

True/False - No explanation needed. (For each: 1 point if correct, 0 points if not answered, -1 points if incorrect)

1. For any six sided die, regardless of whether it is fair or unfair, the probability that an odd number is rolled is $p(1) + p(3) + p(5)$, where $p(i)$ is the probability the number i is rolled.
True/False

True, as the probability of an event is the sum of the probability of the outcomes in the event.

2. The formula for the n th Fibonacci number $\frac{1}{\sqrt{5}}(\phi^n - \bar{\phi}^n)$, where $\phi = \frac{1+\sqrt{5}}{2}$ and $\bar{\phi} = \frac{1-\sqrt{5}}{2}$, comes from the fact that ϕ and $\bar{\phi}$ are the two solutions to $x^2 - x - 1 = 0$.
True/False

True. We replace each element in the sequence f_n with x^n , which then simplifies to $x^2 = x + 1$. The roots of this are ϕ and $\bar{\phi}$.

Problems - Need justification.

1. Set $a_1 = 1$ and $\log_2(a_{n-1} + 5) = a_n$ for $n \geq 2$. Show that $a_n < 3$ for every n .

We proceed by induction. $a_1 < 3$ by definition, giving the base case.

Assume that $a_n < 3$. Then

$$a_{n+1} = \log_2(a_n + 5) < \log_2(8) = 3$$

finishing the inductive step.