Market Structure and the Direction of Technological Change

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Abstract

We ask how market structure affects the direction of technological change. We study a model with endogenous innovation in two dimensions. Innovation comes in two varieties: improvements to existing products, and new products that expand the scope of a technology. We study two market structures. The first is canonical from endogenous growth literature, where innovations can be developed by anyone, and developers market their own innovations. We then consider a more concentrated industry, where all innovation and pricing for a given technology is monopolized. We fully characterize equilibrium dynamics and argue that although the market structure has ambiguous impact on the quantity of innovation, it has an unambiguous effect on the direction of innovation. We also use our model to revisit two classic questions: how to subsidize innovation and how an upstream monopolist should design a downstream market.

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1 Introduction

How does market power affect what gets invented? That is, what is the relationship between market structure and direction of technological change? We argue that market structure should have a clear impact on the direction of research (even though its impact on volume of innovation can be ambiguous). The economic force we focus on is that firms with market power internalize “business stealing” effects of innovations. As a result, market power leads to less innovation in products that are substitutes on the demand side and more innovation in products that are either neutral or compliments to current products and/or have more future potential substitutes. In particular, firms with market power are more likely to direct innovation to new product categories with a lot of potential future business stealing while more competitive firms are expected to invest more in improvements of existing product lines.

To illustrate this intuition, in Section 2 we introduce a model with two endogenous dimensions of technological change: efficiency improvements to existing uses of the technology (which we term efficiency innovations) and developments of new uses for the technology (scope innovations). The main differentiating feature is that efficiency innovations are characterized by larger "business stealing" that is, the profits accrued to new, improved versions come with decreased profits of old products. On the other hand, new uses have relatively little impact on the profits of existing lines (but are subject to future improvements). We assume that developing a new use is more costly than developing an improvement to an existing use.

Even though we cannot claim that this is the most general model of innovation, it illustrates well the trade-offs we intuitively discussed. The model is tractable (despite its richness), captures the dynamics of sequential innovation, and is flexible to allow for numerous extensions.

There are many ways in which innovations differ in practice. The differences we focus on can be illustrated via the following examples. In the market for computer operating systems, some innovations are improvements to existing functions, which allow a more efficient execution of given tasks, while other innovations are new functions that allow the computer to do new tasks. A similar distinction can be made for business applications or video games. In the market for pharmaceuticals some innovation comes in the form of an improvement to an existing cure, but also often researchers come up with a cure for a previously uncured ailment. In general, the relevant distinction comes from the demand side rather than from the technological side: we care about how close substitutes the products are for the customers and not about how similar is the production process.¹

¹We focus on how market power interacts with demand, rather than with cost externalities (cost of research or production) - the technological side can be important as well, but these effects are relatively well understood as they depend less on the dynamic interactions between firms.
For simplicity we study two polar market structures. On one hand, we consider in Section 3 a competitive innovation case that mirrors the common setup of the endogenous growth literature. Each efficiency level is monopolized, but all innovations compete with one another, and there is free entry into innovation. Innovation starts slowly but as more and more uses are developed the total intensity of research increases. At the same time, there is a shift of research from scope innovations to improvements (i.e. efficiency innovations) and eventually the market reaches a steady-state in which further research is done solely on improvements. In the long run the competition leads to a narrow scope but innovation never dries out.

On the other hand, in Section 4 we consider a monopolist who controls all possible innovation, both in terms of efficiency levels and scope of the technology. Such a situation leads to expansion in scope for the reason described above: the monopolist seeks to avoid displacing his own profits on existing products and does not have to worry about having its profits taken over by a competitor. Over time, the amount of research done by the monopolist decreases, as the physical limits to the number of possible product lines are approached.

Overall, the ranking of innovation intensity is ambiguous across the two market structures: innovation in competition usually starts much slower than in monopoly, as the returns to the first innovation are usually higher for the monopolist, but once the monopolist reaches the limits to the scope of technology the ranking of research activity gets reversed. However, as far as direction of innovation is concerned, the differences are stark and unambiguous – market power leads to a very different set of innovations. Which market structure is hence better for innovation (in terms of welfare, a classic IO question) depends on the relative importance of the different types of innovation.

A firm like Microsoft might have great incentive to expand the scope of a PC, but relatively little incentive to debug existing applications. Competitive research into pharmaceuticals might generate ever increasing quality of a relatively fixed set of cures, but may lack the incentive to pay extra costs to pioneer new cures.

For clarity, and in line with the endogenous growth theory, we assume that the potential for improvements is unbounded, whereas we allow for physical barriers on the number of possible uses. It is a modeling simplification that tries to capture the relative limits on the technology, as it allows us to contrast cases where potential scope is relatively important or unimportant.

In Section 5 we use the model to revisit two classic questions. First we study a standard question in the growth literature, whether government should subsidize innovation in the competitive structure. Our answer is that it should (a standard result), but in particular it should encourage scope innovations more than efficiency improvements. This difference is precisely what our model allows us to study.
Further, our analysis of optimal research subsidy provides a rationale for paying attention to whether support for innovation takes place on the supply or demand side, as Romer (2000) stresses. Since different types of innovations have different social benefits, it may be that the government has an interest in guiding through targeted support (such as NSF support for "basic" research, or extra incentive to invent drugs that currently have no effective cure) a particular type of innovation.

We also consider a standard IO question: whether an upstream monopolist should keep downstream monopoly or open a downstream market for competition. Our angle is that the upstream monopolist sells a durable product and its value to customers depends on future innovation in the downstream market. The monopolist cannot commit to future innovation but can commit to a downstream market structure. In such an environment the optimal strategy depends on the relative importance of the different directions of innovation. If the technology affords relatively little possibility of scope innovations, the upstream monopolist should choose a competitive downstream market; if there is great potential for scope, the monopolist will have sufficient incentives to innovate in these many areas.

This trade-off between market structures is not new; the commitment benefit of opening downstream markets has been developed in other contexts. For example, Shepard (1987) showed that licensing technologies to multiple competing firms can serve as a commitment device for a monopolist seeking to deliver high quality. Similarly, Economides (1996) discusses how a firm selling a product with network benefits might allow competition as a way to commit to a large customer base.

We introduce to this question the incentives to innovate. Since we assume that the monopolist cannot commit, our paper is also related to the long literature on durable goods monopolies (including Coase (1972), Stokey (1981), and Bulow (1982)), where a monopolist can sell an object up-front, but faces commitment problems in future actions. The relative importance of the two types of innovation determines how severe the commitment problem is and hence the optimal strategy depends on the nature of the industry.

The relationship between market structure and incentives to innovate is a fundamental topic in Industrial Organization. Most of the existing research in IO focuses attention on the impact of market structure on the amount of innovation – in contrast, we ask about the impact on the direction of innovation. The importance of such directed technical change has been stressed in recent research such as Acemoglu (2002); we add to this literature the interplay between market structure and direction.

We also provide some insights into the relationship between scale and amount of innovation (see, for instance, Scherer (1980), and, more recently, Aghion, et al. (2005)). One way to interpret the two market structures we study is that it entails a single large firm, with
more market power, versus an industry with many smaller firms. If one measured the relationship between firm size and innovation using data from our two polar market structures, there would be a variety of pitfalls. First, the industry life-cycle would matter; initially the closed standard (large firms) do more innovating, but later the open standard overtakes it. Moreover, the closed standard does different sorts of innovation than the open standard, so comparing conventional measures of innovation like patent counts might not be a consistent comparison across market structures.

In terms of modeling we build upon Grossman and Helpman (1991), but of course the same ideas could be embedded in a model along the lines of Aghion and Howitt (1992). Our model adds endogenous variety, and in that sense is similar to a long line of growth theory papers such as Romer (1987). As we use a modern Schumpeterian model to answer policy issues, the paper is related to Schmalensee (2000) who argues that the industry’s Schumpeterian character is important in discussing antitrust issues in industries like computer software.

2 The Model

2.1 Standards and Applications

We study a continuous time, infinite horizon model. In each instant, consumers derive utility from $\tilde{N}$ functions that are related under a common standard. The standard allows a function to be accomplished via a specialized application. A given application $j$ for function $i$ has quality $q^j_i \geq 1$ per physical unit. Without an application, the consumer can accomplish the function directly at a fixed quality $q^0_i = \phi \leq 1$ per physical unit. If the standard were a particular type of computer, examples of directly accomplishing a function, instead of using a specialized application, would be using pencil and paper instead of a spreadsheet or a typewriter instead of a word processor. Except for Section 5.2, we assume that applications are directly consumed by consumers.

We define the standard to be open if each application is owned by a different firm. The standard is closed if all applications are owned by the same firm and only this firm can innovate to obtain new innovations.$^2$

We refer to an innovation that increases $q^j_i$ to $q^{j+1}_i$ for $j > 0$ as an efficiency innovations or a quality improvement or an improvement to an existing application. We call innovation that introduces $q^1_i$ for some $i$ a scope innovation or an invention of a new (or frontier) application.

$^2$In the future we hope to consider a mixed setup with both multi-application and single-application firms competing in a market.
2.2 Preferences

For a given function $i$, the representative consumer consumes $d_i^j$ physical units of application $j$. This leads to $e_i$ efficiency units of the function, where

$$e_i = \sum_j q_i^j d_i^j$$

The representative consumer’s instantaneous utility from a bundle of efficiency units $\{e_i\}$ of the various functions is

$$u(\{e_i\}) - \sum_i \sum_j p_i^j d_i^j$$

where $p_i^j$ is the price paid for application $j$ on function $i$ and $h$ is the amortized cost of the hardware.

In equilibrium consumers will choose only one quality level, the highest quality, denoted simply $q_i$, per function. Therefore, given consumption of $d_i$ units of that quality level of application $i$, utility is

$$u(\{q_i d_i\}) - \sum_i p_i d_i$$

To simplify the analysis, we parameterize the utility function to be logarithmic:

**Assumption 1:** $u(\{q_i d_i\}) = \sum_i \ln(q_i d_i)$.

Log utility is common in the growth literature because it generates stationarity with constant percentage-increases in quality levels, as we assume below. Here it also delivers us, in a stark way, what we want to assume about business stealing effects: more business stealing for quality improvements than from the invention of new applications. Demands are independent across different functions and the representative consumer spends a constant share of his income on every application ladder $i$. In particular, if the representative consumer buys application $q_i^j$ at price $p$, his demand is $d_i^j = 1/p$. As a result, new frontier applications generate no business stealing, while quality improvements do generate business stealing.

2.3 Output Production: Firms and Competition

Qualities fall on a ladder with rungs of size $\lambda > 1$; i.e., for the $j$th $> 1$ quality level on ladder $i$, $q_i^j = \lambda q_i^{j-1}$. The first application has quality $q_i^1 = \lambda$. Each physical unit requires one unit of labor to be produced, so the marginal cost of production, per physical unit, is normalized to 1.

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3 The preference structure for a given application follows Grossman and Helpman (1991).
Competition Between Qualities in an Open Standard  First, following the endogenous growth literature such as Aghion and Howitt (1992) and Grossman and Helpman (1991), suppose that each quality application is monopolized, perhaps due to a patent or a trade secret. There is a Bertrand competition between qualities within a ladder. Non-leading-edge qualities price at marginal cost; to match this price per efficiency unit, the leading edge quality charges \( p_j^1 = \lambda \) for \( j > 1 \) and \( p_1^1 = \lambda / \phi \).

Given demand \( x_i \) on ladder \( i \), profit flows for the leader are \( d_i(p_i - 1) \), where under Assumption 1, \( d_i = 1/p_i \). For instance, if \( \phi = 1 \), \( d_i = 1/p_i = 1/\lambda \) and profits are \( \frac{\lambda - 1}{\lambda} \equiv \pi \). Although our assumption of no dependence of profits on the number of applications is an extreme one, it is simply an expedient abstraction to the idea that there is less business stealing on the creation of new applications than on the creation of quality improvements.

Pricing in a Closed Standard  If a single firm controls pricing on all applications, it faces only the limit price from consumers’ direct accomplishment of the task. Therefore it can set price at most \( p_j^1 = \lambda^j / \phi \). Note that with the Assumption 1, since unit elastic demand implies an infinite monopoly price, this limit price is always optimal.

2.4 Innovation  Innovation comes through research. Research can be done on either developing new applications (i.e. applications for functions with no applications yet) or on improvements to existing applications (i.e. creating applications of higher quality for functions with existing applications). In both cases, research takes place continuously and innovations arrive according to a Poisson process. The arrival rate is proportional to the amount of research intensity, denoted \( x_e \) for existing applications and \( x_f \) for new (frontier) applications.

We assume that research intensity comes from one input, researchers, and that the pool of this input is heterogenous in their skills. A researcher of type \( \theta \) can provide one unit of research intensity, at a cost flow of \( \theta \) for an existing application, and a cost flow of \( \theta + \eta \) for a frontier application. The inclusion of \( \eta > 0 \) means that new applications are more costly to research. Researchers’ types are distributed on \([\theta_l, \infty)\) according to the cumulative distribution function \( F(\theta) \), with \( \theta_l > 0 \). We normalize the outside option of researchers to zero. In the open-standard case researchers may enter freely into research at any instant. In the closed-standard case the researchers are hired by the firm controlling all applications at a uniform wage.\(^4\) In order to keep the model from trivially having no innovation, we

\(^4\)In the open-standard case, WLOG the researchers can also be hired by a continuum of firms that have a free entry to the innovation process and compete for the researchers.
assume that there are types \( \theta \) smaller than the value of profits of the leading application, or 
\[ F(\pi/r - \eta) > 0. \]

The critical feature of our model that ties different applications together is the fact that they draw researchers from a common pool of scarce talent. As a result, innovative effort on one application has an impact on the marginal cost of innovation for all of the applications using the common factor. This equilibrium effect is what gives rise to all of the results about the dynamics across different applications that we develop below, and the differences that develop between open and closed standards.

In the open-standard regime, when a researcher finds an innovation he forms a firm and markets it (becoming one of the producers in the open standard). In the closed-standard regime, workers are hired by the monopolist at a fixed research wage \( w \). Researchers (and firms hiring them) maximize the expected sum of discounted profits/wages net of research costs and use a common discount rate \( r \).

3 Innovation Dynamics in Open Standard

In this section we consider the equilibrium of the economy when the standard is open. We keep Assumption 1 and disregard the bound \( \bar{N} \) but instead establish an endogenous bound on the number of functions for which under the open standard applications will ever be developed (and hence assume that \( \bar{N} \) is higher than this bound).

3.1 Analytic Results

Given the profits \( \pi \) from the Bertrand pricing game, consider the reduced-form game with competition among researchers. We focus on a recursive equilibrium, in which the decision rules of the researchers depend only on the current state of the industry. Since obtaining an improvement over an existing application yields the same profit flows regardless of the identity of the function or the current quality level of the application, in such an equilibrium the strategies of the researchers (and expected profits of the producers) depend only on the current number of functions with existing applications, which we denote by \( N \).

For simplicity we take \( \phi = 1 \) and hence profit flow for the current highest-quality application for any function is 
\[ \pi = \frac{\lambda - 1}{\lambda}, \]
independently of the quality level. We discuss the case \( \phi < 1 \) in Appendix B.

Assumption 2: \( \phi = 1 \).

We start by introducing some notation that allows to make the model more tractable. Denote total research by 
\[ x(N) \equiv x_e(N) + x_f(N). \]
Since the benefits of research are independent
of type, researchers follows a cutoff rule: if type $\theta$ does research on one of the applications, all types $\theta' < \theta$ do as well (although they can do it on some other application). By definition, the cutoff $\bar{\theta}(N)$ solves:

$$F(\bar{\theta}(N)) = x(N) \quad (1)$$

Total intensity $x(N)$ (and hence the cutoff $\bar{\theta}(N)$), as well as the allocation across the two activities is determined by free-entry conditions: the cutoff type must be indifferent between researching any of the existing applications (unless $x_e(N) = 0$), researching a frontier application (unless $x_f(N) = 0$) and opting out of research.\(^5\)

Define $c(x) = F^{-1}(x)$, the cost of the marginal researcher in the existing application search, so that $\bar{\theta}(N) = c(x(N))$. By (1), $c(x)$ is increasing. Let $\rho(N) = \frac{x_e(N)}{x(N)}$ be the probability that conditional on new innovation arriving, it is for an existing application. Let the random variable $\tau(N)$ be the arrival time of an innovation given the aggregate research intensity, $x(N)$. The expected discount factor can be calculated using the Poisson distribution as:

$$\delta(N) \equiv E[e^{-r\tau(N)}|x(N)] = \frac{x(N)}{x(N) + r}$$

Note that it depends on $N$ because so does $x(N)$.

Let $V(N)$ be the value of an incumbent firm just as a new innovation arrives, but without knowing what application it is for. It is defined recursively by

$$V(N) = \rho(N)(1 - \frac{1}{N})((1 - \delta(N))\pi + \delta(N)V(N)) + (1 - \rho(N))((1 - \delta(N + 1))\pi + \delta(N + 1)V(N + 1)) \quad (2)$$

This allows us to define the flow of expected benefit to the two research activities:

$$V_e(N) = (1 - \delta(N))\pi + \delta(N)V(N) \quad (3)$$

$$V_f(N) = (1 - \delta(N + 1))\pi + \delta(N + 1)V(N + 1)$$

$V_e(N)/r$ and $V_f(N)/r$ are the expected total profits of a researcher conditional on achieving one of the corresponding innovations when the current state is $N$. By the properties of Poisson distribution they also represent the flow of expected profits from innovation. Therefore, the free-entry conditions for the researchers are:

\(^5\)I the whole paper the equilibria we construct pin down only the share of active researchers working on the particular tasks but not the allocation of particular types. If the difference in costs of the two types of research depended on $\theta$ we would obtain this additional characterization.
\[
\begin{align*}
V_e(N)/r & \leq \bar{\theta}(N) \\
V_f(N)/r & \leq \bar{\theta}(N) + \eta
\end{align*}
\] 

with equality whenever the corresponding task is undertaken by a positive mass of researchers.

Formally, the recursive equilibrium requires that agents optimize according to equation (4) given (2) and (3), and these individual decisions agree with aggregate variables in (1).

The following proposition summarizes the characterization of the equilibrium.

**Proposition 1** There exists some \( N^* \) such that \( x_f(N) > 0 \) and \( x_e(N) > 0 \) for \( N < N^* \). Further, \( x_f(N^*) = 0 \) and \( x_e(N^*) > 0 \). Finally, \( x(N) \) is increasing in \( N \).

Before we prove it with a series of lemmas (and provide a sufficient condition for uniqueness of the equilibrium), we discuss the economic intuition behind the results.

First, it might seem surprising that in equilibrium competitive innovators can ever be willing to pay the additional cost to develop a new application. After all, we have assumed that the profit flows, \( \pi \), are the same from both types of innovation! Still, in equilibrium if \( \eta \) is not too large (and the supply of researchers, \( c(x) \) is not too elastic) the steady-state \( N^* > 1 \). The reason is that, due to the increasing cost of researchers, research intensity on existing applications rises less than proportionally to the number of existing applications. Therefore \( x_e(N)/N \) - the amount of research on improvements per application - declines in \( N \), and the value of having a marketable application rises in \( N \). If it rises enough from \( N \) to \( N+1 \), it is worth paying the extra cost \( \eta \). However, since \( 1/N \) is convex and \( x_e(N) \) is weakly increasing, \( x_e(N)/N \) drops by very little for large \( N \). Eventually the increase in value of an application gets small and is insufficient to draw research in new applications. The steady state \( N^* \) is reached, at which point there is only research in existing applications.

Second, anytime there is research on new applications, there must also be research on improving existing applications. If there were only research on new applications for a given \( N \), then developing an improvement would make more profits than a new application: it would earn profits until the next new application, at which point the continuation value would be as much as the new application would have made. This is of course impossible since improvements are less expensive. We get the following picture of innovation. For \( N \) below the steady state, there are both types of research, with total research intensity increasing due to the rising value of a leading edge application.

Although the model is admittedly stylized and we have isolated only one particular reason for competitive research leading to increases in scope (the reduction in \( x_e(N)/N \)), other
reasons could be easily incorporated as well. For instance, new products (the first application) might initially be harder for others to improve on, or initial innovations might be more valuable ($\phi < 1$; see Appendix B for a discussion of this case). Initial innovation might lower future research costs or provide a private benefit in research for the pioneering firm. Whatever the other forces, as long as the supply of researchers, $c(x)$ is increasing, by the same economic intuition a higher $N$ is most likely to yield a higher $x(N)$ (so that innovation is accelerating over time). Also, as we argue in the next section, the incentives to innovate are quite different across the two dimensions in an open and closed standard and these differences are likely to be robust to many changes in the model.

We now turn to a construction of the equilibrium. Suppose there are some states $N$ and $N+1$ in which both research dimensions are active in equilibrium. Using the free entry condition we can characterize the equilibrium aggregate research intensity for these states. Note that (3) implies $V_e(N+1) = V_f(N)$. Combining it with the free entry conditions we get:

$$c(x(N+1)) = c(x(N)) + \eta$$

(5)

This allows us to show that aggregate research is increasing in $N$:

**Lemma 1** As long as both research tasks are active, aggregate research effort $x(N)$, and value $V(N)$ are increasing in $N$.

**Proof.** Monotonicity of $x(N)$ follows directly from (5) and monotonicity of $c(x)$.

Regarding $V(N)$, from the free-entry conditions (4) we have

$$V_e(N+1)/r - V_e(N)/r = \eta$$

$$\downarrow$$

$$(\delta(N) - \delta(N+1))\pi + \delta(N+1)V(N+1) - \delta(N)V(N) = r\eta$$

$$\downarrow$$

$$\left(\delta(N) - \delta(N+1)\right)(\pi - V(N)) + \delta(N+1)(V(N+1) - V(N)) = r\eta$$

where $(\delta(N) - \delta(N+1)) < 0$ because we have proven that $x(N)$ is increasing. As the first element on the LHS is negative, we must have $V(N+1) > V(N)$ for the equality to hold.

Next, we establish the existence of a steady-state and that before steady-state both research tasks are indeed active:

**Lemma 2** For any $N$, $x(N) > 0$. For any $N > 0$ such that $x_f(N) > 0$, it must be that $x_e(N) > 0$. Finally, there exists $N^*$ such that for all $N \geq N^*$, $x_f(N) = 0$.  

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Proof. We start with the second claim. Suppose \( x_e(N) = 0 \) and \( x_f(N) > 0 \). Then, combining (2) and (3) we get \( V_e(N) = V_f(N) \), which contradicts the free entry conditions (4).

Now, suppose that there exists an \( N \) such that \( x(N) = 0 \). Then \( \delta(N) = 0 \) and \( V_e(N) = \pi \). Since there are researchers with \( \theta < \pi / r \) (which we assumed to get any innovation in equilibrium), that violates free entry condition.

Finally, suppose that for all \( N \), \( x_e(N) \) and \( x_f(N) \) are positive. From the analysis before we know that this would imply \( V_e(N+1) = V_e(N) + r\eta \) for all \( N \). But that is not possible as \( V_e(N) \in (0,\pi) \).

We assume that \( N^* < \bar{N} \) (the total number of feasible functions). In a steady-state \( N^* \), the values and research intensity can be easily determined as they satisfy the Bellman equation and the free-entry condition:

\[
V(N^*) = \left(1 - \frac{1}{N^*}\right) \left((1 - \delta(N^*)) \pi + \delta(N^*) V(N^*)\right)
\]

(6)

\[
\frac{(1 - \delta(N^*)) \pi + \delta(N^*) V(N^*)}{V_e(N^*)} = r c(x_e(N^*))
\]

(7)

Call the solution to these two equations, for arbitrary \( N \), \( \hat{V}(N) \) and \( \hat{x}_e(N) \), with the associated \( \hat{\delta}(N) \).

Lemma 3 \( \hat{V}(N) \) is increasing in \( N \); \( \hat{x}_e(N) \) is increasing in \( N \).

Proof. Pick any \( N \) and consider \( N' = N + 1 \). \( \hat{V}(N) \) increasing; suppose not. Then \( \hat{x}_e(N) \) would weakly decrease to satisfy (7) But that implies that \( \hat{\delta}(N) \) would weakly decrease; since \( V(N) < \pi \) (the maximum flow payoff), (6) implies that \( V(N^*) \) is increasing.

Finally, suppose \( \hat{x}_e(N) \) is weakly decreasing. That would require that \( \delta(N^*) \) is weakly decreasing and would violate (7) since we have already established that \( \hat{V}(N) \) is increasing.

In order for \( N^* \) to be a steady state, it must be not profitable to search for a new application at \( N^* \), even if no new applications were researched afterwards. In other words, at the steady state \( N^* \), the following inequality holds:

\[
(1 - \hat{\delta}(N+1))\pi + \hat{\delta}(N+1) \hat{V}(N+1) \leq r (c(\hat{x}_e(N)) + \eta)
\]

(8)

where

\[
\hat{\delta}(N+1) = \frac{\hat{x}_e(N+1)}{r + \hat{x}_e(N+1)}
\]
is the expected discount factor if at state $N + 1$ the research is $x_e = \hat{x}_e(N + 1)$ and $x_f = 0$. Condition (8) can be simplified to

$$c(\hat{x}_e(N + 1)) - c(\hat{x}_e(N)) \leq \eta$$

If $c(\hat{x}_e(N))$ is concave, there clearly exists exactly one "crossing point" which provides a sufficient condition for uniqueness of the steady-state and equilibrium:

**Lemma 4** Suppose $c(\hat{x}_e(N))$ is concave in $N$. Then the steady state $N^*$ is unique.

It can be verified directly that $c(\hat{x}_e(N))$ is in fact concave for many distribution functions $F$, for example a linear one. The intuition why we should expect it to hold is as follows: start with a constant $\delta(N)$. Then the solution $V(N)$ to (6) is concave because $\left(1 - \frac{1}{N}\right)$ is concave. Now, for (7) to hold, $c(\hat{x}_e(N))$ and $\delta(N)$ have to increase in $N$. This adjustment has to be larger the more $V(N)$ increases, so it is smaller for larger $N$. Therefore, for a lot of shapes of $c(x)$ we would obtain a concave $c(\hat{x}_e(N))$.

Once we find $N^*$ and $x_e(N^*)$, we can solve for the rest of the equilibrium in the open-standard model by working from the eventual steady state. In particular, iterating on (5) we get:

**Lemma 5** Aggregate research effort $x(N)$ is increasing in $N$ according to $c(x(N + 1)) = c(x(N)) + \eta$.

Given $x(N)$ for all $N \leq N^*$ and $V(N^*)$, we can use equation (4) to calculate $V(N)$. Finally, to compute the individual values $x_e(N)$ and $x_f(N)$, given $x(N)$ and $V(N)$, we can use equation (2). Note that this construction is unique for a given $N^*$, so that a concave $c(\hat{x}_e(N))$ implies a unique equilibrium.

### 3.2 Numerical Example

In order to see how the competitive case evolves in a numerical example, we assume that $F$ is linear, $F(\theta) = (\theta - \theta_i)/a$, and so $c(x) = ax + \theta_i$. Further, we consider the following parameters: $r = 5\%$ (annual interest rate) $a = 0.01$, $\theta_i = 0.2$, $\eta = \theta_i/25$, $\lambda = 1.5$ (so quality increases by 50%). Then $N^* = 9$. In the steady-state $x(N^*) \approx 9.66$, which is also the average number of improvements per year. The research intensities are shown in the figure as a function of $N$: the top line is total investment, the decreasing line is the investment in new applications, and the third line is investment in existing applications.
Note that the difference between the extra cost $\eta$ of a new application is at most four percent of the cost of researching an improvement (for $\theta = \theta_l$), but yet differences between intensities research intensities new and existing applications are large.

The expected time to the next frontier application is drawn in the next figure:

It takes less than 5 months for the first application to be invented. The time till next frontier application stays below one year only until 3 ladders exist and finally reaches over 49 years (not shown on the figure) for the move from $N = 8$ to $N^*$. Note that the difference between the extra cost $\eta$ of a new application is at most 4% of the cost of researching an improvement (for $\theta = \theta_l$), but yet differences between intensities research intensities new and existing applications are great.


4 Innovation Dynamics in a Closed Standard

4.1 Analytic Results

We now turn to the analysis of the closed standard market structure. Recall that quality improves at rungs of $\lambda > 1$ and let $j_i$ denote the number of rungs that the best application has over the default level $\phi = 1$, so that $q_i^j = \lambda^j$. If there is no application for a given function, then let $j_i = 0$. As we argued before, the optimal price of the best application for function $i$ is $\lambda^j_i$. At that price demand is $x_i = 1/p_i = 1/\lambda^j_i$ and hence the monopolist obtains a flow of profits

$$\pi_j = 1 - \frac{1}{\lambda^j}$$

Since $\pi_j$ is increasing and concave in $j$, the optimal research strategy of the monopolist depends not only on $N$ but on the whole state of the applications, i.e. not only on how many functions have application but also on the quality of existing applications. Let $Q = \{j_1, ..., j_N\}$ denote the state of the applications. The monopolist chooses research activities $x_e(Q)$, $x_f(Q)$ to maximize expected profits.

The profit gain from a unit increase in quality is

$$\Delta_j = \pi_{j+1} - \pi_j = \frac{1}{\lambda^j} \left(1 - \frac{1}{\lambda^j}\right)$$

which is decreasing in $j$ and $\Delta_j \to 0$. For now, assume $\Delta_0 - \eta > \Delta_1$, so that the monopolist prefers to invest in frontier applications before improving existing ones (we discuss later how the optimal innovation strategy differs if this condition is not satisfied).

We summarize the optimal monopoly strategy in a proposition:

**Proposition 2** If $j_i = 0$ for any $i$, then $x_f > 0$ and $x_e = 0$. Existing applications are improved only if $j_i = \min Q$. Total innovation $x$ is strictly decreasing in the number of innovations achieved. There exists a level $j^*$ such that all research stops once all applications reach $j^*$.

We prove this proposition through a sequence of lemmas and at the same time provide a more detailed characterization. We first argue that the optimal strategy is to continue research only up to a level $j^*$ and to put research activity only at the current lowest-level application, i.e. one that maximizes $\Delta_j$. As a result, research is done in layers: at any time the monopolist works on the lowest-level applications until he brings all of them to the next level. This continues until he reaches level $j^*$ with all of them.
Lemma 6 Optimal research strategy in the closed standard satisfies:

a) In the long run research stops once all applications reach a level \( j^* \) which is the smallest integer s.t. \( \Delta_{j^*} < r \theta_l \)

b) In the long run all functions have applications.

c) In any time the research is done on a product with the highest \( \Delta_j \) (i.e. lowest \( j_i \)).

Proof. See appendix.

This partial characterization allows us to reduce the state space to relevant points with applications on two levels only. Abusing slightly notation, let \( Q \) now denote a pair \((j,k)\) where \( j \) is the lowest level of applications across \( i \) and \( k \) is the number of functions on this level. We use a convention that \( Q + 1 \equiv (j,k - 1) \) and \( (j,0) \equiv (j + 1,N) \). Finally, let \( C(x) = xc(x) \) denote the total cost of hiring \( x \) mass of researchers, each at wage \( c(x) \), where \( c(x) = F^{-1}(x) \) if \( x_f = 0 \) and \( c(x) = F^{-1}(x) + \eta \) if \( x_f > 0 \).

We can prove the following monotonicity result.

Lemma 7 Total innovation \( x \) is strictly decreasing in the number of innovations achieved, that is \( x(Q + 1) < x(Q) \).

Proof. See appendix.

The intuition is that the rewards to research decrease for two reasons. First, the immediate rewards, \( \Delta_j \), are weakly decreasing. Second, as there is only a finite number of rewards, the increase in continuation-payoffs are strictly decreasing: finding the first innovation brings closer the profits from all the subsequent innovations, an effect missing for the last innovation.

We can now determine the optimal \( x(Q) \). The value function of the monopolist is defined recursively through the optimization problem:

\[
V_M(Q) = \max_x \left[ \frac{x}{x + r} (\pi(Q) - C(x)) + \frac{x}{x + r} V_M(Q + 1) \right] \tag{9}
\]

The first order condition is

\[
\frac{V_M(Q + 1) - (\pi(Q) - C(x))}{(x + r)} = C'(x) \tag{10}
\]

From lemma 6 we can find \( j^* \) as the closest integer higher than the solution to \( \Delta_j = r \theta_l \) (which can be written as \( j = \ln((\lambda - 1) / (r \lambda \theta_l)) / \ln \lambda \)). Given this \( j^* \), let \( Q^* = (j^*,N) \). The value in this steady-state is:

\[
V_M(Q^*) = N \pi_{j^*}
\]

We can use it to solve recursively for \( x(Q) \) and \( V_M(Q) \) by iterating on (10) and (9). That finishes the description of the equilibrium.
Remark 1  We have assumed that $\Delta_0 - \eta > \Delta_1$. If that does not hold, the optimal research policy varies only slightly. Let $j'$ be the smallest $j$ such that $\Delta_0 - \eta > \Delta_1$. If $j' > 1$, then the optimal strategy is to develop first the $j'$ applications for a given function before developing a frontier application for a new function.

Remark 2  Given the symmetry across applications, all strategies with a given level of aggregate $x(Q)$ that put all the research activity at the lowest-quality applications, but differ in the division of $x(Q)$ among them are payoff equivalent and lead to the same evolution of applications.

To summarize, the equilibria in the closed and open standards differ in several ways. First and foremost, the firms in the two different market structures view the two directions of technological change very differently. Firms in the open standard are more eager than the monopolist to invest in the improvements as they do not internalize the business stealing from current producers (though they are discouraged by future business stealing, what makes the ranking of innovation intensity in the short run ambiguous). On the other hand, since in the open standard firms foresee future business stealing by other firms, they are less likely than the monopolist to spend the extra costs to expand the scope of the technology. Even though the model is somehow limited, we argue that these trade-offs are likely to be present in other models of multi-dimensional endogenous innovation, for instance ones that incorporate other motives for new ladders. Second, the dynamics are quite different: under competition innovation accelerates, while under monopoly it slows down. We think that differences of that sort are natural in this type of environment, but we would argue that this comparison is a function of details of our specification. For instance, if scope was potentially unbounded but quality improvements bounded, one would expect the roles to be reversed, with competitive innovation leading to slower and slower innovation and the monopolist keeping a constant pace.

4.2  Numerical Example

We keep the parameters from the open-standard example: $r = 5\%$ (annual interest rate), $a = 0.01$, $\theta_l = 0.2$, $\eta = \theta_l/25$, $\lambda = 1.5$. The new parameter we need to specify is $\overline{N}$. We will compare two values: $\overline{N} = 9$ and $\overline{N} = 100$. Given these parameter values, $j^* = 8$ so the monopolist will research up to 8 applications per ladder. The figures show the dynamics of innovation as a function of $J = \sum j_i$ for $\overline{N} = 9$ on the left and for $\overline{N} = 100$ on the right.

The intensity of innovation is huge in the beginning (as compared with the open standard): for $\overline{N} = 9$ it takes on average less than a month in between the first 30 innovations (in the
open standard in the steady-state the innovations come on average 1.2 months apart and the speed of innovation is much slower in the beginning - the first innovation arrives an order of magnitude faster in the closed standard). Note also that for $N = 100$ research intensity starts almost 3 times as large!

Note further that one can interpret these differences as differences between innovation by large firms (the monopolized closed standard) and small firms (the open standard). However, in this case, the model suggests that measurement of these differences is difficult. First, whether you observe more innovation by large or small firms depends on the point in the product cycle; in the beginning, it appears that large firms do more innovating, but later on the pattern reverses. Moreover, our model suggests that different market structures lead to different types of innovation, so simply measuring a one dimensional innovation variable for each is not an apples-to-apples comparison.

5 Other Implications

Now that the model’s dynamics have been described, we turn to two topics in the study of market structure and innovation. In the first, we ask whether the competitive market structure provides sufficient incentives to innovate. Here we add to the standard discussion a new dimension: if the government should subsidize innovation, should it subsidize any particular direction?

In the second subsection we discuss a hardware monopolist whose product relies on future developments of software. Without the power to commit to investment in software, but with the power to commit to a market structure for software, the monopolist may seek to encourage valuable software by providing an open standard. Our model suggests that one consideration
for such a monopolist is the relative importance of scope versus quality improvements.

5.1 Subsidizing innovation

The equilibria we have characterized above do not achieve social first-best along many directions: the prices of the final products are above marginal cost of production, too little innovation takes place and in the closed standard case, the monopsony power in the market for researchers creates inefficiency there. Hence there are many ways a government can intervene to improve efficiency. We now focus on one possibility: subsidizing innovation in an open standard and ask how the social returns compare between subsidizing frontier or existing applications.

For clarity, we focus on a one-time unexpected subsidy for the marginal researcher in the steady state of the open standard to avoid crowding-out of private innovation caused by an anticipated future government subsidy. In equilibrium the steady state innovation level \( x^* \) is socially inefficient, in the sense that the total surplus would increase if additional researchers joined the innovation effort. The reason is that the private return is equal to \( \frac{r}{x^*+\pi} \) while the social return is equal to \( CS_\Delta/r \), where \( CS_\Delta \) is the extra flow consumer surplus generated by the extra quality improvement. Typically, the second number is higher because the social benefits accrue forever, while the private returns occur only until the firm gets replaced by an improvement; additionally, given our demand structure, the increase in flow of consumer surplus is higher than the profit flow: \( CS_\Delta = \ln \lambda > \frac{\lambda-1}{\lambda} = \pi \). Therefore a subsidy to research increases total welfare. Any such policy would have to decide whether to favor improvements of existing applications or development of frontier applications.

To model this, we simply ask what the planner’s payoff would be to one arrival of each type of innovation. For a quality improvement, the benefit is just \( CS_\Delta/r = (\ln \lambda)/r \). Innovation in a frontier application would have two effects. First, it creates additional profit flow, without business stealing, hence a return to firms of \( \pi/r \) (assuming that the innovation is sold to a firm that then sells it at profit maximizing price; if instead the price is set at marginal cost, then the return is larger, but we want to focus on intervention in innovation alone). Second, a frontier innovation increases \( N \) and hence the steady state intensity of research from \( \hat{x}_e(N^*) \) to \( \hat{x}_e(N^*+1) \) (recall from Section 3 that \( \hat{x}_e(N) \) is defined as the equilibrium research intensity if no further frontier applications are expected in the future). Since increase in \( x \) increases the growth rate of future welfare, the second effect is going to dominate in the long run, even

\[ CS_\Delta \] denotes an increase of equilibrium consumer surplus flow of having one additional application: in the notation of the next subsection, \( CS_\Delta = CS(J+1,N) - CS(J,N) \).

\[ CS_\Delta = \ln \lambda > \frac{\lambda-1}{\lambda} = \pi \] We assume that \( c(\hat{x}_e(N)) \) is concave in \( N \) to make sure that \( N^*+1 \) will be indeed a steady state, see Lemma 4. There is also a third effect that the frontier application costs more, but compared to all future benefits it is likely to be small and hence we ignore \( \eta \) in this section.
though flow of benefit from inventing an improvement to existing applications is higher than from inventing a frontier application. Therefore a sufficiently patient planner would prefer subsidizing frontier applications.

Formally, the steady-state free-entry condition is:

$$\frac{\pi}{r + \hat{x}_e(N)/N} - c(\hat{x}_e(N)) = 0$$

This expression is increasing in $N$ and decreasing in $\hat{x}_e$, hence unless $c(x)$ is vertical at $\hat{x}_e(N^*)$, it must be the case that $\hat{x}_e(N^* + 1) > \hat{x}_e(N^*)$. Furthermore, notice that this expression is decreasing in $\hat{x}_e$ faster if $c(x)$ is increasing faster (to the right of $\hat{x}_e(N)$).

One might be concerned about the behavior of $N^*$ as $r$ gets small; the following lemma shows that $N^*$ converges to a finite number, and so, for large enough $\bar{N}$, an additional frontier application is feasible even for small $r$.

**Lemma 8** $\lim_{r \to 0} N^* < \infty$.

**Proof.** See Appendix. ■

With $N^*$ well-defined for small $r$, we can state formally the comparative static in $r$:

**Proposition 3** Suppose $c(x)$ is finite for all $x$. For sufficiently low $r$, the planner obtains a higher social return from (one-time, unexpected) investment in frontier applications than in existing applications.

**Proof.** As we argued above, the total social return to one-time creation of one additional existing application is simply a constant flow of one additional "step" in consumer surplus: $CS_\Delta/r$. When instead a new frontier application is invented, customers do not gain any additional surplus immediately (as we assumed that the product will be sold by a monopolist) but they will enjoy a faster rate of arrival of future innovations: $\hat{x}_e(N^* + 1)$ instead of $x(N^*)$. (In terms of total surplus there is also an increase of profits for the industry, but it is sufficient to compare the consumers’ gain). That leads to a total gain of:

$$\frac{CS_\Delta}{r^2} (\hat{x}_e(N^* + 1) - x(N^*))$$

The ratio of the total returns to customers from the two types of innovation is hence

$$\frac{\hat{x}_e(N^* + 1) - x(N^*)}{r}$$

As $r \to 0$, condition (11) converges to:

$$N\pi = \hat{x}_e c(\hat{x}_e(N))$$
hence even in the limit \( \hat{x}_e(N) \) is strictly increasing in \( N \). Therefore, for sufficiently small \( r \) the return to customers (and total social return) is higher if the planner subsidizes frontier applications.

To understand the role of the shape of \( c(x) \) on the planner’s preference, consider the following comparative static: fix \( c(x) \) for \( x \leq x(N^*) \), but, for \( x > x(N^*) \), let \( \hat{c}(x) < c(x) \), so that \( \hat{c}(x) \) is flatter (more elastic) than \( c(x) \). Under \( \hat{c}(x) \), a new frontier application creates a higher social benefit (because it induces a larger increase of \( x \) than if the costs were given by \( c(x) \), according to (12)). But, by contrast, an additional frontier application is less attractive to the innovator under \( \hat{c}(x) \), for the same reason; the high rate of future arrivals discourages frontier research. As a result, for both cost functions, the steady state is \( N^* \) with research \( x(N^*) \), but the social benefit of an additional ladder is greater under \( \hat{c}(x) \).

This logic is summarized in the following proposition.

**Proposition 4** Let \( \hat{c}(x) = c(x) \) for \( x \leq x(N^*) \), but let \( \hat{c}(x) < c(x) \) for \( x > x(N^*) \). Then the planner has a greater preference for frontier research under \( \hat{c}(x) \)

One interpretation of scope innovations versus improvements is the contrast between basic research and applied developments. The model suggests both a rationale for governmental support for basic research (the greater research intensity that they foster), and an intuition for when this impact is likely to justify government involvement. For flat \( c(x) \), the private benefit to frontier research is small, but the social benefit from an additional frontier application is large. Both are driven by the fact that a flat \( c(x) \) leads to a big impact of a new ladder on equilibrium research intensity.

### 5.2 Software Applications with Open and Closed Standards

The analysis so far focused on *stand-alone applications* and took the market structure as given. We now use the model to consider *software applications*, where applications only function as software for a common piece of hardware, to better understand the trade-offs in choosing a closed or open standard regime.

In particular, we suppose that hardware is sold by a monopolist. The hardware in our model has no use other than to use the applications. We assume that hardware is infinitely durable and bought in the beginning of the game, in anticipation of future applications (that assumption is unrealistic, as hardware always comes with some applications immediately available and continues to be sold even as applications are developed, but that simplification is not affecting the economics we describe). Before hardware is sold, the monopolist commits to a market structure in the downstream market (and we assume that it cannot commit to future research). For simplicity, in this section we focus on a linear case \( F(\theta) = (\theta - \theta_l) / a \).
The consumers are willing to pay up to their expected surplus for the hardware, which equals their total utility minus the utility that they could obtain from directly performing all the functions. In the closed standard, the monopolist always follows limit pricing of applications to the direct option. Therefore, there is zero expected surplus from the hardware, all profits come from sales of applications.\(^8\)

If the standard is open, prices are \(p_i = 1/\lambda\), quantity is \(1/\lambda\) and consumer surplus (over the outside option) from good \(i\) with quality \(q_i^j = \lambda^j\) is

\[
CS(j) = (j - 1) \ln \lambda
\]

Therefore the hardware seller can extract up to the expected present value of \(CS(j)\) summed over all applications. To calculate this price, let \(J = \sum j_i\) be the number of quality improvements summed over all applications. Then the current flow of surplus given \(J\) is

\[
CS(J, N) = (J - N) \ln (\lambda)
\]

In the steady-state the expected total consumer surplus for a current \(J\) is:

\[
U(J, N^*) = (1 - \delta^k (N^*)) \sum_{k=0}^{\infty} \delta^k (N^*) CS(J + k, N^*)
\]

\[
= \ln (\lambda) \left( J - N^* + \frac{\delta (N^*)}{1 - \delta (N^*)} \right)
\]

In a state \(N < N^*\), the total surplus is:

\[
U(J, N) = (1 - \delta) CS(J, N) + \delta (\rho U(J + 1, N) + (1 - \rho) U(J + 1, N + 1))
\]

(note that \(\delta = \delta (N)\) and \(\rho = \rho (N)\)). Since research intensity for the open standard depends only on \(N\), we can rewrite this as

\[
U(J, N) = \ln (\lambda) (1 - \delta) \sum_{k=0}^{\infty} (\delta \rho)^k (J - N + k) + (1 - \rho) \sum_{k=1}^{\infty} (\delta \rho)^{k-1} U(J + k, N + 1)
\]

It is straightforward to show that this recursion is solved by the function

\[
U(J, N) = \ln (\lambda) (J - N + a_N)
\]

\(^8\)In other words, if the software market is monopolized, the hardware can be sold by competitive firms.
where
\[ a_N = \frac{a_{N+1} \delta (N) (1 - \rho (N)) + \delta (N) \rho (N)}{1 - \delta (N) \rho (N)} \]  
(13)

Since we have found before \( a_{N^*} = \frac{\delta (N^*)}{1 - \delta (N^*)} \), we can use (13) to calculate \( a_N \) for all \( N < N^* \). Then we can calculate

\[ U (0, 0) = \ln (\lambda) a_0 \]

and obtain the optimal hardware price:

\[ \frac{\ln (\lambda) a_0}{r} \]

We argue that the relative importance of the two directions of innovation determines which market structure is optimal for the hardware monopolist. To show that, we first develop a result for the case where the future is relatively important (low \( r \)) \( \bar{N} \) is high and \( \eta \) is small. In this case, it is better to extract surplus through monopolizing applications, since by internalizing business stealing the monopolist can create higher value.

**Proposition 5** If \( r < \frac{\pi}{\theta_i + 4a \ln \lambda} \), then there exists \( \eta^* > 0 \) such that for all \( \eta \leq \eta^* \) if \( \bar{N} \) is sufficiently large, the monopolist prefers to have a closed standard rather than an open one.

**Proof.** See Appendix. ■

**Remark 3** For a given \( \eta \) we can calculate a bound on the interest rates:

\[ r^* = \frac{\theta_i + \eta + 2a \ln \lambda - 2 \sqrt{\eta a \ln \lambda + a^2 \ln^2 \lambda}}{\left( \theta_i + \eta \right)^2 + 4a \theta_i \ln \lambda} \]

Then, if \( r < r^* \) we can find \( \bar{N}^* \) such that for all \( \bar{N} \geq \bar{N}^* \) the closed standard is more profitable.

This bound converges to the one in proposition as \( \eta \to 0 \). For example, if \( \theta_i = a = 0.1 \), \( \eta = \theta_i / 10 \) and \( \lambda = 1.5 \) (so that \( \pi = \frac{1}{3} \)) then the bound is \( r^* = 118\% \). Any discount rate smaller than that makes the monopolist prefer a closed standard for sufficiently high \( \bar{N} \).

By contrast, if \( \bar{N} \) is small and \( r \) is small, then the open standard choice is more profitable. The intuition is that when scope innovations are limited, an open standard leads soon to more innovation and hence a higher consumer surplus which can be extracted by setting the hardware price appropriately.

**Proposition 6** If \( \pi > \eta \frac{\theta_i + \eta}{a} \), then there exists \( r^* > 0 \) such that if \( r < r^* \) and \( \bar{N} \) is sufficiently small, the open standard is more profitable to the hardware owner than the closed standard.
Proof. See Appendix. ■

In the proposition we need a sufficiently large $\pi$ to guarantee that the open standard will develop at least one application - if $\eta$ is too large compared with $\pi$, then the open standard will fail to develop even the first application. Then the closed standard is necessary to overcome the public-good aspect of the higher cost of developing a new application and clearly the closed standard will be more profitable. On the other hand, if the hardware comes with the first application, then this additional restriction is not necessary.

To see the impact of $\bar{N}$, we return to our two numerical examples. As $N^* = 9$, then as long as $\bar{N} \geq 9$, the price of hardware is independent of $\bar{N}$. Contrary, the profits in the closed standard depend crucially on $\bar{N}$. In the figure below we compare the profits from hardware in the open standard (the horizontal line) with the profits from software in the closed standard regime (the increasing curve). As the discussion in this section points out, for small $\bar{N}$ the profits are higher from an open standard and when $\bar{N}$ is large they are higher from the closed standard case (in this example the cutoff is $\bar{N} = 91$).

Note that our discussion of surplus extraction by a monopolist is very similar to a planner maximizing total surplus; in fact, the only thing left out is surplus of the researchers. In examples such as this one, that contributes very little to total surplus, so the planner and hardware monopolist would make similar choices between open and closed standards.

6 Conclusion

We have shown that analyzing two margins for innovation (improvements versus new products), the trade-off between different market structures is not so much about quantity of
innovation as about allocation of innovative efforts. Closed standards/firms with market dominance are good at coming up with new products while open standards/markets with many small firms are better at improving products. The welfare consequences depend crucially on the fundamentals of the technology, like possible scope of applications.

The models suggests that market structure impacts not only the amount but also the direction of technological change. Considering other ways that market structure affects the direction of technological change, as introduced by Acemoglu (2002), seems to be an interesting way to extend the basic structure.

The model could be applied in other ways. Klepper and Thompson (2006) show that a variety of empirical facts about firm dynamics can be explained through a model of submarkets, where submarkets arrive exogenously. One can view the arrival of new applications as a sort of new submarket; one would only have to add obsolescence for it to match Klepper and Thompson’s notion more directly. We have focused on two market structures which correspond to some real world situations. Considering other market structures, including an optimal one for extraction of surplus for the monopolist, is a topic for future research.

7 Appendix A: Proofs

Proof of lemma 6. a) the benefit to innovation at a given product is at most $(1 - \pi_j)/r = \frac{1}{rN}$. As the cost of hiring researchers is at least $c(0) = \theta_l$, for high enough $j$ the expected benefit becomes larger than marginal cost, making further research unprofitable. Now, suppose that the research stops at some $j$. If $\Delta_j > r\theta_l$, then a profitable deviation is to hire researchers with $\theta \in [\theta_l, \theta_l + \varepsilon]$ for some small $\varepsilon$. If $\Delta_{j-1} < r\theta_l$ then a profitable deviation is to reduce research activity to zero once the application reaches level $j - 1$.

b) That follows directly from the assumption that $F(\pi/r - \eta) > 0$ (or equivalently, $\pi_1/r > \theta_l + \eta$), so the expected benefit of the first application is higher than the cost of hiring the most efficient researchers.

c) Given a researcher is hired, it is optimal to put his activity in a product with the highest expected return. The immediate return, $\Delta_j$, is the highest for the goods with the lowest $j$ and the continuation return (from subsequent innovations, $\Delta_{j+1}\ldots\Delta_{j+r-1}$) is also highest for those goods. Hence such strategy is optimal (see the proof of lemma 7 for a more detailed calculation).

Proof of lemma 7. Rewrite the closed standard problem in the following way. Let $p_Q = (\pi(Q) - \pi(Q - 1))/r$, the incremental benefit of the $Q$th innovation, where there are $Q^* = j^*N$ innovations. Note that $p_Q$ is weakly decreasing. Let $C(x) = xc(x)$.
The closed standard problem is choosing \( x_n, n \in \{1, 2, \ldots, Q^*\} \) to maximize

\[
\frac{x_1}{r + x_1}p_1 - \frac{r}{r + x_1}C(x_1) + \frac{x_1}{r + x_1}\left(\frac{x_2}{r + x_2}p_2 - \frac{r}{r + x_2}C(x_2)\right) + \frac{x_1}{r + x_1}\frac{x_2}{r + x_2}\left(\frac{x_3}{r + x_3}p_3 - \frac{r}{r + x_3}C(x_3)\right) + \ldots
\]

\[
\left(\prod_{i=1}^{N-1} \frac{x_i}{r + x_i}\right)\left(\frac{x_N}{r + x_N}p_N - \frac{r}{r + x_N}C(x_N)\right)
\]

Let

\[
G(Q) = \max_{\{x_Q, \ldots, x_{Q^*}\}} \sum_{i=Q}^{Q^*} \left(\frac{x_j}{r + x_j}\left(\frac{x_i}{r + x_i}p_i - \frac{r}{r + x_i}C(x_i)\right)\right)
\]

Note that the solution to the whole problem is \( G(1) \). Denote the solution to this problem for any \( Q \) by \( \{x_Q^*\} \). Note further that \( G(Q) \) is strictly decreasing, since, for \( G(Q) \), choosing \( \{x_Q^*, x_{Q+1}^*, \ldots, x_{Q^*-1}^*, x_{Q^*}^*\} = \{x_{Q+1}^*, x_{Q+2}^*, \ldots, x_{Q^*}^*, 0\} \) gives at least as high a payoff as \( G(Q+1) \), and increasing the last term from zero makes the payoff strictly higher.

The first order condition for \( x_n \) in the full problem is

\[
p_n + G(n + 1) = C'(x_n)(r + x_n) - C(x_n)
\]

If \( C'(x_n)(r + x_n) - C(x_n) \) is increasing, then it is immediate that, since the right hand side is strictly decreasing, the left hand side must be strictly increasing. Extending the proof to arbitrary \( C \) is a direct application of simple ironing techniques; letting \( R(x) = C'(x_n)(r + x_n) - C(x_n) \), and defining the

\[
\kappa(x) = \int_0^x R(x)dx
\]

\[
\bar{\kappa}(x) = \text{conv}(\kappa(x))
\]

the virtual value for the right hand side is defined almost everywhere by \( \frac{d\bar{\kappa}}{dx} \), and extended by right limits everywhere else.

Proof of lemma 8. Recall that \( \hat{x}_e(N) \) is defined by the solution to (6) and (7) which yields

\[
c(\hat{x}_e(N)) = \frac{\pi}{r + \hat{x}_e(N)/N}
\]
So \( \hat{x}_e(N) \) is decreasing in \( r \) and increasing in \( N \). As \( r \to 0 \), this condition becomes:

\[
c(\hat{x}_e(N)) \hat{x}_e(N) = N \pi
\]  

(14)

so \( \hat{x}_e(N) \) converges to a number. Now, to see that the steady-state \( N^* \) is bounded away from \( \infty \) as \( r \to 0 \), suppose \( c'(x) \) is bounded from above. Suppose that for every \( N \),

\[
c(\hat{x}_e(N + 1)) - c(\hat{x}_e(N)) > \eta
\]

(the opposite inequality is our condition for \( N^* \), see discussion before Lemma 4). Then, in (14) the LHS is growing at least linearly, while the RHS is growing much slower than linearly. So it cannot work. Therefore, even as \( r \to 0 \), \( N^* \) is bounded. ■

**Proof of Proposition 5.** We require two lemmas:

**Lemma 9** In the linear case, \( F(\theta) = (\theta - \theta_l)/a \), as \( N \to \infty \), the value from the closed standard converges to \( \frac{1}{2} \left( \frac{\pi - r(\theta_l + \eta)}{ar^2} \right)^2 \) (in flow terms)

**Proof.** Note that, for the closed standard,

\[
V_M(Q) = \max_x (1 - \delta) (N \pi - xc(x)) + \frac{x}{x + r} (V_M(Q + 1))
\]

As \( N \to \infty \), \( x \) converges to a constant that maximizes

\[
x \pi/r - xc(x)
\]

The FOC is:

\[
\pi/r = xc'(x) + c(x)
\]

in the linear case, \( F(\theta) = (\theta - \theta_l)/a \Rightarrow c(x) = ax + \theta_l + \eta \), the optimal choice is:

\[
x = \frac{1}{2} \frac{\pi - (\theta_l + \eta) r}{ar}
\]  

(15)

and the sum of expected discounted profits:

\[
V_M(0) = (1 - \delta)(0 - xc(x)) + \delta(1 - \delta)(\pi - xc(x)) ... \\
= (1 - \delta) \sum_{k=0}^{\infty} (\delta^k k \pi) - xc(x) \\
= \frac{\pi x}{r} - xc(x)
\]
(the last equality uses \( \frac{x}{1 - \delta} = x/r \)).

Substituting the optimal \( x \) from (15):

\[
V_M(0) = \frac{1}{4} \left( \frac{\pi - r (\theta_l + \eta)}{ar^2} \right)^2
\]

Lemma 10 The hardware price for the open standard is bounded above by \( \ln (\lambda) \frac{\pi - r \theta_l}{r} \) (in flow terms).

Proof. The hardware price (in terms of payoff flow) for an open standard is bounded by the steady state discounted sum of consumer surplus, which is

\[
\ln (\lambda) \max \left\{ 1, \frac{\delta(N^*)}{1 - \delta(N^*)} \right\}
\]

In order to calculate \( \frac{\delta(N^*)}{1 - \delta(N^*)} = \frac{x(N^*)}{r} \), calculate:

\[
\begin{align*}
V(N^*) &= \left( 1 - \frac{1}{N^*} \right) ((1 - \delta(N^*)) \pi + \delta(N^*) V(N^*)) \\
\downarrow \\
V(N^*) &= \frac{\pi r}{N^*} \frac{N^* - 1}{N^* r + x(N^*)}
\end{align*}
\]

Now, the free-entry condition (7) is:

\[
\frac{r}{x(N^*) + r} \pi + \frac{x(N^*)}{x(N^*) + r} V(N^*) = r (ax(N^*) + \theta_l)
\]

Combining with \( V(N^*) \) yields:

\[
x(N^*) = \frac{1}{2a} \left( \sqrt{(arN^* + \theta_l)^2 + 4aN^* (\pi - r \theta_l) - (arN^* + \theta_l)} \right)
\]

For any \( r \), \( x(N^*) \) is increasing in \( N^* \) and

\[
\lim_{N^* \to \infty} x(N^*) = \pi - r \theta_l
\]

which is a uniform bound on \( x^*(N^*) \), that allows us to bound the price of hardware without finding the steady state. The hardware price (in terms of payoff flow) is at most:\( h \leq \ln (\lambda) \frac{\pi - r \theta_l}{r}. \)
We now finish the proof of the proposition. From the above calculations for open and closed standards, we can calculate a ratio that bounds the relative profits under the two market structures:

\[
\frac{1}{4} \cdot \frac{(\pi - r (\theta_l + \eta))^2}{ar^2} \cdot \left( \ln (\lambda) \frac{\pi - r \theta_l}{r} \right)
\]

which is continuously decreasing in \( \eta \). At \( \eta = 0 \) the ratio is:

\[
\frac{1}{4ar \ln \lambda} (\pi - r \theta_l)
\]

This expression is positive and decreasing in \( r \). Solving for it to be equal 1 yields the bound on \( r \) stated in the proposition. Given \( r \) less than this bound we can find an \( \eta^* > 0 \) such that ratio is still less than 1. Then, for any \( \eta \leq \eta^* \) we can find \( N \) large enough that the payoffs under the closed standard are arbitrarily close to the calculated bound (for this and all larger \( N \)).

**Proof of Proposition 6.** Once the first application is developed, the free-entry condition is:

\[
(1 - \delta) \frac{\pi}{r} = c(x)
\]

which in the linear case yields that steady-state \( x^* \) solves:

\[
\frac{\pi}{r + x^*} = ax^* + \theta_l
\]

(16)

Direct calculation shows \( x^* \) is decreasing in \( r \).

Note that the first application is developed if

\[
\eta < \frac{\pi}{r + x^*} - \theta_l
\]

the above inequality holds for small \( r \) if

\[
\pi > \eta \frac{\theta_l + \eta}{a}
\]

Consumer surplus for \( j > 0 \) is \( CS(j) = (j - 1) \ln \lambda \), and \( CS(0) = 0 \). Let \( \delta = \frac{x^*}{x^* + r} \). The expected surplus after the first application is developed is:

\[
U(1, 1) = (1 - \delta) CS(1, 1) + \delta (1 - \delta) CS(2, 1) + \delta^2 (1 - \delta) CS(3, 1) ...
\]

\[
= (1 - \delta) \ln (\lambda) \sum_{k=0}^{\infty} k \delta^k = \ln (\lambda) \frac{x^*}{r}
\]
Using condition (16) we can find that \( \lim_{r \to 0} x^* > 0 \). so \( U(1, 1) \) is on the order \( O \left( \frac{1}{r} \right) \) for small \( r \).

We can now bound \( U(0, 0) = h \). The free entry condition at \( N = 0 \) is

\[
(1 - \delta) \pi + \delta V(1) = r (ax_f + \theta_l + \eta)
\]

which yields:

\[
x_f = \left( \frac{\pi}{r + x^*} - (\theta_l + \eta) \right) / a \tag{17}
\]

Assuming \( \pi > \eta \theta_l + \eta \), \( x_f \) converges to a strictly positive number as \( r \to 0 \). Therefore the price of hardware (in payoff flows) is

\[
h = U(0) = \frac{x_f}{x_f + r} U(1) \to O \left( \frac{1}{r} \right)
\]

In the closed standard the profit flow is at most \( \pi_{j \to \infty} = 1 \). ■

8 Appendix B: \( \phi < 1 \)

Throughout the paper we have maintained Assumption 2 that \( \phi = 1 \). How would the analysis change if instead:

**Assumption 2B: \( \phi < 1 \).**

The fundamental change is that profits earned by a frontier application will be higher than profits earned by subsequent improvements, that is \( \pi_1 > \pi_{j \geq 1} = \pi \). How is it going to affect the dynamics of innovation?

In the open standard case, if \( \pi_1 - \eta < \pi \), then the equilibrium changes only slightly. In particular, equation (2) for non-frontier \( (j \geq 2) \) applications has to be supplemented by a value of frontier applications \( (j = 1) \):

\[
V_1(N) = \rho(N) \left( 1 - \frac{1}{N} \right) (1 - \delta(N)) \pi_1 + \delta(N) V_1(N) + (1 - \rho(N)) (1 - \delta(N + 1)) \pi_1 + \delta(N + 1) V_1(N + 1) \tag{18}
\]

Also, we need to modify the benefit to \( x_f \):

\[
V_f(N) = (1 - \delta(N + 1)) \pi_1 + \delta(N + 1) V_1(N + 1)
\]
and the free-entry condition changes accordingly. The $N^*$ has to be calculated with the new value of $V_f(N)$ and a lower $\phi$ clearly leads to a (weakly) higher $N^*$. If $\pi_1$ is very large (in particular, sufficiently larger than $\pi - \eta$), then the equilibrium can change qualitatively as well: it is possible that for some $N$ all research activity will be in the frontier applications, that is $x_f > 0 = x_e$. The reason Lemma 2 can be overturned is that when $\pi_1 - \pi$ is larger than $\eta$, paying more for a frontier application can be more than compensated by the higher profits (for this to happen it is not sufficient that $\pi_1 - \eta > \pi$, since due to business stealing returns to innovation are smaller than $\pi_j/r$).

In the closed standard the changes are less dramatic: we have already taken into account that the $C(x)$ function differs for frontier and non-frontier applications by $\eta x$. Now we have to allow for $\pi(Q)$ to vary as well and increase $\Delta_0$ accordingly. A higher $\pi_0$ will clearly imply a more intensive research on the frontier applications, but once $j = 1$ is reached for all $\overline{N}$, it will not have any further effects.

Finally, in terms of the choice between closed and open standard, the impact of a higher $\pi_1$ is ambiguous. On one hand, if $\overline{N} > N^*$ the closed standard will enjoy the higher $\pi_1$ from a larger number of products. On the other hand, a larger $\pi_1$ may tip an increase in $N^*$ and hence yield a large increase in the price of hardware.

References


