21. Advice

Financial Advisors

If there is any conclusion to be reached after reading the prior twenty chapters it is this: comprehending the range of possible future scenarios from any retirement income strategy is very difficult indeed, and choosing one or more such strategies, along with the associated inputs, seems an almost impossible task. At the very least, retirees will need some help.

Enter the Financial Advisor. Ideally, he or she will have a deep background in the economics of investment and spending approaches, sufficient analytic tools to determine the ranges of likely outcomes from different strategies, and an ability to work with clients to find approaches that are suitable, given their situation and preferences. Moreover, the amount charged for providing such advice should be well below its added value. Tall orders indeed.

In the United States, there are many designations for financial advisors, based on completion of educational programs, sufficient scores on examinations, etc.. Some states place restrictions on those offering financial advice, but there are no uniform standards and in many cases no significant requirements for those who wish to provide such advice.

An important sub-category is described in Wikipedia:

*A Registered Investment Adviser (RIA) is an investment adviser(IA) registered with the Securities and Exchange Commission or a state's securities agency. The numerous references to RIAs within the Investment Advisers Act of 1940 popularized the term, which is closely associated with the term investment advisor (spelled "investment adviser" in U.S. financial law). An IA is defined by the Securities and Exchange Commission as an individual or a firm that is in the business of giving advice about securities.*
Importantly, an RIA must meet a specific standard when giving advice. Again, from Wikipedia:

An IA must adhere to a fiduciary standard of care laid out in the US Investment Advisers Act of 1940. This standard requires IAs to act and serve a client's best interests with the intent to eliminate, or at least to expose, all potential conflicts of interest which might incline an investment adviser—consciously or unconsciously—to render advice which was not in the best interest of the IA's clients.

This “client best interest” standard is not required for those associated with security brokerage firms. Instead:

Registered Representatives (RRs) affiliated with a Broker Dealer are ... required to recommend securities that are deemed "suitable" for non-institutional clients.

In the years leading up to 2017, the U.S. Department of Labor solicited comments on regulations that would require anyone giving financial advice related to retirement savings to conform with the fiduciary standard, thus acting in the client's best interest. The final version was expected to become effective in spring, 2017. However, in one of its early actions, the Trump Administration postponed the date to allow for further review, thus requiring only recommendations that are “suitable”. Of course with regulation, the devil is always in the details, and many pages of regulatory documents and court decisions are devoted to both the “best interest” and “suitable” standards.

Whatever the required standard may be for a Financial Advisor, it behooves clients to request relevant information concerning compensation. Does the advisor charge an annual fee that is a percentage of the client's total assets? A simple hourly fee? Or is the advisor compensated by firms providing financial vehicles, such as annuities, mutual funds, etc.? And, in the latter cases, how are the amounts determined? At the very least, the client needs information essential for determining whether the provider of advice may have some biases related to compensation.

It is all too easy for a client to underestimate the impact of financial advisory fees on expendable retirement income. A fee of 1% of total assets each year may seem small, but this can reduce spendable lifetime retirement income by as much as 20%.
It is important that advisory fees be taken into account when making income projections. As will be discussed later, the most appropriate approach would use and, if needed, modify the RISMAT software to include such fees so that their impact will be shown in the present value pie chart.

In some cases, a simpler approach could still provide relevant projected incomes. For example, if an advisor charges x% of assets each year, the expected real returns on the market portfolio and Tips could be decreased by that amount. However, this would completely obscure the true cost of the advisory fees and should not be considered an acceptable practice. Fees consume a portion of retirees' savings and the client needs to know the present value of their cost.
**The Family Doctor Analogy**

A common trope holds that a good Financial Advisor is like a fine family doctor (or an internal medicine specialist or possibly a geriatric physician). Such a doctor has deep scientific knowledge, can assess client needs, habits and willpower, is able to provide (or have provided by others) scientific diagnoses, and can communicate results of such analyses to the client in simple terms so that the best treatments can be applied.

While this view may be overly optimistic about many family doctors as well as financial advisors, it can serve as an aspiration (in the non-medical sense of the term) for both.

Here is a somewhat forced analogy. A radiologist is able to analyze in detail the results of an X-ray, MRI or CT scan. He or she can forward to a family doctor some images plus a summary of findings and possible treatments. The family doctor then can describe the diagnosis to the patient, show and explain some of the images, then discuss possible treatments.

Now consider the functions of a financial advisor dedicated to helping retirees choose among a bewildering array of possible sources of future income and their associated parameters. The RISMAT software is designed to help in this process. In a small firm, there could be one or more technology specialists with detailed knowledge of the software's functions and the ability to adapt or augment them to include additional income sources and/or relevant aspects not included in the versions included in the original version. Or such specialists could be employed by a separate firm. In either case, the person or persons working directly with clients could focus more on communicating possible outcomes, helping the clients understand the options, then implementing some or all of the chosen approaches. Such a “family retirement doctor” could help each client or pair of clients understand relevant graphs, discuss alternatives, then make informed choices.

Some will take umbrage at this analogy. In most countries, medical doctors must complete years of arduous education and training, be certified and can be denied the right to practice in there is evidence of malfeasance or incompetence. Sadly, some or all of these conditions are missing for the practice of financial advice, although some certifications are available. This said, many diligent, well-educated and dedicated financial advisors focus on helping retirees make intelligent retirement income choices. It is the author's hope that the RISMAT software could help them do so more even more effectively.
Financial Education

Some financial advisors have undergraduate degrees in Finance or Economics. Others have MBA (Masters in Business Administration) degrees, possibly with an emphasis on Finance. Yet others have degrees in other fields, have learned the requisite skills on the job, or have taken courses in programs designed specifically for those planning to provide financial advice.

In the United States, anyone not employed by a broker or dealer who wishes to give investment advice must conform with the previously described fiduciary standard. However, as indicated earlier, Wikipedia tells us that:

Section 202(a)(11)(C) of the Investment Advisers Act of 1940 exempts from the definition of an Investment Adviser (and therefore the associated fiduciary standard) "any broker or dealer whose performance of such services is solely incidental to the conduct of his business as a broker or dealer and who receives no special compensation therefor."

The United States Securities and Exchange Commission (SEC) requires registration for most, but not all, those who provide investment advice. Excluded are professions whose advice is “solely incidental” to the firm or individual's main business. Examples include broker-dealers, lawyers, accountants, and teachers. Others must register: those managing assets totaling less than $100 million with the securities agency of the state with their principal place of business, and those with more assets with the SEC. Representatives of a firm registered with the SEC who provide investment advice must pass an examination (the Series 65 Uniform Investment Adviser Law Examination) or hold an approved designation. According to Wikipedia, in 2017, approved certifications were: Certified Financial Planner (CFP), Chartered Financial Consultant (ChFC), Personal Financial Specialist (PFC), Chartered Financial Analyst (CFA) and Chartered Investment Counselor (CIC).
The requirements for some of these certifications can be substantial. For example, to obtain the Certified Financial Planner (CFP) designation, one must have a bachelor's degree (or higher) from a certified accredited college or university, pass an examination, conform with ethics guidelines and have completed 4,000 or more hours of qualified experience. In the CFP board's 2015 study, seventy-two Principal Knowledge Topics were identified, falling into eight groups:

- Professional Conduct and Regulation
- General Financial Planning Principles
- Education Planning
- Risk Management and Insurance Planning
- Investment Planning
- Tax Planning
- Retirement Savings and Income Planning
- Estate Planning

Many of the detailed topics are addressed in previous chapters, but there are important exceptions, some of which we will identify in a later section. Suffice it to say here that to give useful retirement income advice, one needs many skills. That said, it is the author's belief that an understanding of the issues covered in this book, an ability to make useful projections of the ranges of possible future outcomes and their properties, plus the ability to effectively communicate such projections to a client could significantly enhance the ability of a Financial Advisor to help clients make appropriate retirement income plans.

One of the seventy-two Topics (G.61) relates to this book: Retirement income and distribution strategies. But it is unlikely that all or even a majority of those offering education and training for Financial Advisors include extensive instruction in matrix operations, valuation of uncertain outcomes over many future years, and some of the more technical matters that we have covered. This suggests that a firm offering financial advice to retirees might be well served by augmenting its financial advisors with an employee or consultant with deep technological skills.
Enter the International Association for Quantitative Finance, devoted to the field of Financial Engineering. From the IAQF web site:

*Financial engineering is the application of mathematical methods to the solution of problems in finance. It is also known as financial mathematics, mathematical finance, and computational finance.*

*Financial engineering draws on tools from applied mathematics, computer science, statistics, and economic theory.*

*Investment banks, commercial banks, hedge funds, insurance companies, corporate treasuries, and regulatory agencies employ financial engineers. These businesses apply the methods of financial engineering to such problems as new product development, derivative securities valuation, portfolio structuring, risk management, and scenario simulation.*

*Quantitative analysis has brought innovation, efficiency and rigor to financial markets and to the investment process. As the pace of financial innovation accelerates, the need for highly qualified people with specific training in financial engineering continues to grow in all market environments.*

The web site also provides lists of degree programs in the field. Again, from the IAQF web site:

*There are dozens of financial mathematics masters degree programs around the world. The majority of these programs are in the United States, but there are programs in Canada, the United Kingdom and continental Europe as well. Masters programs usually run from one to two years in length with the major differences between programs being the curriculum’s distribution between mathematical, statistical and computational techniques, and financial theory and its applications. This is further reflected in the choice of faculty, and in particular the balance between tenured professors and practitioners.*
A non-scientific review of information on students graduating from some programs indicates that a substantial majority list MATLAB proficiency on their vitae sheets. And the inclusion of *scenario simulation* in the above list of applications suggests that retirement income scenario matrix analysis should not provide a major challenge to typical holder of such a degree. Perhaps an course could be included in the curriculum that uses our material as a base (*hint!*).

That said, the list of types of employers provided by the IAQF does not include financial planners, nor does the list of applications include retirement income analysis. Perhaps this will be rectified in future years.
Robo-Advisors

The second decade of the twenty-first century has seen the introduction and growth of financial advisory firms that deliver information, analysis and recommendations over the internet and, in many cases, implement the recommendations by purchasing securities (usually exchange-traded fund shares) and/or mutual fund shares. Some provide optional contact with human advisors, usually by phone or internet exchanges; others do not.

Most such robo-advisors focus primarily on the accumulation phase of retirement saving, but some also cover the decumulation phase. Fees, often charged as a percentage of assets under management, are typically no more than 0.50% (50 basis points). In some cases, the expense ratio also depends on the total value of assets managed for an account, with lower percentages for larger amounts managed. In addition to the advisor's fees, the client pays the expenses charged by fund providers, although robo-advisors tend to favor passively managed index funds or ETFs with relatively low expenses.

In early 2017, the range of robo-advisor offerings was growing, with considerable diversity among providers. Vanguard offered its Vanguard Personal Advisor Service® using “low-cost Vanguard funds” for a fee of 0.30% (30 basis points) per year (there is also a minimum account size and a cap on total fees paid). Users of the service can call a personal advisor, exchange emails or video chat with him or her for no extra charge. Charles Schwab offered Schwab Intelligent Portfolios® stating that “no advisory fees, no commissions and no account service fees are charged” and that “The operating expenses you'll pay on the exchange-traded funds (ETFs) in your portfolio are the same as those you'd pay if you invested in them on your own.” In 2017, Schwab added Schwab Intelligent Advisory, which includes virtual meetings with a Planning Consultant for a total cost of 0.28% of assets (excluding cash accounts). Fees charged by other robo-advisors varied considerably.

Such advisors usually recommend investment in several index funds. A skeptic might argue that the use of a number of funds is adopted to give the appearance of greater customization to the client's needs and/or to suggest the need for special skills applied to construct a multi-asset portfolio. Most advisors tailor portfolios to a client's risk tolerance (either self-described or inferred from answers to questions about choices in risky settings). Some robo-advisors also take into account the differential taxation of different sources of income and the tax effects of realizing capital gains.
Most robo-advisors focus on asset classes, each of which can be represented by an index mutual fund or ETF. Often, proprietary methods are utilized to estimate one-period asset class expected returns and standard deviations of return as well as the correlations of returns among such asset classes. Then, given these estimates, as well as measures specific to a particular client, the most efficient combination is found – typically one that will maximize an objective function with portfolio expected return and standard deviation as arguments plus a parameter indicating the client's willingness to accept risk in order to increase expected return. Such approaches generally assume (explicitly or implicitly) that the investor's utility is a quadratic function of one-period investment return.
Investing versus Betting

Anyone who recommends a particular set of investments for an individual, couple, or institution should have a rationale for the recommendation. For example, why hold more than market proportions of investment A? Market proportions of investment B? Less than market proportions of investment C? And none of investment D?

One approach is to start with the assumption that security prices are consistent with an equilibrium in which all investors share a set of probabilistic forecasts, but due to differences in preferences and circumstances they should hold different portfolios. Recommendations made in such a context can be considered investment.

Another approach is to reject this premise, holding that some securities or classes of securities are “overpriced” and others “underpriced”, due at least in part to ignorance among some investors about some information, psychological impediments, or other such elements of the human condition. In this view, causal effects are not randomly distributed among investors, leading to the conclusion that there may be consistent and predictable errors in security prices. Recommendations made to exploit such presumed errors can be considered betting.

For those using the Markowitz one-period mean-variance model, a formal way to approach the task of investment in such a setting is to use an equilibrium model to derive a set of asset expected returns, risks and correlations that are consistent with equality between the sum total of investors' optimal holdings and the amounts available. The Capital Asset Pricing Model is such a model, although more complex models of efficient markets exist. If one accepts the premises of the CAPM, it is possible to make estimates of asset class risks and correlations, derive the beta of each asset class relative to the market, estimate the expected returns of any two assets, then compute expected returns for the other assets by assuming that the expected return of every asset falls on a line in a diagram with expected return on the vertical axis and beta on the horizontal axis (this process is sometimes termed reverse optimization). Given these estimates, optimal portfolios for clients with different degrees of risk tolerance, tax statuses, horizons, etc. can then be determined. As indicated earlier, recommended holdings using such estimates could be considered investment.
Some approaches assume that capital markets are more complex but that securities are still priced efficiently. In such models, investors may differ in not only risk in tolerance, but also in location, tax status, or other respects. In such a model, in equilibrium different investors may hold significantly different portfolios but the sum total of investors' appropriate holdings should equal that available (the market portfolio). The resulting recommendations for particular clients would still be considered investing, rather than betting.

In contrast, some advisors implicitly or explicitly estimate asset class expected returns based on a combination of historic statistics and their views of market inefficiencies. As described in Chapter 7, some do this formally, using a technique developed by Fischer Black and Robert Litterman, first published in their article “Asset Allocation, Combining Investor Views with Market Equilibrium”, in the September 1991 Journal of Fixed Income. The Black-Litterman approach starts with the determination of equilibrium expected returns using reverse optimization based on the CAPM. Next, the advisor creates his or her own “views” about asset expected returns. Finally, Bayesian statistical methods are used to create a final set of asset expected returns that takes into account both the set of expected returns, asset risks and correlations, and the advisor's confidence in its views.

Some robo-advisors (including Wealthfront) make public their historic and assumed asset expected returns, standard deviations and correlations, but many do not. Such inputs undoubtedly vary from advisor to advisor. However, a cursory examination of example asset mixes recommended by different robo-advisors suggests that they have varying views about the relative desirabilities of different asset classes, likely due in large part to varying views about expected returns. This is not unusual. The table below shows some such forward-looking long-term capital market assumptions made by four large investment firms (none a robo-advisor) in early 2017. Each estimate is of an estimated compounded long-term geometric nominal return, which incorporates both annual expected returns and the standard deviation of annual returns. The estimates vary widely. It would not be surprising to find similar variations among those providing advice to individual investors.

<table>
<thead>
<tr>
<th></th>
<th>Firm A</th>
<th>Firm B</th>
<th>Firm C</th>
<th>Firm D</th>
</tr>
</thead>
<tbody>
<tr>
<td>US Large Cap Stocks</td>
<td>6.25</td>
<td>4.70</td>
<td>5.00</td>
<td>5.90</td>
</tr>
<tr>
<td>US Small Cap Stocks</td>
<td>7.00</td>
<td>4.80</td>
<td>7.20</td>
<td>5.90</td>
</tr>
<tr>
<td>Developed non-US Stocks</td>
<td>6.75</td>
<td>9.70</td>
<td>2.70</td>
<td>6.50</td>
</tr>
<tr>
<td>Emerging Market Stocks</td>
<td>9.25</td>
<td>8.60</td>
<td>5.80</td>
<td>5.70</td>
</tr>
<tr>
<td>US Aggregate Bonds</td>
<td>3.00</td>
<td>3.30</td>
<td>2.50</td>
<td>3.10</td>
</tr>
<tr>
<td>Tips</td>
<td>3.50</td>
<td>2.60</td>
<td>1.80</td>
<td>2.70</td>
</tr>
</tbody>
</table>
Yield Tilts


Then (and now), for stocks not held in tax-exempt or tax-deferred accounts, the United States federal government includes dividends in taxable income for the year in which they are received. On the other hand, taxes on capital gains on stocks in such accounts may be deferred until a security is sold; and even then, a lower tax rate will generally apply. In tax-deferred accounts, such as 401Ks and IRAs, neither dividends nor capital gains are taxed until funds are withdrawn, at which time the entire amount withdrawn is taxed. And in tax-exempt accounts, neither dividends nor capital gains are taxed. These differences in tax treatment led to the conjecture that security prices might conform with an equilibrium in which a stock’s expected return is a function of both its beta relative to the market portfolio and its dividend yield. Studying returns from 1936 through 1977 on all stocks listed on the New York Stock Exchange, Litzenberger and Ramaswamy found that overall, high-yield stocks had returned more for given beta values.

Given such differential taxation, security prices may adjust so that an equilibrium is attained in which a tilt towards higher dividend yield may be favorable for tax-exempt or tax-deferred accounts and a tilt towards capital gains for taxable accounts. In the sense of our definitions, such strategies would be considered investing, not betting.

In 1978, based on earlier versions of this research, Wells Fargo Investment Advisors registered a trademark for the term Security Market Plane, describing a three-dimensional graph in which beta relative to market and dividend yield are plotted on horizontal axes and expected return on the vertical axis, with the latter increasing with both beta and yield. Shortly thereafter, the bank offered a Yield-tilt Fund for tax-exempt investors. As it happened, high-yield stocks significantly underperformed comparable low-yield stocks for some while thereafter. Ultimately the fund was closed. Accessed in 2017, the Trademarkia.com site indicated that the current status of the trademark is “Continued Use Not Filed Within Grace Period, Un-Revivable”. Nonetheless, the basic idea lives on.
**Value and Growth Tilts**

In 1992, Eugene F. Fama and Kenneth R. French provided a set of extensive empirical analyses of stock returns in “The Cross Section of Expected Stock Returns”, published in the *Journal of Finance*. A key finding was that in the United States, stocks selling at prices that were low relative to their accounting book values per share tended to have better returns than would be expected, given their beta values. A subsequent paper by Carlo Capaul, Ian Rowley and William F. Sharpe, “International Value and Growth Stock Returns”, published in *The Financial Analysts Journal* in 1993, found similar results for stocks in other countries. Numerous subsequent studies and articles have found similar possible anomalies at various times for other ratios of market prices to fundamental accounting measures.

These results gave rise to the notion that when constructing a portfolio, it may be desirable to overweight stocks selling at low prices relative to fundamental measures. Subsequent analyses of historic data have led some to believe that tilts towards other factors may also be fruitful. The resulting approaches are often termed “smart Beta” strategies although a more appropriate term would be “factor tilts”.

Definitions of “value stocks” vary. For example, the University of Chicago Booth Center for Research in Security Prices (CRSP) “classifies value securities using the following factors: book to price, forward earnings to price, historic earnings to price, dividend-to-price ratio and sales-to-price ratio.” CRSP maintains indices for value stocks using these measures. Moreover, ETFs tracking the CRSP US Large-Cap, US Medium-Cap and US Small-Cap Value indices are available, with expense ratios between 0.08% and 0.09% per year.
Some Robo-Advisors provide graphs of portfolios recommended for clients of different ages and/or with different risk tolerances. Here an example for tax-deferred accounts (IRA or 401(k)) for a 65-year old, from the Betterment\textsuperscript{TM} web site, accessed on March 18, 2017:

Note the significant investment in three US value stock categories stocks (for which ETFs tracking the relevant CRSP indices are utilized), in addition to investment in the overall US stock market.

Examining the numeric values in the graph shows that for a portfolio with a stock allocation worth 60% of the total value, the proportions in the four value asset classes are:

<table>
<thead>
<tr>
<th>Asset Class</th>
<th>Weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>US Total Stock Market</td>
<td>11.6%</td>
</tr>
<tr>
<td>US Large-Cap Value</td>
<td>11.6%</td>
</tr>
<tr>
<td>US Mid-Cap Value</td>
<td>3.7%</td>
</tr>
<tr>
<td>US Small-Cap Value</td>
<td>3.7%</td>
</tr>
</tbody>
</table>

This clearly represents a substantial tilt towards Value, with more money invested in value stocks within the market than in the market as a whole. And the graph shows that proportional relationships among the four asset classes are similar for different total amounts invested in stocks.
A similar strategy is utilized for taxable accounts:

Note that the green (stock) sections of the two graphs are quite similar. This suggests that the value tilt is likely to be motivated by an assumed market inefficiency, rather than differential taxation of dividends and capital gains. The major differences are in the bond investments, with greater investment in municipal bonds for taxable accounts, presumably due to their preferred tax treatment for personal income taxes.
Some robo-advisors take an opposite view. Here are asset allocations recommended in March, 2017 by *Wealthfront* for an investor with risk tolerance equal to the average for its clients:

Here too, the differences in bond holdings between taxable and tax-deferred (retirement) accounts are substantial. In the taxable account, 18% is invested in municipal bonds; in the retirement account, none. In contrast, the retirement account includes no municipal bonds, replacing them with real estate, corporate bonds and emerging market bonds. Differences in taxation are likely to explain at least part of these changes.
Also of interest is the inclusion in both portfolios of the Vanguard Dividend Appreciation ETF (VIG) which “seeks to track the performance of a benchmark index that measures the investment return of common stocks of companies that have a record of increasing dividends over time”. Using industry parlance, this could be considered a growth fund. Note that of the total amounts invested in the U.S. equity market, 16.7% of the taxable account and 42.9% of the retirement account are invested in the growth stock ETF, with the rest in the market as a whole. In this case, at least, Wealthfront favored substantial growth tilts.

The following table contrasts key aspects of the three Vanguard ETFs used by one or both firms:

<table>
<thead>
<tr>
<th>Name</th>
<th>Ticker</th>
<th>Price/Book Ratio</th>
<th>Dividend Yield</th>
<th>Expense Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dividend Appreciation</td>
<td>VIG</td>
<td>4.6</td>
<td>2.16%</td>
<td>0.09%</td>
</tr>
<tr>
<td>Total Stock Market</td>
<td>VTI</td>
<td>3.0</td>
<td>1.90%</td>
<td>0.05%</td>
</tr>
<tr>
<td>Large Capitalization Value</td>
<td>VTV</td>
<td>2.3</td>
<td>2.49%</td>
<td>0.08%</td>
</tr>
</tbody>
</table>

As intended, they differ considerably in price/book ratios. However, both the growth (VIG) and value (VTV) ETFs have higher dividend yields than the fund representing the stock market as a whole. They also both have slightly higher expense ratios than the ETF providing securities from the entire market (VTI). Both of these robo-advisors seemed to be making bets against the market at the time, but they were, in a sense, on opposite sides, one with a value tilt (price/book = 2.3), the other with a growth (price/book = 4.6) tilt.
The following chart, using data from Yahoo Finance, shows the cumulative values of these three ETFs (with dividends reinvested) from the earliest available date at which all three were available (in mid-2006) through March 1, 2017.

As can be seen, the series are correlated, but not perfectly so. The correlations of monthly returns between growth stocks and the market was 0.996, that between value stocks and the market was 0.993, and that between value and growth stocks, 0.982. Over this period, the cumulative performance of growth stocks was best at many points but slightly below the market at the end. Value stocks did slightly better than the market at the outset but their cumulative performance over the total period was below that of the market and growth stocks. If another range of dates had been chosen, the relative order of cumulative returns could easily have been different. But over this period, at least, there was little evidence of a clear superiority of either approach for betting against the market.
These particular ETFs are only two of many available investment funds that tilt holdings away from those of the market portfolio in order to increase exposure to some fundamental or statistical factor while decreasing exposure to another such factor. But it is important to emphasize that if one investor chooses increased exposure to a factor relative to the market, some other investor must accept decreased exposure relative to the market. And they cannot both beat the market. Over any given period, some tilt strategies will do better than the market, while others will do worse than the market. Investors who wish to bet by adopting factor tilts should remember the old idiom: “you pays your money and takes your chances”.

Our approach differs. We assume only investment in markets, with no intentional or unintentional betting. Our investment alternatives include only two asset types: (1) a highly diversified capitalization-weighted global market portfolio and (2) inflation-protected government bonds with little or no default risk. If multiple mutual funds or ETFs are required to represent a global portfolio of bonds and stocks, proportions invested in such funds are maintained at or close to relative market values. We assume that markets are sufficiently efficient that it is difficult to identify securities or classes of securities that are habitually underpriced or overpriced. In short, we choose investment rather than betting.

But this neglects both differential taxation of income sources and many other differences among investors. A more complex approach might account for some of these aspects of the real world by positing an equilibrium in which different investors rationally choose portfolios with different proportions of risky securities, tailoring holdings to accord with differences between their characteristics or circumstances and those of the “average investor”. But such a view would still assume that it is impossible to identify in advance, asset classes that are “underpriced” or “overpriced”; thus betting remains an undesirable activity. We return to this possibility later in this chapter.
This Book and the RISMAT Software

This work is made available as an online book (more succinctly, an ebook). The Matlab functions described in it are also available online, along with a few sample Matlab scripts. All this material is offered for any use conforming with its Creative Commons License (Attribution 4.0 International). Here is a summary of some of the key conditions (from the link provided with the book's Table of Contents):

You are free to:

- Share — copy and redistribute the material in any medium or format
- Adapt — remix, transform, and build upon the material
  for any purpose, even commercially.

The licensor cannot revoke these freedoms as long as you follow the license terms.

Under the following terms:

- Attribution — You must give appropriate credit, provide a link to the license, and indicate if changes were made. You may do so in any reasonable manner, but not in any way that suggests the licensor endorses you or your use.

- No additional restrictions — You may not apply legal terms or technological measures that legally restrict others from doing anything the license permits.

It is important that users of the book and/or the software conform with these terms (and read the detailed terms of the license). None of this material should be construed as investment advice provided by the author, who is an academic economist, not an investment advisor. The book and software are designed primarily for academic use, with the ideas and programs made available as well for those who may wish to employ them for providing retirement income or helping others to do so.
Program Structure

The RISMAT software combines some features of functional programming with others of object-oriented programming. The basic approach is for the user to write a Matlab script that calls a series of Matlab functions. Some of these functions create data structures with multiple elements. For example:

\[ \text{client.p1Name} \]

In Matlab terminology, \textit{client} is a data structure, and \textit{p1Name} an element of that data structure. Most of our data structures have multiple elements, which may contain numbers, strings, matrices, etc..

Other terms that one could apply to elements are: parameter (e.g. the standard deviation parameter of the market), attribute (e.g. the joint and survivor percentage attribute of a fixed annuity), or setting (e.g. the figure delay setting for animated graphs)

Object-oriented programming systems use a similar construct, with similar notation (separation with a period). Thus one might have a client object with a series of properties such as p1Name. We have avoided this terminology, since object-oriented programming advocates (and undoubtedly the producers of Matlab) would say that Matlab does not offer some of the more advanced features of a truly object-oriented programming language, in particular the ability to embed within an object, methods that perform computations. That said, the advantages of the object.property approach are great and we have used it extensively. Users may be forgiven if they choose to refer to “the p1Name property of the client object”.

As indicated, an important feature of truly object-oriented programming approaches is the ability to embed computations within an object. In such a language, an object can include functions that can alter its properties when invoked. Our approach keeps computations in functions. Some create new data structures (objects) with default elements (properties). Others use elements of data structures, alter or augment some of the elements, then return revised versions of one or more data structures. In this sense our approach is similar to functional programming.
**RISMAT Functions used in Scripts**

To provide an analysis using the RISMAT software, one would generally write a simple Matlab script (for example `BobSueSmith.m`). This could be saved in a directory visible to Matlab, then invoked by typing its name at the Matlab command line. Note: in earlier times, this would be called the “main program”.

This section provides examples of the use of four types of such functions that could be called in such a script, along with some comments on the manner in which they might be utilized.

Note: the order in which arguments are provided to Matlab functions matters, so the examples need to be followed carefully. Unfortunately, a uniform convention was not followed as the author wrote the functions over an extended period of time *Mea Culpa*, and my apologies.
**Basic functions**

Three types of function must be invoked in every RISMAT analysis:

Create and process a client data structure:

```plaintext
client = client_create( );
client = client_process(client);
```

Create and process a market data structure:

```plaintext
market = market_create( );
market = market_process( market, client );
```

Create and process an analysis data structure:

```plaintext
analysis = analysis_create( );
analysis_process ( analysis, client, market );
```

To use any of these pairs of functions, a script can execute the first statement to create a structure, then add statements to modify or assign new values to one or more of the structure's elements. Then (or later) the corresponding process function can be executed. This function will use as inputs the variables within its parentheses, then return modified versions of any data structure or structures that precede the equal (=) sign.

A typical script will begin with a `client_create` statement, followed by some statements assigning new values to the client elements and then a `client_process` statement. Next will come a `market_create` statement, possibly followed by some statements assigning new values to the market elements, then a `market_process` statement. Next will be a series of statements designed to add values to the `client.incomesM` and `client.feesM` matrices. Finally, the script will typically end with an `analysis_create` statement, some statements to modify its elements to determine the particular analyses to be shown, then an `analysis_process` statement. However, if one wishes to customize the sequence of analysis outputs, it may be desirable to repeat these three steps, selecting different element values each time.
**Lockbox Functions**

Some of the functions that provide income utilize a set of lockboxes and thus need information on the relative dollar values of Tips and/or market portfolio in each lockbox. The software provides three ways to construct such lockboxes. The functions are:

Create and process lockboxes to provide income distributions that approximate the distribution from investment in the market portfolio for a specified number of years \(n\):

\[
\text{AMDnLockboxes} = \text{AMDnLockboxes\_create}(\;) ; \\
\text{AMDnLockboxes} = \text{AMDnLockboxes\_process( AMDnLockboxes, market, client )};
\]

Create and process lockboxes that provide income distributions consistent with a constant client marginal utility function:

\[
\text{CMULockboxes} = \text{CMULockboxes\_create( } ) ; \\
\text{CMULockboxes\_process} = \text{CMULockboxes\_process( CMULockboxes, market, client )};
\]

Create and process lockboxes that are weighted combinations of two other types of lockboxes:

\[
\text{combinedLockboxes} = \text{combinedLockboxes\_create( } ) ; \\
\text{combinedLockboxes} = \text{combinedLockboxes\_process( combinedLockboxes, market, client )};
\]

The *create* statement in each pair is typically followed by statements assigning different or new values to some or all of the data elements, then the *process* statement is executed. Each of the first two sets of statements provides a matrix of proportions of Tips and the market portfolio in each future year. The last set can combine any two or more such sets after their proportions matrices have been assigned to the appropriate elements of \textit{combinedLockboxes}. 

**Income Functions**

The functions in the next two sections do the main work of the system: creating income (and possibly fee) matrices, then adding their values to those in the corresponding elements of the client data structure matrices. A script must use at least one pair of these functions, but may use as many pairs as desired.

This section includes functions for single income sources; the next describes functions that use one source for a period of years, then another for the remaining years. When processed, each of these sources adds values to the client matrix of incomes and, if relevant, another set of values to the client matrix of fees. Each uses information from its own data structure as well as the client and market data structures.

The functions are:

Create and process a social security data structure:

```
iSocialSecurity = iSocialSecurity_create(  );
client = iSocialSecurity_process( iSocialSecurity, client, market );
```

Create and process a fixed annuity data structure:

```
iFixedAnnuity = iFixedAnnuity_create(  );
client = iFixedAnnuity_process( iFixedAnnuity, client, market );
```

Create and process a lockbox designed to purchase of a fixed annuity at a specified future date (or provide payment to the estate in case neither client survives until that date). More simply put: a Future Annuity Purchase:

```
iFAPlockbox = iFAPlockbox_create(  );
client = iFAPlockbox_process(client, iFAPlockbox, market );
```

Create and process a data structure for a strategy in which a given amount is invested, with a constant real amount spent in each future year until there is no money left or the remainder is paid to the estate (whichever happens first):

```
iConstSpending = iConstSpending_create(  );
client = iConstSpending_process( iConstSpending, client, market );
```
Create and process a data structure for a strategy in which a given amount is invested, with a pre-specified proportion of the remaining value spent in in each future year until there is no money left or the remainder is paid to the estate (whichever happens first):

```c
iPropSpending = iPropSpending_create( );
client = iPropSpending_process( iPropSpending, client, market );
```

Create and process a data structure for a strategy in which a set of lockboxes is employed – each to be used for spending in a designated future year. In the event that the estate is processed before that year, the contents of any remaining lockboxes are sold and the proceeds paid one to the estate. Unlike the previous functions, this returns a revised version of its structure by adding a new matrix providing the lockbox proportions produced by taking into account the clients' bequest motive. For this reason, two outputs – the revised client structure and the revised iLockboxSpending structure – are shown on the left side of the equal sign, enclosed in square brackets:

```c
iLockboxSpending = iLockboxSpending_create( );
[client, iLockboxSpending] = iLockboxSpending_process( iLockboxSpending, client, market );
```

Create and process a data structure for a type of Guaranteed Lifetime Withdrawal Strategy that combines (1) the purchase of shares in a mutual fund with (2) an insurance rider that guarantees that if a specified minimum nominal amount or less is withdrawn each year, withdrawals will continue as long as at least one beneficiary is alive. Moreover, the guaranteed withdrawal may increase in one or more years if returns on the mutual fund are sufficiently high, allowing for nominal income to ratchet up in some cases:

```c
iGLWB = iGLWB_create( );
[client iGLWB] = iGLWB_process ( client, market, iGLWB );
```
Income Combination Functions

These functions create and process data structures that provide incomes and fees using lockboxes for spending in a number of years, followed by income provided by an annuity if anyone is alive at the time. Each uses functions listed in the previous section.

Create and process a strategy that combines the use of lockboxes to provide incomes for a number of years with a fixed annuity purchased at the outset that will provide incomes after the last lockbox year, but only if someone is alive at the time. As usual, any lockboxes remaining when the estate is processed will be sold, with the proceeds paid to the estate:

```plaintext
iLBSplusDFA = iLBSplusDFA_create();
[client, iLBSplusDFA] = iLBSplusDFA_process( client, iLBSplusDFA, market );
```

Create and process a strategy that combines lockboxes used to provide incomes for a number of years with an additional lockbox to be used to purchase a fixed annuity thereafter if someone is alive at the time. Here too, any lockboxes remaining when the estate is processed will be sold, with the proceeds paid to the estate, and this will include the lockbox designed for the future annuity purchase:

```plaintext
iLBSplusFAP = iLBSplusFAP_create();
[client, iLBSplusFAP] = iLBSplusFAP_process( client, iLBSplusFAP, market );
```
**Functions Used for Analyses**

In addition to functions that can be used directly in a script, the RISMAT software includes nine that can be used by the `analysis_process` function. Their names, which indicate their functions, are:

- `analPlotEfficientIncomes`
- `analPlotIncomeDistributions`
- `analPlotIncomeMaps`
- `analPlotPPCSandIncomes`
- `analPlotRecipientPVs`
- `analPlotScenarios`
- `analPlotSurvivalProbabilities`
- `analPlotYearlyPVs`
- `analPlotYOYIncomes`

Users of the software need not concern themselves with these functions. But it is imperative that they be included in the directory containing the `analysis_process` function.
The functions described in the preceding section can be easily obtained from a web site that contains this book. Clicking on Matlab Programs in the table of contents should download a compressed file (RISMATcode.zip). Many computer operating systems provide a simple way to uncompress this in order to obtain the Matlab (.m) files. Separate programs that can uncompress such files are also available.

With these files in a directory accessible from the Matlab system, one can write simple scripts, then concentrate on analyzing alternative strategies for providing retirement income to retirees such as Bob and Sue Smith.
If the RISMAT software is to be used by non-programmers, it may be useful for someone (but
not this author) to produce a graphic user interface (GUI) to make the system and any
extensions or modifications more user-friendly. Matlab provides necessary ingredients. One can
write programs and functions using a construct called a `uicontrol`. Moreover, an interface called
`GUIDE` (Graphic User Interface Development Environment) may be available. Mathworks also
offers a set of software called `App Designer`.

Such an approach could be part of a system with multiple clients and reports. Each client could
have its own `client_create` function – e.g. `BobSueSmithClient_create()`. Standard sets of
income creation and analysis reports could be available. And so on.

There is also the possibility of producing a similar set of functions written in another
programming language. The Python programming language allows for a relatively similar
structure and is open-source (free). There are also open-source matrix operation libraries for
use with Python. They should be able to perform most of the matrix functions in the RISMAT
system. But Matlab has many advantages, including the ability to perform operations on very
large matrices with blinding speed. It is also widely available at no cost for use by students and
faculty at many colleges and universities. And, in the author's opinion, for many those
providing financial advice for retirees the purchase of a MATLAB license, should be well
worth the cost.
**The Need for Further Research**

Many academic research articles and reports end with some variation on the phrase: *More Research is Needed.* This very much applies here. We conclude with brief discussions of some of the features of real life that could be added to the RISMAT system.

The key idea behind this project is that prospective retirement income should viewed as a multi-period probability distribution represented as large matrix of income values, with each row representing a possible scenario and each column a future time period. Thus one row shows incomes in each year for a possible future, another row shows incomes for another possible future, and so on. Looked at from the other viewpoint, each column shows the range of possible incomes in a given year, as viewed from the present time. The daunting task for retirees is to choose among a number of possible such matrices. Worse yet, each matrix should be thought of as very large. Our examples have used 100,000 scenarios and up to 50 or more years – providing as many as 5 million possible income values. Moreover, other elements are important. Who will be alive in each year in a scenario? What will be the market portfolio's return in each year of each scenario? And so on. To deal with such factors requires additional matrices, each containing millions of numbers.

This provides a number of challenges.

First, a computer and programming language must be employed that can handle calculations with many millions of values very rapidly. Amazingly, Matlab can often perform the needed calculations for a typical case on a home computer in a fraction of a minute. It may take a little longer to produce complex graphs, especially animated graphs designed to show information one future a year at a time and do so at a speed that will not overwhelm the viewer.

Second, there must be a way to help retirees understand the key aspects of a given strategy for producing retirement income, then compare those results with those for other possible strategies. The Financial Advisor's task is to deal with all these problems in order to help each retiree or pair of retirees make informed financial decisions. Behavioral research has shown that human beings often make illogical choices among alternatives and that this is especially true when there is uncertainty about outcomes. We need to know more about useful ways to show retirees the ranges of possible future outcomes in a manner that can lead to intelligent choices.

Finally, as we have indicated in earlier chapters, many important aspects of the retiree's problems are not taken into account explicitly in the current version of the RISMAT software and need to be considered. We next discuss five of the more important.
In *The Political History of the Devil* (1726), Daniel Defoe argued that “Things as certain as death and taxes, can be more firmly believed.” Our software deals with the former (we account for uncertainty about the time of death, and also its inevitability), but not the latter. This is a major fault. Taxation is ubiquitous around the globe, different forms of income are often taxed differently, tax laws change from time to time and, worse yet, there is considerable uncertainty about possible future tax regimes.

Despite this complexity, when providing probabilistic forecasts of possible future income it may well be desirable to incorporate some taxation rules than to ignore this fact of life entirely, as we have done. Some investment and withdrawal strategies may be more tax-efficient than others and differential taxation may dictate the choice of investments that differ from our world market portfolio and Tips.

Many robo-advisors advertise the ability of their software to choose investments, saving rates and withdrawal strategies in ways that take taxation into account. Many argue their approach is “tax-efficient”, “tax-aware”, or some other appellation. Few make long-range predictions or evaluate alternative strategies for providing income over many future years. But many recommend strategies based on the argument that taxation should affect both asset allocation (e.g. high-yield versus low-yield stocks, bonds versus stocks) and also asset location (e.g. in which type of account to include holdings of municipal bonds).

We agree, and accept the charge that this book and set of software are best suited for a world in which taxation affects all sources of income in the same manner. In this sense, it provides a setting that focuses on the basic economics of the subject without taking into account the effects of institutions that differ in different countries and change through time in ways extremely difficult to predict.

That said, many of the results obtained without taking taxes into account may be valuable, and judicious selection of parameters might approximate after-tax results reasonable well in many cases. But it could be worthwhile to enhance the software to deal with at least key aspects of the current tax regime in a country or region, while still retaining key fundamental economic and financial attributes – a daunting task the author leaves to others.
Home equity

The current RISMAT software also fails to take into account the role played for many retirees by equity in their own home at the time of retirement. For many people, home equity is second only to Social Security in value at the time of retirement, with fungible savings often third. At retirement or any time thereafter, home equity can be converted to spendable assets by selling the property outright and paying off any outstanding mortgages. Spendable cash may instead be generated by taking out a second (or third) mortgage on the property. And, in the United States it is often possible to take out a Reverse Mortgage that can provide spendable income at the present time and possibly thereafter, while guaranteeing that the owners may remain in the home as long as they live. Such contracts are highly regulated and likely to be complex. But, however utilized, home equity can provide retirement income when desired.

It should not be overly difficult to include at least some aspects of such equity in software to produce retirement income scenario matrices. A particularly promising approach would pair the generation of income from home equity with unexpected changes in personal states, such as those described next.
Long-term Care

We have limited our analysis to five personal states: both retirees are alive (state 3), only the first named retiree is alive (state 1), only the second named retiree is alive (state 2), the last retiree died in the prior year and the estate is distributed among the relevant recipients (state 4) and the estate has already been distributed (state 0). But this taxonomy fails to take account of the fact that many retirees reach a point in later years in which they need significant assistance to cope with the exigencies of daily life.

Here is Wikipedia on the subject:

*Long-term care* (LTC) is a variety of services which help meet both the medical and non-medical needs of people with a chronic illness or disability who cannot care for themselves for long periods.

It is common for long-term care to provide custodial and non-skilled care, such as assisting with normal daily tasks like dressing, feeding, and using the bathroom. Increasingly, long-term care involves providing a level of medical care that requires the expertise of skilled practitioners to address the multiple chronic conditions associated with older populations. Long-term care can be provided at home, in the community, in assisted living facilities or in nursing homes. Long-term care may be needed by people of any age, although it is a more common need for senior citizens.

This information is probably more than one might have wanted to know, and truly depressing.

It is possible to purchase Long-term Care Insurance that will make payments up to some specified amount for up to some number of months, depending on the terms of the policy. To qualify for payments, the insured must usually be able to show that he or she is incapable of performing a designated number of “acts of daily living” without assistance. According to the U.S. government [LongTermCare.gov](#) site, typical “ADL”s are: Bathing, Dressing, Using the Toilet, Transferring (to or from a bed or chair), Caring for Incontinence, and Eating. Moreover, the site warns that “almost 70% of people turning age 65 will need long-term care at some point in their lives.” Welcome to the golden years!

In the United States, Medicare insurance may cover up to 100 days of assistance after a hospital stay, and, for those without adequate financial resources, Medicaid may cover some or all of the cost of a long term care facility. Unfortunately, many people consider the authorized Medicaid facilities below their standards and hope to have sufficient resources to avoid ending their lives in such surroundings.
To expand our approach in order to incorporate this unpleasant prospect, one could increase the set of possible personal states. Thus one might have personal state 1.5 in which only person 1 is alive and he or she needs long-term care, personal state 2.5 in which only person 2 is alive and he or she needs long-term care, personal state 3.25 in which both are alive with person 1 needing long-term care, state 3.50 in which both are alive with person 2 needing long-term care and personal state 3.75 in which both are alive with both needing long-term care. It might even be possible to obtain actuarial estimates for the probabilities of such combinations in various future years. Arduous, at best, but potentially worth the effort.

Fortunately, the failure to consider both the need for long-term care or the advantages of home equity may have some positive aspects. Many retirees borrow against their home equity to cover costs of care while in their homes. Others use some or all of their home equity to pay at least a portion of the expenses associated with moving to a retirement community (which may include long-term care at no extra cost). Failing to include home equity may thus at least partially offset the failure to include the need for long-term care. A rationalization, to be sure. But some solace for those who might wish to use the RISMAT software in its initial form.
Advisory Fees

As we have indicated, it is not unusual for an advisor to charge retirees annual fees based on the assets for which advice or management is provided. Such fees can be as high as 1% or more of asset value each year (although many advisors use a sliding scale, with $x1\%$ of the first $y1$ assets, $x2\%$ of the next $y2$, etc.). The RISMAT software allows the inclusion of fees charged (explicitly or implicitly) by insurance companies, mutual funds, ETFs, etc.. But no provision is made for the expenses associated with advisors, either human or robo-. This is a serious shortcoming, for such the present value of such fees can equal as much as 15% or 20% of retirees' initial savings.

If an advisor charges retirees only an initial fee for consultation, it is a simple matter to reduce the amount to be invested by a corresponding amount, although the fee would not be included in our present value pie chart. But if there are to be continuing fees, especially if they are based on the value of invested assets, it would be desirable for the RISMAT software to compute their magnitudes in different scenarios and years, and include the results in the client fees matrix so their magnitudes can be examined and the overall present value computed and shown. This would be especially useful if a comparison is to be made between annuities and spending approaches, so that the impact of fees can be taken into account for both alternatives, not just one.

As we have argued, an advisor acting in the the best interest of his or her clients should disclose relevant fees and their impact on future retirement income. We have not attempted to include such fees in the present version of the software, but an advisor acting in the best interests of a client should certainly do so.
Optionality

Dictionary.com defines optionality as: *left to one's choice; not required or mandatory*. In this sense, the possible future incomes provided by following a *proportional spending rule* are optional. In any year retirees may choose to spend more or less than the amount provided by spending the pre-specified proportion of the current value of their retirement portfolio. The rule may be followed, but it may not. It thus has optionality.

In a similar manner, a *constant spending rule* has optionality. One may spend the same real amount each year (unless the money runs out), but one does not have to do so.

A lockbox spending rule also has optionality. In any year, it is possible to spend only a portion of the value of the assigned lockbox. Or one may spend the entire value and “break into” one or more lockboxes designed for future years in order to spend more. Here, too there is optionality.

At the other end of the spectrum lie Social Security and annuities.

Once payments begin, Social Security will provide income each year to a beneficiary according to its rules. The amounts may vary, but one must take the income while alive. It is of course, possible to save some of the amount to be used in the future, but there is no explicit way to obtain more income from the Social Security Administration. One can of course save some income for use in a later year, but options are very limited.

Fixed Annuities are similar, with little or no optionality. If purchased at the outset, the amounts to be received each year will be determined by the terms of the contract; it is generally not possible to alter the amount paid at any given time to suit the desires of the recipient. This is true whether or not the annuity includes a deferral period. As with Social Security, one can of course save some annuity income for use in a later year, but there is relatively little optionality.

A strategy that sets aside money in a lockbox to be used to purchase an immediate annuity in some future years represents a mixed case. There are many options for spending the money or using it for some other purchase up to the year in which the annuity is purchased. Thereafter, optionality is highly limited.
Investment in mutual funds with purchase of a Guaranteed Lifetime Withdrawal Benefit income rider from an insurance company also represents a mixed case. In a sense, it might be considered to have complete optionality, since any amount up to the remaining value of the fund holdings may be withdrawn at any time. However, this may significantly reduce future incomes. In order to retain or increase the full guaranteed income, no more than the amount allowed by the insurance contract can be be withdrawn in any given year. In this sense, it offers limited optionality. GLWB strategies thus offer options, but at possibly large costs.

Our analyses do not take optionality, or the lack thereof, into account in any explicit manner. Yet the ability to adapt to unpredictable circumstances is a valuable attribute of any income strategy. Wade D. Pfau, Joseph A. Tomlinson and Steve Vernon in an article “Retirement Income Programs: The Next Step in the Transition from DB to DC Retirement Plans,” published in the Winter 2017 Journal of Retirement, propose the inclusion in analyses of a “Measure of Accessible Wealth”. For each year and scenario, the amount of savings that a retiree could withdraw and deploy to other purposes (accessible wealth) is determined and the median value found. Then each annual median value is weighted by the probability of survival to that age to obtain the single value. The result provides a measure for the concept that we have called optionality. This, or some other such value, might be incorporated in the RISMAT software and considered when alternative strategies are evaluated. The best approach is not obvious, but qualitatively or quantitatively, optionality should be taken into account when evaluating alternative retirement income strategies.
**The Investor and the Advisor**

We have developed software to make probabilistic estimates, show them in different manners, relate incomes to prices, etc. etc. The goal is to let a retiree or pair of retirees get some sense of properties of alternative retirement income strategies, then choose the strategy or combination of strategies that seems most suitable: cost-efficient if possible, and consistent with their own preferences concerning probabilistic outcomes.

But even a probability distribution of possible incomes to be received at a single time is difficult for many people to evaluate. And a set of such distributions, one for each of a number of future time periods is even more daunting. Comparing one such set with another in order to make an informed choice could well be beyond the skills of many retirees. Cue the financial advisor. He or she should be able to help retirees make the difficult decisions associated with providing future income.

Some day Bob and Sue may be able to sit at their dining table with an on-line program and make good retirement income choices by themselves. But it seems likely that for some while an experienced, unbiased, and frugal human advisor may be well worth the added cost, at least at the outset.

Following tradition, we conclude with the admonition that more research is needed (and probably more programming) -- tasks that the author, having reviewed actuarial tables, chooses to leave to others.