

Not Only What but Also When: A Theory of Dynamic Voluntary Disclosure[†]

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We examine a dynamic model of voluntary disclosure of multiple pieces of private information. In our model, a manager of a firm who may learn multiple signals over time interacts with a competitive capital market and maximizes payoffs that increase in both period prices. We show (perhaps surprisingly) that in equilibrium later disclosures are interpreted more favorably even though the time the manager obtains the signals is independent of the value of the firm. We also provide sufficient conditions for the equilibrium to be in threshold strategies. (JEL D21, D82, G32, L25)

We study a dynamic model of voluntary disclosure of information by a potentially informed agent. The extant theoretical literature on voluntary disclosure focuses on static models in which an interested party (e.g., a manager of a firm) may privately observe a single piece of private information (e.g., Grossman 1981; Milgrom 1981; Dye 1985; and Jung and Kwon 1988) or dynamic models in which the disclosure timing does not play a role (e.g., Shin 2003, 2006) as the manager's decision is what to disclose but not when to disclose it. Corporate disclosure environments, however, are characterized by multi-period and multi-dimensional flows of information from the firm to the market, where the information asymmetry between the firm and the capital market can be with respect to whether, when, and what relevant information the firm might have learned. For example, firms with ongoing R&D projects can obtain new information about the state of their projects, where the time of information arrival and its content is unobservable to the market. This is common, for example, in pharmaceutical companies that get results of a drug's clinical trial (prior to FDA approval). Such results are not required to be publicly disclosed in a timely manner and investors' beliefs about the result of a drug's clinical trial may have a great effect on the firm's price. The multidimensional nature of the disclosure game (multi-period and multi-signal) plays a critical role in shaping the equilibrium; e.g., when deciding whether

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to disclose one piece of information the agent must also consider the possibility of learning and potentially disclosing a new piece of information in the future.

In order to study a dynamic model of voluntary disclosure, we extend Dye's (1985) and Jung and Kwon's (1988) voluntary disclosure model with uncertainty about information endowment to a two-period, two-signal setting. We describe the potentially informed agent as a manager of a publicly traded firm. In our model, the manager cares about stock prices in both periods and he may receive up to two private signals about the value of the firm. In each period, the manager may voluntarily disclose any subset of the signals he has received but not yet disclosed. Our model demonstrates how dynamic considerations shape the disclosure strategy of a privately informed agent and the market reactions to what he releases and when. Absent information asymmetry, the firm's price at the end of the second period is independent of the arrival and disclosure times of the firm's private information. Nevertheless, we show that in equilibrium, the market price depends not only on what information has been disclosed so far, but also on when it was disclosed. In particular, we show that the price at the end of the second period given disclosure of one signal is *higher if the signal is disclosed later* in the game. This result might be counterintuitive, as one might expect the market to reward the manager for early disclosure of information, since then he seems less likely to be "hiding something."

The intuition behind this result can be explained through a variant of the original static voluntary disclosure model of Dye (1985). In Dye (1985) the agent either learns nothing or learns a single signal with some probability. The equilibrium in that model is a threshold strategy where the price upon non-disclosure matches the price from disclosing the threshold type. Consider a variant of this game in which the disclosure is done in two steps. In the first step the agent follows an arbitrary exogenously determined disclosure policy. As a result, the set of informed agents who have not disclosed the signal is some set \mathcal{B} . In the second step the agent optimizes so that he reports if and only if doing so improves his payoff. Again, the equilibrium prices upon non-disclosure match the price if the threshold type is disclosed. But these prices are affected by disclosure in the first step, i.e., by the set \mathcal{B} . We shall see that the smaller is \mathcal{B} (in set-inclusion sense), the *higher* the equilibrium non-disclosure price (see Lemma 2 for a proof). For example, consider two possible cases for the first step where $\mathcal{B}' \supset \mathcal{B}''$, that is, in case of \mathcal{B}'' the agent follows a more aggressive disclosure in the first step. Our result is that even if all signals in $\mathcal{B}' \setminus \mathcal{B}''$ are higher than all signals in \mathcal{B}'' , the non-disclosure price in case of \mathcal{B}' is lower. Moreover, if some signals in $\mathcal{B}' \setminus \mathcal{B}''$ are smaller than the second-step threshold in case of \mathcal{B}' , the ranking of prices given non-disclosure is strict. In words, the more aggressive disclosure policy results in a higher inference for those who did not disclose in the second step.

How is this related to our result? Consider two histories (on the equilibrium path) in which the manager discloses only one signal, x . In history 1, the manager disclosed x at $t = 1$ while in history 2 he disclosed x at $t = 2$. Suppose it is now $t = 2$. The equilibrium price depends on the market belief about the value of the other signal y . In both histories, in period 2 once x is revealed, the agent reveals y if it increases current price. This corresponds to the second step in our hypothetical game. The first step in the hypothetical game corresponds to the disclosure policy of y at $t = 1$. Our proof relies on comparing the aggressiveness of the disclosure policy

of y in the two histories. An event that plays a key role in our proof is the following: if the agent reveals x at $t = 1$, investors know that he could not have known only y at $t = 1$. However, if he reveals x at $t = 2$, they cannot rule out that at $t = 1$ he knew only y . Moreover, investors know that if the agent knows only one of the signals at $t = 1$, he discloses it if it is high enough. So when x is disclosed at $t = 2$, it implies that some possible realizations of y can be excluded. Hence, there is more disclosure of y at $t = 1$ in history 2. As we argued in our description of the hypothetical game, even though the excluded values are relatively high realizations, it still leads to a positive inference, especially if the revealed x is high.

In Section II, we formalize and extend this intuition to establish the main result of the paper. We argue that later disclosure receives a better interpretation provided that the equilibrium is monotone and symmetric. To further characterize the strategic behavior and market inferences in our model, in Section III we discuss the main strategic considerations in equilibrium and establish existence of threshold equilibria under suitable conditions.¹

Related Literature.—The voluntary disclosure literature goes back to Grossman and Hart (1980); Grossman (1981); and Milgrom (1981), who established the “unraveling result,” which states that under certain assumptions (including: common knowledge that the agent is privately informed, disclosing is costless, and information is verifiable) all types disclose their information in equilibrium. In light of companies’ propensity to withhold some private information, the literature on voluntary disclosure evolved around settings in which the unraveling result does not prevail. The two major streams of this literature are: (i) assuming that disclosure is costly (pioneered by Jovanovic 1982 and Verrecchia 1983) and (ii) investors’ uncertainty about information endowment (pioneered by Dye 1985 and Jung and Kwon 1988). Our model follows Dye (1985) and Jung and Kwon (1988) and extends it to a multi-signal and a multi-period setting.

As mentioned in the introduction, in spite of the vast literature on voluntary disclosure, very little has been done on multi-period settings and on multi-signal settings.²

To the best of our knowledge the only papers that study multi-period voluntary disclosure are Shin (2003, 2006); Einhorn and Ziv (2008); and Beyer and Dye (2012). The settings studied in these papers as well as the dynamic considerations of the agents are very different from ours. Shin (2003, 2006) studies a setting in which a firm may learn a binary signal for each of its independent projects, where each project may either fail or succeed. In this binary setting, Shin (2003, 2006) studies the “sanitization” strategy, under which the agent discloses only the good (success) news. The timing of disclosure does not play a role in such a setup. Einhorn and Ziv (2008) study a setting in which in each period the manager may obtain a single signal about the period’s cash flows, where at the end of each period the realized cash flows are publicly revealed. If the agent chooses to disclose

¹In most of the existing voluntary disclosure literature (e.g., Verrecchia 1983; Dye 1985; Acharya, DeMarzo, and Kremer 2011), the equilibrium always exists, is unique, and is characterized by a threshold strategy. In our model, due to multiple periods and signals, existence of a threshold equilibrium is not guaranteed, and therefore we provide sufficient conditions for existence (similar to Pae 2005).

²For example, this gap in the literature is pointed out in a survey by Hirst, Koonce, and Venkataraman (2008, p.315), who write “much of the prior research ignores the iterative nature of management earnings forecasts.”

his private signal, he incurs some disclosure costs. Acharya, DeMarzo, and Kremer (2011) examine a dynamic model in which a manager learns one piece of information at some random time and his decision to disclose it is affected by the release of some external news. They show that a more negative external signal is more likely to trigger the release of information by the firm. Perhaps surprisingly this clustering effect is present only in a dynamic model and not in a static one. Given that the firm may learn only one piece of information the effect that we study in our paper cannot be examined in their model. Finally, Beyer and Dye (2012) study a reputation model in which the manager may learn a single private signal in each of the two periods. The manager can be either “forthcoming” and disclose any information he learns or he may be “strategic.” At the end of each period, the firm’s signal/cash flow for the period becomes public and the market updates beliefs about the value of the firm and the type of the agent. Importantly, the option to “wait for a better signal” that is behind our main result is not present in any of these papers.

Our paper also adds to the understanding of management’s decision to selectively disclose information. Most voluntary disclosure models assume a single signal setting, in which the manager can either disclose all of his information or not disclose at all. In practice, managers sometimes voluntarily disclose part of their private information while concealing another part of their private information. To the best of our knowledge, the only exceptions in the voluntary disclosure literature in which agents may learn multiple signals are Shin (2003, 2006), which we discussed above, and Pae (2005). The latter considers a single-period setting in which the agent can learn up to two signals. We add to Pae (2005) dynamic considerations, which are again crucial for creating the option value of waiting for a better signal.

Bhattacharya and Ritter (1983) examine another aspect of disclosure to capital markets. In their model multiple firms compete in an R&D race. A firm may have information about better technology that enables it to advance faster. The trade-off in that paper is that revealing information about this technology leads to better financing terms but at the same time reduces the firm’s technological advantage over competing firms.

I. The Model

Consider the following dynamic voluntary disclosure game. There is an agent, who we refer to as a manager of a publicly traded company, and a competitive market of risk neutral investors. The value of the company is a realization of a random variable, V , and V is not known to the market or the manager. All agents share a common prior over the distribution of V . There are two signals of V , which we denote by X and Y . Conditional on V these signals are identically and independently distributed over \mathbb{R}^2 according to some atomless distribution. We assume that the support of the conditional distribution of a signal is independent of the realization of V and that the density of that distribution is positive on this support.

We denote the expected value of V given the realizations of the two signals, (x, y) , by

$$E[V|X = x, Y = y] = P(x, y) = P(y, x).$$

We assume that P is continuous and strictly increasing in both arguments.

The game has two periods, $t \in \{1, 2\}$. At the beginning of period 1 the manager *privately* learns each of the signals with probability p . Learning a signal is independent across the two signals, so that the probability of learning both signals at $t = 1$ is p^2 . Learning a signal is also independent of the value of any of the signals or the value of the company. In the beginning of period 2 the manager learns with probability p any signal that he has not yet learned in period 1.³

Each period, after potentially learning some signals, the manager decides whether to reveal some or all of the signals he has learned and not yet disclosed: disclosure is voluntary and can be selective. We follow Grossman (1981); Milgrom (1981); and Dye (1985) and assume that: (i) the agent cannot credibly convey the fact that he did not obtain a signal, and (ii) any disclosure is truthful (or verifiable at no cost) and does not impose a direct cost on the manager or the firm.

A public history at time t contains the set of signals that the agent has revealed and the time each signal was revealed, (t_x, t_y) . We denote the public history by H_t^P and let $\mathcal{H}_t^P = \{\emptyset, (x, t_x), (y, t_y), (x, y, t_x, t_y)\}$ denote the set of potential public histories, where \emptyset denotes a history in which no disclosure has been made. The market does not know when the agent has learned a signal. For example, if the agent reveals a signal x in period 2, the market cannot directly observe whether the manager learned that signal in period 1 or 2.

Investors observe only the public history. The agent observes both the public and a private history. Agent's private history at the beginning of period 1 is the signals he has learned so far, $H_1^A \in \mathcal{H}_1^A = \{\emptyset, x, y, (x, y)\}$. At the beginning of period 2 a private history is the signals that the agent has learned and when he has learned them, $H_2^A \in \mathcal{H}_2^A = \{\emptyset, (x, \tau_x), (y, \tau_y), (x, y, \tau_x, \tau_y)\}$, where (τ_x, τ_y) denote the times the agent has learned the signals X and Y , respectively. We denote by $\tau_x, \tau_y > 2$ the case that the agent did not learn the corresponding signal.

A (behavioral) strategy of the agent is a disclosure policy which is a mapping from histories (public and private since the agent observes both) into a decision whether to reveal any of the signals he has observed so far and not disclosed yet.

We model investors in a reduced form: given the public history, they form beliefs about the value of the firm and set the market price at time t equal to

$$P_t(H_t^P) \equiv E[V | H_t^P] = E[P(x, y) | H_t^P].$$

Note that conditional on the agent revealing both signals, the market price is $P_t(x, y, t_x, t_y) = P(x, y)$ and it is independent of when the signals were disclosed. This follows from the fact that upon revealing both signals there is no information asymmetry about V .⁴ However, in other cases investors form beliefs based on the equilibrium strategy of the agent and will infer that the agent might have learned

³ All the model's analysis and results are robust to the introduction of a third period in which the private signals learned by the manager are publicly revealed.

⁴ Recall the assumption that τ_x and τ_y are independent of V .

some signals and decided not to reveal them. For example, when only one signal, e.g., x , has been revealed the price will be

$$P_t(x, t_x) = E_y[P(x, y) | H_t^P = (x, t_x)],$$

where the beliefs over the second signal, y , are formed consistently with Bayes' rule and the equilibrium strategy of the agent, whenever possible.

We assume the manager maximizes a payoff function:

$$U(P_1(H_1^P), P_2(H_2^P)),$$

that is continuous and strictly increasing in both prices. The interpretation is that the manager's compensation is increasing in each period's stock price (and/or that the probability of losing the job is decreasing in each period's stock price and the manager strictly dislikes being fired). In Section III we analyze the model with additional distributional assumptions and with $U(P_1(H_1^P), P_2(H_2^P)) = (P_1(H_1^P) + P_2(H_2^P))$ to provide additional results.

A (perfect Bayesian) equilibrium is a profile of disclosure policies of the agent and a set of price functions $\{P_t(\emptyset), P_t(x, t_x), P_t(y, t_y), P(x, y)\}$ (both on and off the equilibrium path) such that the agent optimizes given the price functions and the prices are consistent with the strategy of the agent by applying Bayes' rule whenever possible. The equilibrium is *monotone* if the price function $P_t(x, t_x)$ is increasing in x for all t and t_x . We restrict our analysis to *symmetric monotone equilibria in pure strategies* that is, monotone equilibria in which $P_t(x, t_x) = P_t(y, t_y)$ (i.e., the price does not depend on which signal has been revealed) and the agent's disclosure policy is deterministic (i.e., on the equilibrium path the agent does not randomize whether to reveal a signal or not given the history).

Remark 1: We assume that investors can tell which signal (X or Y) is disclosed. This applies to many real world applications where signals correspond to different dimensions of the firm's business. For example, signals may correspond to information about revenues and costs, represent information about two different markets, or correspond to two different projects of the firm. That said, given the symmetric setup and our focus on symmetric equilibria, the equilibrium outcomes we describe coincide with equilibrium outcomes in a game where investors cannot tell which signal is disclosed. For example, X and Y may be two signals about future sales of the company and the agent may obtain them over time.

Figure 1 summarizes the sequence of events in the model.

II. Later Disclosures Receive Better Responses

In this section we present our main result: if we compare two public histories in which only one signal is revealed but at different times, the market price is higher in the history with later disclosure.⁵ In other words, the market forms its beliefs based

⁵Since we have only two signals, to show the effect of time of disclosure on equilibrium prices, we have to focus on the histories with one signal revealed.

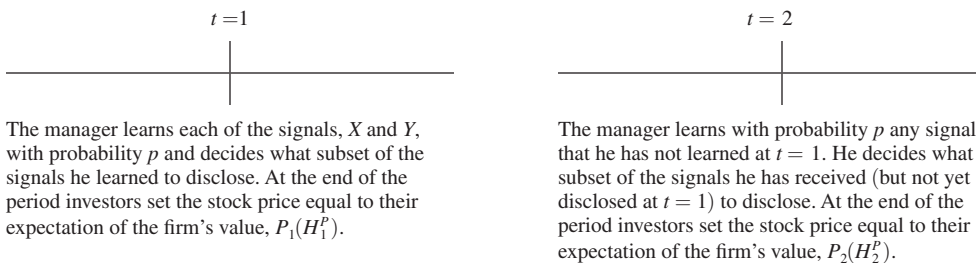


FIGURE 1. TIMELINE

on what is revealed *and also when it is revealed* despite the value V being independent of the times the agent learns the signals.

As mentioned above, we focus on symmetric monotone equilibria in pure strategies. Without loss of generality, we focus on histories such that either X is disclosed before Y or both signals are not disclosed, that is, $t_x \leq t_y$.

THEOREM 1: *Consider any symmetric monotone PBE in pure strategies in which public histories $H_2^p = (x, 1)$ and $\hat{H}_2^p = (x, 2)$ are on the equilibrium path. Then,*

$$P_2(x, 2) \geq P_2(x, 1),$$

i.e., in period 2 the price upon revelation of only one signal is higher if that signal was revealed later.

Theorem 1 characterizes a property of any symmetric monotone PBE in pure strategies. In the rest of this section we refer to this class of PBE as “equilibrium.” In Section III we demonstrate the existence of a threshold equilibrium that has all these assumed properties. Moreover, we show in Section III that the effect of later disclosure on the price at $t = 2$ is strict for a range of signals; that there exists an x' such that $P_2(x, 2) > P_2(x, 1)$ for all $x > x'$ (and $(x, 1), (x, 2)$ are public histories on the equilibrium path).

We prove Theorem 1 via a series of lemmas. Some of the proofs are in the Appendix, but we try to present the main intuition in the remainder of this section.

We start by noting that at $t = 2$, since this is the last period, an agent that revealed one signal is myopic and reveals the second signal if and only if it improves the agent's payoff at $t = 2$ relative to non-disclosure of the second signal. That is:

LEMMA 1: *In any equilibrium, conditional on revealing x (at any time), the manager reveals y at $t = 2$ if and only if $P(x, y) \geq P_2(x, t_x)$.⁶*

⁶To simplify the exposition, throughout this section we assume that an agent who is indifferent will disclose his information, but it is without loss of generality.

Given that $P(x, y)$ is increasing in y , the above lemma implies that at $t = 2$ the agent follows a threshold strategy by disclosing $y > Y_{x,t_x}$, where Y_{x,t_x} is defined by $P_2(x, t_x) = P(x, Y_{x,t_x})$.⁷

A key concept that we define and use is that of *potential disclosers*. The set of potential disclosers is defined as the set of types who, in equilibrium, learned Y either at $t = 1$ or $t = 2$, disclosed $X = x$ at either $t = 1$ or $t = 2$ and did not disclose Y at $t = 1$, i.e., types such that $\{t_x \leq 2, \tau_y \leq 2 \text{ and } t_y > 1\}$. This is the set of agents whose behavior is described by Lemma 1. The set of potential disclosers can be obtained by starting with the set of *informed* (who learned Y) and eliminating types that, on the equilibrium path, would have: (i) disclosed y at $t = 1$, (ii) disclosed y but not x , and (iii) preferred to disclose nothing given x and y . In Sections IIB and IIC we characterize the set of potential disclosers for H_2^P and \hat{H}_2^P , respectively.

Our proof of Theorem 1 follows from the comparison of the sets of potential disclosers for the two histories. Why are the sets of potential disclosers important for comparing $P_2(x, t_1)$ and $P_2(x, t_2)$? Prices at $t = 2$ are determined as follows. For any of the histories, start with two possibilities: either the agent does not know y , i.e., he is *uninformed* (which happens with an ex ante probability $(1 - p)^2$), or he is *informed* (and learned y either at time 1 or 2). Then, in case he is informed, exclude all realizations of Y that are inconsistent with equilibrium behavior given the history. This can be done in two steps: first exclude all types other than the potential disclosers and then apply Lemma 1 to remove additional types. That leaves only types (τ_y, y) that are consistent with the history and equilibrium strategies and we can compute the price as the expected value of $P(x, y)$ over these types (given the disclosed value of X and the conditional distribution of Y given X). We describe this procedure in greater detail in Section IID. A difficulty in computing the equilibrium $P_2(x, t_x)$ is that it is a solution to a fixed-point problem: the price depends on the disclosure policy and vice versa (i.e., Y_{x,t_x} and $P_2(x, t_x)$ are interdependent). This is why it is useful to divide the exclusion of types after the two histories into the identification of the potential disclosers and the application of Lemma 1, where the second step captures the fixed-point reasoning at $t = 2$.

A. Generalized Minimum Principle

In this subsection we introduce Lemma 2, which is an extension of the minimum principle in Acharya, DeMarzo, and Kremer (2011), and which will help us characterize the equilibrium prices.⁸

Given sets \mathcal{A} and \mathcal{B} , and an increasing continuous function g , define $\mathcal{S}_{\mathcal{A},\mathcal{B}}$ as

$$\mathcal{S}_{\mathcal{A},\mathcal{B}} \equiv \mathcal{A} \cup \{\mathcal{B} \cap \{y : g(y) < E[g(y) | y \in \mathcal{S}_{\mathcal{A},\mathcal{B}}]\}\}.$$

Let us explain this definition since the notation is somewhat non-standard. Let the sets \mathcal{A} and \mathcal{B} be subsets of \mathbb{R} and have corresponding measures (not necessarily probabilistic) over these elements, $F_{\mathcal{A}}$ and $F_{\mathcal{B}}$ respectively. The notation

⁷Existence and uniqueness of Y_{x,t_x} follows from (i) $P(x, y)$ is increasing and continuous in y ; (ii) $P_2(x, t_x)$ is the expected value of $P(x, y)$ conditional on the equilibrium beliefs about y so it is in the range of $P(x, \cdot)$.

⁸Acharya, DeMarzo, and Kremer (2011) established a claim that is similar to (0) and (i) of Lemma 2.

$\mathcal{A} \cup \mathcal{B}$ represents a set of \mathbb{R} with a measure $F_{\mathcal{A} \cup \mathcal{B}}$ that is the sum of these measures ($dF_{\mathcal{A} \cup \mathcal{B}} = dF_{\mathcal{A}} + dF_{\mathcal{B}}$). For example, suppose \mathcal{A} is the set $[-10, 10]$ and \mathcal{B} is $[0, 20]$ where $F_{\mathcal{A}}$ and $F_{\mathcal{B}}$ have a constant density over these intervals. The set $\mathcal{A} \cup \mathcal{B}$ corresponds to the interval $[-10, 20]$ with measure $F_{\mathcal{A} \cup \mathcal{B}}$ that is twice as high on the interval $[0, 10]$ as compared to $[-10, 0]$ and $[10, 20]$. The expectation $E[g(y) | y \in \mathcal{S}_{\mathcal{A}, \mathcal{B}}]$ is computed given the set $\mathcal{S}_{\mathcal{A}, \mathcal{B}}$ by normalizing the corresponding measure of $\mathcal{S}_{\mathcal{A}, \mathcal{B}}$ to be probabilistic. $\mathcal{B} \cap \{y : g(y) < E[g(y) | y \in \mathcal{S}_{\mathcal{A}, \mathcal{B}}]\}$ means that we are removing from set \mathcal{B} all elements that are higher than the average y in $\mathcal{S}_{\mathcal{A}, \mathcal{B}}$. When we remove elements from \mathcal{B} , we do not change the measure of the remaining elements (so that the total measure of the set \mathcal{B} drops by the measure of the removed elements): the re-normalization of measures happens only at the time when we compute the overall average. In this way, as we remove more and more elements from \mathcal{B} , the overall average assigns higher and higher weight to the elements in \mathcal{A} .

Here are some important properties of $\mathcal{S}_{\mathcal{A}, \mathcal{B}}$:

LEMMA 2 (Generalized Minimum Principle):

- (0) $\mathcal{S}_{\mathcal{A}, \mathcal{B}}$ exists and is unique.
- (i) $E[g(y) | y \in \mathcal{A} \cup \mathcal{B}] \geq E[g(y) | y \in \mathcal{S}_{\mathcal{A}, \mathcal{B}}]$, with equality if and only if every (up to measure zero) $y \in \mathcal{B}$ satisfies $g(y) < E[g(y) | y \in \mathcal{S}_{\mathcal{A}, \mathcal{B}}]$.
- (ii) Suppose that $\mathcal{B}' \supseteq \mathcal{B}''$. Then $E[g(y) | y \in \mathcal{S}_{\mathcal{A}, \mathcal{B}''}] \geq E[g(y) | y \in \mathcal{S}_{\mathcal{A}, \mathcal{B}'}]$.
- (iii) Suppose that $\mathcal{B}' \supset \mathcal{B}''$. Then $\mathcal{S}_{\mathcal{A}, \mathcal{B}''} = \mathcal{S}_{\mathcal{A}, \mathcal{B}'}$ if and only if $g(y) > E[g(y) | y \in \mathcal{S}_{\mathcal{A}, \mathcal{B}''}]$ for all $y \in \mathcal{B}' \setminus \mathcal{B}''$.⁹

To see the intuition behind the existence of $\mathcal{S}_{\mathcal{A}, \mathcal{B}}$, consider an iterative procedure of constructing $\mathcal{S}_{\mathcal{A}, \mathcal{B}}$. In each step we remove from \mathcal{B} some types that are higher than the previous average, which decreases the average. This is obviously a converging procedure, which stops at the latest after all types in \mathcal{B} are removed (and then we are left with the set \mathcal{A} to compute the expectation).

To see the intuition behind (ii) and (iii), take $g(y) = y$ and note that whether the expectation of Y conditional on $y \in \mathcal{S}_{\mathcal{A}, \mathcal{B}}$ increases or decreases as we remove types from \mathcal{B} depends on whether the removed types are higher or lower than the conditional average. In particular, when we exclude from \mathcal{B} some y that are higher than the conditional average, this average does not change because these values are removed anyhow in the construction of $\mathcal{S}_{\mathcal{A}, \mathcal{B}}$. However, when we remove from \mathcal{B} realizations of y that are lower than the conditional average (even if these are above-average elements of \mathcal{B}) then the conditional average goes up because these are below-average types in the original set $\mathcal{S}_{\mathcal{A}, \mathcal{B}}$. The proof of (ii) and (iii) and the formalization of (0) and (i) are in the Appendix.

In our application, \mathcal{A} corresponds to the set of uniformed agents and \mathcal{B} corresponds to the set of potential disclosers. The function $g(y)$ corresponds to $P(x, y)$

⁹Note that (ii) and (iii) imply that if there are elements $z \in \mathcal{B}' \setminus \mathcal{B}''$ such that $z < E[y | y \in \mathcal{S}_{\mathcal{A}, \mathcal{B}''}]$ then $E[y | y \in \mathcal{S}_{\mathcal{A}, \mathcal{B}''}] > E[y | y \in \mathcal{S}_{\mathcal{A}, \mathcal{B}'}]$.

given the revealed x . F_A is the probability distribution of Y conditional on $X = x$ times the ex ante probability that the agent does not know Y , $(1 - p)^2$. F_B is more complicated since \mathcal{B} is a union of sets of potential disclosers that correspond to the different times that the agent could have learned Y and X , as we describe below. The plan of the proof is to compare the sets of potential disclosers after the two histories in question and apply Lemma 2 to establish the ranking of prices.

B. The Set of Potential Disclosers when X is Disclosed at $t = 1$

In the next two subsections we describe in detail the sets of potential disclosers after the two histories. We start with the set after X has been revealed at $t = 1$. From Lemma 1 we know that at $t = 2$ the agent follows a myopic threshold policy. This implies that no disclosure of Y at $t = 2$ is bad news as compared to no disclosure of Y at $t = 1$ because investors know that the agent is informed with a higher probability at $t = 2$. Hence, the corresponding prices drop over time, i.e., $P_1(x, 1) \geq P_2(x, 1)$. As a result, an informed agent who has revealed X at $t = 1$ would reveal also Y if and only if it increases the current price (i.e., he follows a myopic strategy). The following lemma summarizes these observations:

LEMMA 3: *In equilibrium, conditional on disclosure of X at $t = 1$:*

- (i) *if the agent does not reveal Y , prices drop over time, that is: $P_1(x, 1) \geq P_2(x, 1)$,*
- (ii) *the agent's optimal disclosure strategy for Y is myopic at $t = 1$. That is, conditional on disclosing X at $t = 1$ an informed agent reveals also Y at $t = 1$ if and only if $P(x, y) \geq P_1(x, 1)$.*

If the agent revealed X at $t = 1$, investors know that $\tau_x = 1$, but there are ex ante three possibilities regarding the time he learned Y , $\tau_y \in \{1, 2, 3\}$. We decompose the set of potential disclosers when X is disclosed at $t = 1$, \mathcal{B}_1 , into two disjoint subsets, $\mathcal{B}_1 = \mathcal{B}_1^1 \cup \mathcal{B}_1^2$, based on when the agent has learned y . These subsets are given by¹⁰

$$\mathcal{B}_1^1 = \{y | \tau_x = \tau_y = 1, y \text{ is consistent with only } x \text{ being revealed at } t = 1\},$$

$$\mathcal{B}_1^2 = \{y | \tau_x = 1, \tau_y = 2, y \text{ is consistent with } x \text{ being revealed at } t = 1\}.$$

Let \mathcal{A}_1 denote the y coordinate of the set of uninformed agents. Before we remove from \mathcal{B}_1^1 types which are not consistent with the public histories of potential disclosers, the sets \mathcal{A}_1 , \mathcal{B}_1^1 , \mathcal{B}_1^2 have measures given by the conditional distribution of Y given $X = x$, multiplied by $\{(1 - p)^2, p, p(1 - p)\}$, respectively.

¹⁰By "is consistent with" we mean the realizations of Y and that are consistent with the equilibrium path and the public histories $(x, t_x = 1)$ and $(x, y, t_x = 1, t_y = 2)$.

Using the notation introduced in the previous subsection, Lemma 1 implies that

$$(1) \quad P_2(x, 1) = E[P(x, y) | y \in \mathcal{S}_{\mathcal{A}_1, \mathcal{B}_1}],$$

where

$$\mathcal{S}_{\mathcal{A}_1, \mathcal{B}_1} \equiv \mathcal{A}_1 \cup \{\mathcal{B}_1 \cap \{y : P(x, y) < E[(P(x, y) | y \in \mathcal{S}_{\mathcal{A}_1, \mathcal{B}_1})]\}\}.$$

If X is disclosed at $t = 1$ and Y was learned only at $t = 2$ then no realization of Y can be ruled out. Therefore, \mathcal{B}_1^2 is the whole domain of Y .

On the other hand, \mathcal{B}_1^1 can be described as the intersection of three conditions, $\mathcal{B}_1^1 = \mathcal{C}_1(x) \cap \mathcal{C}_2(x) \cap \mathcal{C}_3(x)$ where

- $\mathcal{C}_1(x)$: At $t = 1$, the agent prefers to reveal x instead of revealing both x and y . By Lemma 3, this condition is that y satisfies $P(x, y) \leq P_1(x, 1)$.
- $\mathcal{C}_2(x)$: At $t = 1$, the agent prefers to reveal x rather than y . Monotonicity of the equilibrium implies that this condition is $y \leq x$.
- $\mathcal{C}_3(x)$: At $t = 1$, the agent prefers to reveal x rather than to hide both x and y .

It is hard to fully pin down the equilibrium implications of the last condition. However, as the next lemma shows, if Theorem 1 did not hold, that is if $P_2(x, 2) < P_2(x, 1)$, we obtain a simple way to express equilibrium prices.

LEMMA 4: *Suppose that in equilibrium $P_2(x, 2) < P_2(x, 1)$. Then, there exists $y^*(x) \geq Y_{x,2}$ and the corresponding set $\mathcal{B}'_1 \equiv \{y | \tau_y = 1, y \leq \min\{x, y^*(x)\}\}$, such that if we replace \mathcal{B}_1^1 with \mathcal{B}'_1 in equation (1), the resulting price is still $P_2(x, 1)$.*

C. The Set of Potential Disclosers when X is Disclosed at $t = 2$

When X is disclosed at $t = 2$, investors in general do not know whether $\tau_x = 1$ or $\tau_x = 2$. This could make it more complicated to describe the price $P_2(x, 2)$ since there would be four cases to consider for the potential disclosers: $(\tau_x, \tau_y) \in \{1, 2\}^2$. However, as we prove in the Appendix, if Theorem 1 did not hold, that is if $P_2(x, 2) < P_2(x, 1)$, we could rule out that $\tau_x = 1$ if $t_x = 2$.

LEMMA 5: *Suppose that in equilibrium $P_2(x, 2) < P_2(x, 1)$. Then the public history with $t_x = 2$ and $t_y > 1$ is consistent only with $\tau_x = 2$ (i.e., if the agent reveals X at $t = 2$, investors infer that the agent must have learned X at $t = 2$).*

The intuition is that the contradictory assumption, $P_2(x, 2) < P_2(x, 1)$, provides stronger incentives for an agent who has learned $X = x$ at $\tau_x = 1$ to disclose at $t_x = 1$ instead of waiting with disclosure till $t_x = 2$. The details of the proof are in the Appendix.

Since our proof of Theorem 1 is by contradiction, from now on we maintain the assumption that after the history $\hat{H}_2^p = (x, 2)$ investors assign probability 1 to $\tau_x = 2$. This allows us to decompose the set of potential disclosers \mathcal{B}_2 analogously

to the decomposition of \mathcal{B}_1 above. In particular, we decompose \mathcal{B}_2 into two disjoint subsets, $\mathcal{B}_2 = \mathcal{B}_2^1 \cup \mathcal{B}_2^2$:

$$\mathcal{B}_2^1 = \{y \mid \tau_x = 2, \tau_y = 1, y \text{ is consistent with } x \text{ being revealed at } t = 2\},$$

$$\mathcal{B}_2^2 = \{y \mid \tau_x = \tau_y = 2, y \text{ is consistent with } x \text{ being revealed at } t = 2\}.$$

The set \mathcal{A}_2 is the y coordinate of the uninformed agents. The three sets have the same corresponding measures as in the case of \mathcal{A}_1 and \mathcal{B}_1 .

Using the notation from Section IIA we can write

$$(2) \quad P_2(x, 2) = E[P(x, y) \mid y \in \mathcal{S}_{\mathcal{A}_2, \mathcal{B}_2}],$$

where

$$\mathcal{S}_{\mathcal{A}_2, \mathcal{B}_2} \equiv \mathcal{A}_2 \cup \{\mathcal{B}_2 \cap \{y : P(x, y) < E[(P(x, y) \mid y \in \mathcal{S}_{\mathcal{A}_2, \mathcal{B}_2})]\}\}.$$

What realizations of Y are not consistent with equilibrium? First, for both sets, \mathcal{B}_2^1 and \mathcal{B}_2^2 we need to exclude types $y > x$ because, given our assumption that the equilibrium is symmetric and monotone, the agent would prefer to reveal y and not x in period 2 in those cases. This is in fact the only exclusion we can make in case of \mathcal{B}_2^2 , so $\mathcal{B}_2^2 = \{y \mid \tau_x = \tau_y = 2, y \leq x\}$.¹¹

Regarding \mathcal{B}_2^1 , we need to also exclude realizations of y that would have been disclosed at $t = 1$ if the agent knew only y at $t = 1$. Therefore: $\mathcal{B}_2^1 = \{y \mid \tau_x = 2, \tau_y = 1, y \leq x, y \in \mathcal{ND}\}$ where \mathcal{ND} is the set of values of y that are not disclosed at $t = 1$ when the agent only knows y at $t = 1$.

D. Proof of the Main Theorem

Suppose by contradiction that $P_2(x, 2) < P_2(x, 1)$. Following Lemma 5 we can assume that the time when an agent discloses X coincides with when he has learned it, so that $t_x = \tau_x$. We divide the type space based on when the agent learns his information. Let $\mathcal{L}_{\tau_x, \tau_y}$ denote the set of types who learn X at τ_x and Y at τ_y (recall our convention that $\tau_y = 3$ means that the agent did not learn Y). In constructing the sets of uninformed and potential disclosers, we first condition on τ_x and then impose additional equilibrium conditions by removing certain types to obtain the set of potential disclosers.

When X is disclosed at $t = 1$, conditioning on $\tau_x = 1$ leads to $\mathcal{L}_{1,1} \cup \mathcal{L}_{1,2} \cup \mathcal{L}_{1,3}$, where the set of uninformed, \mathcal{A}_1 , corresponds to the y coordinate of $\mathcal{L}_{1,3}$ while the set of potential disclosers, \mathcal{B}_1 , corresponds to the y coordinate of a subset of $\mathcal{L}_{1,1} \cup \mathcal{L}_{1,2}$.

When X is disclosed at $t = 2$, conditioning on $\tau_x = 2$ leads to $\mathcal{L}_{2,1} \cup \mathcal{L}_{2,2} \cup \mathcal{L}_{2,3}$ where the set of uninformed, \mathcal{A}_2 , corresponds to the y coordinate of $\mathcal{L}_{2,3}$ while

¹¹ We also know that y is such that the agent prefers to reveal x over keeping both x and y hidden. This can be ignored for computation of prices because it implies only that $P_2(\emptyset) \leq P_2(x, 2)$, which is independent of y .

the set of potential disclosers, \mathcal{B}_2 , corresponds to the y coordinate of a subset of $\mathcal{L}_{2,1} \cup \mathcal{L}_{2,2}$.

When we condition on $\tau_x = 1$ the measure of $\mathcal{L}_{1,3}$ is the same as the measure of $\mathcal{L}_{2,3}$ when conditioning on $\tau_x = 2$. Since the conditional distribution of Y given X is independent of τ_x , when we project on the Y coordinate both cases lead to the same set; this implies that $\mathcal{A}_1 = \mathcal{A}_2$. In constructing \mathcal{B}_1 and \mathcal{B}_2 we start with $\mathcal{L}_{1,1} \cup \mathcal{L}_{1,2}$ and $\mathcal{L}_{2,1} \cup \mathcal{L}_{2,2}$, respectively. When we condition on $\tau_x = 1$ the measure of $\mathcal{L}_{1,1} \cup \mathcal{L}_{1,2}$ is the same as the measure of $\mathcal{L}_{2,1} \cup \mathcal{L}_{2,2}$ when conditioning on $\tau_x = 2$. Since the conditional distribution of Y given X is independent of τ_x , when we project on the Y coordinate both cases lead to the same set.

We now eliminate types based on the equilibrium strategy to obtain the set of potential disclosers in both cases. From the previous two subsections, under the contradictory assumption that $P_2(x, 2) < P_2(x, 1)$, we can see that $\mathcal{B}_1^2 \supseteq \mathcal{B}_2^2$.

Regarding the comparison of \mathcal{B}_1^1 and \mathcal{B}_2^1 , define a set $\mathcal{B}'_2 = \{y \mid \tau_x = 2, \tau_y = 1, y \leq \min\{x, y^*(x)\}, y \in \mathcal{N}\mathcal{D}\}$ where $y^*(x) \geq Y_{x,2}$ was introduced in Lemma 4 (in the definition of \mathcal{B}'_1), so that $\mathcal{B}'_2 \subseteq \mathcal{B}_2^1$. Using part (iii) of Lemma 2, if we replace \mathcal{B}_2^1 with \mathcal{B}'_2 in (2), then the resulting price is still $P_2(x, 2)$ (because we are only adding to the set \mathcal{B}_2^1 types above $P_2(x, 2)$).

Combining these two comparisons, we get that $\mathcal{B}_2^1 \subseteq \mathcal{B}_1^1$ for the sets used in the equations characterizing equilibrium prices, (1) and (2). This leads to a contradiction based on the Generalized Minimum Principle (Lemma 2): given the ranking of the sets of potential disclosers, it must be that $P_2(x, 2) \geq P_2(x, 1)$.

III. A Threshold Equilibrium

In this section we discuss additional properties of the equilibrium and then, under additional assumptions (linear payoff as well as normally distributed signals and V), we show (by construction of a threshold equilibrium) that the assumptions in Theorem 1 are non-vacuous and that for large enough values of x the inequality in the last part of the theorem is strict, so that later disclosures receive strictly better interpretation.

To see the difficulties in fully characterizing equilibria in our game, consider first a one-signal benchmark: the agent can only learn X and has a positive probability of learning it in any period. In equilibrium of that model the agent follows a myopic threshold strategy: he reports x if and only if it increases current price (and that price is by assumption increasing in the revealed signal). The reason is that the price upon non-disclosure is decreasing over time (as investors assign a higher and higher probability that the agent is informed) and hence there is no option value from waiting (for details see Acharya, DeMarzo, and Kremer 2011).

It is quite different in our model. Consider an agent who learned only one signal, $X = x$. In period 2, assuming he has not disclosed anything yet, the agent is myopic and hence will disclose if and only if

$$(3) \quad P_2(x, 2) \geq P_2(\emptyset).$$

In a model with only one signal, $P_2(x, 2)$ is uniquely pinned down and is independent of the disclosure time. In contrast, in our model $P_2(x, 2)$ depends on investors'

beliefs about Y and these depend on the equilibrium strategy. It leads to two complications. First, we have to specify off-equilibrium beliefs when the agent discloses a value of x that is off the equilibrium path (and the freedom to pick off-path beliefs leads to multiplicity of equilibria).¹² Second, even though $E[P(x, y) | X = x]$ is increasing in x , it does not guarantee that $P_2(x, 2)$ is increasing because the disclosure policy for Y depends on the realized value of x (and depending on the realized x , investors assign a different probability to the agent being informed about Y). If $P_2(x, 2)$ were decreasing, agent's optimal strategy might fail to be a threshold one. A sufficient condition for prices to be increasing in x is that p is small because then the term $E[P(x, y) | X = x]$ corresponding to uninformed agents dominates.

The incentives to disclose for an agent who learned only one signal are even more complicated at $t = 1$. That agent discloses $X = x$ if and only if

$$(4) \quad E_y[U(P_1(x, 1), \max\{P(x, y), P_2(x, 1)\}) | X = x] \\ \geq E_y[U(P_1(\emptyset), \max\{P_2(\emptyset), P(x, y), P_2(x, 2), P_2(y, 2)\}) | X = x].$$

The left-hand side is the expected payoff if the agent discloses today, which takes into account that he may learn the other signal at $t = 2$ and then decide to disclose it.¹³ The right-hand side is the expected payoff if the agent decides not to reveal the signal today, which takes into account that in the next period he will have the option to either reveal nothing, reveal only x , or, if he learns the other signal in the meantime, he can reveal y or both signals. Condition (4) illustrates the main difficulty in constructing an equilibrium: both sides of the inequality depend on x . Even if all prices are increasing in x (which, as we discussed above, is not always guaranteed), whether agent's best response is a threshold strategy depends on whether the difference between the right-hand side and the left-hand side of (4) crosses zero only once; that in turn depends on the slopes of the different price functions and U .

Condition (4) also shows that a forward-looking agent benefits in two ways from delay of disclosure. First, he takes into account that he may learn $Y = y$ so large that

$$P_2(y, 2) > \max\{P_2(\emptyset), P(x, y), P_2(x, 2)\} \geq P_2(x, 1),$$

in which case he will disclose only the second signal at $t = 2$. Moreover, if the inequality in Theorem 1 is strict, i.e., $P_2(x, 1) < P_2(x, 2)$, then it creates a second benefit from delay of disclosure.¹⁴ These benefits from delay imply that the equilibrium price for disclosure of a single signal has to be strictly higher than the non-disclosure price, i.e., $P_1(x, 1) > P_1(\emptyset)$ for all x disclosed on the equilibrium path. This is in contrast to equilibrium prices if the agent cares only about the first-period

¹²Multiplicity of equilibria can also be caused by multiple fixed points between the beliefs of the market and the agent's best response to these beliefs.

¹³We abuse notation here for brevity: the second-period maximization problem of the agent depends on whether he learns the second signal or not.

¹⁴Similar considerations appear when we analyze the incentives of an agent who knows both signals at $t = 1$ to disclose, since if he is planning not to disclose Y , he benefits from delaying disclosure because $P_2(x, 2) > P_2(x, 1)$. See the online Appendix for details.

price or in a model with only one signal. For example, if the agent cared only about the first-period price (i.e., if $U(P_1, P_2)$ was constant in P_2), then the agent would reveal x in period 1 if and only if $P_1(x, 1) \geq P_1(\emptyset)$ and hence there would not need to be a uniform gap between disclosure and non-disclosure prices.¹⁵ The more the agent cares about the second-period prices, the higher is his best-response threshold for disclosure (keeping the equilibrium prices fixed).

The strategy of the agent who knows both signals and has not yet revealed any of them is difficult to describe since it is a function of two variables and the incentives to disclose the higher signal depend on the value of the lower one (it is even difficult at $t = 2$ when the agent is myopic, as shown in Pae 2005). Still, one property has to hold in equilibrium: suppose $\mathcal{N}\mathcal{D}_t$ is the set of x such that if the agent knows only x at t he does not disclose it (in a threshold equilibrium, these are realizations below the threshold for an agent who knows one signal). Then, if the agent knows both x and y , in equilibrium he does not disclose only x if $x \in \mathcal{N}\mathcal{D}_t$ (but he could disclose either both or none of the signals). Otherwise, investors would infer that he must know the other signal and by an unraveling argument the agent would be better off disclosing both signals.

A. Normal Model

As this discussion illustrates, even in a two-signal, two-period model, the equilibrium conditions are quite complicated. To show existence of a well-behaved equilibrium we make additional assumptions.

Suppose that the value of the firm, V , is normally distributed and (without loss of generality), V has zero mean, i.e., $V \sim N(0, \sigma^2)$. The private signals that the manager may learn are given by $X = V + \tilde{\varepsilon}_x$ and $Y = V + \tilde{\varepsilon}_y$, where $\tilde{\varepsilon}_x, \tilde{\varepsilon}_y \sim N(0, \sigma_\varepsilon^2)$ and $\tilde{\varepsilon}_x, \tilde{\varepsilon}_y$ are independent of V and of each other. Finally, we assume that the manager maximizes the sum of prices in two periods: $U(P_1(H_1^P), P_2(H_2^P)) = P_1(H_1^P) + P_2(H_2^P)$.

The joint normal distribution of the signals implies the following conditional expectations:

$$E[V|X = x] = E[Y|X = x] = \beta_1 x,$$

$$P(x, y) = E[V|X = x, Y = y] = \beta_2(x + y),$$

where $\beta_1 = \frac{\sigma^2}{\sigma^2 + \sigma_\varepsilon^2}$ and $\beta_2 = \frac{\sigma^2}{2\sigma^2 + \sigma_\varepsilon^2}$.¹⁶

Next, we define a threshold strategy. Without loss of generality, consider the case when the agent learns X either at $t = 1$ or $t = 2$, so that $\tau_x \leq 2$, and that $x \geq y$ in case the agent learned both.

¹⁵If the agent cares only about the second-period price (i.e., if $U(P_1, P_2)$ is constant in P_1), there exists an equilibrium with no disclosure in the first period.

¹⁶Note that $\beta_2 < \beta_1 < 2\beta_2 < 1$ and $\beta_2(1 + \beta_1) = \beta_1$. Also, note that $E[V|X = x] = \beta_1 x$ would be the price after disclosure of x if investors knew that the agent does not know Y . However, since investors assign a positive probability to the agent knowing Y and hiding it, equilibrium prices are lower and in general more complex.

DEFINITION 1: *Suppose that H_2^A and H_2^A are two private histories such that H_2^A differs from H_2^A only in the value of X , which equals x' under H_2^A and equals x under H_2^A . We say that a strategy is a threshold strategy if for any such H_2^A and H_2^A with $x' > x$ the following holds: if x is disclosed at time $t_x \in \{1, 2\}$ then x' is also disclosed at t_x or earlier. The equilibrium is a threshold equilibrium if the agent follows a threshold strategy.*

The following proposition states the main result of this section and is proven in the online Appendix.

PROPOSITION 1: *For $p < 0.77$ there exists a threshold equilibrium, characterized by a threshold x^* , in which*

- (i) *an agent who at $t = 1$ learns only one signal discloses it at $t = 1$ if and only if it is greater than x^* . If the agent learns both signals at $t = 1$, and one of them is greater than x^* , then he discloses at $t = 1$ either the highest signal or both signals. Disclosing a single signal $x < x^*$ at $t = 1$ is not part of the equilibrium disclosure strategy;*
- (ii) *there exists $x' \geq x^*$ such that $P_2(x, 2) > P_2(x, 1)$ for any $x \geq x'$ and both public histories, $(x, 2)$, $(x, 1)$, are on the equilibrium path;*
- (iii) *for public histories on the equilibrium path, $P_1(x, t_x)$ is increasing in x (so the equilibrium is monotone).*

In words, in our equilibrium there is a single threshold x^* such that if the agent knows only one of the signals at $t = 1$, he reveals it if and only if it is above x^* (there is also a threshold at $t = 2$ not described in the proposition). Moreover, if the agent knows both signals, he never discloses only one of them if it is lower than x^* (but he may disclose both). If at least one of the signals is above x^* , the agent reveals either one or both signals (and we do not rule out that the lower of the two revealed signals is below x^*).

The proof of Proposition 1 is complex and long, so it is delegated to the online Appendix. The proof focuses on the incentives to disclose at $t = 1$ since this is where the dynamic considerations play a crucial role. The road-map of the proof is as follows. We first assume that the manager follows a threshold disclosure strategy. Then, for each public history we show properties of the equilibrium prices given any threshold strategy. In particular, using the assumption that by period 2 the agent learns signal Y with probability $p + p(1 - p) < 0.95$ (which is implied by $p < 0.77$) we identify upper and lower bounds to the slopes of equilibrium prices $P_2(x, 2)$, $P_2(x, 1)$ and $P_1(x, 1)$ (as a function of x).¹⁷ We use these bounds to show that the manager's expected payoff upon disclosure of a single signal is increasing faster in his signal as compared to his expected payoff upon non-disclosure (for example, that the left-hand side of (4) is increasing faster in x than the right-hand

¹⁷ We conjecture that a threshold equilibrium exists also for values of p greater than 0.77; however, for tractability reasons we restrict the values of p since it simplifies the proof.

side, and similarly for all other private histories). This implies that it is indeed optimal for the manager to follow a threshold strategy in the first period, consistent with the initial assumption. We finish the proof by describing off-equilibrium beliefs and arguing that there exists a (fixed-point) threshold such that investors' beliefs and the agent's best response coincide.

IV. Conclusions

The vast literature on voluntary disclosure models focuses on static models in which an interested party (e.g., a firm's manager) may privately observe a single piece of private information (e.g., Dye 1985 and Jung and Kwon 1988). However, there are many real life circumstances in which, investors are uncertain about the time in which a firm observes value-relevant information and the disclosure of such information is voluntary. For instance, firms that have ongoing R&D projects can obtain new information about the state of their projects, where the time of information arrival and its content are unobservable to the market. Moreover, such information is not required to be publicly disclosed. One such example is pharmaceutical companies that get results of drug clinical trials. Investors' beliefs about a drug's clinical trial often have a great effect on the firm's price and may also affect investors' beliefs about the prospect of other projects of the firm. In such a setting, our model predicts that when the firm discloses the results of only part of its ongoing projects, a later disclosure gets a more positive market reaction (when keeping the disclosed information constant). Another related example is firms that apply for patents. After the initial application, the firm first waits to receive a notice of allowance (NOA) from the US Patent and Trademark Office (USPTO) for each of the applications, which indicates that the patent is near approval. Typically, patent applications may include many claims to be covered under the patent and the NOA informs the firms which of the claims have been approved and which have not been approved. Following the NOA, the firm waits for the formal issuance, indicating that the USPTO has formally bestowed patent protection.¹⁸ As Lansford (2006, p. 5) indicates: "It is important to note that firms enjoy wide discretion as to when to announce a patent event." Lansford (2006) documents that firms indeed time the disclosure of NOA strategically. In such circumstances, a manager deciding whether to disclose one piece of information must take into account the possibility of learning and potentially disclosing a new piece of information in the future. In this paper we have analyzed equilibrium consequences of such strategic considerations.

Our main result is that, in contrast to dynamic models with a single signal, the equilibrium reaction to voluntarily disclosed information depends not only on what is disclosed but also when, and that later disclosures receive a more favorable reaction even though the time the agent learns the signal is not informative per se.

Our discussion of condition (4) additionally suggests that the more the agent cares about the first-period prices (relative to the second-period prices) the more likely he should be to reveal information early. Multiplicity of equilibria makes it hard to precisely make/prove such a claim, but the intuition follows from (4): if we

¹⁸It typically takes a few months between the NOA and the time at which the patent is published in the USPTO website.

keep price functions as given, the agent’s best response is to disclose a larger set of signals at $t = 1$ if he cares more about current prices. Higher weight assigned by the manager to the first period’s price can reflect, for example, managers who face higher short-term incentives, managers of firms that are about to issue new debt or equity, a higher probability of the firm being taken over, a shorter expected horizon for the manager with the firm, etc. This intuition suggests a direction for new empirical investigation of how timing of voluntary disclosure by managers correlates with their long-term incentives.

APPENDIX

PROOF OF LEMMA 2:

- (0) For a constant c let $\mathcal{S}_{\mathcal{A},\mathcal{B}}^c = \mathcal{A} \cup \{\mathcal{B} \cap \{y: g(y) \leq c\}\}$. For $c \rightarrow -\infty$ we have that $E[g(y)|y \in \mathcal{S}_{\mathcal{A},\mathcal{B}}^c] = E[g(y)|y \in \mathcal{A}] > c$ and for $c \rightarrow \infty$ we have that $E[g(y)|y \in \mathcal{S}_{\mathcal{A},\mathcal{B}}^c] = E[g(y)|y \in \mathcal{A} \cup \mathcal{B}] < c$. From continuity we can find c^* for which $E_y[\mathcal{S}_{\mathcal{A},\mathcal{B}}^{c^*}] = c^*$. This establishes existence. Now suppose by way of contradiction that there are multiple solutions. Specifically, assume there are $c' < c''$ so that $E[g(y)|y \in \mathcal{S}_{\mathcal{A},\mathcal{B}}^{c'}] = c'$ and $E[g(y)|y \in \mathcal{S}_{\mathcal{A},\mathcal{B}}^{c''}] = c''$. When we compare $\mathcal{S}_{\mathcal{A},\mathcal{B}}^{c'}$ to $\mathcal{S}_{\mathcal{A},\mathcal{B}}^{c''}$ we note that $\mathcal{S}_{\mathcal{A},\mathcal{B}}^{c''} \supset \mathcal{S}_{\mathcal{A},\mathcal{B}}^{c'}$ and that for $y \in \mathcal{S}_{\mathcal{A},\mathcal{B}}^{c''} \setminus \mathcal{S}_{\mathcal{A},\mathcal{B}}^{c'}$ we have $g(y) < E[g(y)|y \in \mathcal{S}_{\mathcal{A},\mathcal{B}}^{c''}]$. This implies that $\mathcal{S}_{\mathcal{A},\mathcal{B}}^{c''}$ can be represented as a union of $\mathcal{S}_{\mathcal{A},\mathcal{B}}^{c'}$, with the average $c' < c''$, and a set of types that are lower than c'' . This however, implies that $E_y[\mathcal{S}_{\mathcal{A},\mathcal{B}}^{c''}] < c''$ and we get a contradiction.
- (i) When comparing $\mathcal{S}_{\mathcal{A},\mathcal{B}}$ to $\mathcal{A} \cup \mathcal{B}$ we note that we have excluded above average types for which $g(y) > E[g(y)|y \in \mathcal{S}_{\mathcal{A},\mathcal{B}}]$. This results in lower average type.
- (ii) Suppose first that there exists $y \in \mathcal{S}_{\mathcal{A},\mathcal{B}'} \setminus \mathcal{S}_{\mathcal{A},\mathcal{B}''}$. Since $\mathcal{B}' \supseteq \mathcal{B}''$ it must be that these $y \in \mathcal{B}' \cap \mathcal{B}''$. From the definition of $\mathcal{S}_{\mathcal{A},\mathcal{B}}$, since $y \in \mathcal{S}_{\mathcal{A},\mathcal{B}''}$ we conclude that $E[g(y)|y \in \mathcal{S}_{\mathcal{A},\mathcal{B}''}] > g(y)$. Since $y \notin \mathcal{S}_{\mathcal{A},\mathcal{B}'}$, we conclude that $E[g(y)|y \in \mathcal{S}_{\mathcal{A},\mathcal{B}'}] < g(y)$, which implies the claim. Hence, we will assume that $\mathcal{S}_{\mathcal{A},\mathcal{B}'} \supseteq \mathcal{S}_{\mathcal{A},\mathcal{B}''}$ and we consider $y \in \mathcal{S}_{\mathcal{A},\mathcal{B}'} \setminus \mathcal{S}_{\mathcal{A},\mathcal{B}''}$; this implies $g(y) < E[g(y)|y \in \mathcal{S}_{\mathcal{A},\mathcal{B}'}]$. Hence, all the elements $y \in \mathcal{S}_{\mathcal{A},\mathcal{B}'} \setminus \mathcal{S}_{\mathcal{A},\mathcal{B}''}$ have below-average $g(y)$ in $\mathcal{S}_{\mathcal{A},\mathcal{B}'}$ which implies that $E[g(y)|y \in \mathcal{S}_{\mathcal{A},\mathcal{B}''}] \geq E[g(y)|y \in \mathcal{S}_{\mathcal{A},\mathcal{B}'}]$.
- (iii) Consider the set $\mathcal{S}_{\mathcal{A},\mathcal{B}''}$, and note that it satisfies the definition for $\mathcal{S}_{\mathcal{A},\mathcal{B}'}$. Hence, the claim follows from uniqueness that was proven in (0).

PROOF OF LEMMA 4:

Given the assumption that $P_1(x, 1) \geq P_2(x, 1)$, we can apply part (iii) of Lemma 2 and ignore condition $\mathcal{C}_1(x)$ in the definition of \mathcal{B}'_1 because by doing that we only add types s.t. $P(x, y)$ is above the equilibrium price.

The constraint $C_3(x)$ can be described as $\Pi_x \geq \Pi_\emptyset$ where

$$\Pi_\emptyset = U(P_1(\emptyset), \max\{P(x, y), P_2(x, 2), P_2(y, 2), P_2(\emptyset)\})$$

$$\Pi_x = U(P_1(x, 1), \max\{P(x, y), P_2(x, 1)\}).$$

Π_\emptyset is the expected payoff of a type that knows X and Y at time 1 and decides to reveal nothing; and Π_x is the payoff of the same type that decides to reveal x only. Since x is revealed alone on the equilibrium path at time $t = 1$, the inequality $\Pi_x \geq \Pi_\emptyset$ needs to hold. We also know that on the equilibrium path x is being disclosed at $t = 2$ which implies that $P_2(x, 2) \geq P_2(\emptyset)$. Condition $C_2(x)$ implies already that $y \leq x$ and by monotonicity of equilibrium $P_2(x, 2) \geq P_2(y, 2)$. So, without changing the intersection of $C_1(x) \cap C_2(x) \cap C_3(x)$ we can define $C_3(x)$ by replacing Π_\emptyset with

$$\Pi'_\emptyset = U(P_1(\emptyset), \max\{P(x, y), P_2(x, 2)\}).$$

If $\Pi_x \geq \Pi'_\emptyset$ for all y then the constraint $C_3(x)$ can be ignored by defining $y^*(x) = \infty$. If this condition does not hold for any y then the agent does not disclose x at $t = 1$ if he knows both signals. This can be ruled out as an agent who only knows x decides to disclose it at $t = 1$. If for each realization of Y he would have preferred to keep quiet then this would be the case also when he does not know Y .

So we can focus on the case where $\Pi_x < \Pi'_\emptyset$ holds for some but not all y . Since we assumed $P_1(x, 1) > P_2(x, 2)$, this requires $P_1(x, 1) < P_1(\emptyset)$. In turn, that implies: (i) for y such that $P(x, y) > P_2(x, 1)$ the \max in both Π_x and Π'_\emptyset is attained at $P(x, y)$ and hence $\Pi_x < \Pi'_\emptyset$ in that range; (ii) for y such that $P(x, y) < P_2(x, 2)$ both Π_x and Π'_\emptyset are independent of y and it has to be that $\Pi_x \geq \Pi'_\emptyset$ in that range since otherwise there would be no y for which $\Pi_x \geq \Pi'_\emptyset$; (iii) for y such that $P(x, y) \in [P_2(x, 2), P_2(x, 1)]$ we have that Π'_\emptyset is constant while Π_x is increasing in y . Hence, there exists a unique y^* such that $P(x, y^*) \in [P_2(x, 2), P_2(x, 1)]$ for which $\Pi_x = \Pi'_\emptyset$. This y^* defines $C_3(x)$.

PROOF OF LEMMA 5:

We first rule out the possibility that the agent has learned X at $t = 1$ but not Y . Suppose by contradiction that in equilibrium the manager does not reveal $X = x$ at $t = 1$ when he knows only this signal (here we are using the restriction to pure strategy equilibria). Since, as we assumed, the public history $(x, 1)$ is on the equilibrium path, investors after that history would infer that the agent must know Y . The standard unraveling argument leads to a contradiction.

Next, we rule out the possibility that the agent learns both signals at $t = 1$. Let $\Pi_D(x)$ denote the payoff of an agent who knows only $X = x$ at $t = 1$ from disclosing x at $t = 1$; and let $\Pi_N(x)$ denote his payoff from not disclosing at $t = 1$. We have,

$$\Pi_D(x) = pE_y[\Pi_D(x, y) | X = x] + (1 - p)U(P_1(x, 1), P_2(x, 1))$$

$$\Pi_N(x) = pE_y[\Pi_N(x, y) | X = x] + (1 - p)U(P_1(\emptyset), \max\{P_2(x, 2), P_2(\emptyset)\}),$$

where

$$\Pi_D(x, y) = U(P_1(x, 1), \max\{P_2(x, 1), P(x, y)\}),$$

$$\Pi_N(x, y) = U(P_1(\emptyset), \max\{P_2(x, 2), P_2(y, 2), P(x, y), P_2(\emptyset)\}).$$

Since, as we argued in the beginning of this proof, if the agent knows only $X = x$ at $t = 1$ in equilibrium he discloses it, we have $\Pi_D(x) \geq \Pi_N(x)$. Consider now an agent who knows both signals at $t = 1$ and prefers to disclose just x at $t = 2$. Such an agent knows at time $t = 1$ that he will disclose x and not disclose y at $t = 2$. It must be that $\Pi'_N(x) \geq \Pi'_D(x)$ where

$$\Pi'_D(x) = U(P_1(x, 1), P_2(x, 1))$$

$$\Pi'_N(x) = U(P_1(\emptyset), P_2(x, 2)).$$

We claim that this leads to contradiction because $P_2(x, 1) > P_2(x, 2)$ implies that if $\Pi'_D(x) - \Pi'_N(x) \leq 0$ then $\Pi_D(x) - \Pi_N(x) < 0$.

To show this, note that $\Pi_D(x) - \Pi_N(x)$ is a weighted average over possible information sets of the agent in period 2. In case the agent does not learn Y in period 2, then trivially:

$$0 \geq \Pi'_D(x) - \Pi'_N(x) \geq U(P_1(x, 1), P_2(x, 1)) - U(P_1(\emptyset), \max\{P_2(x, 2), P_2(\emptyset)\}).$$

For the harder case that the agent learns Y in period 2, start with the observations that for any increasing function U and any constants $\alpha_1, \alpha_2, \beta_1, \beta_2, \beta_3, \beta_4$, if

$$U(\alpha_1, \beta_1) - U(\alpha_2, \beta_2) \leq 0$$

$$\text{and } \beta_1 > \beta_2$$

then

$$U(\alpha_1, \max\{\beta_1, \beta_3\}) - U(\alpha_2, \max\{\beta_2, \beta_3, \beta_4\}) \leq 0,$$

and the inequality is strict if $\beta_3 > \beta_2$.

Applying it to our problem, we get that

$$\Pi'_D(x) - \Pi'_N(x) \leq 0$$

$$\text{and } P_2(x, 1) > P_2(x, 2)$$

implies that

$$\Pi_D(x, y) - \Pi_N(x, y) \leq 0$$

for all y and the inequality is strict for $P(x, y) > P_1(x, 1)$. Taking the average over possible information sets in period 2 completes the reasoning.

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