2 and f_Z is estimated by kernel density estimation of Z. I then estimate the coefficients on the linear spline basis functions in a manner similar to the estimation of $F_{P_0^A}$.

Here are the estimation details. For the left hand side of equation (3), I pick a set of evenly spaced grid points $p_1, \ldots, p_{M^P}$ for p and run the local linear regression on those points. In particular, I pick p_1 and p_{M^P} as the 0.1 percentile and 99.9 percentile of $\widetilde{P}^A$ and $M^P = 200$. I use normal kernel with the asymptotically optimal bandwidth by Ruppert et al. (1995). Let the estimated probabilities be $\widehat{P}_{\text{lhs}}(p)$ on each grid point of p.

To evaluate the right-hand side of equation (3), I use a spline approximation of F_{S_0}:

$$F_{S_0}(s) = B^S(s)'\theta^S + \mathbf{1}(s > s_{K^S+1}),$$

where $B^S(s) = \begin{bmatrix} B_1^S(s) \\ \vdots \\ B_{K^S}^S(s) \end{bmatrix}$ and $\theta^S = \begin{bmatrix} \theta_1^S \\ \vdots \\ \theta_{K^S}^S \end{bmatrix}$. The functions $B_k^S(\cdot)$ are the linear spline basis functions on knots $\{s_k\}_{k=0}^{K^S+1}$ in the same fashion as in Section 4.3. In the estimation, I set the knots evenly spaced, with s_0 as the 0.1 percentile of $\widetilde{P}^A$ minus the sum of the 99.9 percentile of $\widetilde{Y}$ and the 99.9 percentile of Z, s_{K^S+1} as the 99.9 percentile of $\widetilde{P}^A$, and $K^S = 200$.

Furthermore, the probability density f_Z of Z is obtained by kernel density estimation:

$$\widehat{f}_Z(z) = \frac{1}{N_Z}\sum_{i=1}^{N_Z} K_h(z - Z_i),$$

where N_Z is the number of observed auction-day pairs. I use normal kernel and Scott's rule-of-thumb bandwidth. Estimates for the remaining distributions F_{R_0} and $f_{\widetilde{Y}}$ are already obtained when accounting for game-level unobserved heterogeneity.

To evaluate the integration in the right-hand side of equation (3), I use Gauss-Hermite quadrature with 10 nodes. The nodes on which to evaluate integration over y are rescaled by applying the method of Liu and Pierce (1994) to $f_{\widetilde{Y}}$, and the nodes on which to evaluate integration over z are rescaled by applying the same method to f_Z. Let the right hand side of equation (3) as a function of p and θ^S be $\widehat{P}_{\text{rhs}}(\theta^S; p)$.

The parameter θ^S is estimated by

$$\min_{\theta^S} \sum_{m=1}^{M^P} \left(\widehat{P}_{\text{lhs}}(p_m) - \widehat{P}_{\text{rhs}}(\theta^S; p_m)\right)^2$$

$$\text{subject to } 0 \le \theta_1^S \le \theta_2^S \le \cdots \le \theta_{K^S}^S \le 1.$$

4.5 Secret Reserve Price as a Function of Seller Value

The private component of seller's secret reserve price $R_0(S_0)$ can be obtained, using Assumption (A9), by:

$$R_0(S_0) = F_{R_0}^{-1} \circ F_{S_0}(S_0).$$

5 Model Estimates

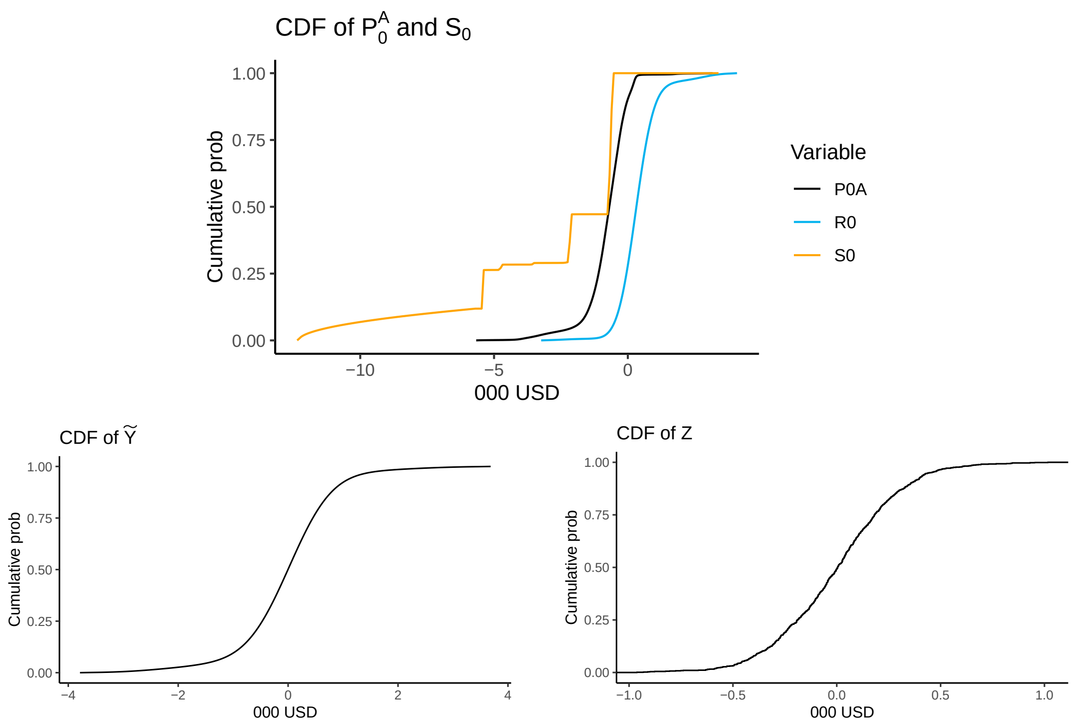

Figure 1: Estimated CDFs for the dealer sample. The units are $1,000.

Figure 1 shows estimated CDFs for the dealer sample, and Table 2 shows the moments and percentiles for those estimated CDFs, together with moments of $\boldsymbol{x}'\widehat{\boldsymbol{\beta}}$ (estimated observed heterogeneity). All are shown in units of $1,000. The values of P_0^A, S_0, R_0, $\widetilde{Y}$, Z can be negative as they are relative to $\boldsymbol{x}'\boldsymbol{\beta}$, the "fair price" explained by the observable characteristics. For instance, if $\boldsymbol{x}'\boldsymbol{\beta}$, $\widetilde{Y}$, Z, and S_0 are at their medians ($5,629, −$2, $4, and −$747), the corresponding seller value of the item is the sum of them, $4,884.

The average seller valuation is substantially below the average auction price: the average value of the item to the sellers ($\mathbb{E}[S_0 + \boldsymbol{x}'\boldsymbol{\beta} + \widetilde{Y} + Z]$) is estimated to be

\$4,203, while the average auction price ($\mathbb{E}[P_0^A + \boldsymbol{x}'\boldsymbol{\beta} + \widetilde{Y} + Z]$) is \$6,253, which is 49 percent above average seller value. On the other hand, the average secret reserve price $\mathbb{E}\left[R_0(S_0) + \boldsymbol{x}'\boldsymbol{\beta} + \widetilde{Y}\right]$ is \$7,405, even higher than the auction price. The plots indicate that the private component of seller values S_0 is first-order stochastically dominated by the second order statistic of bidders' private values P_0^A, which is again first-order stochastically dominated by $R_0(S_0)$. Despite this, the supports of P_0^A and S_0 overlap quite a bit, which means that there are cases where the seller would be better off keeping the item herself than trading.

The unobserved heterogeneity component $\widetilde{Y}$ has a standard deviation of \$852 and an interquartile range of \$920. These are larger than the corresponding values for P_0^A (standard deviation of \$769 and interquartile range of \$771). This implies that accounting for unobserved heterogeneity explains a substantial chunk of correlation between reserve prices and auction prices and has a substantial impact on the model estimates. The mean of $\widetilde{Y}$ is $-$\$33, but this is the result of normalization at $\mu_{\widetilde{Y}} = 0$ in equation (1) (recall that, without this normalization, $\widetilde{Y}$, $P_0^A + Z$, and R_0 are identified only up to constant shifts).

The market hotness Z has a standard deviation of \$284, which is 37% of that of P_0^A. The interquartile range is \$370, again less than half of the corresponding number for P_0^A. This implies that Z plays a modest role in the learning process by the seller.

	Mean	SD	pt25	pt50	pt75
$\boldsymbol{x}'\boldsymbol{\beta}$	7.069	4.975	3.185	5.629	9.863
$\widetilde{Y}$	−0.033	0.852	−0.470	−0.002	0.450
S_0	−2.833	3.213	−5.391	−0.747	−0.642
$R_0(S_0)$	0.369	0.722	−0.052	0.319	0.706
P_0^A	−0.783	0.769	−1.095	−0.693	−0.324
Z	0	0.284	−0.188	0.004	0.182

Table 2: Moments (mean, standard deviation) and percentiles (25%, 50%, 75%) of estimated CDFs for the dealer sample. The units are \$1,000.

Figure 2 shows the estimated function R_0. The function is always above the 45-degree line: the reserve price is always above the seller's value of the item when $Z = 0$.

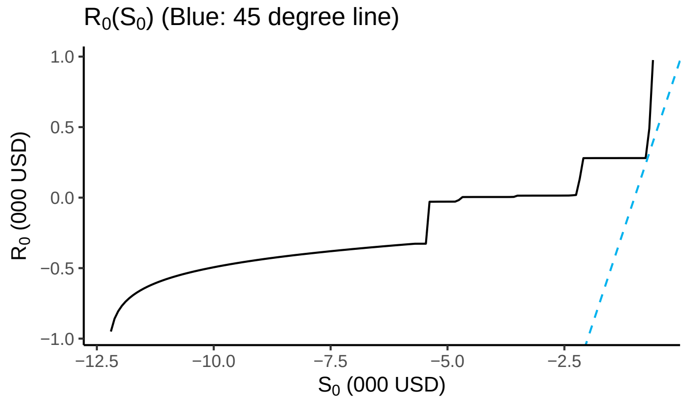

Figure 2: Estimated function R_0. The blue line indicates the 45 degree line.

6 Counterfactuals

6.1 Scenarios

I simulate the following:

1. Non-binding secret reserve prices (status quo) as a benchmark.
2. Infinite non-binding secret reserve prices. This is equivalent to having no reserve prices and having the seller decide whether to accept or reject the auction price P^A, regardless of the value of P^A.
3. Binding secret reserve prices. In this case, the binding secret reserve price solves

$$\max_R \mathbb{E}\left[\mathbf{1}(P^A \geq R)P^A + \mathbf{1}(P^A < R)S \,\middle|\, S_0, Y\right]. \tag{4}$$

The scenarios differ in their consequences in the following manner. Under infinite secret reserve prices (second scenario), the seller accepts P^A if and only if $P^A \geq S$. In contrast, the status quo (first scenario) could generate a "mistaken sale", i.e., sale of the item when $P^A < S$, if the auction price P^A is above the secret reserve price R. This does not necessarily mean that sale is inefficient: it is efficient if the winning bidder's value is above S, as P^A is the second highest value among biders and is lower than the winning bidder's. This happens when $P_0^A < S_0$ and the realization of Z is a high value and, consequently, P^A is so high it is above R, which does not depend on the realization of Z. Under the third scenario of binding secret reserve prices, I could see both "mistaken sale" (sale when $R \leq P^A < S$) and "mistaken no-sale" (no sale when $R > P^A > S$). Note that the distribution of P^A is the same across all scenarios, as each bidder's weakly dominant strategy is to drop out of the ascending auction when the price equals his value of the item.

For each scenario, I simulate a dataset of the same size as the original. I start with $\boldsymbol{x}'\widehat{\boldsymbol{\beta}}$ in the original dataset, and simulate random draws of S_0, P_0^A, $\widetilde{Y}$, and Z. For the first scenario, I simulate the reserve price as $R = R_0(S_0) + \boldsymbol{x}'\boldsymbol{\beta} + \widetilde{Y}$ using the estimated function $R_0(\cdot)$. For the third scenario, it can be shown that the optimal reserve price R^* is additively separable with respect to Y, i.e., $R^* = R_0^*(S_0) + Y$, and estimate R_0^* by computing the seller's expected payoff on a grid of the value of R_0 and picking the value that maximizes the expected payoff.

6.2 Simulation Results

	(1) Non-Binding RP	(2) RP = ∞	(3) Binding RP
Pr(trade)	0.717	0.711 (−0.6pp)	0.709 (−0.8pp)
Avg Seller Payoff ($000)	6.436	6.438 (+0.0%)	6.425 (−0.2%)
Avg Revenue ($000)	4.658	4.620 (−0.8%)	4.612 (−0.9%)

Table 3: Counterfactual simulation results (Columns 2 and 3) contrasted with status quo (Column 1). For Columns 2 and 3, the parentheses indicate the change of the probability of trade in percentage points compared to the status quo (first row) and the change of average seller payoff and average revenue relative to the level under the status quo (second and third rows).

Table 3 shows the simulated probability of trade, average seller payoff and average revenue for the status quo (Column 1) and two counterfactual scenarios (Columns 2 and 3). The results change very little. Under the scenario of infinite reserve prices (Column 2), the probability of trade goes down by 0.6 percentage points compared to the status quo. This corresponds to the mistaken sales under the status quo. Under the scenario of binding reserve price (Column 3), the probability of trade goes down by 0.2 percentage point compared to the second scenario. This consists of 3.7 percent of "mistaken sale" subtracted by 3.9 percent of "mistaken no-sale." As the probability of trade changes little, so do the seller payoff and revenue: they change by less than 1 percent relative to the status quo:

With infinite reserve prices, the average seller payoff goes up by less than 0.1% while average revenue goes down by 0.8% compared to the status quo. With binding reserve prices, average seller payoff goes down by 0.2%, while average seller revenue goes down by 0.9% compared to the status quo.

6.3 Discussions

As seen, the results are similar to each other under all scenarios. This tells us that the rationale for the auctioneer to implement the status quo may lie in reasons that are more subtle than seller payoff and revenue. Here, I offer several remarks regarding what may motivate the mechanism choice in practice. Presumably, the auctioneer finds it costly to implement infinite reserve prices as they find it costly to contact sellers for approval of sale for all values of the auction price. On the other hand, they may not wish to implement binding reserve prices (even though doing so would dispense them from the cost associated with contacting sellers after auctions) because the sellers find it costly to commit to reserve price and risk "mistaken sale" or "mistaken no-sale."

The simulation results may look different if I use an alternative specification of Z. Currently, Z has a smaller variation than that of P_0^A in terms of standard deviation and interquartile range as seen in Section (5). This makes the learning by the seller of her value play a limited role. If I choose another specification of Z that exhibits more variation — for instance, the average auction price within a finer segmentation than at the auction house-day level — I could potentially observe larger changes across formats.

7 Conclusion

This paper addresses why some auctions allow for non-binding secret reserve prices, using the dataset on wholesale market for used cars where such practice is prevalent. To that end, I estimate a structural model of ascending auctions. I introduce a component of seller's value that the bidders observe perfectly but the seller does not observe until after the auction, as in Andreyanov and Caoui (2020), to justify the observation that the auction prices often fall below the secret reserve prices but are accepted by the sellers nonetheless. I then develop the estimation method to deconvolve the observed and unobserved game-level heterogeneity and the component of seller's value that the seller learns over time.

I simulate two counterfactuals in addition to the status quo of non-binding secret reserve prices: (i) infinite non-binding secret reserve prices and (ii) binding secret reserve prices. I find that the results are very similar (the difference is by less than 1%), and I argue that the motivation for the current format may be a compromise between auctioneers' desire to reduce transaction costs in the after-auction process and sellers' concern about committing to a reserve price in advance.

A Proofs of Propositions

A.1 Proof of Proposition 1

Proof. The seller's payoff by accepting P^A is P^A, and her payoff by rejecting it is S. The proposition follows because the seller observes $S = S_0 + Y + Z$ at this stage. □

A.2 Proof of Proposition 2

Proof. Bidder i's payoff when the auction price is P^A is $B_i - P^A$ if he successfully buys the item and 0 otherwise. The result follows from the standard argument on ascending auctions: Compare bidder i's strategy to drop out when the price equals B_i (Strategy A) with another strategy B. Suppose that Strategy B causes bidder i to drop out when the price is at $P \neq B_i$. There are two possibilities: (i) $P < B_i$ and (ii) $P > B_i$. In Case (i), if all other bidders drop out when the price is below P, then bidder i wins at auction price P, and both Strategies A and B yield the same payoff. If all other bidders will not drop out until the price reaches B_i, both strategies result in zero payoff to bidder i. The result is different when the last bidder other than i drops out at a price $P^A \in (P, B_i)$. In such case, bidder i will obtain positive payoff $B_i - P^A$ with positive probability under Strategy A but zero payoff under Strategy B. Thus, Strategy A weakly dominates Strategy B. Similarly, in Case (ii), the strategies diverge in their consequences only when $P^A \in (B_i, P)$. Strategy A results in zero payoff of Bidder i, while Strategy B yields negative payoff $B_i - P^A$ with positive probability. □

A.3 Proof of Proposition 3

Proof. 1. The seller's ex ante expected payoff can be rewritten as:

$$\begin{aligned}
&\pi^S(R \mid S_0, Y) \\
&\quad = \mathbb{E}_{Z,P_0^A}\Big[\mathbf{1}(P^A \geq R)(P_0^A + Y + Z) + \mathbf{1}(P^A < R)\big(\max\{P_0^A + Y + Z, S_0 + Y + Z\} - c\big) \,\Big|\, Y, S_0\Big] \\
&\quad = Y + \mathbb{E}[Z] + \mathbb{E}_{Z,P_0^A}\Big[\mathbf{1}(P^A \geq R)P_0^A + \mathbf{1}(P^A < R)\big(\max\{P_0^A, S_0\} - c\big) \,\Big|\, Y, S_0\Big] \\
&\quad = Y + \mathbb{E}[Z] + \mathbb{E}_{Z,P_0^A}\Big[\mathbf{1}(P_0^A + Z \geq R_0)P_0^A + \mathbf{1}(P_0^A + Z < R_0)\big(\max\{P_0^A, S_0\} - c\big) \,\Big|\, Y, S_0\Big].
\end{aligned} \tag{5}$$

where $R_0 \equiv R - Y$. Maximizing this over R is equivalent to maximizing the last term, and the last term does not depend on Y other than through R_0. As such, the value of R_0 that maximizes the last term is independent of Y.

2. Define $\pi_0^S(R_0 \mid S_0) \equiv \pi^S(R \mid S_0, Y) - Y - \mathbb{E}[Z]$. This can be rewritten as:

$$\pi_0^S(R_0 \mid S_0) = \mathbb{E}_{P_0^A}\Big[\big(1 - F_Z(R_0 - P_0^A)\big)P_0^A + F_Z(R_0 - P_0^A)\big(\max\{P_0^A, S_0\} - c\big) \;\Big|\; S_0\Big].$$

If Z has a continuous distribution, differentiating π_0^S with respect to R_0 yields:

$$\begin{aligned}\frac{\partial \pi_0^S}{\partial R_0} &= \mathbb{E}_{P_0^A}\Big[-f_Z(R_0 - P_0^A)P_0^A + f_Z(R_0 - P_0^A)\big(\max\{P_0^A, S_0\} - c\big) \;\Big|\; S_0\Big] \\ &= \mathbb{E}_{P_0^A}\Big[f_Z(R_0 - P_0^A)\big(\max\{0, S_0 - P_0^A\} - c\big) \;\Big|\; S_0\Big] \\ &= \int_{-\infty}^{S_0} f_Z(R_0 - p_0^A)(S_0 - p_0^A)\, dF_{P_0^A}(p_0^A) - \int_{-\infty}^{\infty} c\, f_Z(R_0 - p_0^A)\, dF_{P_0^A}(p_0^A).\end{aligned}$$

R_0 is obtained by setting this partial derivative to zero and solving for R_0. Furthermore, using Leibniz rule,

$$\frac{\partial^2 \pi_0^S}{\partial S_0\, \partial R_0} = f_Z(R_0 - S_0)(S_0 - S_0) f_{P_0^A}(S_0) + \int_{-\infty}^{S_0} f_Z(R_0 - p_0^A)\, dF_{P_0^A}(p_0^A),$$

which is positive if $R_0(S_0) - S_0$ is in the interior of the support of Z, i.e., $R_0(S_0) - S_0 \in (\underline{z}, \overline{z})$ (note that P_0^A has an unbounded support). By Theorem 1 of Edlin and Shannon (1998), R_0 is strictly increasing in S_0.

3. First, assume $c = 0$. In the last term of equation (5), $\max\{P_0^A, S_0\} - c$ is always weakly larger than P_0^A if $c = 0$. Therefore, increasing R_0 does not reduce the seller's expected payoff, so $R_0 = \infty$ (and hence $R = \infty$) is an optimal strategy for the seller.
If $c > 0$, let $\overline{z} < \infty$ be the upper bound of Z. Compare the two reserve prices (minus Y): $R_0^1 = S_0 + \overline{z}$ and an arbitrary R_0^2 that is strictly greater than R_0^1. Then

$$\pi_0^S(R_0^2 \mid S_0) - \pi_0^S(R_0^1 \mid S_0) = \mathbb{E}_{Z, P_0^A}\Big[\mathbf{1}(R_0^1 \le P_0^A + Z < R_0^2)\big(\max\{0, S_0 - P_0^A\} - c\big) \;\Big|\; S_0\Big].$$

If $R_0^1 \le P_0^A + Z$, then $P_0^A + Z \ge S_0 + \overline{z} \ge S_0 + Z$ by definition of R_0^1, so $P_0^A \ge S_0$. Thus, $\pi_0^S(R_0^2 \mid S_0) - \pi_0^S(R_0^1 \mid S_0) = -c\,\mathbb{E}\big[\mathbf{1}(R_0^1 \le P_0^A + Z < R_0^2)\big] < 0$ (it's strictly negative since P_0^A has a continuous, unbounded support). The seller receives strictly higher expected payoff by setting R_0^1 than by setting any higher reserve price.

4. If Z is permanently 0, equation (5) (minus Y) can be rewritten as:

$$\begin{aligned}
&\mathbb{E}_{P_0^A}\left[\mathbf{1}(P_0^A \geq R_0)P_0^A + \mathbf{1}(P_0^A < R_0)\big(\max\{P_0^A, S_0\} - c\big) \,\Big|\, Y, S_0\right] \\
&= \mathbb{E}_{P_0^A}\left[P_0^A + \mathbf{1}(P_0^A < R_0)\big(\max\{0, S_0 - P_0^A\} - c\big) \,\Big|\, Y, S_0\right] \\
&= \mathbb{E}[P_0^A] + \int_{-\infty}^{R_0} \big(\max\{0, S_0 - p_0^A\} - c\big)\, dF_{P_0^A}(p_0^A).
\end{aligned}$$

The FOC is

$$0 = \big(\max\{0, S_0 - R_0\} - c\big) f_{P_0^A}(R_0),$$

which is satisfied if $R_0 = S_0 - c$, and hence $R = S_0 + Y - c$.

□

A.4 Proof of Proposition 4

Proof. The following holds:

$$\begin{aligned}
&\Pr\big(\widetilde{S} \leq \widetilde{P}^A < \widetilde{R} \mid \widetilde{P}^A = p\big) \\
&= \Pr\big(S_0 + \widetilde{Y} + Z \leq \widetilde{P}^A < R_0 + \widetilde{Y} \mid \widetilde{P}^A = p\big) \\
&= \mathbb{E}_{\widetilde{Y},Z}\left[\mathbb{E}_{S_0}\big[\mathbf{1}\big(S_0 \leq p - \widetilde{Y} - Z \cap p - \widetilde{Y} < R_0\big) \,\Big|\, \widetilde{Y}, Z; \widetilde{P}^A = p\big] \mid \widetilde{P}^A = p\right] \\
&= \mathbb{E}_{\widetilde{Y},Z}\left[\Pr_{S_0}\big(S_0 \leq p - \widetilde{Y} - Z \cap p - \widetilde{Y} < R_0 \,\Big|\, \widetilde{Y}, Z; \widetilde{P}^A = p\big) \mid \widetilde{P}^A = p\right]
\end{aligned}$$

Assuming R_0 is a monotone increasing function of S_0,

$$= \mathbb{E}_{\widetilde{Y},Z}\left[\Pr_{S_0}\big(S_0 \leq p - \widetilde{Y} - Z \cap S_0 > R_0^{-1}(p - \widetilde{Y}) \,\Big|\, \widetilde{Y}, Z; \widetilde{P}^A = p\big) \mid \widetilde{P}^A = p\right]. \quad (6)$$

By independence of S_0 from $\widetilde{P}_0^A, \widetilde{Y}, Z$, the conditioning $\widetilde{Y}, Z; \widetilde{P}^A = p$ can be disregarded in getting $\Pr(\cdot)$. Hence,

$$\begin{aligned}
(6) &= \mathbb{E}_{\widetilde{Y},Z}\left[\max\{F_{S_0}(p-\widetilde{Y}-Z) - F_{S_0}(R_0^{-1}(p-\widetilde{Y})), 0\} \,\middle|\, \widetilde{P}^A = p\right] \\
&= \mathbb{E}_{\widetilde{Y},Z}\left[\max\{F_{S_0}(p-\widetilde{Y}-Z) - F_{R_0}(p-\widetilde{Y}), 0\} \,\middle|\, \widetilde{P}^A = p\right] \\
&= \iint \max\{F_{S_0}(p-y-z) - F_{R_0}(p-y), 0\} f_{\widetilde{Y},Z|\widetilde{P}^A}(y, z \mid \widetilde{P}^A = p)\, dy\, dz \\
&= \iint \max\{F_{S_0}(p-y-z) - F_{R_0}(p-y), 0\} f_{\widetilde{Y}}(y) f_Z(z) f_{P_0^A}(p-y-z)\, dy\, dz \Big/ M(p),
\end{aligned} \tag{7}$$

where $M(p) = \iint f_{\widetilde{Y}}(y) f_Z(z) f_{P_0^A}(p-y-z)\, dy\, dz$.

The second equality uses monotonicity of R_0 and hence $R_0 = F_{R_0}^{-1} \circ F_{S_0}$. The last equality uses the definition of conditional density $f_{\widetilde{Y},Z|\widetilde{P}^A}(y, z \mid p) = \dfrac{f_{\widetilde{Y},Z,\widetilde{P}^A}(y,z,p)}{\iint f_{\widetilde{Y},Z,\widetilde{P}^A}(y,z,p)\, dy\, dz}$ and $f_{\widetilde{Y},Z,\widetilde{P}^A}(y,z,p) = f_{\widetilde{Y},Z,P_0^A}(y,z,p-y-z)$ by change of variable. □

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