Otto von Bismarck, the Chancellor of Germany from 1871 to 1890, is widely credited with creating the first welfare state in order to gain the support from the “working class” who might otherwise favor socialist rivals. Responding to critics of his initial 1881 proposal for a social insurance program, he is said to have replied “Call it socialism or whatever you like. It is the same to me.” Eventually the Reichstag passed The Pensions and Disabilities Act of 1889. According to wikipedia:

The old age pension program, insurance equally financed by employers and workers, was designed to provide a pension annuity for workers who reached the age of 70. ... this program covered all categories of workers (industrial, agrarian, artisans and servants) from the start. Also, ...the principle that the national government should contribute a portion of the underwriting cost, with the other two portions prorated accordingly, was accepted without question. The disability insurance program was intended to be used by those permanently disabled.

At the time, life expectancy in Germany at birth was estimated to be roughly 45 years, so the fiscal burden of the old age pensions could be expected to be relatively small. This changed somewhat in 1916 when the age at which pension payments would begin was lowered to 65.
The portrait of Bismarck painted by Franz von Lenbach in 1890, shows Bismarck in a dress uniform. Socialist or not, he was the father of social retirement insurance programs.
**The United States Social Security Act of 1935**

Franklin Delano Roosevelt (often called FDR) was President of the United States from 1933 through 1945. A Democrat, he served through the depths of the great depression and most of World War Two. Here is a painting by an artist whose name seems to have been lost on the internet.

In January 1934, Roosevelt sent a message to Congress arguing that:

*In the important field of security for our old people, it seems necessary to adopt three principles: First, non-contributory old-age pensions for those who are now too old to build up their own insurance. It is, of course, clear that for perhaps thirty years to come funds will have to be provided by the States and the Federal Government to meet these pensions. Second, compulsory contributory annuities which in time will establish a self-supporting system for those now young and for future generations. Third, voluntary contributory annuities by which individual initiative can increase the annual amounts received in old age. It is proposed that the Federal Government assume one-half of the cost of the old-age pension plan, which ought ultimately to be supplanted by self-supporting annuity plans.*
On August 14, 1935, Congress passed the Social Security Act:

An act to provide for the general welfare by establishing a system of Federal old-age benefits, and by enabling the several States to make more adequate provision for aged persons, blind persons, dependent and crippled children, maternal and child welfare, public health, and the administration of their unemployment compensation laws; to establish a Social Security Board; to raise revenue; and for other purposes.

On the same day, President Roosevelt signed it, as shown in the photograph below.

Thus, after more than four decades, the United States followed Germany's lead, providing retirement and other benefits for its citizens. With some exceptions, all workers in the U.S. are required to participate in the Social Security system, the key exceptions being people employed by state and local governments that provide their own pension plans.

Later in this chapter we will create a function for creating a scenario matrix of incomes from U.S. Social Security payments. But first we discuss the history of the system, its current benefit structure and some issues concerning its future solvency. Other sections will provide information on counterparts to the U.S. Social Security system in other countries as well as pension systems for employees of American state and local governments.
In 2016, the Official Social Security website stated that “Social Security's *Old-Age, Survivors, and Disability Insurance* (OASDI) program and Medicare's *Hospital Insurance* (HI) program are financed primarily by employment taxes. Tax rates are set by law … and apply to earnings up to a maximum amount for OASDI.” The latter amount, termed the *maximum taxable earnings*, can and often does change from year to year. To quote the Social Security administration: “Throughout the last four-plus decades, the SSA has consistently increased the earnings cap on Social Security taxes to reflect the impact of inflation on wages and the average American's cost of living.” The following table provides the figures.

<table>
<thead>
<tr>
<th>Year</th>
<th>Maximum Taxable Earnings</th>
</tr>
</thead>
<tbody>
<tr>
<td>1937 - 50</td>
<td>$3,000</td>
</tr>
<tr>
<td>1951 - 54</td>
<td>3,600</td>
</tr>
<tr>
<td>1955 - 58</td>
<td>4,200</td>
</tr>
<tr>
<td>1959 - 65</td>
<td>4,800</td>
</tr>
<tr>
<td>1966 - 67</td>
<td>6,800</td>
</tr>
<tr>
<td>1968 - 71</td>
<td>7,800</td>
</tr>
<tr>
<td>1972</td>
<td>9,000</td>
</tr>
<tr>
<td>1973</td>
<td>10,800</td>
</tr>
<tr>
<td>1974</td>
<td>13,200</td>
</tr>
<tr>
<td>1975</td>
<td>14,100</td>
</tr>
<tr>
<td>1976</td>
<td>15,300</td>
</tr>
<tr>
<td>1977</td>
<td>16,500</td>
</tr>
<tr>
<td>1978</td>
<td>17,700</td>
</tr>
<tr>
<td>1979</td>
<td>22,900</td>
</tr>
<tr>
<td>1980</td>
<td>25,900</td>
</tr>
<tr>
<td>1981</td>
<td>29,700</td>
</tr>
</tbody>
</table>
Tax rates, which must be set by law, have also increased over time, but more sporadically, as shown in the table below (taken from the Social Security site, absent the footnotes).

<table>
<thead>
<tr>
<th>Calendar year</th>
<th>Tax rates as a percent of taxable earnings</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Rate for employees and employers, each</td>
</tr>
<tr>
<td></td>
<td>OASDI</td>
</tr>
<tr>
<td>1937-49</td>
<td>1.000</td>
</tr>
<tr>
<td>1950</td>
<td>1.500</td>
</tr>
<tr>
<td>1951-53</td>
<td>1.500</td>
</tr>
<tr>
<td>1954-56</td>
<td>2.000</td>
</tr>
<tr>
<td>1957-58</td>
<td>2.250</td>
</tr>
<tr>
<td>1959</td>
<td>2.500</td>
</tr>
<tr>
<td>1960-61</td>
<td>3.000</td>
</tr>
<tr>
<td>1963-65</td>
<td>3.625</td>
</tr>
<tr>
<td>1966</td>
<td>3.850</td>
</tr>
<tr>
<td>1967</td>
<td>3.900</td>
</tr>
<tr>
<td>1968</td>
<td>3.800</td>
</tr>
<tr>
<td>1969-70</td>
<td>4.200</td>
</tr>
<tr>
<td>1971-72</td>
<td>4.600</td>
</tr>
<tr>
<td>1973</td>
<td>4.850</td>
</tr>
<tr>
<td>1974-77</td>
<td>4.950</td>
</tr>
<tr>
<td>1978</td>
<td>5.050</td>
</tr>
<tr>
<td>1979-80</td>
<td>5.080</td>
</tr>
<tr>
<td>1981</td>
<td>5.350</td>
</tr>
<tr>
<td>1982-83</td>
<td>5.400</td>
</tr>
<tr>
<td>1984 a</td>
<td>5.700</td>
</tr>
<tr>
<td>1985 a</td>
<td>5.700</td>
</tr>
<tr>
<td>1986-87 a</td>
<td>5.700</td>
</tr>
<tr>
<td>1988-89 a</td>
<td>6.050</td>
</tr>
<tr>
<td>1990 and later b, c</td>
<td>6.200</td>
</tr>
</tbody>
</table>

Social Security is a highly politicized subject, and there has not been sufficient agreement between the President and Congress to make any changes in contribution rates since 1990.

While many commentators make a point of differentiating between 6.2% of income paid by a worker and the 6.2% paid by an employer, most economists would consider the total more relevant. The fact is that the total cost of an employee is 12.4% greater because of social security than it would be otherwise; in all likelihood most employers make their hiring and compensation decisions accordingly.
The Social Security web site (accessed on April 14, 2016) included two Benefit Calculation examples for workers retiring in 2016. The two cases are shown below.

<table>
<thead>
<tr>
<th>Year</th>
<th>Nominal earnings</th>
<th>Indexing factor</th>
<th>Indexed earnings</th>
<th>Nominal earnings</th>
<th>Indexing factor</th>
<th>Indexed earnings</th>
</tr>
</thead>
<tbody>
<tr>
<td>1976</td>
<td>$8,627</td>
<td>5.0378</td>
<td>$43,461</td>
<td>$15,300</td>
<td>4.5168</td>
<td>$69,106</td>
</tr>
<tr>
<td>1977</td>
<td>9,173</td>
<td>4.7530</td>
<td>43,599</td>
<td>16,500</td>
<td>4.2614</td>
<td>70,313</td>
</tr>
<tr>
<td>1978</td>
<td>9,932</td>
<td>4.4033</td>
<td>43,734</td>
<td>17,700</td>
<td>3.9479</td>
<td>69,877</td>
</tr>
<tr>
<td>1979</td>
<td>10,835</td>
<td>4.0491</td>
<td>43,872</td>
<td>22,900</td>
<td>3.6303</td>
<td>83,134</td>
</tr>
<tr>
<td>1980</td>
<td>11,848</td>
<td>3.7145</td>
<td>44,010</td>
<td>26,900</td>
<td>3.3303</td>
<td>86,265</td>
</tr>
<tr>
<td>1981</td>
<td>13,081</td>
<td>3.3748</td>
<td>44,146</td>
<td>29,700</td>
<td>3.0257</td>
<td>89,864</td>
</tr>
<tr>
<td>1983</td>
<td>14,663</td>
<td>3.0501</td>
<td>44,419</td>
<td>35,700</td>
<td>2.7346</td>
<td>97,627</td>
</tr>
<tr>
<td>1984</td>
<td>15,467</td>
<td>2.8808</td>
<td>44,557</td>
<td>37,800</td>
<td>2.5828</td>
<td>97,630</td>
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<tr>
<td>1985</td>
<td>16,175</td>
<td>2.7631</td>
<td>44,692</td>
<td>39,600</td>
<td>2.4773</td>
<td>98,100</td>
</tr>
<tr>
<td>1986</td>
<td>16,707</td>
<td>2.6834</td>
<td>44,832</td>
<td>42,000</td>
<td>2.4059</td>
<td>101,046</td>
</tr>
<tr>
<td>1987</td>
<td>17,826</td>
<td>2.5225</td>
<td>44,967</td>
<td>43,800</td>
<td>2.2616</td>
<td>99,059</td>
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<td>1988</td>
<td>18,761</td>
<td>2.4041</td>
<td>45,104</td>
<td>45,000</td>
<td>2.1555</td>
<td>99,996</td>
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<tr>
<td>1989</td>
<td>19,564</td>
<td>2.3126</td>
<td>45,243</td>
<td>48,000</td>
<td>2.0734</td>
<td>99,522</td>
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<tr>
<td>1990</td>
<td>20,529</td>
<td>2.2105</td>
<td>45,379</td>
<td>51,300</td>
<td>1.9818</td>
<td>101,668</td>
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<tr>
<td>1991</td>
<td>21,359</td>
<td>2.1310</td>
<td>45,517</td>
<td>53,400</td>
<td>1.9106</td>
<td>102,027</td>
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<tr>
<td>1992</td>
<td>22,527</td>
<td>2.0266</td>
<td>45,654</td>
<td>55,500</td>
<td>1.8170</td>
<td>100,844</td>
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<tr>
<td>1993</td>
<td>22,789</td>
<td>2.0093</td>
<td>45,791</td>
<td>57,800</td>
<td>1.8015</td>
<td>103,767</td>
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<td>1994</td>
<td>23,470</td>
<td>1.9568</td>
<td>45,927</td>
<td>60,600</td>
<td>1.7544</td>
<td>106,318</td>
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<tr>
<td>1995</td>
<td>24,848</td>
<td>1.8814</td>
<td>46,064</td>
<td>61,200</td>
<td>1.6868</td>
<td>103,233</td>
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<tr>
<td>1996</td>
<td>25,758</td>
<td>1.7937</td>
<td>46,202</td>
<td>62,700</td>
<td>1.6082</td>
<td>100,832</td>
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<tr>
<td>1997</td>
<td>27,342</td>
<td>1.6948</td>
<td>46,339</td>
<td>65,400</td>
<td>1.5195</td>
<td>99,375</td>
</tr>
<tr>
<td>1998</td>
<td>28,858</td>
<td>1.6105</td>
<td>46,476</td>
<td>68,400</td>
<td>1.4439</td>
<td>98,765</td>
</tr>
<tr>
<td>1999</td>
<td>30,566</td>
<td>1.5255</td>
<td>46,613</td>
<td>72,600</td>
<td>1.3677</td>
<td>99,296</td>
</tr>
<tr>
<td>2000</td>
<td>32,340</td>
<td>1.4456</td>
<td>46,749</td>
<td>76,200</td>
<td>1.2960</td>
<td>98,758</td>
</tr>
<tr>
<td>2001</td>
<td>33,209</td>
<td>1.4119</td>
<td>46,887</td>
<td>80,400</td>
<td>1.2658</td>
<td>101,773</td>
</tr>
<tr>
<td>2002</td>
<td>33,640</td>
<td>1.3979</td>
<td>47,024</td>
<td>84,900</td>
<td>1.2533</td>
<td>105,403</td>
</tr>
<tr>
<td>2003</td>
<td>34,663</td>
<td>1.3645</td>
<td>47,161</td>
<td>87,000</td>
<td>1.2234</td>
<td>106,433</td>
</tr>
<tr>
<td>2004</td>
<td>36,275</td>
<td>1.3039</td>
<td>47,298</td>
<td>87,900</td>
<td>1.1690</td>
<td>102,757</td>
</tr>
<tr>
<td>2005</td>
<td>37,711</td>
<td>1.2579</td>
<td>47,435</td>
<td>90,000</td>
<td>1.1278</td>
<td>101,498</td>
</tr>
<tr>
<td>2006</td>
<td>39,558</td>
<td>1.2026</td>
<td>47,572</td>
<td>94,200</td>
<td>1.0782</td>
<td>101,566</td>
</tr>
<tr>
<td>2007</td>
<td>41,473</td>
<td>1.1504</td>
<td>47,710</td>
<td>97,500</td>
<td>1.0314</td>
<td>100,561</td>
</tr>
<tr>
<td>2008</td>
<td>42,549</td>
<td>1.1245</td>
<td>47,847</td>
<td>102,000</td>
<td>1.0082</td>
<td>102,836</td>
</tr>
<tr>
<td>2009</td>
<td>42,027</td>
<td>1.1147</td>
<td>47,983</td>
<td>106,800</td>
<td>1.0236</td>
<td>109,324</td>
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<tr>
<td>2010</td>
<td>43,143</td>
<td>1.1154</td>
<td>48,120</td>
<td>106,800</td>
<td>1.0000</td>
<td>106,800</td>
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<tr>
<td>2011</td>
<td>44,622</td>
<td>1.0815</td>
<td>48,258</td>
<td>106,800</td>
<td>1.0000</td>
<td>106,800</td>
</tr>
<tr>
<td>2012</td>
<td>46,146</td>
<td>1.0487</td>
<td>48,395</td>
<td>110,100</td>
<td>1.0000</td>
<td>110,100</td>
</tr>
<tr>
<td>2013</td>
<td>46,668</td>
<td>1.0355</td>
<td>48,532</td>
<td>113,700</td>
<td>1.0000</td>
<td>113,700</td>
</tr>
<tr>
<td>2014</td>
<td>48,669</td>
<td>1.0000</td>
<td>48,869</td>
<td>117,000</td>
<td>1.0000</td>
<td>117,000</td>
</tr>
<tr>
<td>2015</td>
<td>50,211</td>
<td>1.0000</td>
<td>50,211</td>
<td>118,500</td>
<td>1.0000</td>
<td>118,500</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Highest-35 total</th>
<th>1,628,054</th>
<th>Highest-35 total</th>
<th>3,593,696</th>
</tr>
</thead>
<tbody>
<tr>
<td>AIME</td>
<td>3,876</td>
<td>AIME</td>
<td>8,556</td>
</tr>
</tbody>
</table>
Case A is the more straightforward, since it involves a person born in 1954, who has just reached “.. the year of first eligibility (the year a person attains age 62 in retirement cases)”.
Each of the numbers in the first column shows the person's actual nominal earnings in a year or the maximum taxable earnings for that year, whichever is smaller. Each of the resulting values is then multiplied by an indexing factor for the year to obtain the appropriate value of indexed earnings. This is intended to “bring nominal earnings up to near-current wage levels”. For some reason, the factor will equal 1.0 for the year in which the person attains age 60 and all later years. Otherwise “The indexing factor for a prior year Y is the result of dividing the average wage index for the year in which the person attains age 60 by the average wage for year Y.” (all quoted statements are from the Social Security web site).

Next, the highest 35 years of indexed earnings are summed and the total divided by 35, giving the average indexed annual earnings. This is then divided by 12 to obtain the “averaged indexed monthly earnings” (AIME). In this case, the years excluded for the AIME calculations (shown in red) are the earliest ones, but this need not always be the case.

Case B differs in that the worker, born in 1950, is older than 62 at the time of the calculation. Moreover, he or she has managed to earn the maximum taxable earnings or (most likely) more in each year. By rule, these nominal earnings are only indexed up to the point at which he (or she) turned 60. Once again the 35 highest indexed values are averaged, then the AIME is calculated as before.

The indexing factors are based on ratios of the historic values of an average wage index to its level at the time a worker turns 60. Each value represents the ratio of the current value of a measure of the average wage per worker (including, since 1991, contributions to deferred compensation plans) to its value at the end of the year in question.
The use of a wage index may seem surprising. A more natural approach might have used an index of the cost of living, so that the calculations would be based on real incomes, stated in terms of current purchasing power. Over the years, many people have advocated for a change from “wage indexing” to “price indexing” for computing social security benefits, on the grounds that this would be more reasonable and (importantly) reduce the cost of providing benefits. However, in the last few decades, the differences in the two indices have shrunk. The figure below shows the annual changes in wages and prices from 1976 through 2015.

In years when wages increase more than prices, real wages rise, and in years when wages increase less than prices, real wages fall. As is widely known, real wages for average workers in the United States have been stagnant for many years. For the period covered in the figure, the average change in the nominal wage was 4.3%, while the average change in the cost of living was 3.7%. However, in the latter part of the period, each was lower. From 2000 through 2015, the average rate of increase in wages was 2.69% while prices increased at a rate of 2.16%.
The Social Security Administration's actuaries have studied historical data for both these factors. When making projections for the economic viability of the system, they often consider three possibilities, designated “low cost”, “intermediate”, and “high cost”. Here are the long-term assumed annual increases used for projections in the 2015 report.

<table>
<thead>
<tr>
<th></th>
<th>Nominal Wage</th>
<th>Consumer Price Index</th>
<th>Real Wage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low cost</td>
<td>3.6%</td>
<td>2.0%</td>
<td>1.6%</td>
</tr>
<tr>
<td>Intermediate</td>
<td>4.1%</td>
<td>3.0%</td>
<td>1.1%</td>
</tr>
<tr>
<td>High Cost</td>
<td>4.6%</td>
<td>4.0%</td>
<td>0.6%</td>
</tr>
</tbody>
</table>

Most summaries of the future prospects for the system utilize projections based on the intermediate assumptions, which project that the real interest rate will rise from current low levels to a steady 2.9% per year from 2022 onward. More on this later. Now, back to the calculation of benefits.

The key figure for a social security beneficiary is not the average indexed monthly earnings (AIME), but the “basic Social Security benefit”, called the primary insurance amount (PIA) – “...the benefit a person would receive if he/she elects to begin receiving retirement benefits at his/her normal retirement age.” (more on this also later).
The PIA is a function of the AIME as long as contributions were made in at least 40 quarters, otherwise no benefits will be paid. Otherwise, PIA = f(AIME). To quote the Social Security Administration, in 2016, it was:

(a) 90 percent of the first $856 of his/her average indexed monthly earnings, plus
(b) 32 percent of his/her average indexed monthly earnings over $856 and through $5,157, plus
(c) 15 percent of his/her average indexed monthly earnings over $5,157.

Graphically:

![Graph showing the relationship between Average Indexed Monthly Earnings and Primary Insurance Amount.]

The percentages in the formula (90%, 32%, 15%) are fixed by law and remain the same from year to year. The so-called bend points (in 2016, $856 and $5,157) change each year in proportion to changes in the same national average wage index used to compute average indexed monthly earnings.
Viewed as an investment, the social security program clearly will provide a higher rate of return for those with low incomes than for their more fortunate peers with higher incomes, if the low income workers they live as long or longer. Given this, some argue that the program should be considered a combination of a payroll income tax and an old age benefit system. However, such a view would be highly toxic politically and, if widely accepted, could threaten the system's future. Undoubtedly, the combination is designed to be progressive, benefiting the poor proportionately more than the rich. But the differences tend to be smaller than one might infer from this graph. As shown in Chapter 3, a retiree who earned a higher income is likely to live longer than one who earned a lower income, so the present value of $1 per month of income received from social security is worth more for an upper-income person than for one with a lower income.

An article by Neil Irwin in the New York Times (April 24, 2016) addresses this issue directly. The title and subtitle are “Mr. Moneybags Gets More Out of Social Security; Longer life spans may give the rich an advantage in a system intended to favor the poor”. Irwin's Mr. Moneybags is consistently in the top 1% of earners, starts taking Social Security benefits at 66 and lives to be 87, based on estimates in a research paper by Stanford economist Raj Chetty and seven colleagues. Using a model built by the Times, Irwin calculated that Mr. Moneybags would receive an inflation-adjusted “internal rate of return” of 1.07%. In contrast, his gardener earns $30,000 per year, and men in that income die, on average, at age 78. With this life span, the gardner would obtain a 0.92% real rate of return on his “investments” in Social Security. In this case, at least, the system would be regressive – benefitting the rich proportionately more than the poor.
Irwin's example assumes that each of the two protagonists starts taking social security benefits at age 66. But this is not mandatory. Yes, the Primary Insurance Amount (PIA) is indeed the amount per month that can be received if a participant chooses to begin receiving payments at the full retirement age (now 66). But one can choose to begin payments as early as age 62 and as late as age 70, with the initial monthly payment adjusted according to a standard table. The figure below (from the Social Security web site When To Start Receiving Retirement Benefits, shows the amounts that would be received for someone with a PIA of $1,000 per month whose full retirement age is 66.

This chart applies for those born in 1954, for whom the full retirement age is 66. For those born in later years, the full retirement age will be greater, increasing by 2 months each year until it reaches 67 for those born in 1960 or later (unless, of course, there is a change in the underlying legislation).
Should one take a smaller amount of money each month for a longer period or a larger amount for a shorter period? The answer depends in whole or in part on life expectancy or, more specifically, the mortality probability distribution. The social security site suggests that “If you live to the average life expectancy for someone your age, you’ll receive about the same amount in lifetime benefits. It doesn’t matter if you choose to start receiving benefits at age 62, full retirement age, age 70, or any age between.” This might be true if your life expectancy were the same as that assumed by the Social Security Administration. But as we know, such prospects differ. Moreover, the reductions for retiring prior to 66 and bonuses for retiring after 66 were determined years ago, using actuarial tables and interest rates that were presumed to be relevant at the time but are almost certainly inaccurate now.

We can get some idea of the tradeoffs using the `iFixedAnnuity` data structure developed in Chapter 10. Consider a 62-year old male with a primary insurance amount of $1,000 per month. The actual amounts he will receive each month will depend on the age at which he chooses to start receiving benefits, as shown in the prior figure. If, for example. He begins payments immediately (at age 62), he will receive $750 per month for the next 12 months, then an amount equal to $750 times the ratio of the CPI a year hence to the current value, and so on. For him, Social Security is equivalent to an immediate real annuity. And we have software that can evaluate such an annuity.

Our software uses annual periods, so we need to convert $750 per month to an equivalent value received at the beginning of the year. Since Social Security does not adjust for changes in the cost of living during a year, we can find the present value of a nominal dollar received each month for the next twelve months. To do so, we need to discount the monthly payments using a nominal rate of interest. Given our standard assumptions (a real rate of interest of 1% and a rate of inflation of 2%), the nominal interest rate can be assumed to be 3.02% (1.01*1.02 = 1.0302). Doing the calculations for each month provides the result that the present value of a nominal dollar each month for the next twelve months is $11.8086. Thus $750 each month is equivalent to $ 8,856 (11.8086*$750) each year.

We next use our fixed annuity procedure to find the annual real income obtained with an arbitrary amount invested and a value-to-cost ratio of 1.0. After processing the annuity, we can compute the ratio of the amount invested to the annual income; this indicates the present value of a real annuity of $1 per year. Multiplying this ratio by the annual income (here, $8,856) gives the present value of the real annuity obtained if benefits start at age 62. In this case it is approximately $192,000.
The process is similar for cases in which benefits start in later years. If, for example, benefits are to start in a year, when the beneficiary is 63, the social security payments are equivalent to a deferred fixed real annuity with payments beginning a year hence. We represent this by setting $iFixedAnnuity.guaranteedIncomes$ to $[0]$, indicating zero incomes in the first year. For a deferral to age 64, we set $iFixedAnnuity.guaranteedIncomes$ to $[0 0]$. And so on.

The following graph shows the present values of the annuities obtained for different claiming ages for a 62-year old male recipient with a PIA of $1,000 per month. If one wishes only to maximize the present value of such an annuity, it is best to wait until age 69 to begin benefits. Up to age 70, deferral provides a real benefit each year, but for fewer years. In this case, the increases in annual benefits more than compensate for the fact that they start later and are likely to be paid for fewer years in total. Of course, these results depend on our choice of actuarial tables, the assumed real rates of interest and (to a relatively minor extent) the particular random numbers used to determine mortality in different scenarios. That said, it appears that a male who has never married and has no plans to do so might gain as much as 17% in present value by deferring the start of social security benefits until after the age that Social Security calls the “full retirement age”.

![Graph of Present Values of Social Security Payments](image)
What about a 62 year old female with the same primary insurance amount? The results are shown in the following figure. The present values are higher than those for a male because she is likely to live longer. But deferral can still increase the present value of her annuity (by up to 22%, since greater payments will likely be received for more years than for her male counterpart). If she too has never married and has no intention of doing so, it may well pay to defer starting Social Security payments until after the her “full retirement age”.

Unfortunately, the choices are not as simple for Bob and Sue, since they are married, and Social Security provides for spousal benefits. Such benefits complicate the situation for those currently married and, in some cases, for those with former spouses.

To receive a worker's spousal benefits, a spouse must be 62 or older and the worker must be receiving Social Security benefits. (Before May 2016 there was a loophole that allowed the worker to file for benefits, then suspend them to wait for higher payments, leaving the spousal payments intact. But, with very rare exceptions, this option is no longer available.) There are also benefits for unmarried children and disabled adult children.
If a spouse is at full retirement age or older, his or her spousal benefit is equal to 50% of the worker's benefit. If he or she is younger, the amount is smaller, and as low as 35% for a spouse of age 62. But it is not possible to receive both (1) a spousal benefit and (2) a standard Social Security benefit based one's own employment. In effect, you get the larger of the two amounts (and, if eligible for more than two, the largest of them all). And this greatly complicates the choice of a claiming strategy.

There is more. Social Security also provides survivor benefits. If one partner in a marriage dies, the widow(er) can receive as much as 100% of the amount the decedent was receiving. If the decedent had not yet claimed social security benefits, the widow(er) will typically be eligible to receive 100% of the spouses full retirement amount. But the general rule still applies. If a person is dually eligible for benefits based on his or her work record and any other benefits (spousal or survivor), Social Security will pay only the larger of the two amounts.

Complex? Certainly. Confusing? Undoubtedly. When should each partner in a marriage choose to start receiving Social Security benefits? It very well may be best for one person to start benefits, with the other taking spousal benefits for one or more years, then filing for his or her own benefits at a late age. The Social Security Administration counsels “Each person's situation is different. Make sure you talk to a Social Security representative before you decide to retire.” But it may also be desirable to do some of your own research. For an entertaining and exhaustive guide by an economist and two journalists, see Laurence J. Kotlikoff, Philip Moeller and Paul Solman, “Get What's Yours, The Secrets to Maxing Out Your Social Security,” Simon and Schuster, 2016. For a detailed guide to the intricacies of the system, see Andy Landis, “Social Security, The Inside Story, An Expert Explains Your Rights and Benefits”, 2016.
If this amount of homework is too arduous, a number of online services will test different claiming strategies for you, suggesting one that appears to be best according to some criterion. Many levy charges, but not all. Here is the advice provided for Tom and Mary Jones by Financial Engines' free online Social Security Planner after being told that Tom was born on Jan. 1, 1949, Mary on Jan. 1, 1951 and that each had an average annual salary of $102,672 (12 times the maximum AIME for Case B in the Social Security table shown earlier) and an average life expectancy.

The recommended strategy has Tom beginning his payment of $38,700 per year as soon as possible, with Mary deferring until she can obtain the largest possible amount: $42,300 per year. In the interim, Mary can collect spousal benefits of $16,700 per year, equal to slightly over 43% of Tom's earned benefit. The footnote for the $42,300 per year of Survivor Benefits indicates that this assumes that Tom and Mary each live to be at least 70.

The site indicates that the recommended approach was chosen to maximize “... the average total lifetime benefit you could receive, for you and your spouse/partner, and for surviving spouse (if married). The total is an average based on hundreds of scenarios for your potential lifespan(s), taking into account how likely each scenario is.” The supplemental information indicates that estimates for those with indicated average life expectancies are based on actuarial tables from the Society of Actuaries and a 0% discount rate.
Full disclosure: I was a co-founder of Financial Engines and now hold the honorific title *Director Emeritus*. I completely retired from the firm in 2010 and was not involved in the development of the Social Security software.

The choice to not discount future payments seems slightly inappropriate. But, assuming that all the cash flows are measured in real terms, the optimal strategy might differ little from that obtained had future payments been discounted at the current real rates of 1% per year or less. A more fundamental question concerns the assumption that the goal is to maximize the value of an equivalent annuity. Since it is impossible to sell a Social Security contract, the likely market value at which such a sale could be made may not be the best measure of its value to the beneficiaries. If we had multi-period utility functions for Tom and Mary (both individually and as a couple) plus mortality tables based on their personal histories and current health, we could find the claiming schedule that provides the maximum possible expected utility, taking all the information into account. And the resulting strategy might differ significantly from the one that maximizes the estimated value of a joint and survivor annuity. Strategies found using tools such as the Financial Engines planner should thus be considered interesting possibilities, but the final decisions concerning claiming strategy should be made by the beneficiaries (here, Tom and Mary).

There are other issues. Unfortunately, the amounts provided by calculations such as this do not indicate the *spendable incomes* that most will realize from Social Security payments. In the real world there are both *deductions* and *income taxes*.

Once a person begins receiving benefits, all amounts will be adjusted at the beginning of each calendar year by an amount equal to a prior 12-month change in the consumer price index. In this sense, benefits are intended to remain constant in real terms. However, the net amount actually received will be smaller, since amounts are deducted to help finance part B (for non-hospital costs) of Medicare, the Federal old-age health insurance program. The size of such a deduction depends on the income declared on your income tax two years earlier. For higher-income retirees, such deductions can reduce the amount received by 10% or more.
Finally, there is the matter of income taxes. Here is what Carrie Schwab-Pomerantz, President of the Charles Schwab Foundation had to say in 2014: “To determine whether your Social Security benefits will be taxed, the IRS uses what it calls your “combined income” — which is the sum of your adjusted gross income (AGI), non-taxable interest, and half of your Social Security benefits. If your combined income exceeds a certain limit, 50 to 85 percent of your benefits may be taxed.”

The specific rules are complex (no surprise). In 2016, the base for the computation was “combined income”, which equals adjusted gross income plus nontaxable interest + 0.5* Social Security benefits. To quote the Social Security Administration:

**If you file a federal tax return as an "individual" and your combined income is**
- between $25,000 and $34,000, you may have to pay income tax on up to 50 percent of your benefits
- more than $34,000, up to 85 percent of your benefits may be taxable.

**If you file a joint return, and you and your spouse have a combined income that is**
- between $32,000 and $44,000, you may have to pay income tax on up to 50 percent of your benefits
- more than $44,000, up to 85 percent of your benefits may be taxable.

Perhaps Mr. Moneybags' after-tax relative benefit is not as much greater than that of his gardner as suggested in the New York Times article.
Sustainability of the Social Security System

The Social Security system retains a *Trust Fund* which holds specially issued U.S. Treasury bonds as a reserve. The bonds pay “a market rate of interest” and are redeemed when they reach maturity or are needed to pay benefits. There are two funds, one for the Old Age and Survivors Insurance (OASDI) program, which pays retirement and survivor benefits; the other for the Disability Insurance (DI) program which pays disability benefits. In the month of March 2016, 72.0% of all benefits paid went to retired workers, their spouses and children, 10.1% to survivors of workers, and 17.9% to those receiving disability insurance. Average monthly benefits were $1,300 for retirees and their spouses, $1,114 for those receiving survivor benefits and $1,022 for the disabled.
The figure below shows the historic and projected level of the combined trust funds for both programs (OASDI) as estimated in 2015. There are three alternatives, based on intermediate (II), low cost (I) and high cost (III) sets of assumptions. The Trustees Report provides the details but the following summary from the report suffices for our purposes.

“The low-cost alternative includes a higher ultimate total fertility rate, slower improvement in mortality, a higher real-wage differential, a higher ultimate real interest rate, a higher ultimate annual change in the CPI, and a lower unemployment rate. The high-cost alternative, in contrast, includes a lower ultimate total fertility rate, more rapid improvement in mortality, a lower real-wage differential, a lower ultimate real interest rate, a lower ultimate annual change in the CPI, and a higher unemployment rate.... Actual future costs are unlikely to be as extreme as those portrayed by the low-cost or high-cost projections.”

This is the type of analysis that leads to political rhetoric arguing that “social security is broken”, the system will “run out of money” within 20 years, and it is “unsustainable”. One counterargument is that it would be simple enough to just issue more Treasury bonds and send them to the Social Security Administration. Of course future taxpayers would have to eventually cover interest and principal payments on the bonds so “the problem” would simply be shifted, not “solved”.

![Figure II.D7.—Long-Range OASI and DI Combined Trust Fund Ratios Under Alternative Scenarios](image)
A more nuanced analysis of the solvency of the Social Security system is contained Appendix F of the Trustees Report; unfortunately, it is seldom cited. It includes estimates of the system's infinite horizon unfunded obligation – a somewhat esoteric but highly useful measure. The idea is to find the present values of (1) the inflows (dedicated tax income) for the infinite future and (2) the outflows (future cost) for the infinite future. While there are ways to avoid taking an infinite amount of time to complete such an analysis, it apparently requires enough effort that only the results of computations based on the “intermediate” assumptions are included in the report.

Importantly, the results are separated into those for three groups. Past participants are those no longer alive, current participants are those 15 or older in 2015, and future participants are those under age 15 or not yet born (including those to be born forever and ever). The results below are taken from Table VI.F2 in the 2016 Trustees Report with the term “contributions” substituted for “dedicated tax income” and “benefits” for “future cost”. It also includes shortfalls (present values of benefits minus costs) and summaries for all the measures.

<table>
<thead>
<tr>
<th></th>
<th>PV of Contributions</th>
<th>PV of Benefits</th>
<th>PV of Benefits - Costs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Past Participants</td>
<td>58.6</td>
<td>56.4</td>
<td>-2.2</td>
</tr>
<tr>
<td>Current Participants</td>
<td>30.6</td>
<td>62.5</td>
<td>31.9</td>
</tr>
<tr>
<td>Future Participants</td>
<td>76.2</td>
<td>79.3</td>
<td>3.1</td>
</tr>
<tr>
<td>Total</td>
<td>165.4</td>
<td>198.2</td>
<td>32.8</td>
</tr>
</tbody>
</table>

Taken at face value, this table tells a fascinating tale. For future participants, it indicates that the system is relatively sound. The present value of all future benefits paid is only slightly more than the present value of all the amounts that will be paid into the system. Going forward (in this sense) the system is not broken!

But overall, the system is $32.8 Trillion in the hole. Why? Not because of past participants (long dead), nor future participants (very young or yet to be born), but because of the current participants around us – young and old. Collectively, they have contributed and will contribute amounts that will fail to cover their benefits by $31.9 Trillion unless something is changed. As the protagonist in the Pogo comic strip for Earth Day in 1971 said when contemplating litter in a primeval forest, “We have met the enemy and he (sic) is us”. Of course the saying goes back farther, at least to Commodore Perry in the Battle of Lake Erie, who said “We have met the enemy and they are ours (troops)”. But the point is made.
If right, these numbers are disturbing. Consider the fact that entire Gross Domestic Product of the United States in 2015 was slightly less that $18 Trillion. Based on these estimates, to “make social security solvent” would require devoting the entire output of the country for over 1.8 years to building up the trust fund. Or, as the Report concludes:

To illustrate the magnitude of the projected infinite horizon shortfall, consider that it could be eliminated with additional revenue equivalent to an immediate increase in the combined payroll tax rate from 12.4 percent to about 16.6 percent, or with cost reductions equivalent to an immediate and permanent reduction in benefits for all current and future beneficiaries by about 24 percent.

Not a pretty picture. But even these woeful numbers are likely to be overly optimistic. All the present values are computed by discounting using the “intermediate” real interest rates shown in the following figure.
For the Social Security's infinite horizon calculations, cash flows from 2025 onward are
discounted at a real interest rate of 2.7%. But in recent years even the longest-term TIPS
provided at most a real return of slightly more than 1% per year. Perhaps real rates will return
to levels of 2.0% and above in 2019, reach 2.7% in 2025, then remain at that level thereafter,
as projected in the figure. But present values are usually computed using today's term structure
of interest rates. To be sure, there are no infinitely-lived TIPS in the U.S. But it seems
improbable that if there were, they would have a real return as high as 2.7%. For all these
reasons, the problems for Social Security are likely to be even worse than shown in our
previous table.

How much worse? Comparison of the results obtained in 2016 with those in the 2015 Trustees
Report provides at least a hint. The present values for 2015 were computed using an ultimate
real discount rate of 2.9%, 0.2% (20 basis points) higher than the 2.7% rate for the 2016 present
values. The total shortfall to the infinite horizon in 2015 was estimated to be $25.8 Trillion,
while that in 2016 was $32.1 Trillion – an increase of $6.3 Trillion! Imagine the size of the
problem if one were to use a discount rate closer to current long-term TIPS real returns.

However one does the calculations, it is hard to conclude that Social Security in its present
form is sustainable. If the program it is to be maintained as is, contributions must be increased
and/or benefits cut (most likely, both).

Not surprisingly, political opinions about the Social Security system differ. Conservatives tend
to portray it as a redistributive “tax and spend” system and favor instead some sort of explicit
private or semi-public “save and invest” alternative with benefits proportional to savings.
Progressives tend to consider Social Security a desirable form of protection against old-age
poverty, with funds for lower-income citizens provided at least in part by those with higher
incomes.

In this century there have been calls to replace Social Security in whole or in part with some
other type of retirement savings system. One of the more vocal advocates for such a change
was President George W. Bush. After being elected for a second term in 2004, he told reporters
that he would “fix” Social Security because “I earned capital in this campaign, political capital,
and now I intend to spend it.”
In his subsequent address to Congress on the State of the Union President Bush indicated his desires:

“We must make Social Security permanently sound, not leave that task for another day. We must not jeopardize our economic strength by increasing payroll taxes. We must ensure that lower-income Americans get the help they need to have dignity and peace of mind in their retirement. We must guarantee that there is no change for those now retired or nearing retirement. And we must take care that any changes in the system are gradual, so younger workers have years to prepare and plan for their future.

As we fix Social Security, we also have the responsibility to make the system a better deal for younger workers. And the best way to reach that goal is through voluntary personal retirement accounts. Here is how the idea works:

Right now, a set portion of the money you earn is taken out of your paycheck to pay for the Social Security benefits of today's retirees. If you're a younger worker, I believe you should be able to set aside part of that money in your own retirement account, so you can build a nest egg for your own future.

Here is why the personal accounts are a better deal: Your money will grow, over time, at a greater rate than anything the current system can deliver. And your account will provide money for retirement over and above the check you will receive from Social Security. In addition, you'll be able to pass along the money that accumulates in your personal account, if you wish, to your children and -- or grandchildren. And best of all, the money in the account is yours, and the government can never take it away.

The goal here is greater security in retirement, so we will set careful guidelines for personal accounts: We'll make sure the money can only go into a conservative mix of bonds and stock funds. We'll make sure that your earnings are not eaten up by hidden Wall Street fees. We'll make sure there are good options to protect your investments from sudden market swings on the eve of your retirement. We'll make sure a personal account cannot be emptied out all at once, but rather paid out over time, as an addition to traditional Social Security benefits. And we'll make sure this plan is fiscally responsible by starting personal retirement accounts gradually and raising the yearly limits on contributions over time, eventually permitting all workers to set aside 4 percentage points of their payroll taxes in their accounts.

If this sounds too good to be true, that may be because it is. The claim that “your money will grow at a greater rate than anything the current system can deliver” echoed statements at the time by some economists who should have known better that in the long run a “conservative mix of bonds and stock funds” would certainly outperform Treasury securities (emphasis mine). So much for risk/return tradeoffs. And it remained unclear how the “good options to protect your investments from sudden market swings on the eve of your retirement” would be obtained, guaranteed and priced.
In any event, after calling for the changes, President Bush toured the country in an attempt to build support. But the Gallup organization's polls showed that public disapproval of his handling of Social Security rose from 48 to 64 percent between the address and June. Congressional Democrats uniformly opposed the reforms, with Republicans far less than enthusiastic. Congressional leaders soon dropped the effort and in October the President admitted that his attempts to change the system had failed. Since then, there has been remarkably little political discussion of possible reforms for the Social Security system. The system remains highly popular, particularly among older citizens, higher proportions of whom turn out to vote for candidates for national offices.

That said, many appear to believe that as generous a system will not be available when they retire. Here are the results of an opinion survey conducted by the Pew Research Center in 2014.

<table>
<thead>
<tr>
<th>When you retire, Social Security will provide:</th>
<th>Millennial %</th>
<th>Gen X %</th>
<th>Boomers under 65 %</th>
</tr>
</thead>
<tbody>
<tr>
<td>Benefits at current levels</td>
<td>6</td>
<td>9</td>
<td>26</td>
</tr>
<tr>
<td>Benefits at reduced levels</td>
<td>39</td>
<td>36</td>
<td>42</td>
</tr>
<tr>
<td>No benefits</td>
<td>51</td>
<td>50</td>
<td>28</td>
</tr>
<tr>
<td>Don't know</td>
<td>4</td>
<td>5</td>
<td>4</td>
</tr>
</tbody>
</table>

For their polls, Pew defines Millennials as those age 18 to 34 in 2015, Gen X as those 35 to 50 and Boomers as those 51 to 69 at the time.

Despite these gloomy forecasts, a majority of members of each of the three groups (69% of Boomers, 67% of Gen-X and 61% of Millennials) chose the statement “Benefits should not be reduced” rather than “Some future reductions need to be considered”. No wonder politicians are reluctant to take on the system's problems.
But the problems must be addressed. And reductions in benefits for those with lower net worth do not seem to be an answer. Results from the most recent U.S. Census, published in 2011, included estimates of household net worth based on:

**Assets**

*Interest-earning assets held at financial institutions, stocks and mutual fund shares, rental property, home ownership, IRA and Keogh accounts, 401k and Thrift Savings Plans, vehicles, and regular checking accounts. (but not including any traditional defined benefit pensions)*

**Liabilities**

*Mortgages on own home, mortgages on rental property, vehicle loans, credit card debt, educational loans, and medical debt not covered by insurance.*

The median net worth for households in which the “householder” (the first person listed on ownership or rental documents) is between 65 and 69 years old was $194,226. Excluding any home equity, the net worth was $43,921.

In March 2016, the average payment from Social Security to a retired worker was $1,345 per month, equal to $16,140 per year; and the average payment to the spouse of a retired worker was $695 per month, equal to $8,340 per year. For the typical person or couple retiring in the United States, Social Security is the most valuable asset, followed by home equity, with retirement savings a distant third.

Social Security benefits are not guaranteed. They can be changed by congressional action. In this sense contributions to the system are not equivalent to a default-free fixed income real annuity. For example, if there were an unanticipated increase in longevity (due, say to a breakthrough in preventing and/or curing many forms of cancer) one could imagine a decrease in overall benefits. In this sense, Social Security may have some elements of a Tontine.
Social Retirement Systems Outside the United States

The Organization for Economic Co-operation and Development (OECD) maintains statistics for pension funds in a number of countries. Of particular interest is the net replacement rate, defined as:

“....the individual net pension entitlement divided by net pre-retirement earnings, taking into account personal income taxes and social security contributions paid by workers and pensioners. It measures how effectively a pension system provides a retirement income to replace earnings, the main source of income before retirement.”

The following figure shows such replacement rates in 2015 for seven countries and for workers with six different ratios of pre-retirement earnings to average pre-retirement earnings in each country.


As can be seen, France and Italy provided considerably more generous social retirement payments than the U.S., and the United Kingdom considerably less. With the exception of Italy, the proportions of pre-retirement earnings replaced are lower, the greater the earnings.
Other terms of social pension systems vary across countries. Some have features that differ considerably from those of the U.S. system. A prominent example is the Swedish system, in which 16% of a person's wages or salary will be contributed to an “income pension” and 2.5% to a “premium pension”. Interest earned on the former is based on the growth of wages and salaries in Sweden. But the other portion can be invested in securities chosen (from a specified set of alternatives) by the individual. The system thus combines two mandatory elements – one with benefits partially dependent on the strength of the overall labor economy, the other dependent on the investment returns of a portfolio chosen by the worker. Shades of George W. Bush.

The United States is not the only country in which the fertility ratio is declining, putting tremendous pressure on social retirement insurance systems. It seems likely that many such systems will be modified in future years, with contributions increased, benefits reduced or both. It will be a challenge to insure solvency of such systems and an equitable distribution of retirement wealth among the citizenry.
State and Local Defined Benefit Pensions

As indicated earlier, employees of U.S. State governments and of local governments within states can be exempt from participation in the Social Security system if they are covered by a pension plan offered by their employer. Most such plans are financed to some extent with contributions made by the governmental agency plus mandatory contributions made by employees. In the private sector, there is increasing use of defined contribution plans in which contributions are invested and benefits depend on the performance of the chosen investments. However, most government plans provide specified benefits that are guaranteed by the employer, and are thus termed defined benefit plans. In this sense they are similar to the Social Security system. However, their terms are more stringent than those of Social Security in one regard. The benefits earned by an employee are supposed to be guaranteed by the government employer. In this sense, the employer issues and guarantees an incremental fixed annuity for each employee every year. In most cases, benefits are wholly or partially adjusted for inflation so a state or local government pension may be considered a fixed real annuity that is at least supposed to be default-free.

Unfortunately, for many governments sponsoring defined benefit pension plans, there is a major incompatibility between the promises made and the funds devoted to support those promises. Contributions made by employees and/or taxpayers are placed in a pool of assets, with benefits paid from that pool. Typically the assets are invested in some combination of bonds, stocks, private equity, hedge funds and possibly other exotica. But the obligations are fixed, usually in real terms. A natural question arises. Why not invest the assets in Treasury bonds (mostly or entirely TIPS) with cash flows matched to those promised, thus defeasing the liabilities? Why should the taxpayers of a government be in a position equivalent to borrowing money from employees, then investing the money in a risky portfolio?
The answer appears to be rooted in the manner in which the contributions to such a pension fund are determined. Every year a registered actuary (or actuarial firm) computes the present value of the benefits earned to date. The resulting number (sometimes called an accrued benefit obligation or ABO) purports to measure the fund's accrued liability, based on the cash flows that would need to be paid in each future year if every employee quit immediately. Since such payments are purportedly guaranteed by the full faith and credit of the sponsoring government, one might assume that the cash flows would be valued using either the term structure of relevant government bonds or at least a single discount rate based on the average maturity of the obligations. Actuaries who do such calculations for corporate pension funds select discount rates relatively close to the current and/or historic yields on corporate bonds. But government actuaries do not. Instead, they use a combination of yields on government bonds and the fund's expected return on assets, with far greater emphasis on the latter. Not surprisingly, this gives estimates for the present value of the liabilities that are much lower than those consistent with economic theory and common sense. It also provides an incentive for the people managing the assets in such pension funds to favor investments that do not provide easily observable market values, since for such investments pension funds are allowed to assume much higher expected rates of return, thereby lowering the calculated value of their liabilities.

A vivid example of the disparity between actuarial and market values of liabilities is provided by calculations made by actuaries for the California Public Employees Pension Retirement System (CALPERS) for those cities, counties and other agencies in California which make contributions to and draw benefits from the System. For each such entity, CALPERS computes an actuarial value of liabilities on which annual contributions are based. But it also offers the local agency the choice to terminate their association with the System and pay a lump sum, with CALPERS agreeing to subsequently pay all accrued benefits as they come due. The discount rate used for the calculation is in fact based on bond yields. Here are the details, from CalPERS Circular Letter No. 200-058-11.

The discount rate assumption to be used for actuarial valuations for employers terminating a contract (or portion of a contract) with CalPERS, and for the annual actuarial valuation of the Terminated Agency Pool, will be a weighted average of the 10 and 30 year US Treasury yields in effect on the valuation date. The weighted average percentages will be the weights that when applied to the duration of the 10 and 30 year US Treasury, determined at current spot rates, equal the duration of the expected benefit payment cash flows of the contract (or portion of a contract in the case of a partial termination) being terminated or the terminated Agency Pool. In addition, the inflation assumption used to project the expected benefit payment cash flows of the contract (or portion of a contract in the case of a partial termination) being terminated or the terminated Agency Pool will be the inflation imbedded in the US Treasury Inflation Protected Securities (TIPS) on the valuation date.
To provide an estimate of the value such computations might provide, each year CALPERS computes a present value based a rounded value of the yield on 20-year Treasury bonds. The table below shows aggregate actuarial and market (termination) liabilities results for a group of over 200 California cities, counties and agencies.

<table>
<thead>
<tr>
<th>Year</th>
<th>Actuarial Discount Rate</th>
<th>Market Discount Rate</th>
<th>Actuarial Liability ($Billion)</th>
<th>Market Liability ($Billion)</th>
<th>Market/Actuarial Liability</th>
<th>Duration</th>
</tr>
</thead>
<tbody>
<tr>
<td>2011</td>
<td>7.5%</td>
<td>4.8%</td>
<td>37.7</td>
<td>53.6</td>
<td>1.42</td>
<td>13.9</td>
</tr>
<tr>
<td>2012</td>
<td>7.5%</td>
<td>3.0%</td>
<td>39.5</td>
<td>71.9</td>
<td>1.82</td>
<td>13.9</td>
</tr>
<tr>
<td>2013</td>
<td>7.5%</td>
<td>3.7%</td>
<td>40.8</td>
<td>65.9</td>
<td>1.61</td>
<td>13.4</td>
</tr>
</tbody>
</table>

Given the disparities between actuarial and market discount rates, the ratio of market value to actuarial value varies widely. But in every case, the market value is greater than the actuarial value. And by billions and billions of dollars.

The last column shows duration – a number derived from the discount rates and liabilities. This is often interpreted as a weighted average number of years in the future when payments are to be made, with the weights based on the relative magnitudes of those payments. A more direct interpretation is that the relationship between the liabilities and the discount rates is the same as it would be if there were only one payment, to be made at a date on the number of years hence indicated by the duration (for example 13.4 years from now for the results in 2013).
For example, assume that there is a payment of $X$, to be made in year $d$. The present value of the payment at an actuarial discount rate of $ra$ (e.g. 0.075 for 7.5%) would be:

$$PVa = \frac{X}{(1 + ra)^d}$$

and the present value at a market discount rate of $rm$ (e.g. .048 for 4.8%) would be:

$$PVm = \frac{X}{(1 + rm)^d}$$

The ratio of the present values would then be:

$$\frac{PVm}{PVa} = \frac{(1+ra)^d}{(1+rm)^d}$$

or:

$$\frac{PVm}{PVa} = (\frac{1+ra}{1+rm})^d$$

This provides the market/actuarial liability multiple, given the two discount rates and duration.

Another version can be used to determine duration, given the discount rates and present values. Starting with the previous formula, taking the logarithms of both sides, then rearranging gives:

$$d = \frac{\log(PVm / PVa)}{\log((1+ra)/(1+rm))}$$

This is the formula used to derive the implied durations shown in the table. Interestingly, the three estimates are quite similar. This suggests that one might assume a duration of roughly 14 years would suffice to estimate the market value of a set of liabilities, given the actuarial value ($PVa$), and the actuarial and market discount rates ($ra$ and $rm$).
The market values of liabilities computed by CalPERS for member cities and agencies in California can be found, along with a great deal of other information, at ca.pensiontracker.org, a site maintained by a team at the Stanford Institute for Economic Policy Research. A companion site, at us.pensiontracker.org, provides information about public pensions in every U.S. state. The summary information for the country in 2014, from the latter home page is shown below:

**Pension Debt**

**United States Public Employee Pension Systems**

<table>
<thead>
<tr>
<th>Market Basis</th>
<th>Actuarial Basis</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total Pension Debt: $4.833 trillion</td>
<td></td>
</tr>
<tr>
<td>Pension Debt Per Household: $41,219</td>
<td></td>
</tr>
<tr>
<td>Total Pension Debt: $1.040 trillion</td>
<td></td>
</tr>
<tr>
<td>Pension Debt Per Household: $8,872</td>
<td></td>
</tr>
</tbody>
</table>

The figures are for unfunded liabilities. Using the market value of assets and the actuarial value of liabilities, governmental pension systems are underfunded by slightly over $1 Trillion. Dividing by the number of households in the United States gives a debt of $8,872 per household. But this does not really represent the economic value of the liabilities. Comparing assets with the market values of the liabilities (using our formula for each state with a duration of 14 years) results in a debt of over $4.8 Trillion, equal to $41,219 per household (rich or poor). Not as bad as Social Security, perhaps, but sobering.
Of course, the situation differed from state to state. The following figure shows the values for all of the states, ranked from lowest to highest:

Alaska's situation was the worst, with an unfunded liability of over $113,000 per household. Illinois was next, with almost $78,000 per household, followed closely by California, with a total of $77,000 per household. You can find your state at the web site.
The situation is depressingly similar at national, state and local levels: one finds promises of substantial future incomes coupled with insufficient taxes collected. Why? One can't help but suspect a combination of two elements. First, it is highly useful politically to promise some voters future guaranteed benefits while minimizing the apparent current costs for other voters. This is enabled at the state and local level by complicit actuaries and at the national level by the complexity of the social security system. In both cases, the process is facilitated by the complexity of the relationships between current and future values. To use some political jargon: politicians with relatively short horizons are sorely tempted to kick the long-term pension can down the road.

Despite the complexity of the subject, there is a growing awareness by some members of the general public about the dire state of public pensions. As a result, there have been numerous attempts, both legislative and judicial, to reduce the amounts promised government workers. Public employee unions and others have argued that terms of a government pension plan are sacrosanct and constitute a binding and irrevocable contract made between the employer and employee at the beginning of service. Others argue that this holds only for benefits for those already retired and for benefits already earned (accrued) by current employees. Yet others argue that post-retirement cost of living adjustments are not binding contractually and may be adjusted or eliminated if needed. Perhaps the most stringent position is that taken by the California Supreme Court. In a 1955 decision the court held that “... changes in a pension plan which result in disadvantage to employees should be accompanied by comparable new advantages.” This “California Rule” has been interpreted by the courts to hold that pension benefits already earned and those that would accrue if an employee were to continue working for the government or agency are both protected.

Those advocating for reductions in government pension benefits (already earned and/or to be earned) often claim that the amounts paid employees are excessive. One sometimes reads about a former mid-level civil servant living on an income of hundreds of thousands of dollars per year, fully adjusted, as needed, for increases in the consumer price level. Studies have attempted to determine whether government employees are in any sense “overpaid” relative to those in the private sectors. Overall, the results have varied. But what does seem true is that on average governmental positions provide somewhat lower salaries but somewhat higher post-retirement incomes than comparable positions in the private sector. One rationale for this is the desire to keep civil servants loyal and honest by “back-loading” their total compensation with a reward that can be withdrawn in case of criminal activity, etc.. Perhaps the best test of whether or not government employees are over-compensated, given their skills and work habits, is the number of qualified applicants that apply for each position. Whenever a job is posted, if large numbers of qualified applicants apply, it may well be that working conditions, salaries and/or benefits are indeed over-generous. If not, overall compensation, including benefits, may be appropriate after all.
Corporate Retirement Plans

Historically, most large private corporations in the United States provided employees with defined benefit pension plans. Some such plans remain, but in recent years, many such employers have closed their defined benefit plan (or “frozen” it to cover only employees previously hired), relying instead on the provisions of section 401(k) of the Internal Revenue Code, that provides for deductions from the firm's taxes and deferral of an employee's income tax for money put into an employee's defined contribution plan by the employer or employee. Such plans are known as 401(k) plans in the private sector and 403(b) plans in the public sector, based on the relevant sections of the IRS code. Both allow the resulting funds to reinvest interest and dividends without taxation. Instead, any (and all) withdrawals are included in the taxable income of the recipient.

In a typical defined contribution (DC) plan, there are assets but no liabilities per se (unless funds have been borrowed temporarily prior to retirement). Thus there is no sense in which a DC plan can be considered underfunded. (Some have argued for computing the cost for which a sufficient annuity could be purchased and comparing the asset value of a DC plan with that amount, but this is rarely done.)

Despite major movement towards the adoption of defined contribution plans, the corporate sector in the United States still has some assets and liabilities in defined benefit plans. The status of the corporations in Standard and Poor's 500 stock index was summarized in the 2016 Wilshire Consulting Report on Corporate Pension Funding Levels. Based on the latest available valuations, the DB funds of these firms had $1.32 Trillion of assets and $1.61 Trillion of liabilities, with an unfunded liability of $0.29 Trillion. Of course it is important to know the discount rates on which the liability valuations were based. In this case they varied, but the reported median rate was 4.37%. The Internal Revenue Service rules allow corporations to discount pension liabilities using an historical average yield on high-quality corporate bonds. Generally such yields are higher than those on U.S. Treasury securities such as the rates used in the Pension Tracker calculations. Nonetheless, in the United States the unfunded liabilities of corporate defined benefit plans are dwarfed by those of federal, state and local government plans.

In most cases, it should be possible to model payments from a corporate defined benefit plan using the $iFixedAnnuity$ functions developed in Chapter 10. These functions might also be used to approximate the payments provided by beneficiaries' Social Security plans. But to cover at least some of the nuances of the Social Security system, we will create new functions specifically designed for Social Security benefits.
The iSocialSecurity functions

Given the complexities of Social Security benefits, our functions must accommodate an number of alternative patterns of income. The approach we will take attempts to compromise between a desire for simplicity and one for generality. The victims of this choice are parsimony and beauty for the resulting code.

As usual, we will use one function to create parameters and another to process the information, along with that for the client and market data structures, resulting in a matrix of incomes, to be added to an existing client incomes matrix.

The first element in the iSocialSecurity data structure contains information required to create a vector of incomes for personal state 3. Recall the case of Tom and Mary Jones. In the first four years, if both Tom and Mary are alive, the couple gets Tom's earned benefit of $38,700 plus Mary's spousal benefit of $16,700, for a total of $55,400. For all subsequent years, if they are both alive, they receive Tom's earned benefit of $38,700 plus Mary's earned benefit of $42,300, for a total of $81,000. Here is a statement designed to reflect this.

\[
\text{isocialsecurity.state3incomes} = [55400 \ 55400 \ 55400 \ 55400 \ 81000];
\]

This is considered a partial version of a full vector in which the last element is repeated until the last year in the client matrices. For some cases, a single value may suffice (equivalent to a vector of length 1); the value will then be repeated for every subsequent year.
The function that will process this information is (of course) named `iSocialSecurity_process`:

```matlab
function client = iSocialSecurity_process ( iSocialSecurity, client, market );
```

It begins by setting some local variables:

```matlab
% get number of scenarios and years
[nscen nyrs] = size( client.pStatesM );
% save personal states
pStatesM = client.pStatesM;
% create social security incomes matrix
incomesM = zeros( nscen, nyrs );
```

The next task is to add incomes for personal state 3 to the `incomesM` matrix. But, as we have indicated, the input values will generally be in a partial version of a full vector, with the last element equal to the income that will be received for every subsequent year if both are alive. Here are the statements from the `iSocialSecurity_process` function that create an extended version of the vector.

```matlab
% extend input vector
vec = iSocialSecurity.state3Incomes;
if length( vec ) > nyrs;  vec = vec( 1: nyrs );  end;
lastval = vec( length(vec) );
vec = [ vec  lastval*ones( 1, nyrs-length(vec) ) ];
```

The next section of the function, shown below, starts by creating a matrix, `allIncomes`, with the state 3 incomes vector in each row. Next, a matrix, `states`, is created with a value of 1 (true) in every scenario/year cell in which Tom and Mary are alive (personal state 3). Finally, the value in each cell of the first matrix is multiplied by the value in the corresponding cell of the second matrix. The result is a matrix, `stateIncomes`, that includes the incomes in all instances of personal state 3, with a value of zero in each of the other cells.

```matlab
% create matrix with incomes for personal state 3
allIncomes = ones( nscen, 1 ) * vec;
states = ( pStatesM == 3 );
stateIncomes = states .* allIncomes;
```

This task completed, we add the matrix with the state 3 incomes to the previously empty matrix designed to include the incomes for each of the possible personal states:

```matlab
% add to incomes matrix
incomesM = incomesM + stateIncomes;
```
So much for personal state 3. We turn now to the more complex tasks of describing and processing personal states 1 and 2.

Here is a statement that indicates Tom's potential incomes:

\[
\text{iSocialSecurity.state1Incomes} = [ 
\begin{array}{cccc}
\text{Inf} & 38700 & 38700 & 38700 \\
\text{Inf} & \text{Inf} & 38700 & 38700 \\
\text{Inf} & \text{Inf} & \text{Inf} & 38700 \\
\text{Inf} & \text{Inf} & \text{Inf} & \text{Inf} & 42300 \\
\end{array}
\];
\]

Consider his situation. Tom's earned benefits, already being paid, are $38,700 per year. Mary will collect $42,300 per year if she begins payments in year 5. This is 1.32 times her primary insurance amount (PIA) of roughly $32,050 per year - the annual income she would receive if she were to start payments at her full retirement age. There is no chance that Tom can become a widower in Year 1 since it is the present and they are both alive. But he might be one in year 2. If so, he would be eligible to continue to receive the amount he is now getting ($38,700 per year) or Mary's PIA, whichever is greater. In this case, $38,700 is more than $32,050, so if Tom becomes a survivor in year 2 he will get $38,700 per year for the rest of his life. The first row of the matrix, when extended to the right, indicates this. We use \textit{Inf} (the symbol for infinity) to indicate an irrelevant entry.

Now consider the possibility that Mary will die in year 2 so that Tom becomes a survivor in year 3. Here too, Tom will be able to obtain only $38,700 per year in that year and all subsequent years. This will also be the case if Tom becomes a survivor in years 3 or 4. But if Mary dies in any later year, she will have been receiving a personal benefit of $42,300 per year. And Tom would then be eligible to receive either his own benefit of $38,700 or Mary's of $42,300. Of course he will take the latter. Thus Tom's income from social security will be $42,300 if he becomes a widower in year 5 or any year thereafter. Clearly, the last row in this matrix should have a non-zero value only in the last column (and ours does).

We could extend the matrix for more years but this would be a bother. The last row can be understood to apply to all cases in which Tom becomes a widower in or after year 5.
In this case, the situation is the different if Mary becomes a widow. If Tom dies in years 1 through 4, before she begins collecting her own benefits, she will receive a survivor benefit of $38,700. But if he dies after year 4, she will have started receiving her own benefit, and Social Security will pay her the largest benefit for which she is eligible, which in this case is her own amount of $42,300.

Here is a statement providing the information:

```matlab
SocialSecurity.state2Incomes = [   Inf    38700  38700  38700  42300
   Inf    Inf       38700   38700  42300
   Inf    Inf       Inf        38700  42300
   Inf     Inf       Inf       Inf       42300];
```

This way of representing the potential benefits to a surviving spouse is convoluted, at best. And it is not fully general. If, for example, Mary had an ex-husband or minor children, they might be eligible for benefits after her death but our programs would not be able to represent this. But our goal is only to provide programs that can represent social security benefits in at least some of the more common situations, leaving broader coverage to others.

Now to the use of this information in the `iSocialSecurity_process` function. For parsimony we use a single set of statements to handle both a case in which person 1 is the survivor and one in which person 2 has that distinction. The main section, with some missing statements (to be shown next), has this form:

```matlab
% add incomes for personal states 1 and 2
for s = 1:2

% get input matrix and personal state matrix
if s == 1
    m = iSocialSecurity.state1Incomes;
else
    m = iSocialSecurity.state2Incomes;
end;

% extend input matrix
[nrows ncols] = size( m );
if ncols > nyrs; m = m( :, 1:nyrs ); ncols = nyrs; end;
lastcol = m( :, ncols );
umadd = nyrs - ncols;
if numadd > 0; m = [ m lastcol*ones(1,numadd) ]; end;

% .......
end; % for s = 1:2
```
For each of the two personal states, we save the input matrix as matrix $m$, then extend it to the right in a manner similar to that used earlier for the vector for personal state 3. It remains to describe the statements in the missing sections.

We know that each line in $m$ applies to all scenarios in which a spouse dies in a particular year. For example, the first line in $m$ applies for every scenario in which the personal state equals 3 in year 1 and 1 in year 2. The second line applies for every scenario in which the personal state equals 3 in year 2 and 1 in year 3. And so on, until we come to the last line. It, or the portion remaining, applies for every case in which Tom and Mary are both alive in the first four years and Mary becomes a widow after she has started receiving her own benefits. Conveniently, each such scenario is distinguished by the presence of a 3 in the personal state matrix in column 4.

Here are the statements that process all but the last row in an input matrix:

```matlab
% process all but last row
for i = 1: nrows-1
    % get row from matrix
    incrow = m( i, : );
    % find column for last 3
    last3col = sum ( incrow == Inf ) ;
    % replace Inf with zero in incrow
    for c = 1 : last3col;  incrow(c) = 0 ;  end;
    % create vector with s in pStateM rows with desired sequence of 3 and s
    psrows = ( pStatesM(:, last3col) == 3 )  &  ( pStatesM(:, last3col+1) == s );
    % make matrix with incrow in every eligible row
    mm = psrows * incrow;
    % set all cells with state not equal to s to zero
    mm =  mm .* ( pStatesM == s );
    % add to incomes matrix
    incomesM = incomesM + mm;
end; % for i = 1: nrows-1
```

Each row but the last from the income matrix is processed in turn. We start by finding the last entry with the $Inf$ symbol, since this is the last column that should contain 3 in the personal state matrix. Next we replace all the $Inf$ entries with zeros. The next statement creates a vector with 3's where the personal states should be 3 and the state number in question (s) in all other columns. Next we create a column vector with 1 (true) in every row which has a 3 in the desired column and a value equal to s in the next column and a 0 (false) in every other row. Multiplying this vector by the selected row of from the incomes matrix provides the income vector in only those rows that meet the criterion concerning when Tom or Mary become survivors. Next, we multiply our new matrix by a matrix that has 1's in cells for which the person in question (s) is alive and zeros elsewhere. Then we add the resulting matrix to $incomesM$, which we are using to store all the incomes generated in the function.
We use a similar process for the last row of the matrix, adapting the approach to reflect the fact that it applies to all the cases in which the person in question becomes a survivor after a given year:

```matlab
% process last row
% get row from matrix
incrow = m( nrows, : );
% find column for last 3
last3col = sum( incrow == Inf );
% replace Inf with zero in incrow
for c = 1: last3col; incrow(c) = 0; end;
% create vector with 1 in pStateM rows with >= the number of 3s
psrows = ( pStatesM(:, last3col ) == 3 );
% make matrix with incrow in every eligible row
mm = psrows * incrow;
% set all cells with state not equal to s to zero
mm = mm .* ( pStatesM == s );
% add to incomes matrix
incomesM = incomesM + mm;
```

The approach is similar to that used previously, except that here we are concerned with all rows in which there is a 3 in the chosen column. Thus all the statements are the same except the one that creates the `psrows` vector. Of course we could have combined the two sections, but chose to make them separate to at least slightly reduce the difficulty of following the logic.

These matrix operations are somewhat difficult to understand, but they provide remarkable efficiency. For the Jones case, the total time required to run the `iSocialSecurity_process` function on the author's vintage Macbook pro was less than one second. One shudders to even think about the time the process might have consumed had one used loops to process each of the 100,000 rows of the matrix one at a time. Once again, matrix operations serve us well.
Social Security Present Values

Once we have expended the considerable effort to find the amounts that Social Security will pay Tom and Mary, then put the requisite information in an iSocialSecurity data structure, we can easily examine the benefits using the tools in our analysis function. For example, we could add these statements to a JonesCase script:

```plaintext
analysis.caseName = 'Jones Case';
analysis.plotRecipientPVs= 'y';
```

The result would be a graph showing the present values of possible future benefits:

![Graph showing present values of possible future benefits](image)

The total present value is large: over $1.5 Million. Of course, Tom and Mary received the maximum possible amounts available for any couple their ages, so this may not be a total surprise. We'll return to our less affluent friends Bob and Sue shortly. But before doing so, note that almost 75% of the present value comes from potential payments when both Tom and Mary are alive (shown in green) since neither receives as much when he or she is alone. Of the remainder, considerably more of the present value comes from Mary's payments for two reasons: first, if she survives past 70, her payments will be larger and second, she is younger and female, and thus likely to live longer than Tom. Finally, there are no explicit fees nor have we included any possible amounts that Social Security might pay to other relatives once Tom and Mary are both gone. Thus both the Fees and Estate present values are zero.
Bob and Sue's Social Security Income

Tom and Mary have served us well, but it is time to part company with them and return to Bob and Sue, the main protagonists of this work. We do so with some embarrassment, since they each decided to claim Social Security benefits just before becoming our example retirees. Bob (now 67) did so when he reached his full retirement age. Sue (now 65) did so because she had a medical condition, now resolved. While Bob had done well professionally and contributed substantially to Social Security, Sue had chosen to work in the non-profit sector, earning less and accumulating lower Social Security benefits. The net amounts they will receive each month after deductions for the standard Medicare Part B premium and the income-related monthly adjustment amount (based on their previous year's income tax return) are: $2,500 for Bob and $1,200 for Sue. Multiplying by 12 gives $30,000 per year for Bob and $14,400 per year for Sue. If both are alive (personal state 3), they will receive $44,400 ($30,000 + $ 14,400) per year. If only Bob is alive (state 1), he will get $30,000 (his benefit). And if only Sue is alive (state 2), she will get a survivor benefit of $30,000 (equal to Bob's benefit, which was larger than hers).

To reflect this, we add a few statements to the SmithCase script:

```plaintext
% incomes in state 1, last column repeated for subsequent years
iSocialSecurity.state1Incomes = [Inf 30000]
% incomes in state 2, last column repeated for subsequent years
iSocialSecurity.state2Incomes = [Inf 30000]
% incomes for state 3, last column repeated for subsequent years
iSocialSecurity.state3Incomes = [44000]
% process social security
client = iSocialSecurity_process (iSocialSecurity, client, market);
```
Despite their somewhat modest incomes, Bob and Sue's Social Security benefits are still substantial. Adding these statements to the SmithCase script:

```octave
% create analysis
analysis = analysis_create( );
analysis.plotRecipientPVs = 'y';
% process analysis
analysis_process( analysis, client, market );
```

Produced:

This shows that Bob and Sue's Social Security benefits are worth roughly $950,000 (the exact value will depend on the scenarios generated in a given analysis). If they were to purchase a commercial fixed annuity with similar payments, it would almost certainly cost them well over a million dollars due to the addition of expenses and fees. Social Security annuities can be very valuable indeed.

For a great many reaching retirement age in the United States, the value of Social Security greatly exceeds the amounts in savings and other retirement accounts. No wonder that so many older voters vigorously support its continuation at present levels.

This said, many retirees do have other funds that can provide income in their retirement years. In the remainder of this book we will consider different ways to invest and/or annuitize such wealth. But different income sources should not be treated in isolation. Rather, retirees should consider combinations that include income from other sources and from social security, which will hopefully continue to provide an important anchor for most retirees' consumption.