
Vision, Imaging Science and Technology Activities
(VISTA Lab)
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Stanford Center for Cognitive and Neurobiological Imaging (CNI)
Department of Psychology
Stanford University


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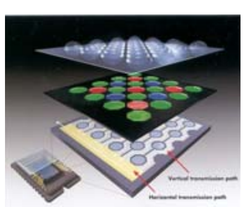
Entrance to Stanford

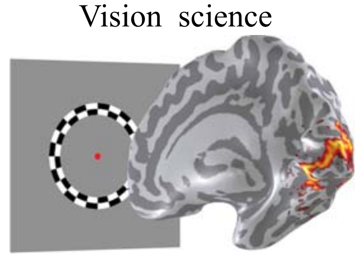
VISTA Lab, CNI, Psychology

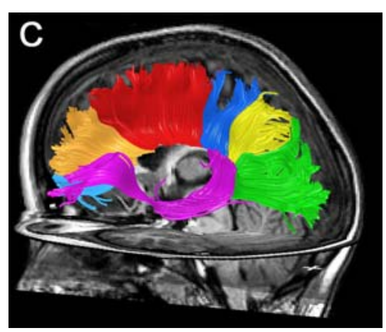
VISTA Lab Research

Image System Engineering



Vision science





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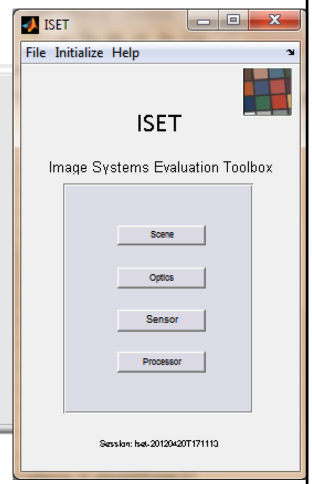
t_ImageFormation.m

Introduction to optics computing

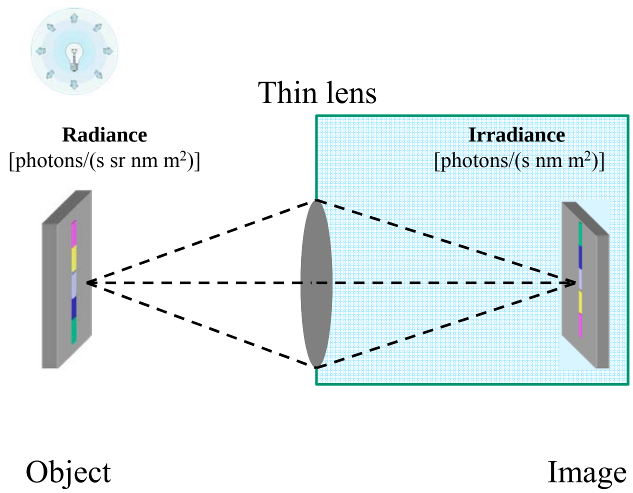


Lenses map scene points to image points

- Pinhole apertures and diffraction
- Thin lenses
- Lens-maker equation
- Depth of focus

Lenses map scene radiance to image irradiance

- A point in object space should map to a unique point in image space
- Capture as many photons from the object space as possible



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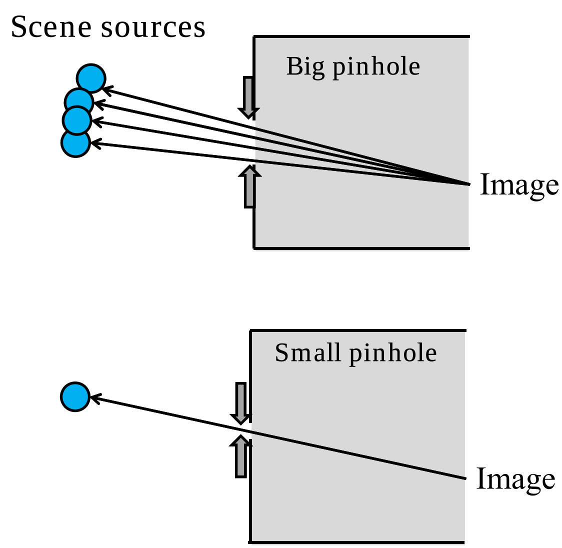
Pinholes and diffraction

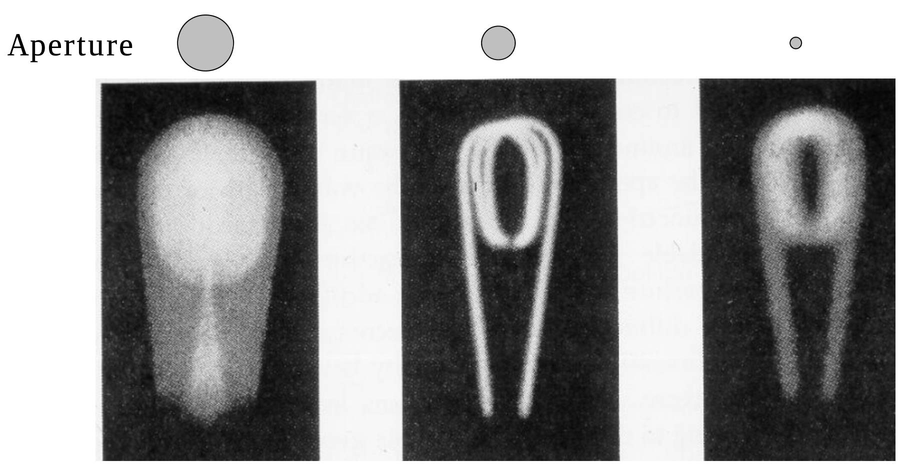
Pinhole optics

- Pinhole optics achieve the point to point mapping by limiting which scene points contribute to an image point
- Small pinholes make sharper images, but fail to capture many photons



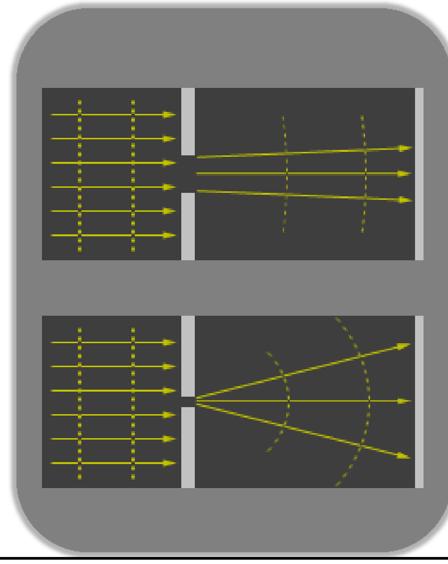
Diffraction limits image sharpness as the pinhole becomes small

(From Jenkins and White)



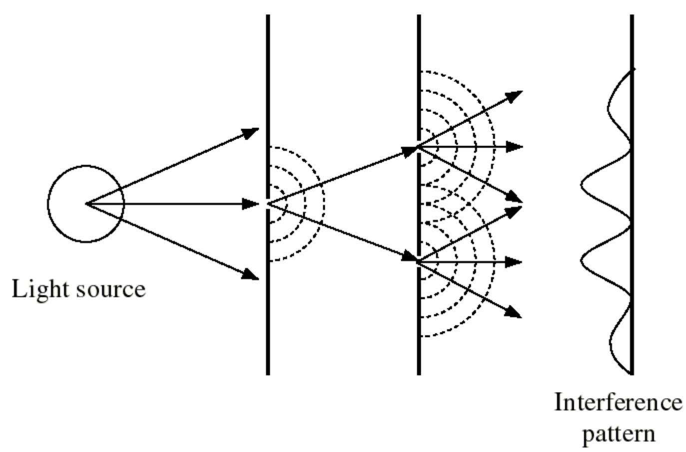
Diffraction: Rays

- Parallel rays incident on an aperture, say rays from a point source at infinity, begin to diverge.
- The smaller the aperture, the larger the divergence.
- This can be explained if we consider light as a wave phenomenon



Light as a wave phenome

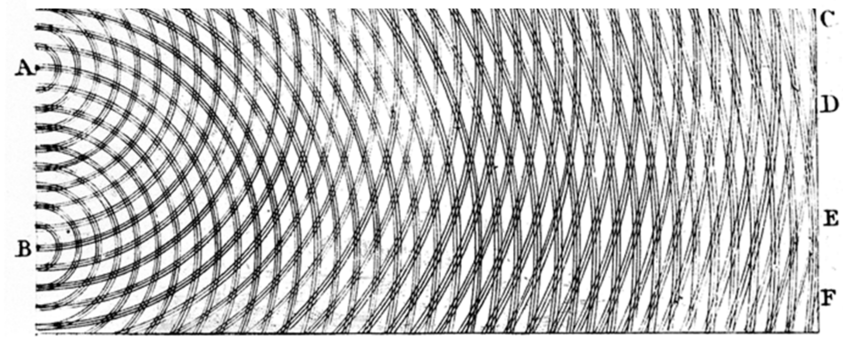
Thomas Young's double-slit experiments



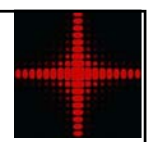
Light as a wave phenome
Thomas Young’s double-slit experiments

Thomas Young's sketch of two-slit diffraction, which he presented to the Royal Society in 1803

http://en.wikipedia.org/wiki/Diffraction



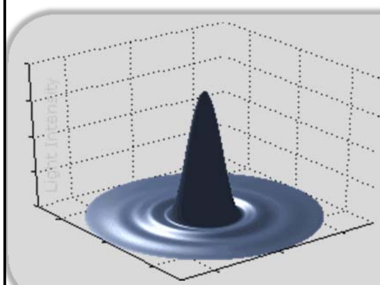
Diffraction in space domain



Square aperture

Diffraction is the explanation for why
an object point-source spreads out to form a finite image spot

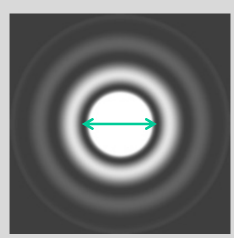
For an optical system with circular aperture
the finite image spot forms an Airy Disk (Point Spread Function)



$$PSF(x) = \frac{[2 \cdot J_1(u)]^2}{u^2}$$

$$\text{with} \quad u = \frac{\pi \cdot x}{\lambda \cdot f/\#}$$

$$\text{and} \quad x_{cuttoff} = 1.22 \cdot \lambda \cdot f/\#$$



Airy disk diameter: 2.44 λ f/#

The diffraction pattern formula for a disk can be calculated from first principles

$$I(\upsilon) = I_0 \left(\frac{2J_1(\upsilon)}{\upsilon} \right)^2$$

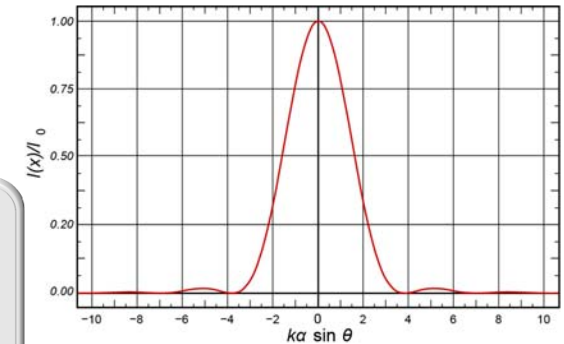
$$\upsilon = 2\pi \left(\frac{NA}{M\lambda} \right) r_i$$

$J_{1:}$ First order Bessel

NA: Numerical aperture

M: Magnification

$r_{i:}$ Radial distance



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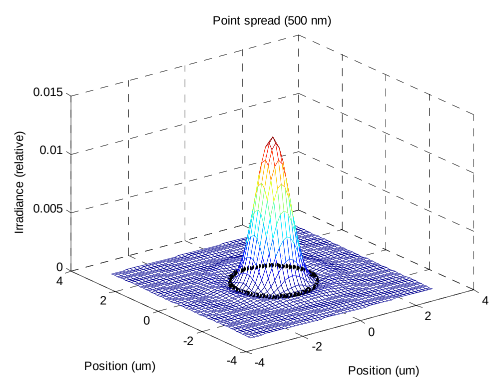


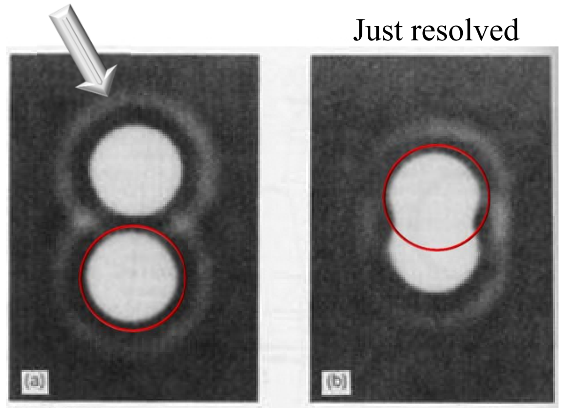
ISET tutorial:
t_airyDisk

Angular resolution criterion for diffraction

- **Rayleigh criterion** is a measure of spatial resolution
- Two point sources are “just resolved” when the diffraction maximum of one image coincides with the first minimum of the other

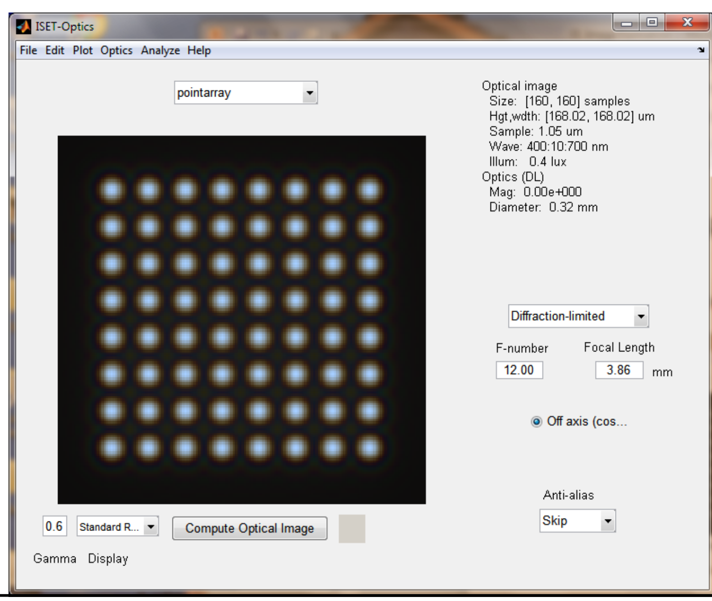
Airy disk

Just resolved



Computing: Diffraction limited demo

t_opticsDiffraction.m

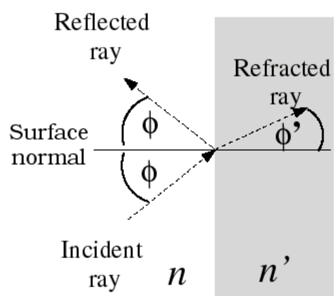


Snell's Law

Willebrord Snellius

Lens Design: Snell's Law

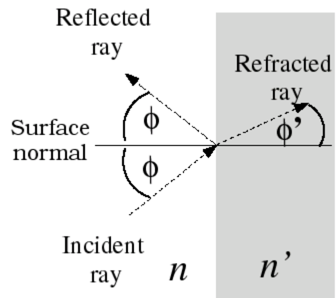
(a)



$$\frac{\sin(\phi)}{\sin(\phi')} = \frac{n'}{n}$$

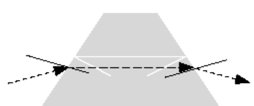
Lens Design: Snell's Law

(a)



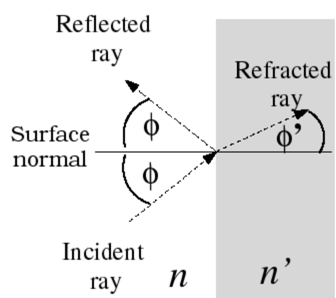
$$\frac{\sin(\phi)}{\sin(\phi')} = \frac{n'}{n}$$

(b)



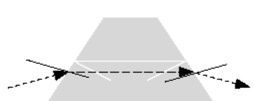
Lens Design: Snell's Law

(a)

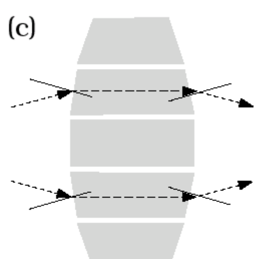


$$\frac{\sin(\phi)}{\sin(\phi')} = \frac{n'}{n}$$

(b)



(c)



Formula: Converting radiance to irradiance

- Irradiance formula

$$I = \frac{\pi}{1 + 4(f\#)^2 (1 + |m|)^2} R$$

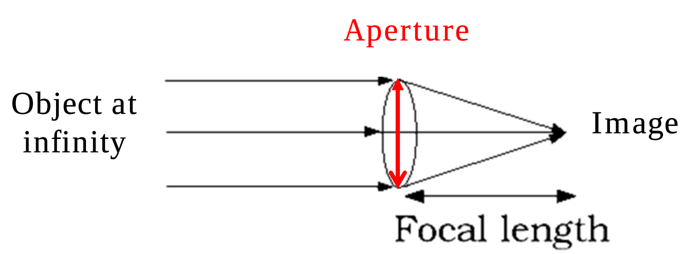
R = *radiance*
$f\#$ = *f-number*
m = *magnification*

- ISET code snippet

```
% This is the basic radiance to irradiance calculation
%
scene = sceneCreate;          % Default scene
oi = oiCreate;                % Diffraction limited optics
oi = oiCompute(scene,oi); % Do the calculation here
vcAddAndSelectObject(oi); oiWindow; % Show the result
```

Thin lens characterization

- Thin lens can be characterized by its aperture and focal length, both in meters

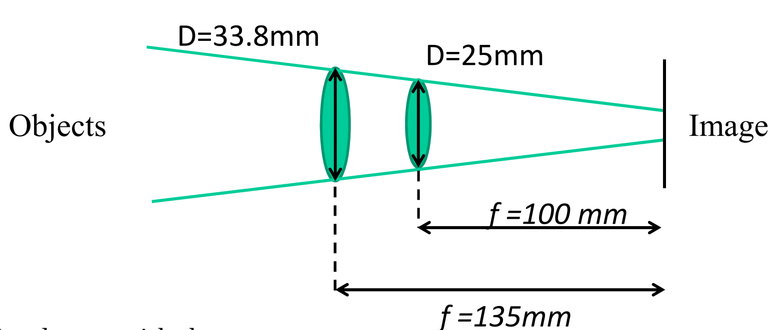


F/# summarizes light sensitivity (speed)

- For many purposes, a thin lens can be summarized by a single number, the ratio of the aperture diameter to focal length, or **F-number** (f/#)
- A value of f/4, for example, means the aperture diameter is 1/4 the focal length of the lens

$$f/\# = \frac{focal\ length}{aperture\ diameter}$$

F/# summarizes light sensitivity (speed)

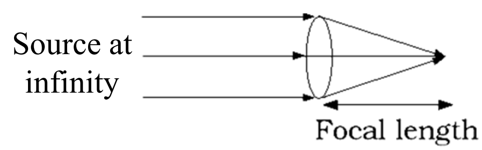


Two lenses with the same f/# (f/4 in this case) produce the same image luminance

Lensmaker's Equation

$$\frac{1}{d_s} + \frac{1}{d_i} = \frac{1}{f}$$

d_s = *source dist*
d_i = *image dist*
f = *focal length*



Lens power (diopters) = 1/(Focal length(m))

Lensmaker's Equation

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d_s = *source dist*
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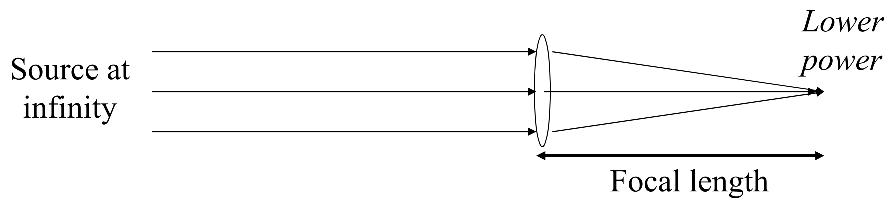

Lens power (diopters) = 1/(Focal length(m))

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ISET: s_opticsDepthDefocus

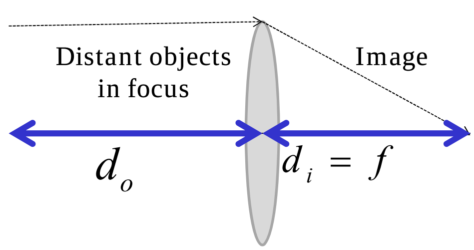
Depth of Field

ISET: s_opticsDepthDefocus

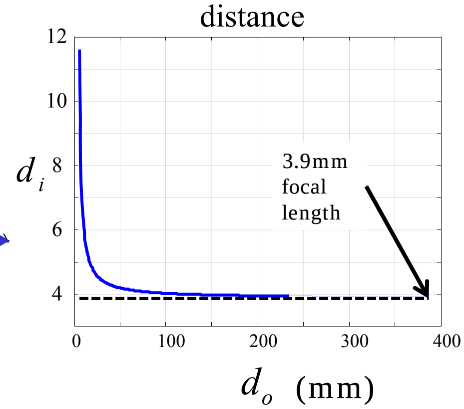
3D Scenes: Depth and Defocus

Lensmaker's equation

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$$



Distance to image plane depends on object distance



ISET: s_opticsDepthDefocus

3D Scenes: Depth and Defocus

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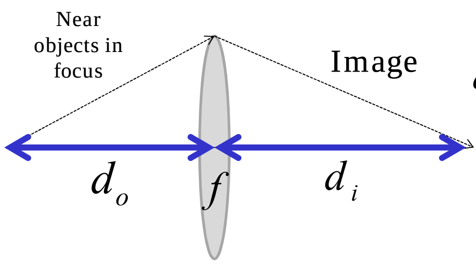
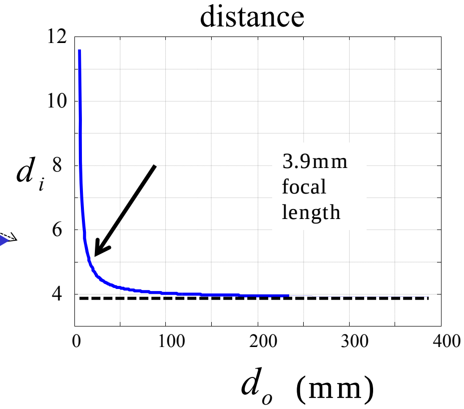


Distance to image plane depends on object distance



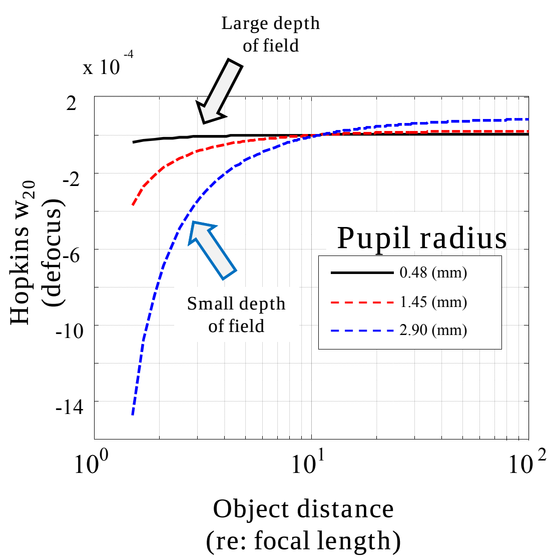
ISET: s_opticsDepthDefocus

Depth of Field

Depth of field is the range of distances with good focus

DOF depends on both
- Object distance
- Pupil radius

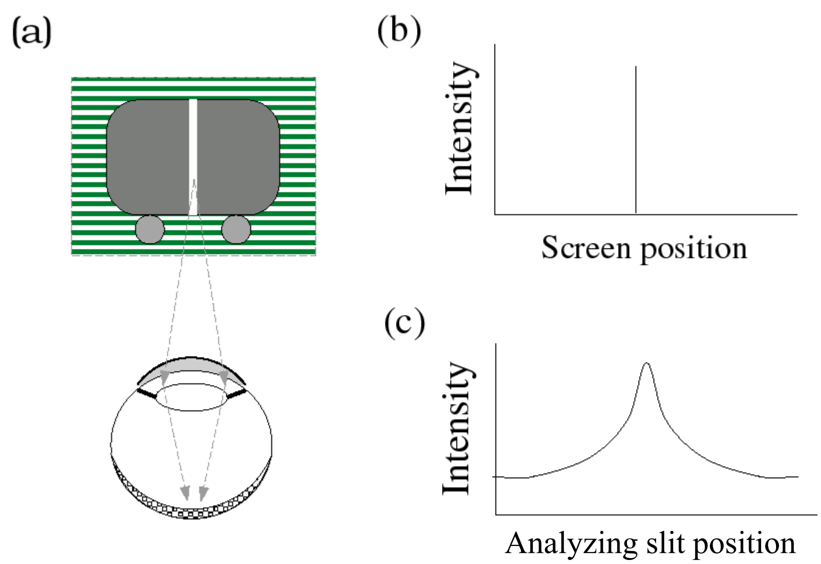
Small radius larger depth of field - and less light



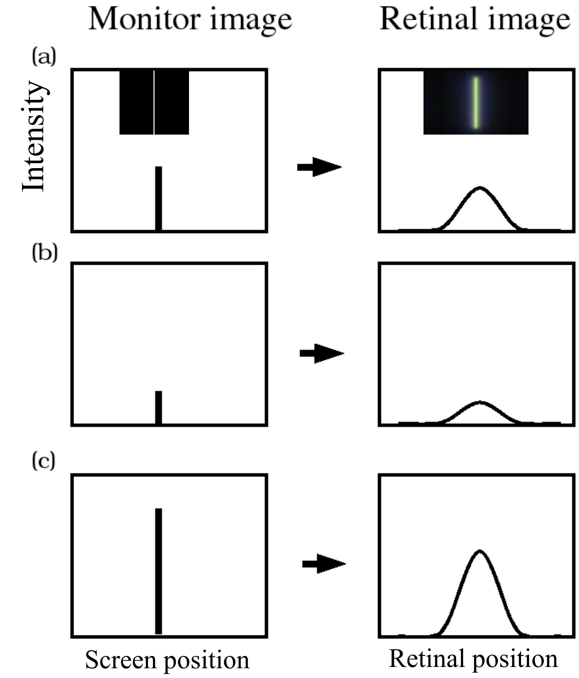
Linear systems and optics

- Linespread function
- Shift-invariant linear systems
- Harmonics and the modulation transfer function

The Linespread Function
A summary of optical quality

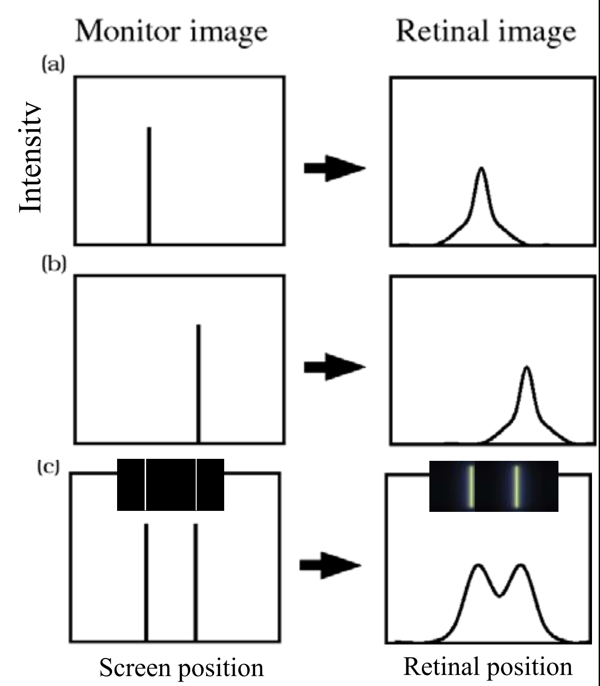


Thin lenses and many typical optical systems satisfy the linear system principle of homogeneity



Graphs for one wavelength

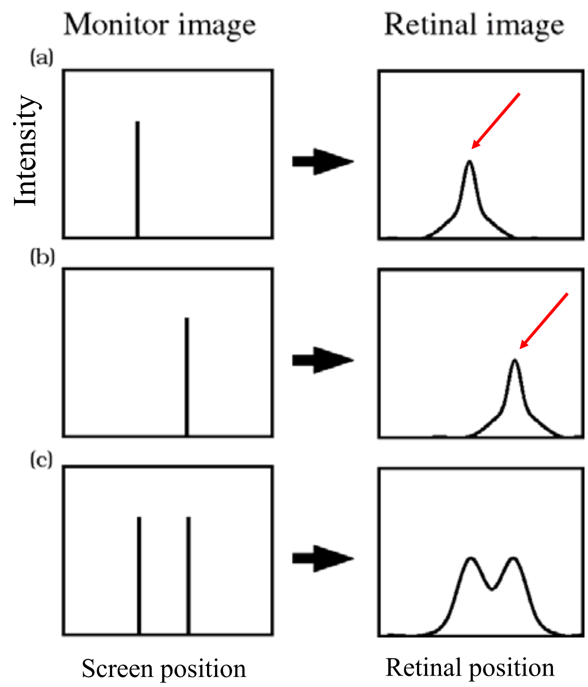
Many typical optical systems are linear systems and thus also satisfy superposition



Graphs for one wavelength

Some linear systems also satisfy shift-invariance; Many satisfy it locally

Isoplanatic *is the term used in optics for a shift-invariant region of the lens; most lenses are only locally shift-invariant*



Inferred line spread (Westheimer)

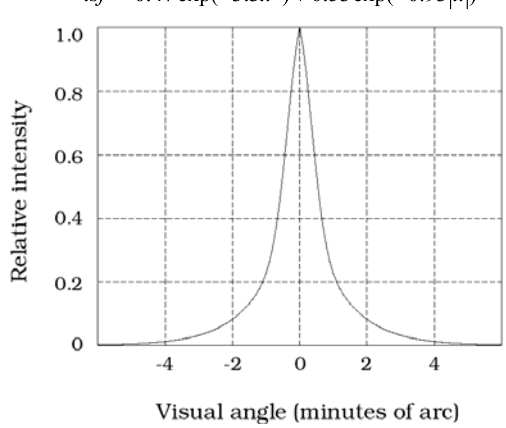
ISET: westheimerLSF

```
% Open window
vcNewGraphWin;

% set spatial samples
xSec = -300:300;

% Plot the line spread
plot(xSec,westheimerLSF(xSec));
grid on
xlabel('Position (arc sec)');
ylabel('Relative intensity');
```

$$lsf = 0.47\exp(-3.3x^2) + 0.53\exp(-0.93|x|)$$



Fourier series

- Linear models
- 1D Fourier Series
- 2D Fourier Series

Linear models

Most branches of science use linear models to summarize and analyze their data

$$S(t) = \sum_{i=1}^{i=N} w_i B_i(t)$$

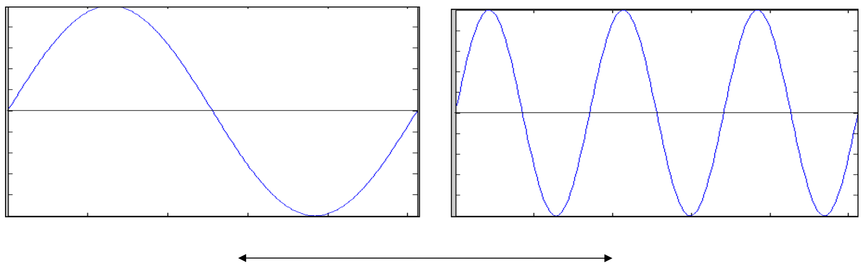
Signal you aim to describe — Weights — Basis functions

Functional MRI uses the general linear model (GLM) routinely for analyzing time series

The Fourier series is a linear model

$$S(x) = M + \sum_f a_f \sin(2\pi fx + \phi)$$

$$= M + \sum_f u_f \sin(2\pi fx) + v_f \cos(2\pi fx)$$



Space

a – Amplitude of the harmonic

f – Frequency of the harmonic

ϕ – Phase of the harmonic

The Fourier series and linear systems

Fourier Series

$$S(t) = \frac{u_0}{2} + \sum_{f=1}^{N} (u_f \sin(2\pi ft) + v_f \cos(2\pi ft))$$

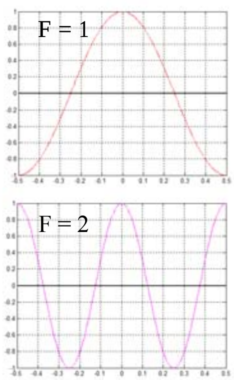
It is very surprising that any function, S(t), can be expressed as the sum of harmonics

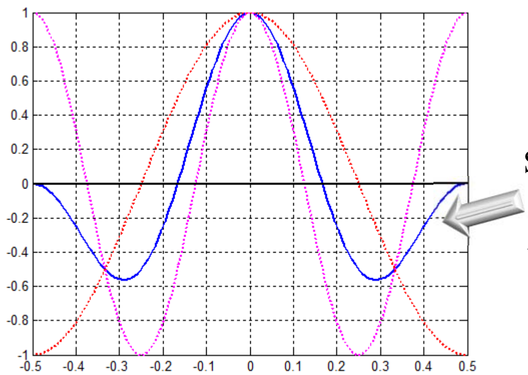
Jean Baptiste Joseph Fourier

A French mathematician and physicist best known for describing the **Fourier series** and their application to problems of heat transfer. The **Fourier transform** and Fourier's Law are also named in his honor. Fourier is also generally credited with the discovery of the greenhouse effect [1]. (Wikipedia)

How can harmonics sum to an impulse?

Summing across frequencies, cosinusoids add at the origin and cancel at all other positions



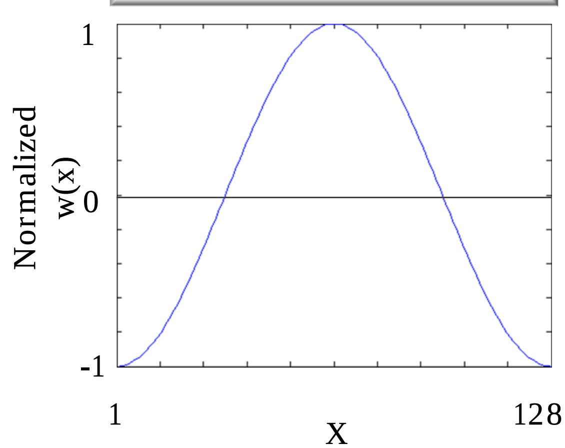


Sum of the first two harmonics

How can harmonics sum to an impulse?

$$w(x) = \left(\frac{1}{N_f}\right)\sum_{f=1}^{f=128}\cos(2\pi fx)$$

At each step, we normalize by the number of frequency terms, N_f



Images and Fourier series

- Image contrast
- Transform space

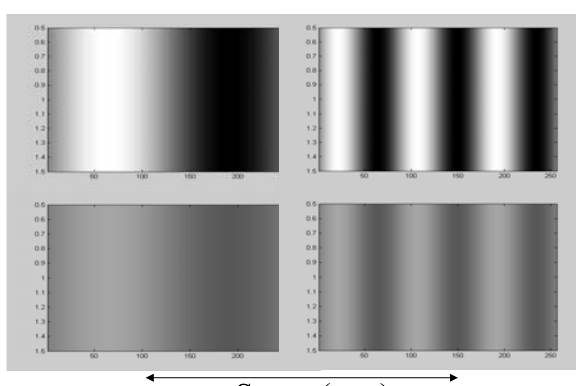
Images, harmonics, and contrast

- Light intensity is always > 0, image harmonics are specified in contrast – variations around the mean
- The contrast harmonic parameters are
  contrast : $-1 < a < 1$
  (dimensionless)
  frequency: f
  (cycles per unit distance)
  phase: ϕ
  (shift in position)

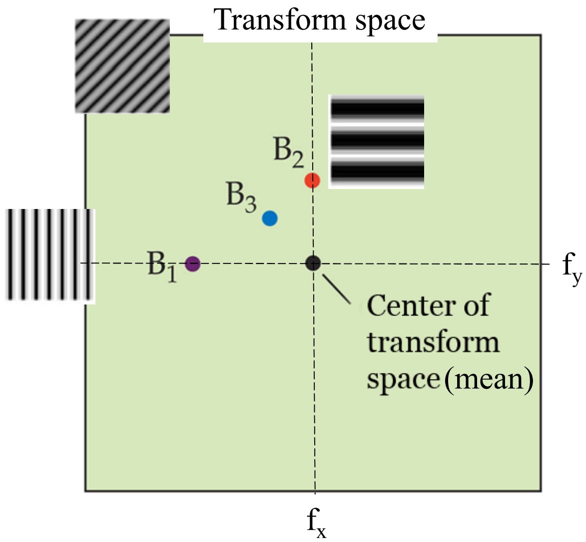
Fourier: Any image can be expressed as a weighted and shifted sum of harmonics

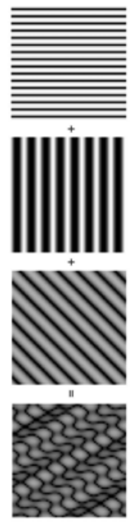
$$c(x) = M(1 + a \ \sin(2\pi fx - \phi))$$

M = mean level, a = contrast, f = spatial frequency



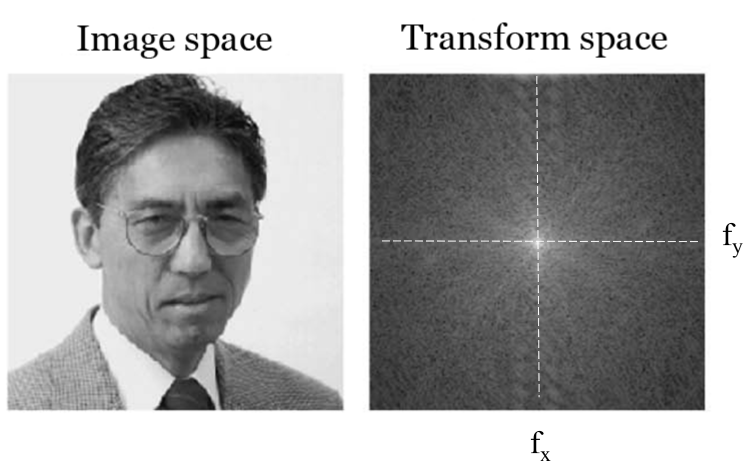
Images can be represented as Fourier series using 2D harmonics





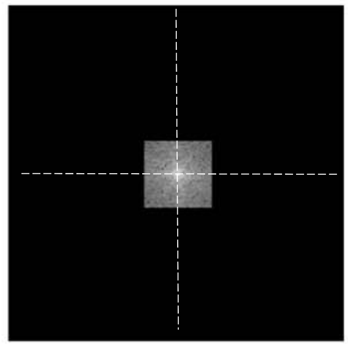
Functional Magnetic Resonance Imaging, Huettel et al. © Sinauer, 2004, 2009

How different parts of Transform space contribute to Image space



Functional Magnetic Resonance Imaging, Huettel et al. © Sinauer, 2004, 2009

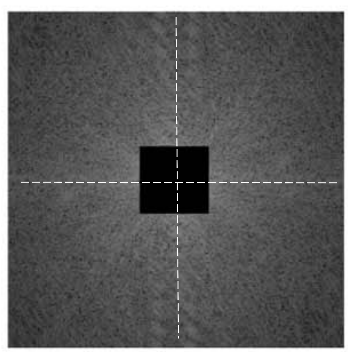
How different parts of Transform space contribute to Image space



The center of Transform space contains **low spatial frequency** information

Functional Magnetic Resonance Imaging, Huettel et al. © Sinauer, 2004, 2009

How different parts of Transform space contribute to Image space



The outer portion of Transform space contains **high spatial frequency** information

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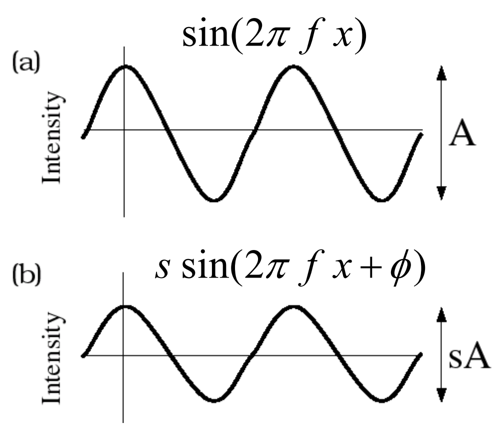
The modulation transfer function (MTF)

- Harmonics are nearly eigenfunctions of linear systems
- The MTF of the human optics
- Pointspread

Harmonics have a special role in shift-invariant linear systems

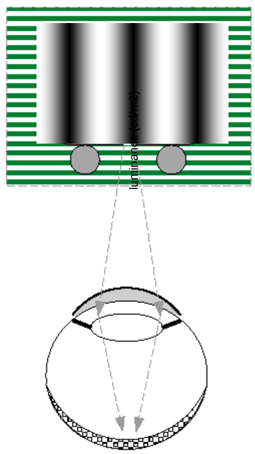
- Harmonics are (almost) eigenfunctions. They pass through at the same frequency, but scaled and shifted in phase.
- The true eigenfunctions are complex exponentials

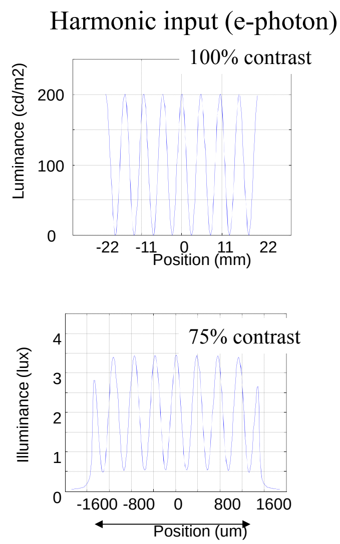
$$Ae^{i\phi} = a\cos(\phi) + ib\sin(\phi))$$



But – light is not negative
What to do?

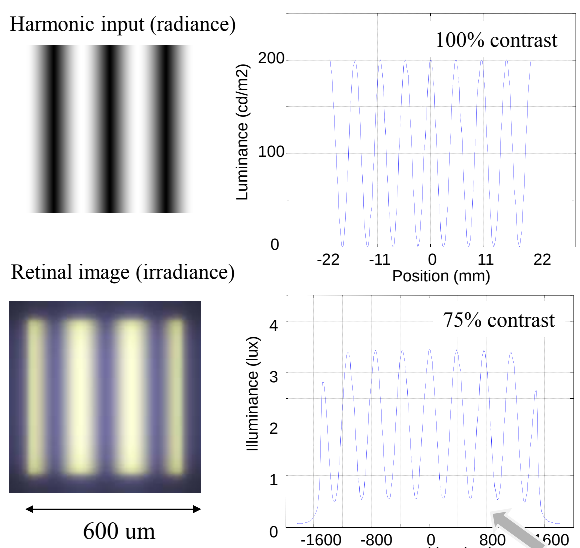
The Modulation Transfer Function:
The contrast reduction at harmonics





The Modulation Transfer Function:
The contrast reduction at harmonics

The contrast reduction varies with the spatial frequency and the wavelength

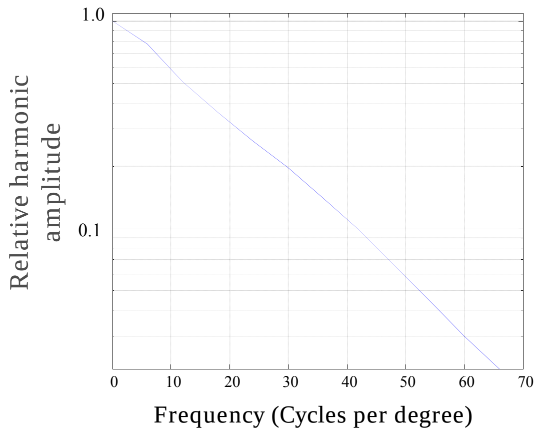


Modulation Transfer Function (MTF)

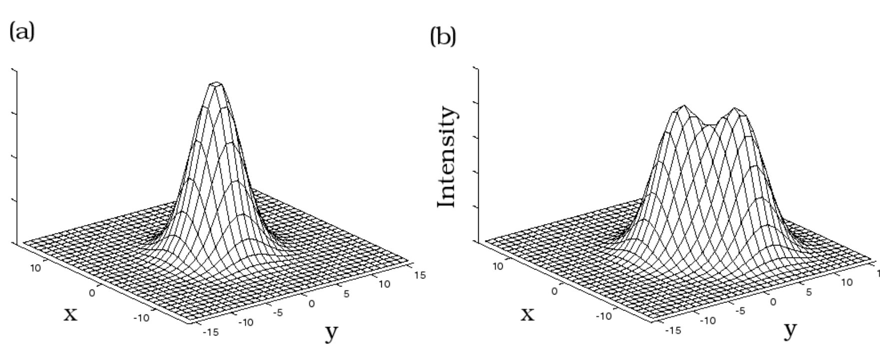
```
xSec = -300:300;    % 600 sec, total = 10 min
westheimerOTF = abs(fft(westheimerLSF(xSec)));

% One cycle spans 10 min of arc,
% so freq=1 corresponds to  6 c/deg

vcNewGraphWin;
freq = [0:11]*6;
semilogy(freq,westheimerOTF([1:12])); grid on;
xlabel('Freq (cpd)'); ylabel('Relative contrast');
set(gca,'ylim',[-.1 1.1])
```



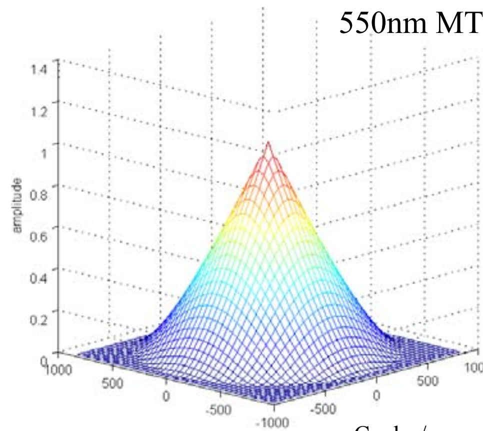
The point spread function generalizes the line spread



The diffraction pattern formula for a disk can be calculated from first principles

Diffraction limited OTF

550nm MTF



Cycles/mm

Cycles/mm

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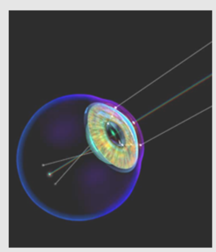
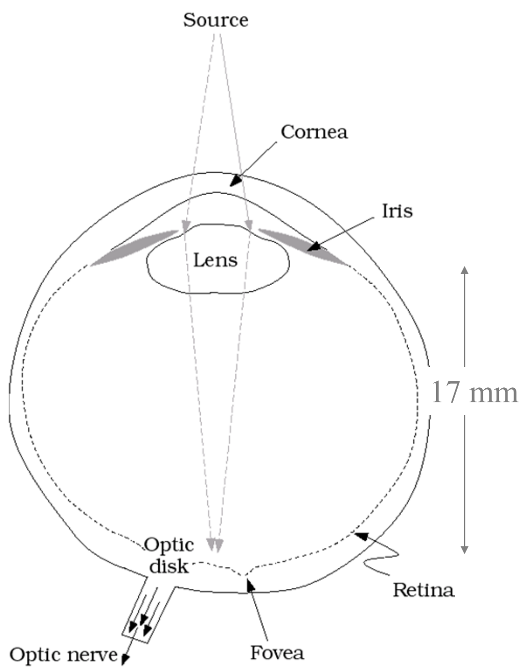
Human Image Formation

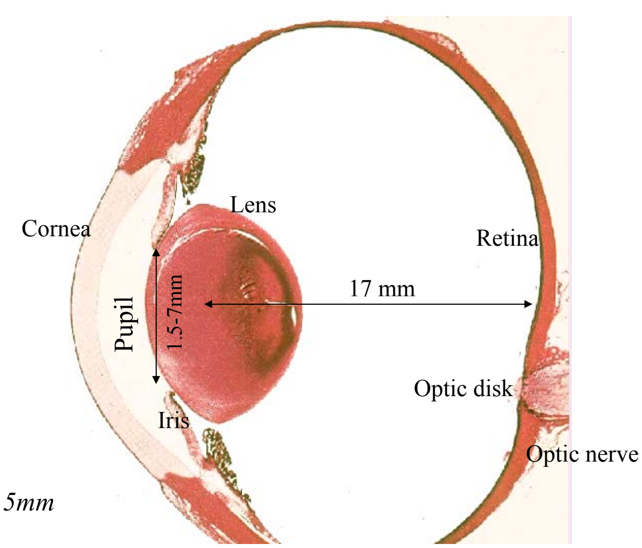
- Human eye and retina: anatomy
- Pupil size and lens accommodation
- Human Linespread and MTF
- Astigmatism



The Cornea and Lens Focus the Image on The Retina



Human Eye in Cross-Section



F-number ~ 2.4-11
Retinal thickness ~ 0.5mm

http://www.optics.rochester.edu/workgroups/cml/opt307/spr06/joe/index.htm

Lens Fiber Hook and Eye

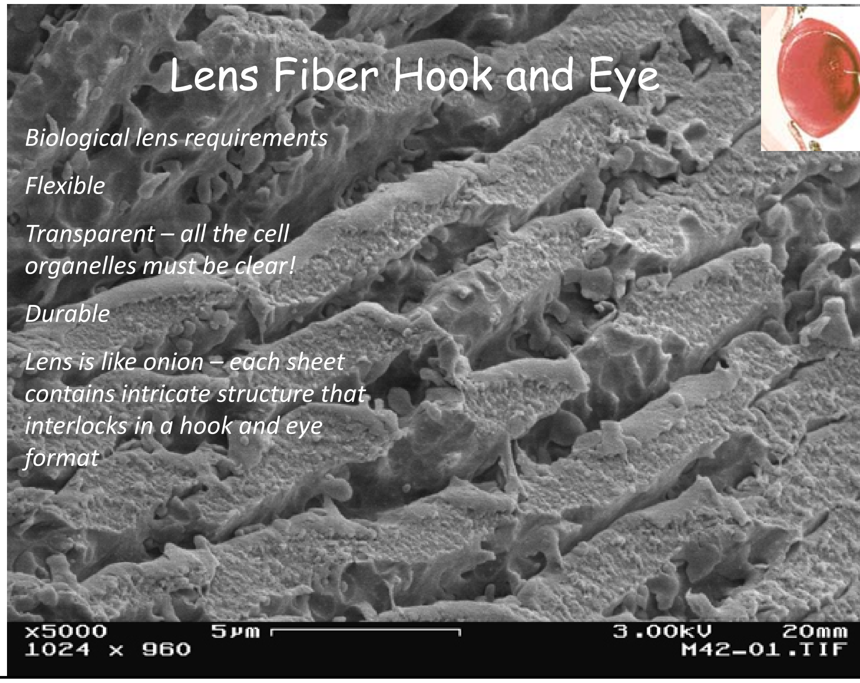
Biological lens requirements

Flexible

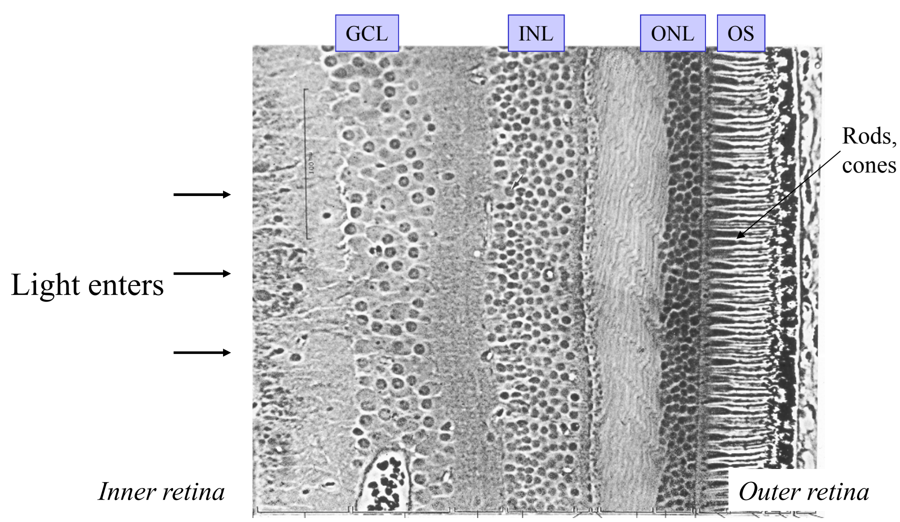
Transparent – all the cell organelles must be clear!

Durable

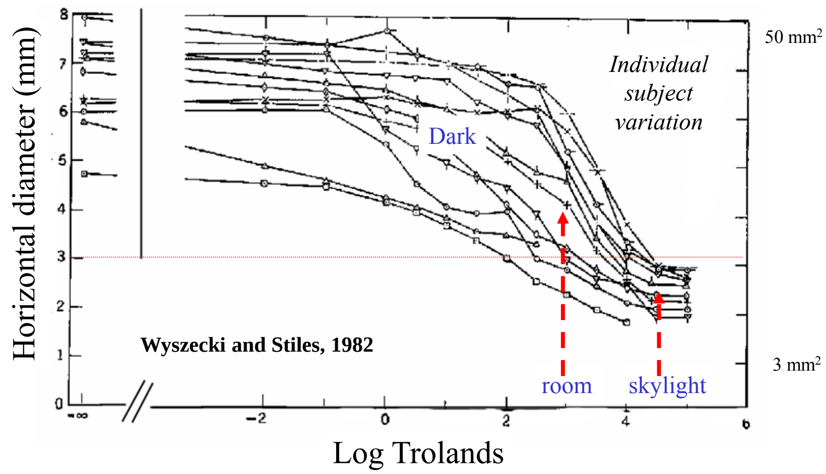
Lens is like onion – each sheet contains intricate structure that interlocks in a hook and eye format



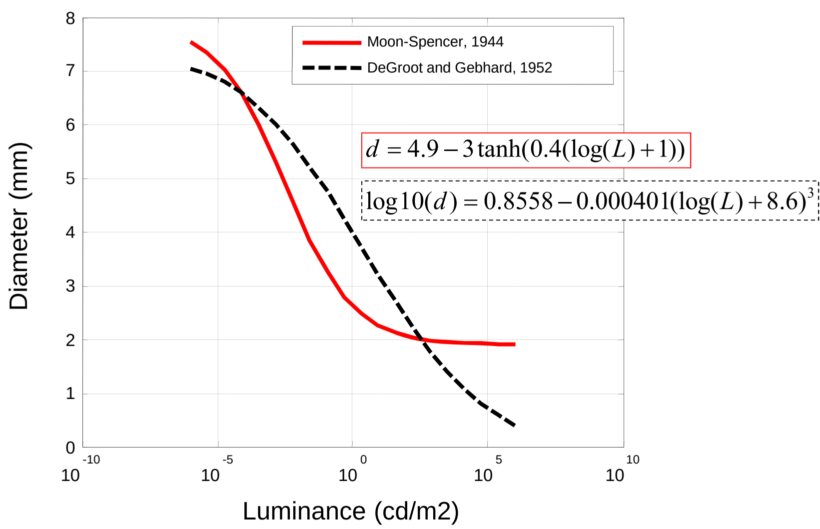
Light Passes Through The Retinal Cells And Is Absorbed by the Rods and Cones



Pupil diameters vary with light level



Pupil Size Changes, Influencing Retinal Illuminance and Acuity

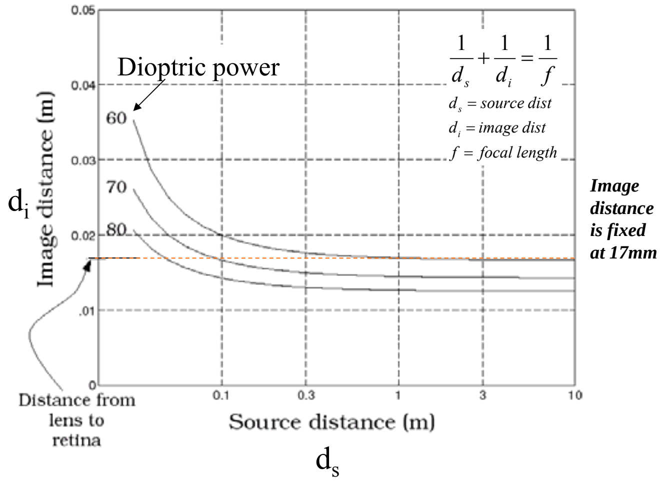


Human accommodation

When the source is distant, muscles in the eye transform the lens to lower power

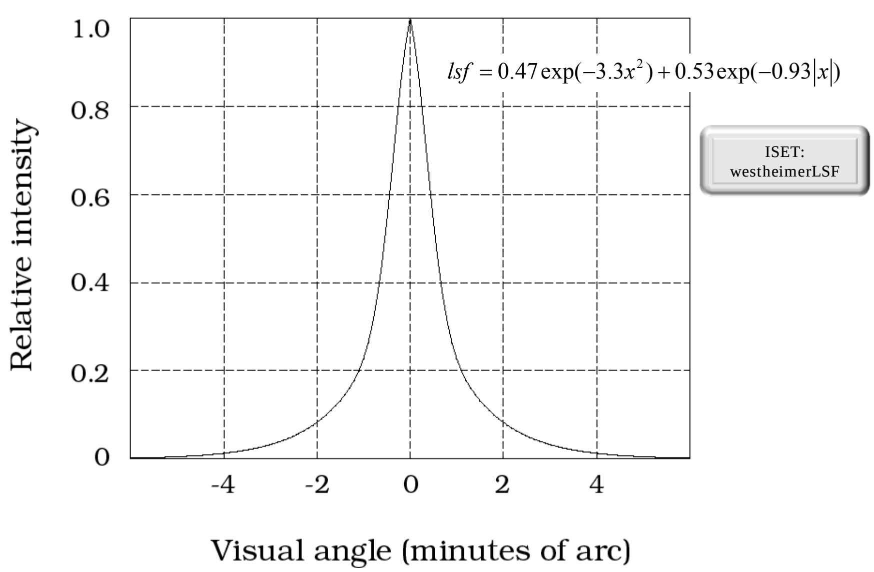
When the source is close, the muscles make the lens higher power

This process is called accommodation

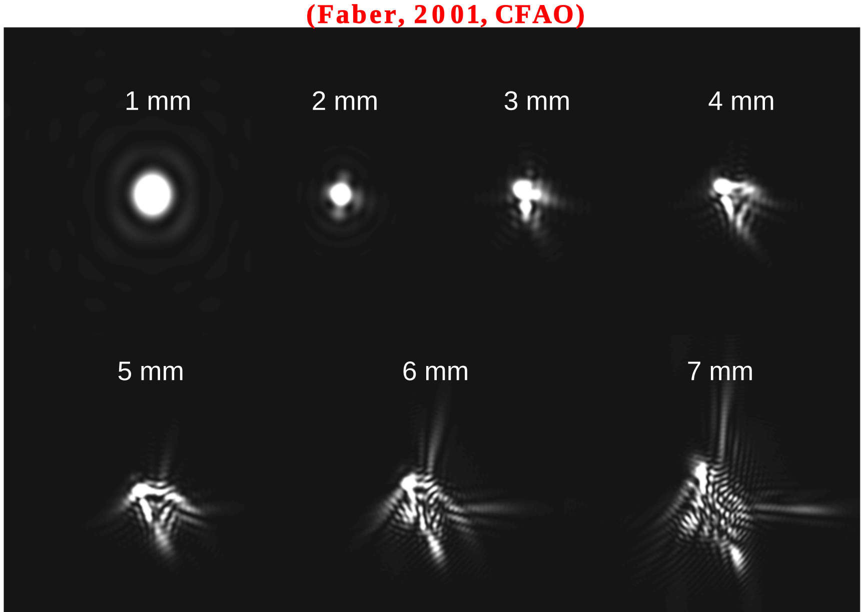


Inferred human line spread (Westheimer)

ISET: westheimerLSF

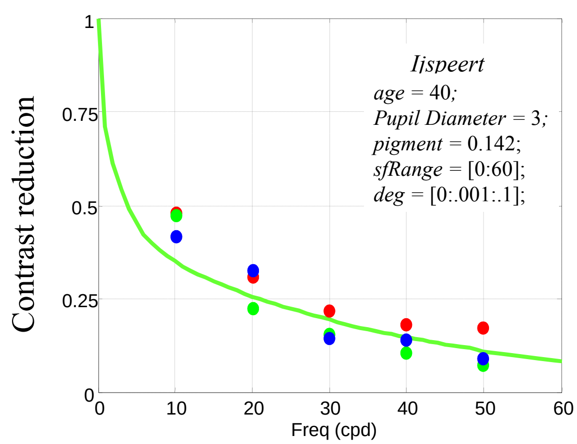


Visual Acuity Is Worse When Pupils Dilate (Faber, 2001, CFAO)

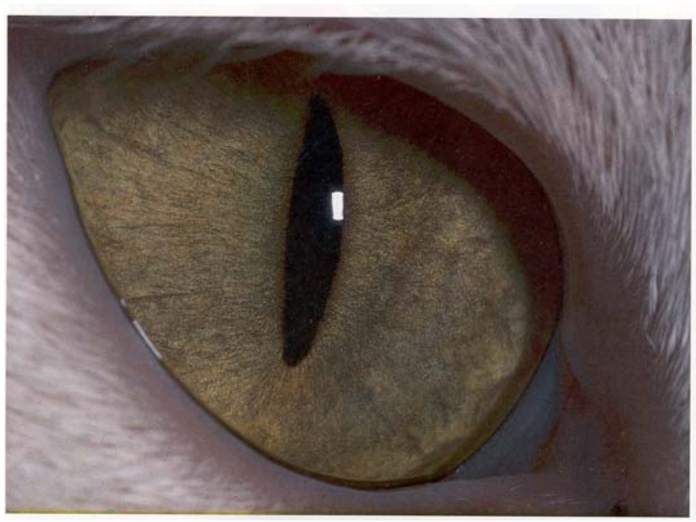


Estimated human pointspread functions

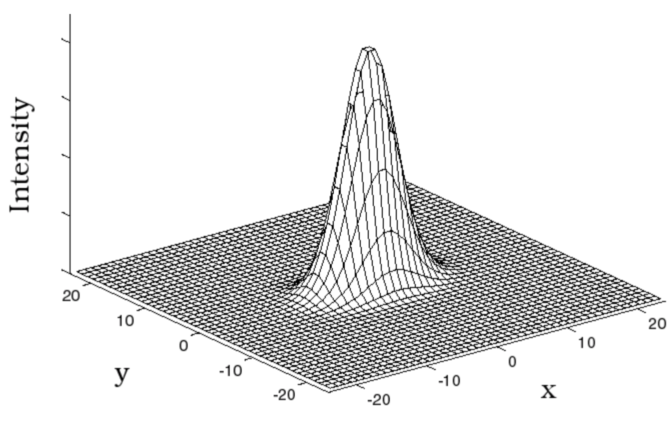
Theoretical modulation transfer function compared with data



Some animals have non-circular pupils: Cats will have asymmetric diffraction



Astigmatism measures the orientation of the point spread function



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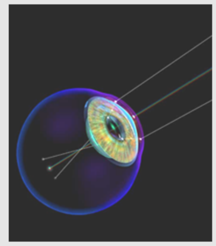
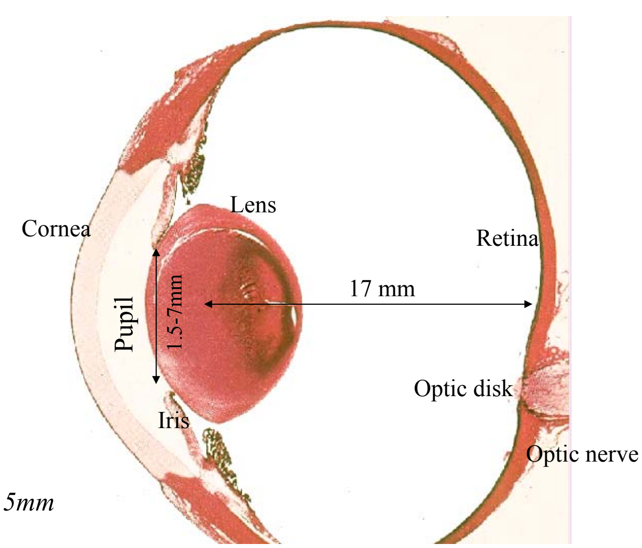
Human Chromatic Aberration

- Focus and wavelength
- Example images
- Dioptric power by wavelength
- MTF and linespread by wavelength

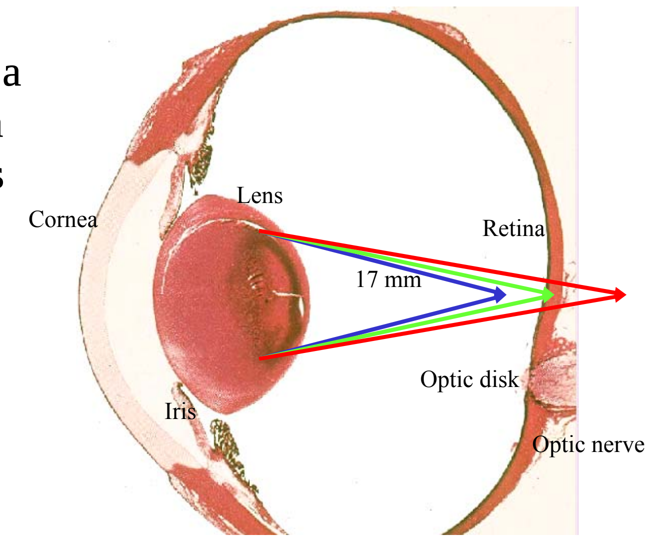


Human eye in cross-section

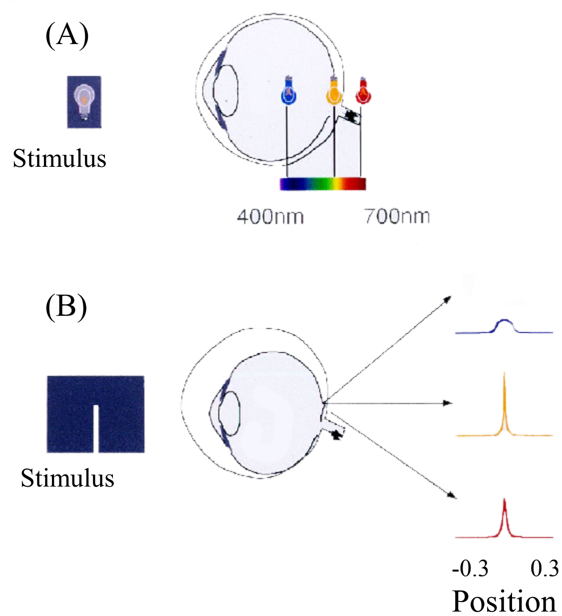


F-number ~ 2.4-11
Retinal thickness ~ 0.5mm

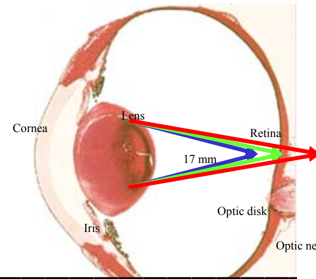
Chromatic aberration is a difference in optical focus across wavelength

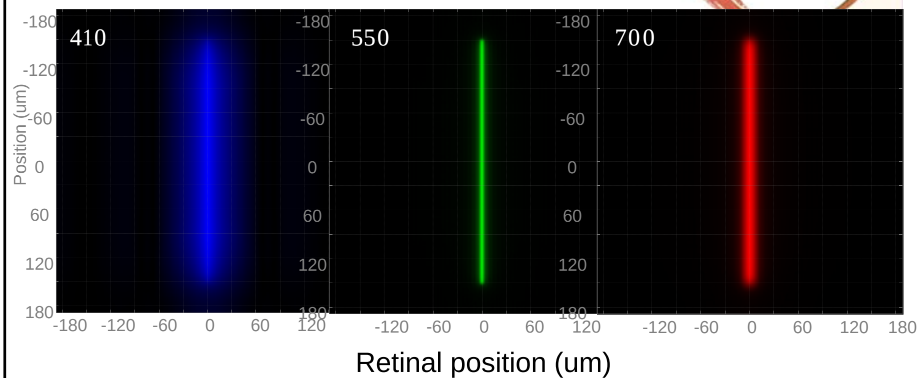


Chromatic aberration is a difference in optical focus for different wavelengths



Defocus differs by wavelength

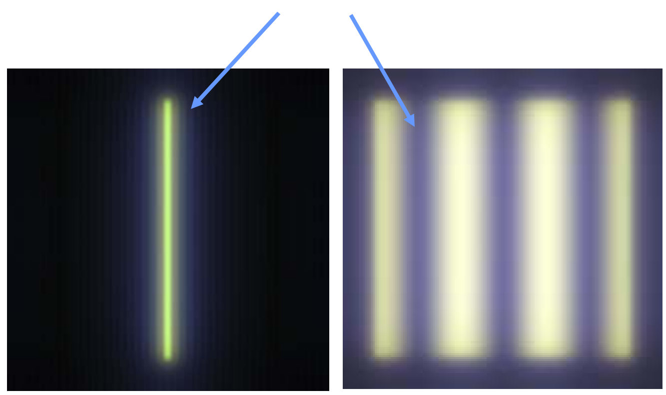




ISET: s_HumanLSF

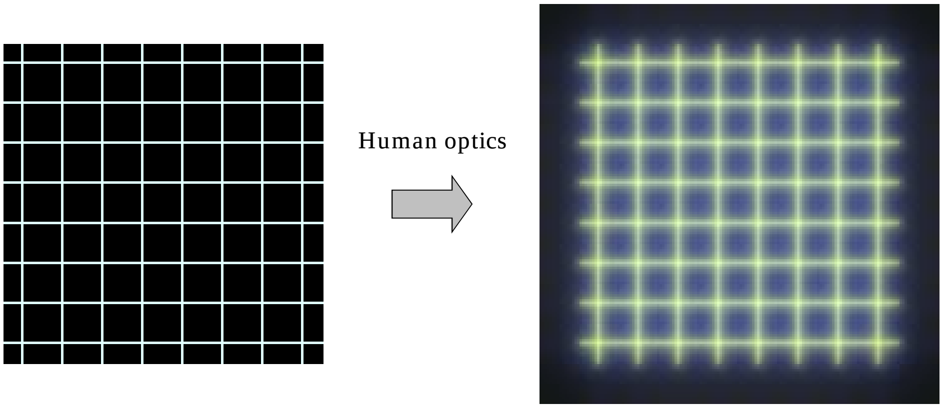
Example: Wavelength-dependent spread

Short-wavelength light spreads more

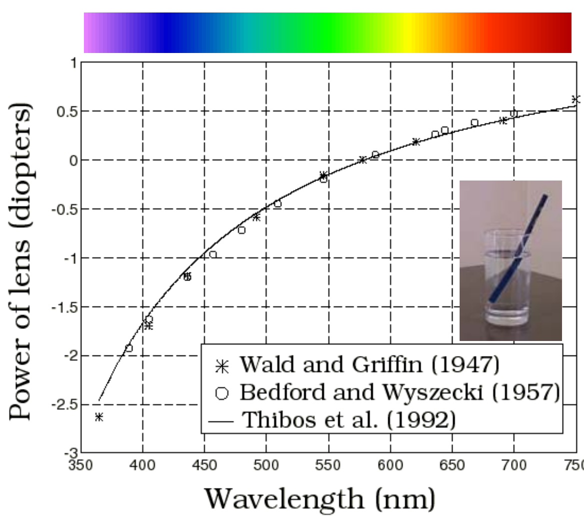


Example: Wavelength-dependent spread

Broadband radiance produces chromatic irradiance

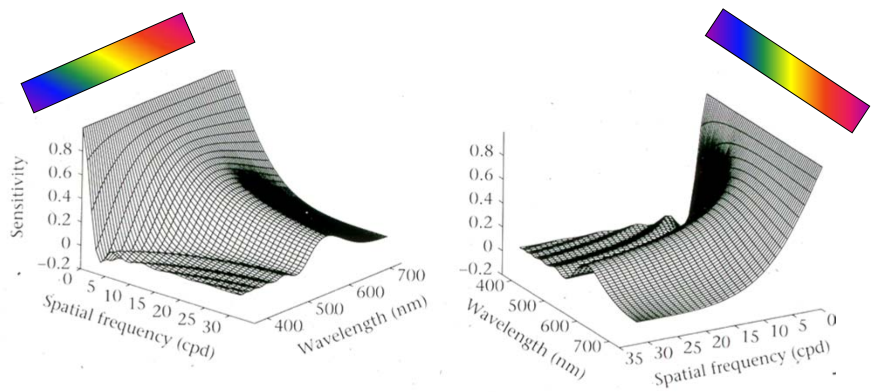


Chromatic aberration is summarized by variation in optical power across wavelength; very similar across people



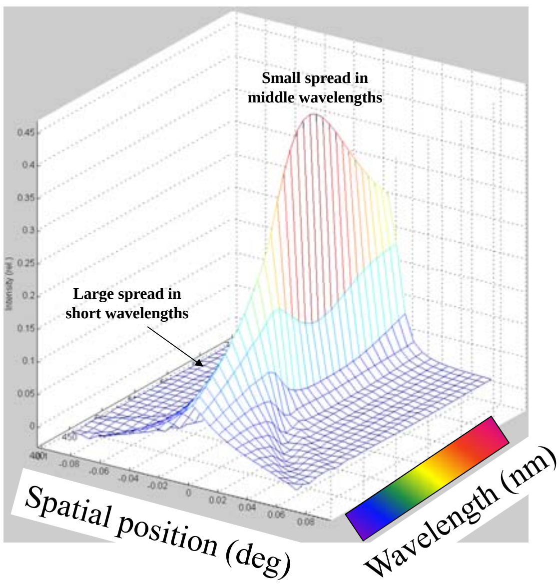
Chromatic MTF of the human optics

(Marimont and Wandell, J. Opt. Soc Am. A., 1994)



Chromatic aberration summarized by the line spread function

(ISET: humanLSF)



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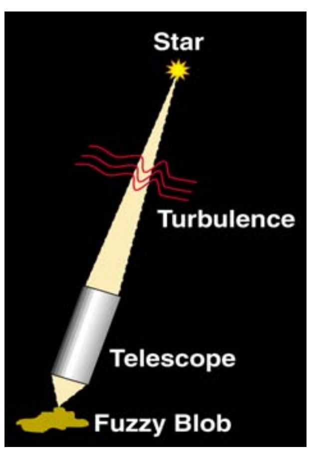
Adaptive Optics

Turbulence in the atmosphere limits the performance of astronomical telescopes

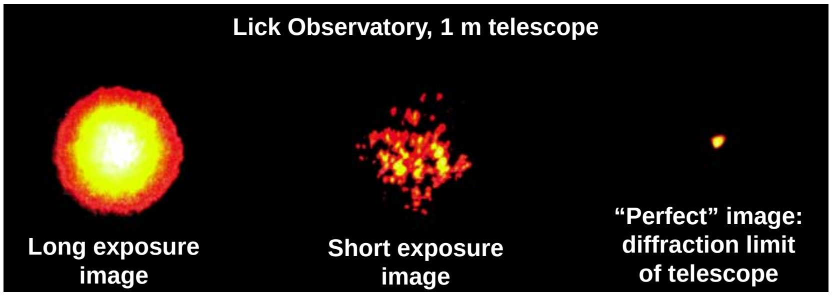
physics.gmu.edu/~hgeller/astr402/AOintro2001.ppt



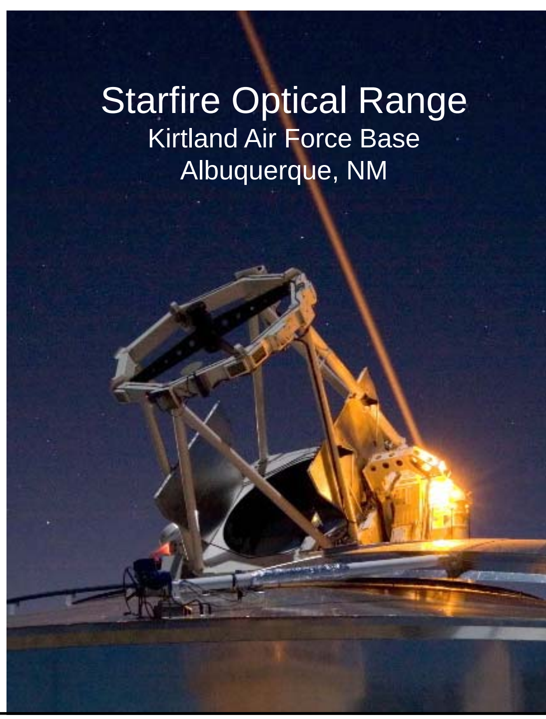
- **Turbulence is the reason why stars twinkle**
- **More important for astronomy, turbulence spreads out the light from a star; makes it a blob rather than a point**

Even the largest ground-based astronomical telescopes have no better resolution than an 8" backyard telescope!

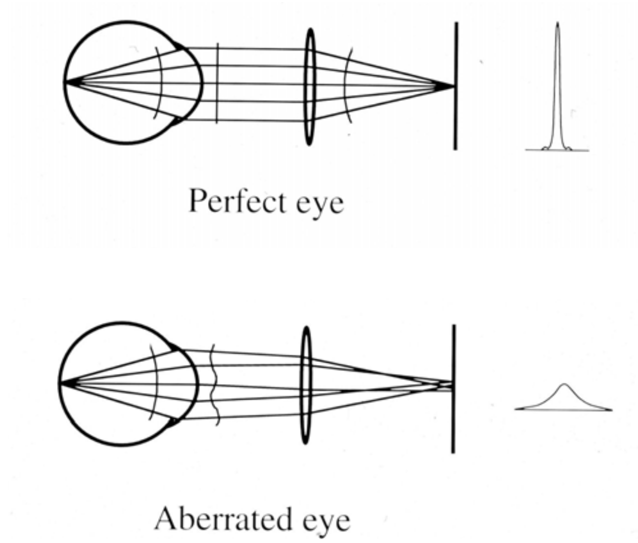
Images of a bright star, Arcturus



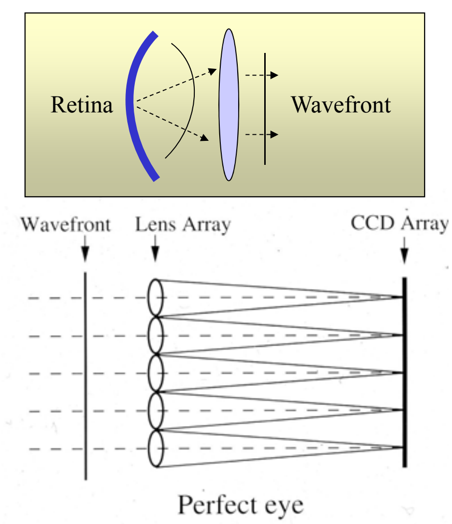
Distant stars should resemble "points,"
if it weren't for turbulence in Earth's atmosphere



Adaptive optics: Reaching the diffraction limit

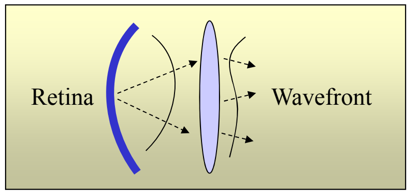


Hartmann-Shack Wavefront Sensor: Measures the optical path imperfections

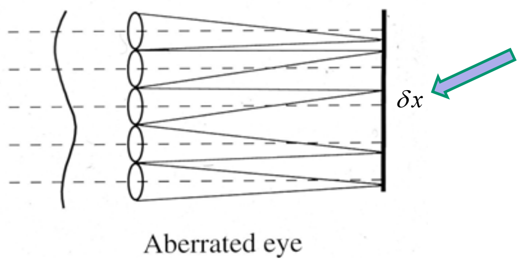


Put a lens here and converge the collimated wavefront to a point

Hartmann-Shack Wavefront Sensor
Measures the image imperfections



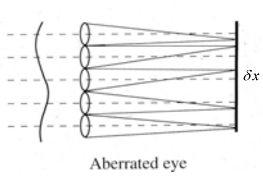
Your lens will not converge the wavefront well

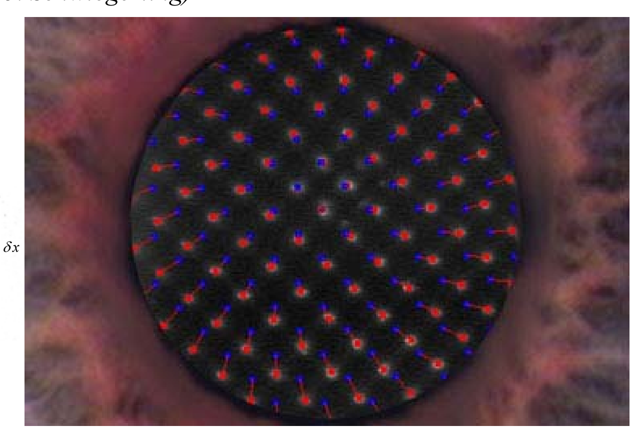


Displacement images at the CCD

Eye with wavefront scanning data superimposed. Data appears as red and blue dots over the eye's iris. Wavefront perturbations cause the red and blue dots to separate.

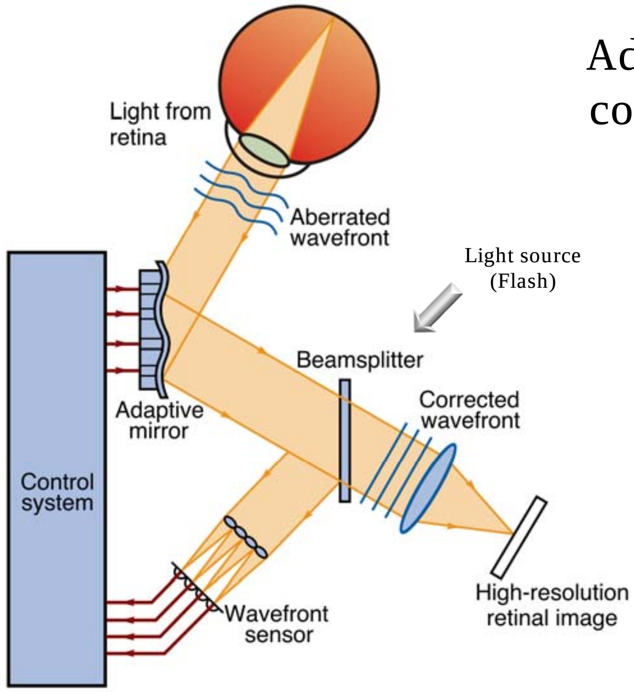
(Credit: J. Schwiegerling)





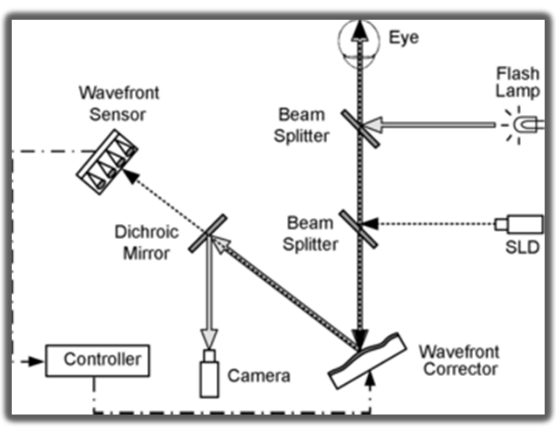
http://www.opticsreport.com/content/article.php?article_id=1005

Adaptive optics corrects retinal wavefront

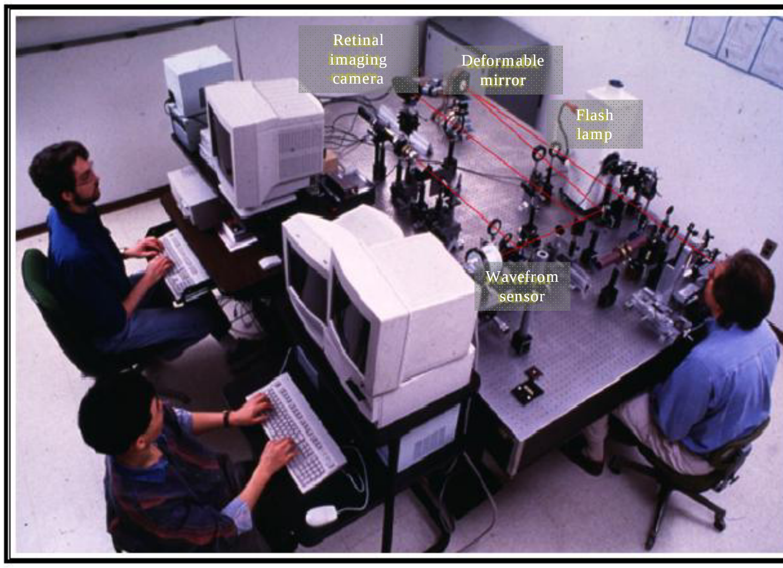


J. Carroll, D. Gray, A. Roorda, D. R. Williams (2005)

(B)



Rochester Adaptive Optics Ophthalmoscope



Adaptive Optics:
Point spread improvements

(Williams lab)

Human point spread

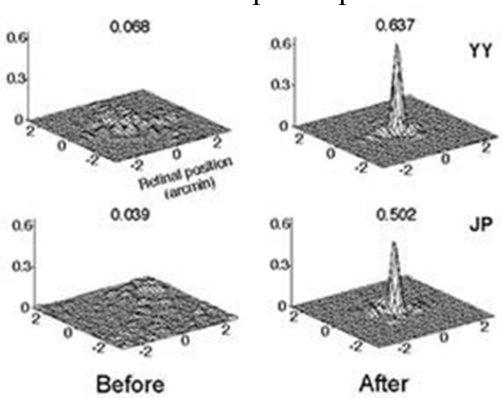
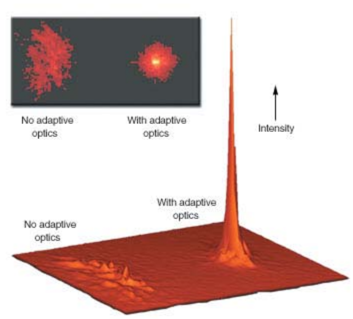


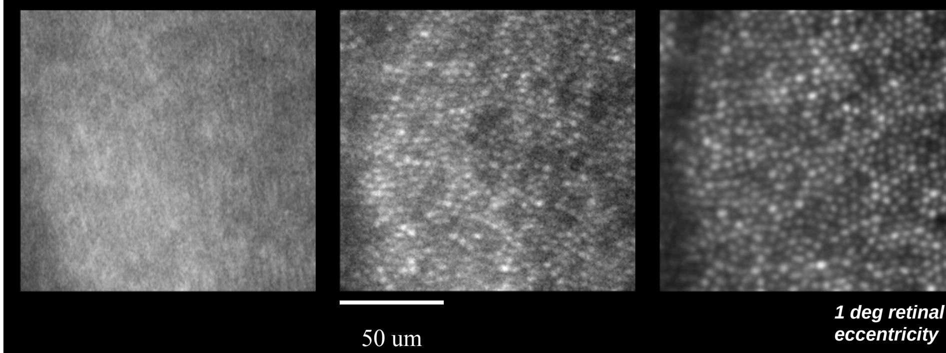
Figure 7b

Astronomy



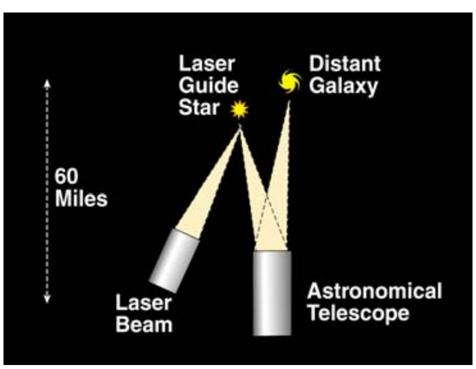
Adaptive optics allows cellular resolution *in vivo*

Without adaptive optics (single image)	With adaptive optics (single image)	With adaptive optics (many images)

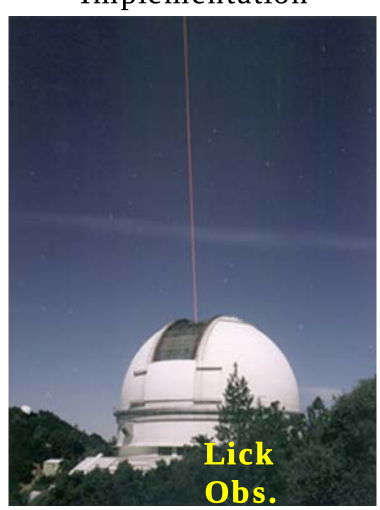


If there is no nearby star, make your own "star" using a laser

Concept

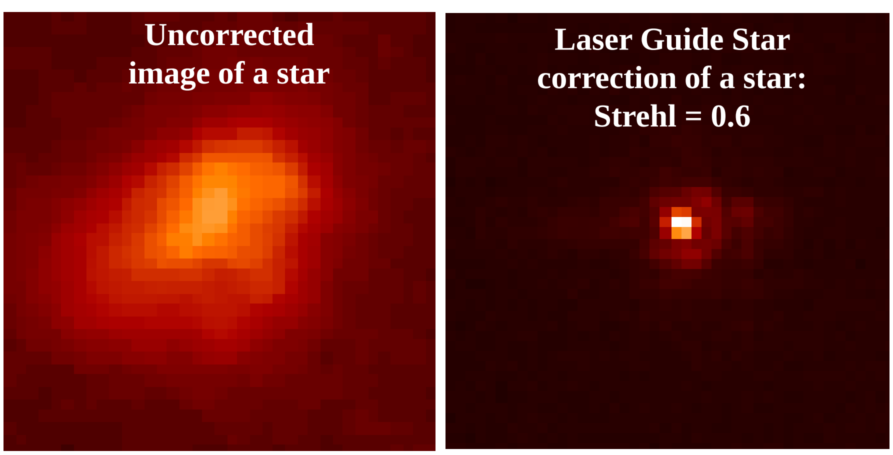


Implementation



Laser in 120-inch dome

Laser guide star adaptive optics at Lick Observatory



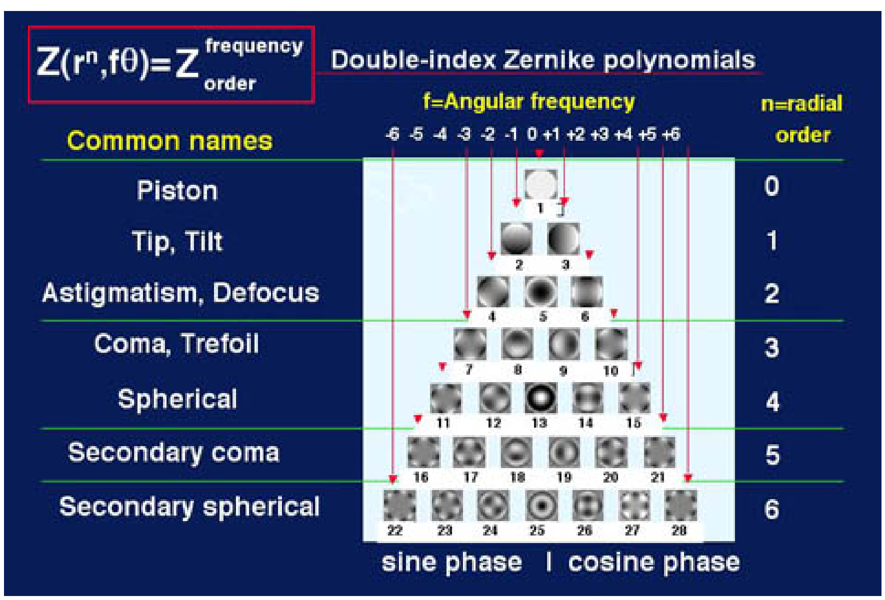
Ircal1129.fits RX J0258.3+1947 10/20/00 2:04 Ks V=15 K=~13.32 20s S=0.6 LGS

Wavefront characterizations

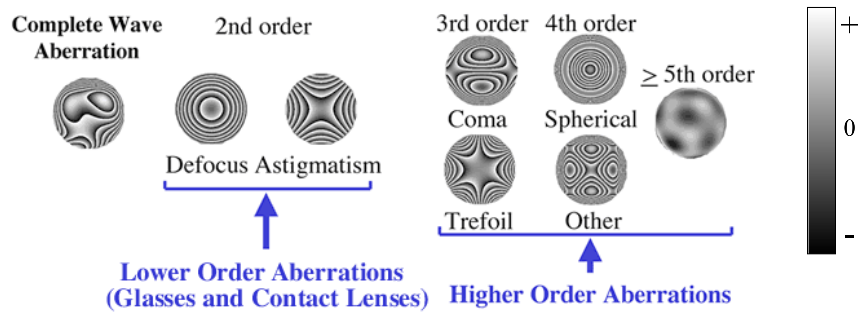
The weighted sum of an orthogonal set of polynomials over the unit circle

Wavefront toolbox on github
t_wvfZernicke.m

Zernike polynomials



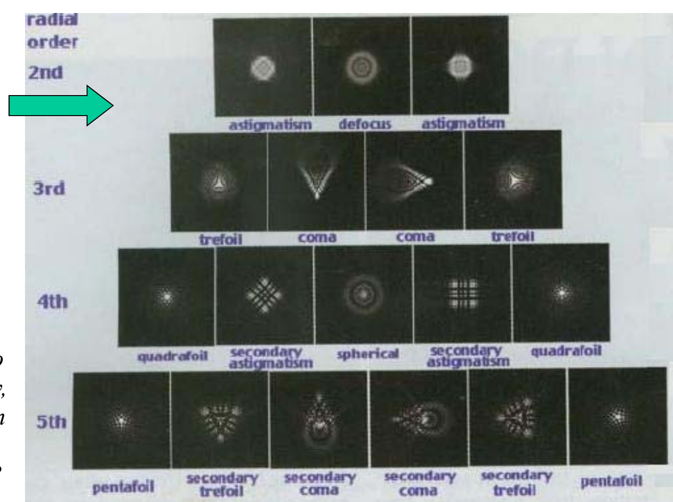
Visualization of Wavefront Error



Common point spread functions

Lower order are typical and diagnosed easily

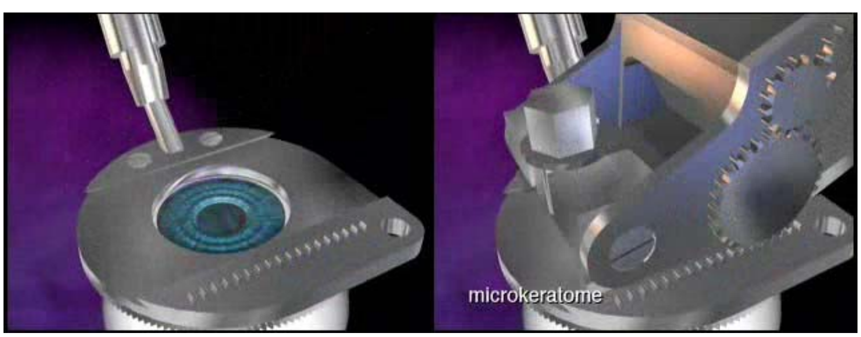
Because the likely aberrations are known, it is possible to estimate the aberrations from the pointspread. Theoretically – not so much. But practically, could be OK (Grissan et al., 2007). Compressed sensing?



Laser Eye Surgery

Laser Eye Surgery

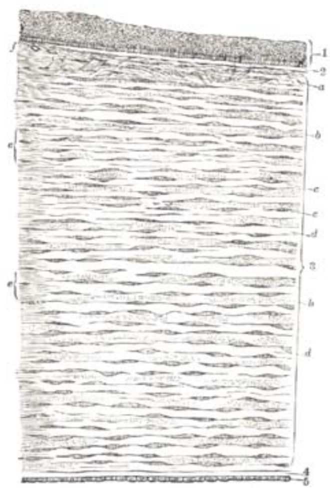
1. Measure the wavefront aberration
2. Cut a flap in the cornea (microkeratome)



Application: Laser Eye Surgery

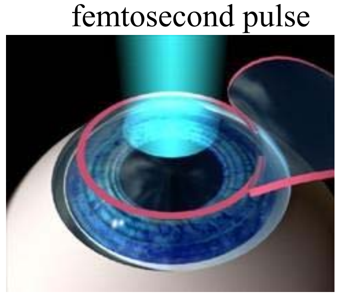
1. Measure the wavefront aberration

2. Cut a flap in the cornea (microkeratome)



Laser Corrective Eye Surgery

3. An excimer laser (193 nm) reshapes the surface of the corneal stroma to improve the wavefront – the tissue (10’s of microns) is vaporized with a femtosecond pulse



Performing the laser ablation in the deeper corneal stroma typically provides for more rapid visual recovery and less pain than the earlier technique, photorefractive keratectomy (PRK).

End Section: Image formation

Adaptive Optics Imaging Images The Human Retina At High Resolution

(Roorda & Williams, 1999; Williams & Roorda, 1999)

Human cones 1 deg eccentricity