A Bilinear Model of the Illuminant's Effect on Color Appearance

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A naive theory of color appearance might attempt to predict the color appearance of a test light from the spectral power distribution of the light. Many color appearance phenomena, including simultaneous color contrast and effects of observer adaptation, falsify this type of theory. In addition to the spectral power distribution of the test light, a theory of color appearance must incorporate some information about the context in which the test light is seen.

In natural viewing, we typically use color names to describe objects. Implicit in the statement "My car is red" is the idea that the redness is a property of the car. The spectral properties of the color signal reflected from an object to the eye are determined both by the spectral properties of the object's surface and by the spectral properties of the ambient illumination. For the sensation of color appearance to be a useful code for object surface properties, the visual system must actively adjust to variation in the illumination to stabilize object color appearance. A visual system is called color constant if it adjusts perfectly, so that the appearance of an object is invariant despite changes in illumination.

It has long been known that the human visual system exhibits imperfect color constancy (Evans, 1943; 1948; Helmholtz, 1896; Helson & Jeffers, 1940; see Boring, 1942; Wyszecki, 1986). For this reason, the goal of classic empirical studies was not to accept or reject human color constancy but rather to measure and model the effect of viewing context on color appearance (Burnham, Evans & Newhall, 1975; Helson & Jeffers, 1940; Helson, Judd & Warren, 1952; Hunt, 1953). To make the measurement problem tractable, these studies used a simple laboratory model where isolated test lights were presented against uniform backgrounds. Within this model, changing the viewing context is accomplished by changing the spectral properties of the background. The classic experiments and associated theories (Brewer, 1954; Burnham, Evans & Newhall, 1957; Jameson & Hurvich, 1964; Judd, 1940) form the basis of our current understanding of the effect of context on color appearance. Although this work provides a successful account of appearance effects within the restricted domain, it is not clear how to generalize the results to natural images.[1]

Natural images are formed when illumination is reflected to the eye from the objects in the image. To understand the difficulty in generalizing from uniform backgrounds to natural images, note that there are many physically distinct ways to vary a natural image. One is to change the spectral properties of the illumination. A second is to change the shape and spectral properties of the object surfaces that compose the image. If a visual system adjusts to changes in illumination, varying the illumination will affect color appearance. If a visual system exhibits contrast effects, perhaps for the purpose of enhancing object edges, then varying the surfaces will also have an effect. Experiments conducted using a uniform background confound these two types of variation. Changes in the spectral properties of the background are legitimately interpreted either as changes in illumination or as changes in surface reflectance.

In this chapter, we describe our recent experimental work designed to isolate the effect of varying the illuminant from the effect of varying the surfaces in the image. In our experiments, the viewing context is controlled by having the observer view a series of images, each of which is a simulation of a set of uniformly illuminated surfaces presented on a cathode ray tube (CRT) display device. The observer judges the color appearance of test lights embedded in these images. When there are multiple surfaces in an image, it is possible for a visual system to estimate the illuminant independent of the choice of surfaces (Buchsbaum, 1980; Maloney & Wandell, 1986). In any experimental condition we hold the simulated illuminant constant; the simulated surfaces are varied to counterbalance against contrast effects. We interpret our data as measuring the visual system's adjustment to the simulated illuminant. Our measurements indicate that the visual system's adjustment to simulated changes in illumination is regular and can be understood with a bilinear model that is motivated by the physics of natural image formation.

Fundamentals

Any quantitative discussion of color appearance must be based on an understanding of the physical properties of light and how the visual system measures these properties. Cornsweet (1970) provides an elementary introduction to the fundamentals of color vision. A detailed modern treatment using linear algebra can be found in Wandell (1987).

Figure 13.1 illustrates how the light that reaches the eye (**c**) arises when an illuminant (**e**) reflects from a surface (**s**). The illuminant is characterized by its spectral power distribution, which specifies how much power it contains at N_λ evenly spaced sample wavelengths λ_n in the visible spectrum. Typically the visible spectrum is sampled between approximately 370 nm and 730 nm with a wavelength spacing $\Delta\lambda$ between 1 nm and 10 nm. The illuminant spectral power distribution can be described graphically, as shown in the figure. We use the symbol **e** to denote illuminant spectral power distributions.

The illuminant reflects off a surface to form the light that reaches the eye. We call the light reaching the eye the color signal and denote it by the symbol **c**. The spectral power distribution of the color signal is determined by the spectral power distribution of the illuminant and the spectral reflectance function of the surface. The surface's reflectance function specifies the fraction of illuminant power that is reflected from the surface at each sample wavelength λ_n.[2] To compute the color signal power at any wavelength, we multiply the illuminant power at that wavelength by the corresponding value of the surface reflectance function.

Color vision begins when the color signal enters the eye and is measured by the visual system. The visual system makes three separate measurements on the color

1. Land's (1964; 1983; Land & McCann, 1971) theory, based on the retinex algorithm, has been popularized as a successful account of color appearance in natural images. It is, however, based on the many of the same fundamental ideas and is less closely tied to empirical data than the earlier models (see Brainard & Wandell, 1986; McLaren, 1986; Shapley, 1986; West & Brill, 1982; also Judd, 1960; Woolfson, 1959).

2. We assume that a surface's reflectance function does not depend on viewing geometry. This is true for diffuse illumination. For point illumination, an object's surface reflectance can depend on the angle at which the illuminant strikes the surface. This phenomenon is important but beyond the scope of this chapter. D'Zmura and Lennie (1986), Lee (1986), Shafer (1985), Tominaga and Wandell (1989), and chapter 14 of this volume contain analyses of the relation between the viewing geometry of an object and its surface reflectance.

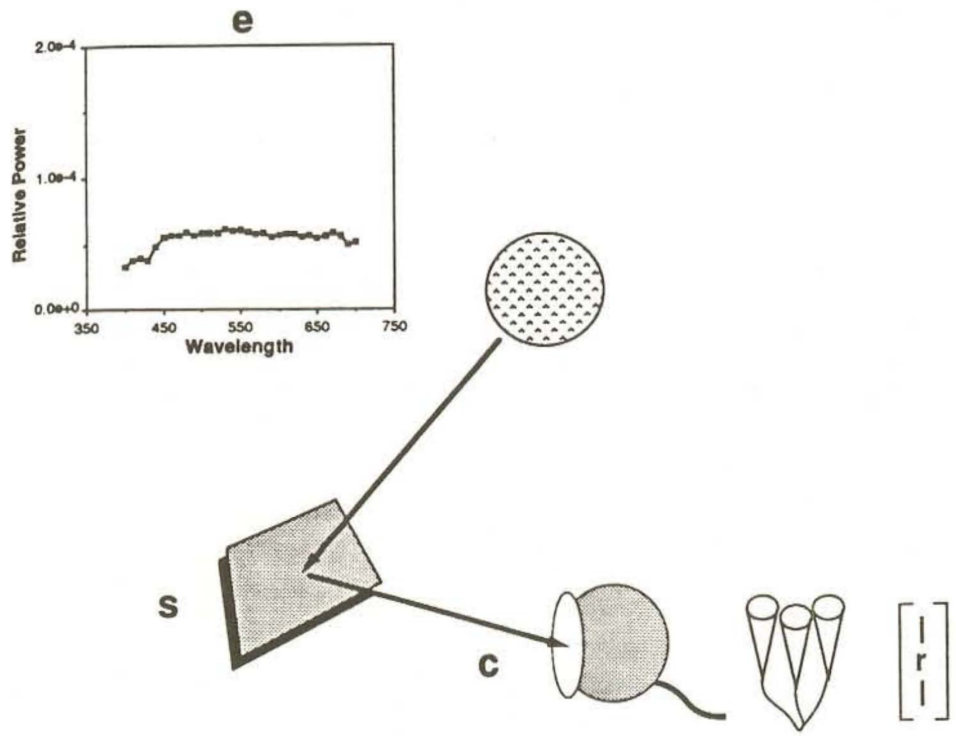

Fig. 13.1
Image formation and initial encoding. An illuminant (**e**) reflects from a surface (**s**) to form the color signal (**c**) that reaches the eye. The visual system represents the color signal by the responses of three classes of cones (**r**). (Adapted from Brainard & Wandell, 1990b.)

signal at each spatial location of an image. These measurements are the responses of three classes of light sensitive photoreceptors on the retina: the long wavelength sensitive (L) cones, the middle wavelength sensitive (M) cones, and the short wavelength sensitive (S) cones. The three cone responses generated by a color signal are called the color signal's cone coordinates.[3] They are the information available to the visual system about the color signal and about the underlying illuminant and surface.

We denote the cone coordinates of a color signal by the vector **r**, whose entries are the responses of the three cone classes. This vector is drawn in the lower right of figure 13.1. The relation between a color signal's spectral power distribution and its cone coordinates is determined by the spectral responsivities of the three classes of cones. The spectral responsivity of a cone class specifies how strongly the cone responds to color signal power at each sample wavelength. To find a cone's quantal absorptions, we multiply the color signal power at each wavelength by the cone responsivity at that wavelength, and then sum these products over wavelength. Because the spectral responsivities of the human cones have been estimated psychophysically (Smith & Pokorny, 1975; see Boynton, 1979) and confirmed physiologically (Schnapf, Kraft & Baylor, 1987), it is possible to compute the cone coordinates to any color signal.

The relation between a color signal **c** and its cone coordinates is conveniently expressed using matrix and vector notation. We represent **c** with an N_λ dimensional column vector whose nth entry is the power at sample wavelength λ_n. Let $\mathbf{R}$ be a matrix with three rows and N_λ columns, where each row describes the responsivity of one class of cones. The first row describes the respon-

3. Strictly speaking, there can be only one photoreceptor at each spatial location. It is convenient, however, to assume that at each location the color signal is sampled by all classes of receptor. Brainard, Wandell, and Poirson (1989) discuss the implications of the fact that there can only be one receptor at each spatial location. The assumption that all three classes of receptors sample the color signal at each location causes no difficulty as long as the color signal is constant across regions that are large compared to the spacing between photoreceptors. In addition to the cones, there is a fourth class of photoreceptor: the rods. The rods are not generally thought to play a role in color vision at the high light levels typical of daylight viewing.

sivity of the L cones: The nth entry of the first row is the responsivity of an L cone at sample wavelength λ_n. The second and third rows describe the responsivity of the M and S cones. When the cone responsivities are described by $\mathbf{R}$, the relation between a color signal's spectral power vector $\mathbf{c}$ and its cone coordinates $\mathbf{r}$ is given by

$$\mathbf{r} = \mathbf{R}\mathbf{c}. \tag{1}$$

Without loss of generality, the choice of wavelength spacing $\Delta\lambda$ is incorporated into equation 1 by absorbing it in the units used to express light power.

It is useful to extend equation 1 to include the physical factors that give rise to the color signal. Let $\mathbf{e}$ be the N_λ dimensional column vector whose nth entry is the power at sample wavelength λ_n. Let $\mathbf{S}$ be the N_λ by N_λ diagonal matrix whose nth diagonal entry is the surface reflectance at sample wavelength λ_n. We then have $\mathbf{c} = \mathbf{S}\mathbf{e}$. Combining this with equation 1 we arrive at a relation between the illuminant spectral power distribution $\mathbf{e}$ and the response vector $\mathbf{r}$ that encodes the color signal reflected from a surface:

$$\mathbf{r} = (\mathbf{R}\mathbf{S})\mathbf{e}. \tag{2}$$

Experimental Method

To study the visual system's adjustment, we ask what happens to the appearance of a test light when we manipulate this adjustment. Our basic experimental strategy was to use a matching paradigm. Matching paradigms have been widely used in psychophysics and offer the advantage that subjects are only required to judge identity of sensation. Such procedures are in general more reliable and less open to multiple interpretations than procedures that require subjects to name, rate, or scale sensation (see Brindley, 1970).

At the start of an experimental session, we trained the subject to remember the color appearance of a *prototype color signal*, which was presented in the context of CRT simulations of surfaces illuminated by a *standard illuminant*. During the course of an experimental session, the subject set *memory matches* to this learned appearance standard. To set a match, the subject used a button box to adjust the spectral power distribution of a color signal emitted from a CRT. We had the subject set memory matches both in the context of CRT simulations of surfaces illuminated by the standard illuminant and in the context of CRT simulations of surfaces illuminated by a *changed illuminant*. The standard illuminant matches measured whether the subject could perform the memory matching task veridically. The changed illuminant matches defined a *matching color signal*. This color signal had the same color appearance when it was viewed in the context of the changed illuminant images as did the prototype color signal when it was viewed in the context of the standard illuminant images. The difference between the prototype and matching color signals was our measure of the effect of the visual system's adjustment on color appearance.

Figure 13.2 illustrates our experimental procedure. The upper half of the figure shows the spectral power distributions of the standard illuminant $\mathbf{e}$ and one possible changed illuminant $\mathbf{e} + \Delta\mathbf{e}$. The lower left of the figure shows the cone coordinates $\mathbf{r}$ of the prototype color signal. The lower right of the figure shows the cone coordinates $\mathbf{r} + \Delta\mathbf{r}$ of the matching color signal. The symbol "$\sim$" is used to indicate the appearance match between $\mathbf{r}$ and $\mathbf{r} + \Delta\mathbf{r}$ across the change in simulated illuminant.

We controlled the state of the visual system by having the subject judge and adjust color signals that were presented in a small region of larger *context images*. The context images were implemented as a CRT simulation of uniformly illuminated matte surfaces.[4] The spatial structure of our simulated images is shown in figure 13.3. In each image, 25 small rectangular surfaces were simulated against a background surface. The figure gives the size of the simulated surfaces in degrees of visual angle. For any given image, the simulated surfaces were chosen by random draw from the Munsell papers (Nickerson, 1957). Subjects viewed the images binocularly in an otherwise dark room. Both head and eye movements were permitted during viewing.

When we wanted the visual system to be adjusted to the standard illuminant, we simulated surfaces illuminated

4. To simulate an illuminated surface, we used equation 2 to compute cone coordinates $\mathbf{r}$ from the illuminant spectral power distribution $\mathbf{e}$ and surface reflectance function $\mathbf{S}$. We then used calibration measurements of our CRT monitor to compute input values for the monitor frame buffer so that the emitted color signal generated these same cone coordinates. Brainard (1989a) describes our display hardware and calibration measurements. Brainard and Wandell (1990a) describe the software that was used to implement the simulations.

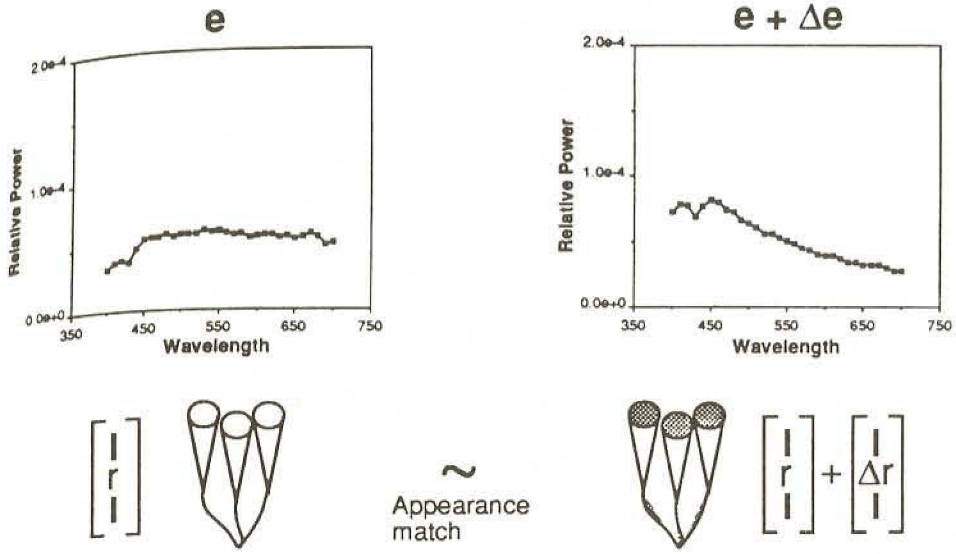

Fig. 13.2
Experimental framework. The experiment measures the cone coordinates **r** and **r** + Δ**r** of color signals that match in appearance across a change of simulated illuminant. The illuminant change is described by Δ**e**. (Adapted from Brainard & Wandell, 1990b.)

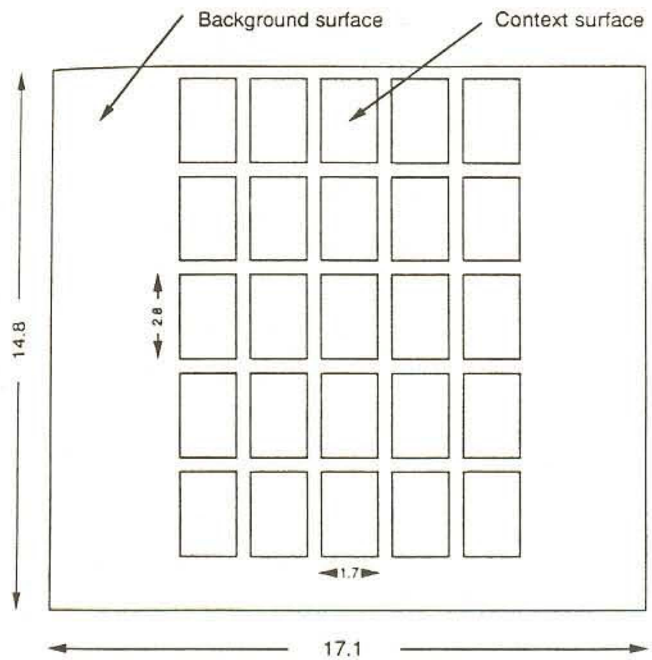

Fig. 13.3
Spatial structure of the simulated images used in the experiment. Twenty-five small rectangular surfaces were simulated against a background surface. The units of the dimensions shown in the figure are degrees of visual angle.

by the standard illuminant. When we wanted the visual system to be adjusted to the changed illuminant, we simulated surfaces illuminated by the changed illuminant. To change the subject's adjustment to the illuminant, the subject viewed a series of images where the spectral power distribution of the spatially uniform illuminant was varied slowly. The subject always viewed images characterized by a single illuminant for an extended time before setting any appearance matches.

Features of the Experimental Method

There are a number of important features about our experimental method. In this section we discuss four of these: our choice of images, how we randomize with respect to contrast effects, how we control observer adaptation, and the use of a CRT to present the images.

Choice of Images

In real world viewing, images can be quite complex: There can be multiple sources of illumination, images can contain different numbers of surfaces, the surfaces can have a huge variety of sizes and shapes, surface reflectance functions can depend on viewing geometry, and the spectral functions that describe illuminant power and surface reflectance can be nearly arbitrary. To bring all of the richness of natural images into the laboratory would make the construction, manipulation, and description of the experimental stimuli intractable. It is necessary to restrict the features of natural images that will be incorporated into the laboratory veiwing situation. Although it is desirable to keep the structure of the images as simple as possible, it is important to consider what features of natural images provide information used by the visual system to adjust to the illuminant.

It would be unreasonable, of course, to measure the visual system's adjustment using images where the relevant information about the illuminant had been removed. Over the past ten years, there has been a great deal of progress in understanding exactly what information about the illuminant is available in simple images. This understanding provides guidance to the experimentalist who wishes to choose a laboratory model that is simple but that retains this information. In particular, the work of Buchsbaum (1980) and of Maloney and Wandell (1986) shows that it is possible for the visual system to estimate the illuminant when the images consist of uniformly illuminated matte surfaces. Although the algorithms differ in detail, they share the property that their estimates of the illuminant improve with the number of surfaces in the image. To show this quantitatively, we implemented the algorithm proposed by Buchsbaum (1980) and simulated its performance on images consisting of 1, 25, and 100 surfaces. The images were created by choosing surfaces by random draw from the set of Munsell papers. We computed the location-by-location cone coordinates for each image when it was uniformly illuminated by the standard illuminant of figure 13.2. We then ran Buchsbaum's algorithm to estimate the illuminant.[5] Figure 13.4 shows the results. When there is a only a single surface in the image, the estimates of the illuminant vary wildly. As the number of surfaces in the image increases, the estimates of the illuminant improve. Clearly, an image with single surface (uniform background) does not provide enough information for the visual system to estimate the illuminant. We chose images of the form shown in figure 13.3 as a compromise between the full complexity of natural images and the simplicity of uniform backgrounds. Our images are simple compared to natural images but contain sufficient information for the visual system to estimate the illuminant.

Contrast Effects

In the introduction we argued that when only uniform backgrounds are used, it is impossible to separate the effect of the visual system's adjustment to the illuminant from other factors such as simultaneous color contrast and color assimilation.[6] An understanding of the influence of spatial and temporal image factors that can vary within a class of images that share a common illuminant must be included in a complete theory of color appearance, but these effects are different from adjustment to the illuminant. Because we wanted to measure only the effect of changing the illuminant, we used many images throughout an experimental session. This randomized our design with respect to local spatial contrast effects. While a subject set a match, for example, the particular surfaces in the image were changed each time the subject pressed a button, as was the location of the color signal being judged. The spectral power distribution of the simulated illuminant remained constant. Because these other factors are randomized, we attribute our results to the effects of the visual system's adjustment to the simulated illuminant.

Control of Adaptation

The early experimentalists were careful to assess color appearance only after they had set the visual system's state of adaptation by having the observer view a uniform field for some minutes before any measurements were made (Burnham, Evans & Newhall, 1957; Helson & Jeffers, 1940; Helson, Judd & Warren, 1952; Hunt, 1953). Similarly, in our experiments, we were careful to adjust the visual system to the illuminant by having the observer view simulated images that shared a common illuminant for several minutes. In many recent experiments designed to study the effect of changing the illuminant on color appearance, the adjustment of the visual system was not under experimental control. Rather the illuminant

5. To implement the algorithm, we had to choose a reference surface and a set of basis functions for the surfaces and illuminants. For the reference surface we used the mean reflectance of the Munsell paper data set. For the surface basis functions, we used the first three prinicipal components of this data set. For the illuminant basis functions, we used the first three principal components of daylight reported by Judd, MacAdam, and Wyszecki (1964). The standard illuminant could be expressed exactly as a linear combination of these illuminant basis functions.

6. Wyszecki (1986) provides a thorough review of such phenomena. Excellent color plates demonstrating color contrast and color assimilation can be found in Evans (1948) and Albers (1975).

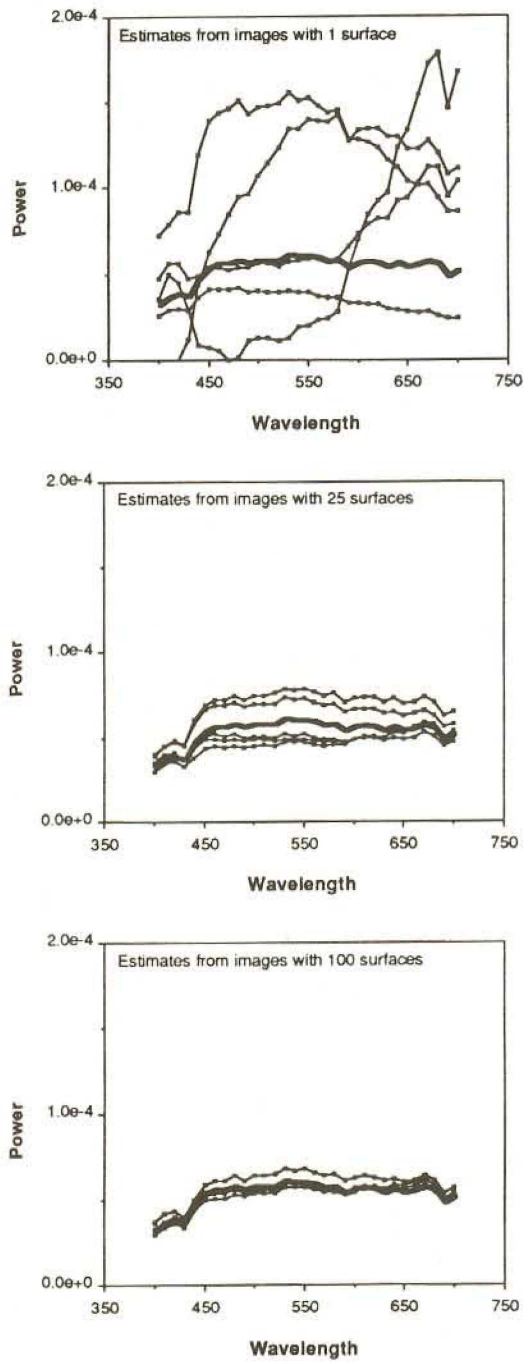

Fig. 13.4

Effect of number of surfaces on estimate of illuminant. *Top,* Illuminant estimates for five randomly chosen images consisting of a single surface. *Middle,* Illuminant estimates for five randomly chosen images consisting of 25 surfaces. *Bottom,* Illuminant estimates for five randomly chosen images consisting of 100 surfaces. In each panel, the heavy solid line shows the actual illuminant.

changed every few seconds as the subject freely compared the appearance of color signals presented in images seen either side-by-side or in quick succession (Arend & Reeves, 1986; Benzschawel, Walraven & Rogowitz, 1987; Blackwell & Buchsbaum, 1988; Hurlbert, Lee & Bülthoff, 1989; McCann, McKee & Taylor, 1976; Valberg & Lange-Malecki, 1987; Walraven, Benzschawel & Rogowitz, 1988). Although there have been some claims that adjustment to the illuminant happens almost instantaneously (Land & Daw, 1962), careful measurements (Ahn & MacLeod, 1990; Fairchild, 1990; Hunt, 1950; Jameson, Hurvich & Varner, 1979) indicate that the adjustment can take from tens of seconds to several minutes. We believe that the results of experimental work on the visual system's adjustment that permits and controls adaptation should be distinguished from the results of experiments that do not.

Use of CRT to Present Images

The use of a computer-controlled CRT to present stimuli offers great advantages in terms of stimulus control. For example, it would be technically difficult to randomize against contrast effects without software control over the images. But the use of a CRT also raises the issue of whether the visual system processes the simulated images in the same way it would have processed reflectance implementations of the same images. We believe that our current understanding of the initial encoding of light by the visual system provides a firm theoretical foundation for the use of CRT simulations: The simulated images are designed to provide the same stimulation of the retinal cone mosaic as the reflectance images. Although variation in the color matching functions between observers and with eccentricity, as well as limitations of CRT resolution and calibration accuracy, require that this match is only approximate, we believe that the simulations provide a reasonable visual match to the reflectance images they are designed to simulate. Presently it is an open empirical question as to whether the quality of simulation obtainable with a CRT is sufficient to cause the visual system to process the simulated images in the same way as it would have processed reflectance images. In adaptation experiments using uniform fields, Fairchild (1990) compared results obtained using a CRT to control adaptation and results obtained using a light booth. He found little difference between the two conditions. On the other hand, Gorzynski and Berns (1990), using a different experimental paradigm, report differences in performance when simulated and reflectance images are used.

As with any scientific research, the question of whether the results obtained in the laboratory generalize must be addressed empirically. Clearly, the use of CRT simulations to display our images is but one of many laboratory simplifications. The images we chose to simulate are themselves much simpler than natural images. In addition to validating the use of a CRT, it is important to validate these underlying simplifications. Indeed, we view the present experiments as serving this role with respect to the earlier experiments conducted with an even simpler laboratory model.

Basic Experimental Results

The measurement conditions for a single session are summarized by two vectors: the prototype color signal's cone coordinates **r** and the illuminant change $\Delta \mathbf{e}$. Implicit in this summary are the standard illuminant spectral power distribution, the spatial structure of the context images, and the surface reflectance functions used in the images. These were held fixed throughout the experiments reported here. Two subjects, one of the authors (D.B.) and a paid undergraduate (S.E.), observed in the experiments reported here. Both had normal color vision as tested with the Ishihara color plates (Ishihara, 1977). Subject D.B. was an experienced psychophysical observer and was aware of the design and purpose of the experiments. Subject S.E. began as a naive observer but became progressively better informed about the experiment over several months of observing.

The raw data from an experimental session are the cone coordinates of the subject's memory matches set when his visual system was adjusted to the standard illuminant and when his visual system was adjusted to the changed illuminant. A single session required approximately one hour of observing. Two sessions were always run for any measurement condition and the subject's responses from both sessions were pooled together. We summarize the data from the two sessions by the mean of 12 matches set under each illuminant.

Figure 13.5 shows the results of measurements of the effect of three illuminant changes on the color appearance of a single prototype color signal for subject D.B. The data are presented in three two-dimensional plots. The top panel shows the M cone coordinates of the matches plotted against the L cone coordinates, the middle panel shows the S vs. the L, and the bottom shows the S vs. the M. The open symbols show the mean of the subject's standard illuminant matches. The closed symbols show the mean of the changed illuminant matches. The cross in each panel shows the cone coordinates of the prototype color signal. The arrows indicate the change in cone coordinates, $\Delta \mathbf{r}$, required to maintain constant color appearance as the illuminant was changed.

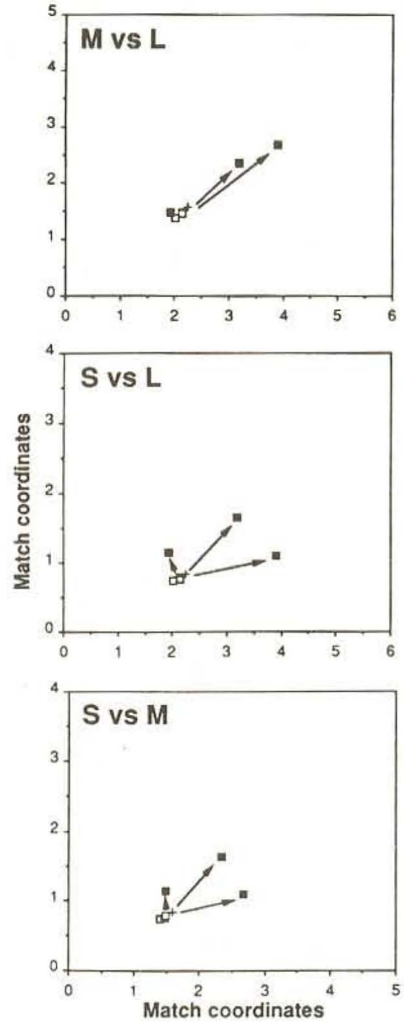

Fig. 13.5
Results of measurements for three illuminant changes. The cross indicates the cone coordinates of the prototype color signal. The open squares show the mean of the standard illuminant matches. The filled squares show the mean of the changed illuminant matches. (Prototype color signal R6 _ 4; subject D.B.)

There are two important features to note about the data. First, the subject's matches set under the standard illuminant are veridical: In all three cases the open symbols lie close to the cross. Our data showed no systematic biases in subject's memory matches. Second, when the illuminant is changed there is a considerable effect on color appearance: The closed symbols are displaced from the open symbols. Each different illuminant change has a different effect on color appearance. A detailed description of our experimental method and complete tabulation of our data is given in Brainard (1989b).

Because subject's matches set under the standard illuminant were veridical, we summarize the results of an experimental condition simply by $\Delta\mathbf{r}$, the change in cone coordinates necessary to preserve color appearance. We say that $\Delta\mathbf{r}$ is the effect of the illuminant change $\Delta\mathbf{e}$ on the appearance of a prototype color signal with cone coordinates $\mathbf{r}$. To understand the visual system's adjustment to the illuminant we measure the relation between the prototype color signal's cone coordinates, $\mathbf{r}$, the illuminant change, $\Delta\mathbf{e}$, and the effect of the illuminant change on color appearance, $\Delta\mathbf{r}$.

To help interpret $\Delta\mathbf{r}$, note that if the visual system made no adjustment to the illuminant change, then $\Delta\mathbf{r}$ would be $\mathbf{0}$. Another possibility is that $\Delta\mathbf{r}$ corresponds to color constant performance. Equation 2, $\mathbf{r} = (\mathbf{RS})\mathbf{e}$, shows how the prototype color signal can be interpreted as a simulation of an illuminated surface. When this same surface is illuminated by $\mathbf{e} + \Delta\mathbf{e}$, the cone coordinates are given by $\mathbf{r} + \Delta\mathbf{r} = (\mathbf{RS})(\mathbf{e} + \Delta\mathbf{e})$. Subtracting these two expressions yields the change that corresponds to color constant performance[7]:

$$\Delta\mathbf{r} = (\mathbf{RS})\Delta\mathbf{e}. \tag{3}$$

A Bilinear Model

Overview

Our experimental method can be used to measure the effect $\Delta\mathbf{r}$ of any illuminant change $\Delta\mathbf{e}$ on the color appearance of any prototype color signal $\mathbf{r}$. Because there are many possible pairs of $\Delta\mathbf{e}$ and $\mathbf{r}$, it is not feasible to make direct measurements for all of them. We need to find some way to organize and understand the effect of illuminant change.

To simplify the modeling task, we begin by considering the effects of the two indpendent variables separately. First we hold the cone coordinates of the prototype color signal constant and consider the effect of varying the illuminant. Then we hold the illuminant change constant and consider the effect of varying the prototype color signal. After we consider each of the variables separately, we can turn to the question of understanding how they interact with each other.

Illuminant Change Linearity

Shepard (1987) has suggested that psychological relations are often internalizations of external physical laws. Recall that equation 3 gives the physical relation between the illuminant change and the change in cone coordinates of a surface as the illuminant is changed. If the visual system were color constant, our measured relation between illuminant change $\Delta\mathbf{e}$ and measured change $\Delta\mathbf{r}$ would be predicted by equation 3. Even though we know the equation 3 will not hold exactly, it still suggests a model for performance.

Equation 3 expresses a linear relation. The linearity of equation 3 stems from the linear relation between illuminants and the cone responses of the reflected color signal, when the prototype color signal (and hence the implicit prototype surface reflectance characterized by $\mathbf{S}$) is held fixed. Because the linearity is fundamental to the physics of lights and surfaces, we might expect to find it in the psychology of color appearance. That is, the linearity of equation 3 suggests a relation between $\Delta\mathbf{r}$ and $\Delta\mathbf{e}$ that ought to hold if the visual system has internalized the physics of reflectance. We can look for a relation between these two measurable quantities of the form

$$\Delta\mathbf{r} = \mathbf{M}_{\mathbf{r}}\Delta\mathbf{e}, \tag{4}$$

where $\mathbf{M}_{\mathbf{r}}$ is a matrix that depends on the prototype color signal but not on the illuminant change $\Delta\mathbf{e}$. The empirical relation expressed by equation 4 can be tested by measur-

7. Because of surface metamerism under the standard illuminant (see Wyszecki & Stiles, 1982) there will be many different surfaces that satisfy $\mathbf{r} = (\mathbf{RS})\mathbf{e}$. Each choice of $\mathbf{S}$ leads to a different color constant $\Delta\mathbf{r}$. In this sense there is not a unique prediction for the behavior of a color constant system. In practice, we handle this ambiguity by examining the color constant prediction for a surface whose reflectance is typical of natural surfaces.

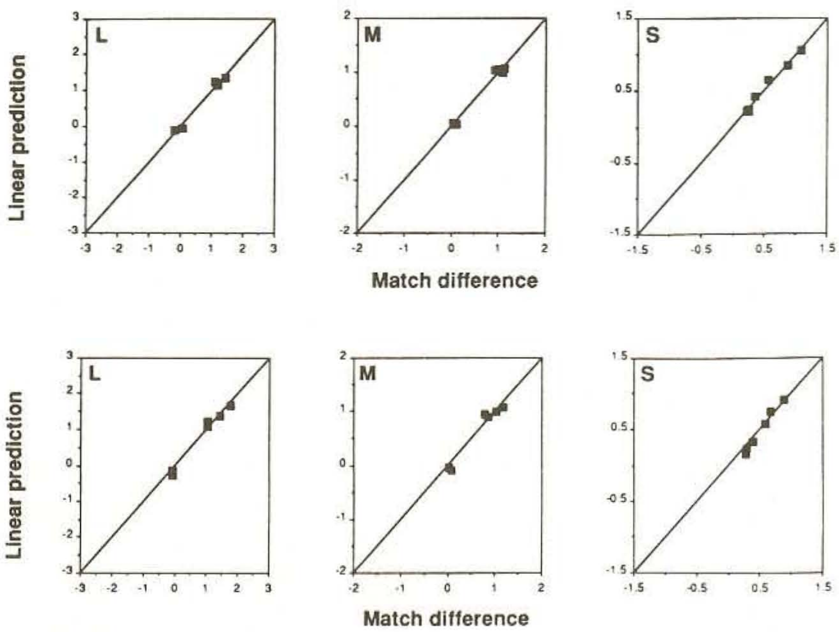

Fig. 13.6
Illuminant change linearity. The figure plots the predicted difference between the changed and standard illuminant matches against the measured difference for the L, M, and S cone components of $\Delta\mathbf{r}$. (*Top*, Prototype color signal N6; subject S.E. *Bottom*, Prototype color signal R6_4; subject D.B.) (Adapted from Brainard & Wandell, 1990b.)

ing the effect of a number of illuminant changes that are a linear combination of a small number of illuminant changes.

Suppose we fix two illuminant changes, $\Delta\mathbf{e}_1$ and $\Delta\mathbf{e}_2$ and measure their effect on color appearance, summarized by $\Delta\mathbf{r}_1$ and $\Delta\mathbf{r}_2$. Now consider any other illuminant change that is physically a linear combinaton of $\Delta\mathbf{e}_1$ and $\Delta\mathbf{e}_2$. That is, consider illuminant changes of the form $\Delta\mathbf{e} = \alpha_1 \times \Delta\mathbf{e}_1 + \alpha_2 \times \Delta\mathbf{e}_2$. If equation 4 describes behavior, then we expect that corresponding to each such $\Delta\mathbf{e}$ we will measure $\Delta\mathbf{r} = \alpha_1 \times \Delta\mathbf{r}_1 + \alpha_2 \times \Delta\mathbf{r}_2$. There is nothing special, of course, about having chosen only two basis illuminant changes. The general implication of a measurement system that obeys equation 4 is that once we measure the effect of a finite number of illuminant changes, we can predict the results for measurements for any linear combination of these illuminant changes. This is of particular interest because Judd, MacAdam, and Wyszecki measured approximately 600 daylight spectral power distributions and concluded that, within their measurement error, all of the variation in daylight could be accounted for by a linear combination of just four basis illuminant changes.

To test whether equation 4 describes performance, we pick two basis illuminant changes, $\Delta\mathbf{e}_1$ and $\Delta\mathbf{e}_2$, and measure the effect of changing the illuminant for many linear combinations of them. We use linear regression to find the best fit to the measured data consistent with equation 4. If equation 4 describes performance, the prediction will be close to the measured data. Figure 13.6 plots, for two subjects, results for six illuminant changes that were all linear combinations of two basis illuminant changes. The plots show the linear prediction plotted against the data. For each panel, the x-axis plots one component (L, M, or S) of the measured change $\Delta\mathbf{r}$, while the y-axis plots the corresponding linear prediction. If illuminant change linearity held perfectly, then these data would fall right along the diagonal. The data support using an equation of the form of 4 to model performance.

The six illuminant changes used to test equation 4 are typical of the spectral variation of daylight. Figure 13.7 shows the CIE chromaticity coordinates of the experimental illuminants and of four CIE illumination standards. The hatched square shows the CIE chromaticity coordinates of the standard illuminant. The filled squares show the CIE chromaticity coordinates of the six changed illuminants. (Two pairs of changed illuminants differ only in intensity and thus have identical CIE coordinates.) The open squares show the chromaticities of four CIE stan-

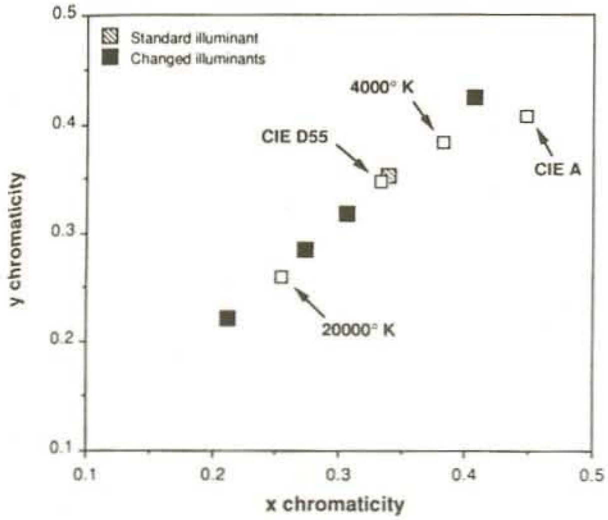

Fig. 13.7

Experimental illuminant chromaticities. The hatched square shows the CIE chromaticity coordinates of the standard illuminant. The four filled squares show the CIE chromaticity coordinates of the six changed illuminants. The open squares show the chromaticities of four CIE standard illuminants, as marked.

dard illuminants, as marked. The standard illuminant is close to CIE standard daylight D55. The changed illuminants clearly span the range of daylight variation between 4000°K and 20,000°K equivalent blackbody radiators. This range is typical of the range between direct sunlight and blue skylight (see Wyszecki and Stiles, 1982, pp. 6–7, p. 761). CIE illuminant A, which is representative of incandescent illumination, lies outside of the range tested. A third basis illuminant change would have to be added to represent the spectral power distribution of CIE illuminant A.

Prototype Linearity

A second part of understanding the effect of the illuminant change is to understand what happens as we vary the test light's cone coordinates. We study this by reversing the roles of $\Delta\mathbf{e}$ and $\mathbf{r}$ in the analysis. We fix the illuminant change and then make measurements of $\Delta\mathbf{r}$ for several different test surfaces with different cone coordinates $\mathbf{r}_1$, $\mathbf{r}_2$, etc. Again, we look for a linear relation, this

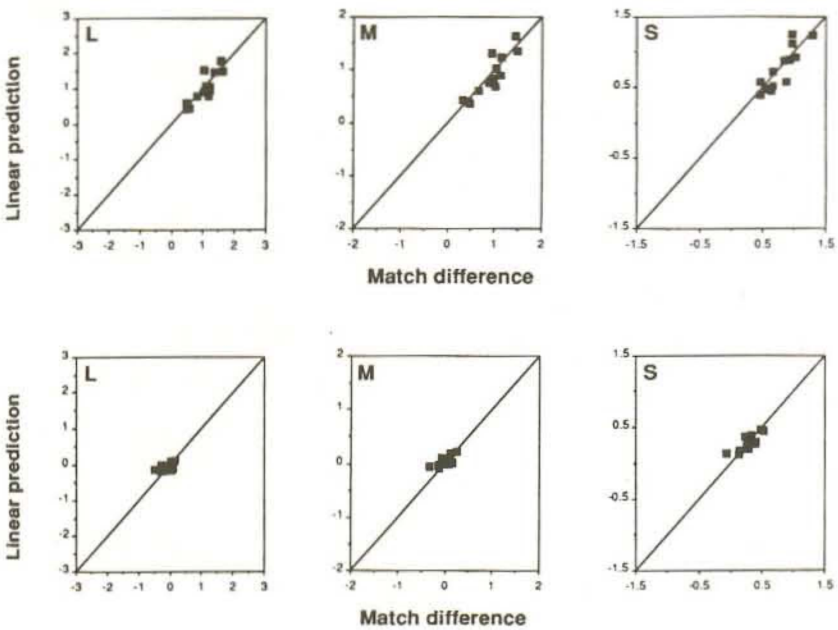

Fig. 13.8

Prototype linearity. The figure plots the predicted difference between the changed and standard illuminant matches against the measured difference for the L, M, and S cone components of $\Delta\mathbf{r}$. (*Top*, Changed illuminant 12; subject S.E. *Bottom*, Changed illuminant 1; subject D.B.) (Adapted from Brainard & Wandell, 1990b.)

time between the cone coordinates $\mathbf{r}$ and the measured change $\Delta\mathbf{r}$. In analogy to equation 4, we look for a relation of the form:

$$\Delta\mathbf{r} = \mathbf{M}_{\Delta\mathbf{e}}\mathbf{r}. \tag{5}$$

The use of equation 5 is also motivated by the linearity of the physics of reflection. We can test this relation in exactly the same way as we tested equation 4. Figure 13.8 shows the linear model prediction $\mathbf{M}_{\Delta\mathbf{e}}\mathbf{r}$ plotted against the measured $\Delta\mathbf{r}$ for data from two subjects. The left panels show the L cone coordinates of the two vectors, the middle panels the M cone coordinates, and the right panels the S cone coordinates. If a linear function perfectly modeled performance, the data in each panel of the two plots would fall on the diagonal lines with unit slope. The data support using an equation of the form of 5 to model performance.[8]

Bilinearity

The results presented above show that $\Delta\mathbf{r}$ is a linear function of both $\Delta\mathbf{e}$ and $\mathbf{r}$ when the other variable is held fixed. When a function with two inputs is a linear function for each, it is said to be bilinear (Hungerford, 1974, p. 211). The fact that the relation between $\Delta\mathbf{r}$, $\Delta\mathbf{e}$, and $\mathbf{r}$ is bilinear means that the two experimental variables $\Delta\mathbf{e}$ and $\mathbf{r}$ interact in a very constrained manner.

One way to understand about the constraint imposed by the bilinearity is as follows. Let P be the dimension of $\mathbf{r}$ and M be the dimension of $\Delta\mathbf{e}$. (Here $P = 3$ and $M = 2$.) Denote the entries of $\mathbf{r}$ by r_i, $i = 1, \ldots, P$ and the entries of $\Delta\mathbf{e}$ by Δe_j, $j = 1, \ldots, M$. Given $\mathbf{r}$ and $\Delta\mathbf{e}$ we can construct a PM dimensional vector, $\mathbf{x}$, where the entries of $\mathbf{x}$ are all of the products $r_i \Delta e_j$, $i = 1, \ldots, P$, $j = 1, \ldots, M$, ordered lexicographically in i and j. Brainard (1989b) shows that the relation between $\Delta\mathbf{r}$, $\Delta\mathbf{e}$, and $\mathbf{r}$ is bilinear if and only if we can write

$$\Delta\mathbf{r} = \mathbf{F}(\mathbf{r}, \Delta\mathbf{e}) = \mathbf{M}\mathbf{x}, \tag{6}$$

where the matrix $\mathbf{M}$ has dimensions 3 by PM. That is, we can re-express the bilinear relation as a simple linear relation using the constructed variable $\mathbf{x}$. Equation 6 makes it clear that a bilinear model has the same advantages as a linear model. Once we make measurements of the effect of a small number of illuminant changes ($\Delta\mathbf{e}_j$, $j = 1, \ldots, M$) on the appearance of a small number of prototype color signals ($\mathbf{r}_i$, $i = 1, \ldots, P$), we can predict the effect of any illuminant change $\Delta\mathbf{e}$ that is a linear combination of the $\Delta\mathbf{e}_j$ on the appearance of any prototype color signal whose cone coordinates $\mathbf{r}$ are a linear combination of the $\mathbf{r}_i$.

Although we have already tested both illuminant change linearity and prototype linearity individually, we can also test bilinearity directly using equation 6. To produce enough measurement conditions for a reasonable test, we pooled the data from our two subjects. There was good consistency between the subjects for the their common measurement conditions. Since pooling the data can only reduce the quality of the bilinear fit, doing so also provides a test of how well a single model can characterize the behavior of multiple subjects. Figure 13.9 shows plots of the bilinear function prediction $\mathbf{M}\mathbf{x}$ against the measured $\Delta\mathbf{r}$ for the entire data set. The figure shows the results of 45 measurements from 37 distinct measurement conditions. (Some conditions are measured for both subjects and subject S.E. replicated measurements for a few conditions). Since each measurement is a three-dimensional vector, there are $3 \times 45 = 135$ degrees of freedom in the data set. The bilinear model has only $3 \times (3 \times 2) = 18$ free parameters and describes the data well.

Discussion

Summary

This chapter has addressed the problem of understanding the visual system's adjustment to changes in illumination. We began by introducing the problem of understanding the effect of the visual system's adjustment. We then reviewed the fundamentals of color vision discussion of color appearance and presented our experimental method.

There were three important features of the experimental method. First, the images used to define the illuminant

8. When evaluating three-dimensional data, it is important to bear in mind that for any given condition, the effects may occur on only one of the three dimensions. This does not mean that the effects are any smaller. For the illuminant change $\Delta\mathbf{e}$ used for subject D.B., the measured effect of illuminant change $\Delta\mathbf{r}$ occurs almost entirely along the S cone coordinate dimension. For this reason, the left and center bottom panels of figure 13.8 do not show much of an effect of illuminant change.

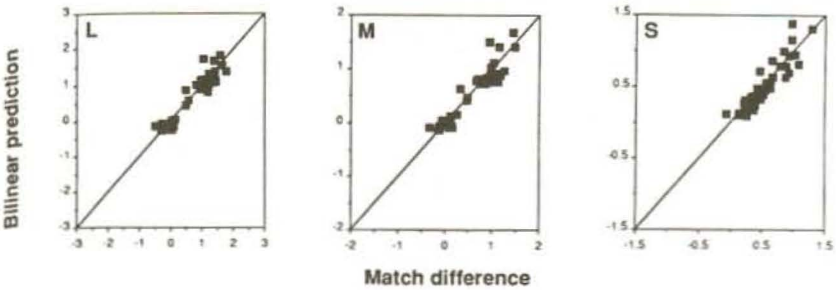

Fig. 13.9
Bilinearity. The figure plots the predicted difference between the changed and standard illuminant matches against the measured difference. (All conditions; subjects D.B., S.E.) (Adapted from Brainard & Wandell, 1990b.)

contained enough information for the visual system to estimate the illuminant. Second, for any illuminant condition we randomized our design with respect to local spatial contrast effects by using many different images that shared a common simulated illuminant. Third, our use of a memory matching paradigm allowed us to control the visual system's adjustment. The subject always viewed images that were simulations of uniformly illuminated surfaces and we assessed color appearance only after the subject had adjusted to the simulated illuminant.

We developed and tested a bilinear model for understanding the visual system's adjustment to the illuminant. The parameters of the bilinear model are determined by a small number of measurements. Once determined, the model allows us to predict the effect of the adjustment to any illuminant.

Color Constancy

The data presented in this chapter may be analyzed to address the question How color constant is performance for our laboratory model? Given a prototype color signal with cone coordinates $\mathbf{r}$, we can find a surface described by $\mathbf{S}$ such that equation 2 holds: $\mathbf{r} = (\mathbf{RS})\mathbf{e}$. We can then ask what *equivalent illuminant change* $\Delta\hat{\mathbf{e}}$ would we have had to have made such that measured change $\Delta\mathbf{r}$ corresponded to the behavior of a color constant visual system. That is, we can find $\Delta\hat{\mathbf{e}}$ such that the measured $\Delta\mathbf{r}$ satisfies

$$\Delta\mathbf{r} = (\mathbf{RS})\Delta\hat{\mathbf{e}}. \tag{7}$$

The equivalent illuminant change $\Delta\hat{\mathbf{e}}$ is interpreted as the portion of the illuminant change to which the visual system actually adjusted.[9]

Figure 13.10 plots two illuminant changes and the corresponding equivalent illuminant changes computed from the data. If the visual system were color constant, the actual and equivalent illuminant changes would be identical. In both cases, we see that the visual system is compensating for approximately the correct relative spectral power distributon of the illuminant change, but that the magnitude of the adjustment is only about half of color constant performance. Figure 13.11 shows this quantitatively. For each of our 45 measurement conditions, we computed the equivalent illuminant change. We then found the scale factor which, when multiplied by the actual illuminant change $\Delta\mathbf{e}$, produced the best fit to the equivalent illuminant change $\Delta\hat{\mathbf{e}}$. Figure 13.11 plots a histogram of these 45 scale factors, which are distributed around a mean value of 0.48.

Laboratory Model

The concept of the equivalent illuminant change is useful for understanding how the results of the present experiments might be applied to natural images. From our bilinear model of performance, we can compute the equivalent illuminant change from a description of how much the illuminant in a natural scene differs from our standard illuminant. This equivalent illuminant change is a description of how much adjustment the visual system will undergo when it views this natural image, and it can be used to predict the color appearance of surfaces in the image.

9. As noted in note 7, because of metamerism, there is no unique prediction for a color constant system. To compute $\Delta\hat{\mathbf{e}}$, we required that both $\mathbf{s}$ and $\Delta\hat{\mathbf{e}}$ lie within three-dimensional linear models of natural surfaces and illuminants. The actual illuminant changes were described by the linear model for illuminants used in the computations.

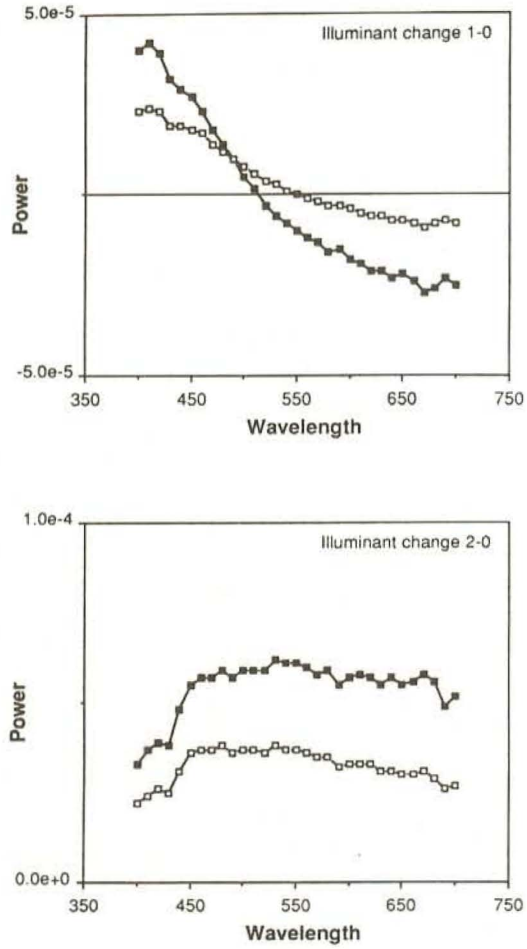

Fig. 13.10
Equivalent illuminant analysis. The closed symbols show two experimental illuminant changes. The open symbols show the corresponding equivalent illuminant changes. Because of metamerism, the computation of equivalent illuminant change from the data is not unique. We constrained our computations by requiring that both **s** and $\Delta\hat{\mathbf{e}}$ lie within three-dimensional linear models of natural surfaces and illuminants. (Prototype color signal R6_4; subject D.B.)

As we discussed in the description of our method, however, the laboratory model used in the experiments does not incorporate many aspects of natural viewing. This means that there is reason for caution in intepreting our results in terms of human performance in the real world. Because the images were presented using a CRT, they were necessarily small and of low luminance. Both of these factors may have had an effect on the magnitude of the measured adjustment, as may have the fact that the images were simulations. And it is possible that the visual system normally relies on image cues not incorporated

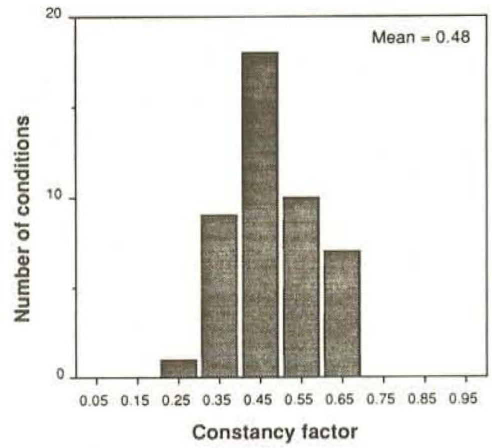

Fig. 13.11
Degree of color constancy. Histogram of the scale factor relating the actual illuminant change to the equivalent illuminant change for all 45 measurement conditions.

in our laboratory model to estimate the illuminant (see D'Zmura & Lennie, 1986; Funt & Drew, 1988; Funt & Ho, 1988; Klinker, Shafer & Kanade, 1988; Lee, 1986). One of the attractive features of the method developed here is that it can be extended to more complex laboratory models. It is possible to vary aspects of the laboratory model to see which ones have an influence on the visual system's adjustment and which ones do not.

Although there is some chance that magnitude of the visual system's adjustment to illuminant changes may depend on the laboratory model, we think it is useful to entertain the hypothesis that the bilinear nature of this adjustment, shown by the present experiments, is a consequence of the mechanisms applied by the visual system to adjust for the illuminant. While incorporating additional factors into the laboratory model may affect the parametric operation of these mechanisms, the basic structure of the adjustment may well remain bilinear.

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