# Retirement Income Analysis with scenario matrices 

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## 3. Longevity

When creating, evaluating or implementing a retirement income strategy, two questions are key:

How much money do you have to provide future income?
How long might you need such income?

With rare and unfortunate exceptions, we can't know with certainty how long we will live. At best, we can estimate the probabilities of living to various future ages. Thus we need to be able to build matrices with scenarios having different longevities, in accordance with the best possible estimates of their relative probabilities. This chapter deals with ways to make such estimates.

## Mortality Tables

Given adequate resources and reasonable health, almost everyone would prefer a longer life. Thus longevity is a positive term. The converse is mortality, generally regarded negatively. But the two are clearly related. For example, if you have a $2 \%$ chance of dying within a year, you have a $98 \%$ chance of living that long. So a table of 1 -year mortality rates can be simply converted to one of 1-year survival (longevity) rates. Letting M and S be the respective matrices:

$$
S=1-M
$$

Most historic and prospective analyses focus on mortality rates, as shall we in this chapter. The good news is that for those in their early retirement years, such rates are low and have been falling over time. The bad news is that the chance of dying with the next year rather inevitably increases as (if) one ages.

This chapter focuses on three key sources useful for predicting future mortality rates: (1) historic mortality in the United States and other countries, (2) projections made by the United States Social Security actuaries, and (3) projections made by the United States Society of Actuaries. Throughout, it is important to remember the advice of a famous U.S. Baseball player, manager and philosopher, Yogi Berra:
"It's tough to make predictions, especially about the future."
$\ldots$ an admonition that applies to much in this book.

## The Human Mortality Database

The standard source for historic data on mortality is the website www.mortality.org, maintained by researchers at both the Department of Demography at the University of California, Berkeley in the United States and the Max Planck Institute for Demographic Research in Bostock, Germany. It has annual data for 37 countries or areas on experienced mortality rates by age, going back as far as the 1700's in some cases. For each covered year, statistics are provided on the mortality of people of different ages, with data for Males, Females and the total population. All the data can be easily downloaded without charge.

We concentrate here on the measure indicating $q(x)$-- the probability of death between a given age x and age $\mathrm{x}+1$. More precisely, we organize the historic data into matrices in which each row represents an age, each column an historic year, and each entry the proportion of people of a given age range who died in that year. For example, for the U.S.:


As will be conventional in this book, proportions are expressed as decimal numbers. Thus 0.02 indicates that $2 \%$ of the people of the specified age died in the year in question.

Since we are interested in people of possible retirement age, it is useful to concentrate on those of age 50 and over. Moreover, since mortality rates for "super-centarians" (those 101 and older) pose problems (to be discussed later), we limit our focus to those from 50 through 100, inclusive. In order to cover relatively recent history and yet select a period for which data are available for most countries, we limit our analysis to calendar years from 1959 through 2009. In effect, we extract a section from the larger matrix for each country:


Which gives a new matrix:


The Human Mortality Database contains mortality rates for such a matrix for the United States and 32 other countries (after removing countries and areas already included in other areas). For simplicity, we take the average of each of the entries for the 32 non-US countries to make a single matrix of mortalities for "non-US countries." These two matrices provide the material for our analyses of historic mortality rates.

## United States Mortality Rates in 2009

The figure below shows mortality rates in 2009 for Males in the United States.


As can be seen, each age group had a greater chance of dying within a year than its predecessor. Moreover, the rates increase more rapidly as one goes from younger to older age groups. The pattern is far from linear. But not if the data is plotted using a logarithmic scale for the $y$-axis (mortality), as shown below.


This is an example of a plot using the semilogy function. The vertical (y) axis plots the logarithm of the value in question (here, the mortality rate), but the values shown are in the original units. Moreover, the horizontal grid lines correspond to equally spaced values in the original units. In this case mortality rates run from $0.001\left(10^{-3}\right)$ to $1.0\left(10^{\circ}\right)$.

The striking feature of this graph is the fact that the data points fall remarkably close to a straight upward-sloping line. The logarithm of the mortality rate increases by an almost constant rate each with age. This is hardly unusual. One of the first to notice such a relationship was Benjamin Gompertz, a British actuary who posited it in 1825 as a "law of human mortality".

A natural way to see how well such a law describes male U.S. mortality rates in 2009 is to fit a straight line to the data using least-squares regression. The figure below shows the result.


The fit is not perfect, but very good indeed, with a R-squared value of 0.9939 , indicating that $99.39 \%$ of the variation in the logarithm of mortality is "explained" by the fitted relationship. Many economists can only dream of getting similar results when testing their theories.

In this case the slope of the line relating the logarithm of mortality to age is 0.0900 , indicating that the mortality rate at any age is equal to $1.0942\left(=\mathrm{e}^{0.0900}\right)$ times that at the prior age. Thus the instantaneous rate of increase is 0.09 and the corresponding annual rate of increase is 9.42\%.

So much for U.S. Males. What about U.S. Females? The figure below shows that they are definitely superior when it comes to longevity. In the United States, at every age, males are more likely to die than females, although the gap narrows as one moves to higher ages.


The figure below shows that a straight line also fits the data for U.S. Female mortality well.


Here too, the fit is also excellent, with an r-squared value of 0.9943 . The slope is somewhat greater 0.0977 (equal to an annual ratio of 1.1026), since the gap between male and female mortality narrows as one moves to higher ages.

Despite the remarkable fits of straight lines in the Male and Female diagrams, there are deviations, and they are not randomly located at different ages. In both cases, actual mortality rates tend to be greater than the fitted values for younger ages (roughly between 50 and 60), below the fitted values for mid-range ages (from roughly 60 to 85 ), and above them for older ages (greater than 85). Is this pervasive or simply an artifact of the data for one year in one country? We shall see.

## Mortality Rates Outside the United States in 2009

To see if the results shown thus far are an artifact of mortality rates for a single country in a single year, we start by comparing the results for 2009 in the U.S. with those in other countries. More specifically, we focus on equally-weighted averages of the mortality rates in the following thirty-one countries or areas: Australia, Austria, Belgium, Bulgaria, Belarus, Canada, Czech Republic, Denmark, East Germany, Estonia, Finland, France, Hungary, Iceland, Ireland, Italy, Japan, Latvia, Lithuania, Netherlands, Norway, Poland, Portugal, Russia, Slovakia, Spain, Sweden, Switzerland, Ukraine, United Kingdom and West Germany.

The figure below shows male mortality rates for the U.S. And the non-U.S. Countries.


As can be seen, the averages of the mortality rates in the thirty-one countries are somewhat higher for all ages than are the corresponding mortalities in the U.S.. But the relationships are both close to linear in this semilog plot. In fact the R-squared value for the non-U.S. Countries is 0.9969 , even higher than that for the U.S. (0.9939).

The next figure provides comparative results for female mortalities.


In this case the mortalities are quite similar, with higher values in the U.S. for younger ages (up to 62 or 63, and lower values thereafter). But again the relationships are close to linear approximations, with an R-squared value of 0.9938 for the non-U.S. Countries, differing only slightly from the value of 0.9943 for the U.S..

Despite these relatively small differences, the greater mortality rates of males is evident outside the U.S, as well as within it, as the following figure shows.


Finally, there is the question of deviations from linear approximations. This next two figures show the actual mortality rates and the fitted linear function for males and females in the nonU.S. countries.



Again, the deviations are relatively small but it is striking that they have the same patterns as those for the U.S. - actual mortalities are somewhat higher than the fitted values in the lowest and highest age ranges and somewhat lower than the fitted values in the median age ranges. This suggest that some sort of function that plots as a curve in a semilog plot might better capture the relationship between mortality and age - an approach that we will illustrate later.

## Cohort Mortality Rates

Thus far we have concentrated on mortality rates for different ages in a given year - that is the rates in each column of an historic mortality matrix. But no real human being was exposed to the rates in a given column (year). Consider a cohort of males, each of whom was of age 50 in 1959. Their mortality rate in that year was given by the entry in the first row and column of our matrix. The mortality rate for those who survived that year were given by the entry in the second row and column, which showed the result for males of age 51 in 1960. Those who survived that year experienced the mortality rate in row 3 and column 3, and so on. So for a given cohort, mortality rates lie along a diagonal. This is illustrated in the figure below for two such cohorts - one of age 50 in 1959, the other of age 74 in 1959.


This means that in order to obtain estimates of future mortality probabilities for people now alive we need a mortality matrix that includes estimates for future years, as illustrated in the following diagram for two people, currently of age 50 and 75 .


In this case, historic rates are available for years through and including 2009. The yellow area includes projections of future mortality rates made at that time. The arrows show the probabilities that would them be used to estimate future mortality rates for two people, one of age 50 in 2009, the other of age 75 at the time.

Clearly, we need some way to project historic mortality rates into the future. To start, it is useful to see how such rates changed over time in the past.

## Historic Changes in Mortality Rates

Projected future mortality rates are normally produced by applying some sort of procedure to the most recent rates (the last blue column in the preceding figure) to obtain a set of future rates (the first yellow column), then applying a different procedure to the rates in that column to obtain the next set of future annual rates (column), and so on. A later section of this chapter will describe a prominent approach of this type. Here we will use our historic matrices for the U.S. and non-U.S. countries. to examine some ways in which mortality rates changed over the period from 1959 to 2009.

The following figures show the results of two calculations for each of our four mortality matrices (U.S. Male, U.S. Female, Non-U.S. Male and Non-U.S. Female). First we take the ratio of each mortality in a matrix to the mortality for the same age in 1959. This provides a matrix of relative values. Next, we take a simple average of all the values in each of the columns, which gives the average improvement across ages in that year relative to 1959. The following figure plots the results for U.S. Males and Females.


As can be seen, mortality rates in the U.S. decreased substantially over the period, with progress more rapid for women initially but with men catching up so that by 2010 the cumulative reductions were similar. Note also that the year-to-year changes differed substantially, with average mortality rates even increasing in some years, raising the cumulative ratios.

The next diagram provides the same information for the non-U.S. Countries.


In this case, women made progress at rates similar to those of their U.S. Counterparts, but the overall improvement for men was somewhat lower. And the variation from year to year was even greater than in the U.S..

It is useful to translate the cumulative changes in average mortality rates to equivalent annual rates, compounded annually. The following table provides the results.

| Group | Annual Ratio of <br> Year over Year Mortalities |
| :---: | :---: |
| U.S. Males | 0.9898 |
| U.S. Females | 0.9901 |
| Non-U.S. Males | 0.9931 |
| Non-U.S. Females | 0.9893 |

Quite remarkably, for each of the four groups, mortality rates decreased at average rates of of close to $1 \%$ per year over the period.

While mortalities tended to decrease by close to $1 \%$ per year overall, the improvements were not the same for different age groups. The following diagram provides the average annual ratios of year-over-year mortality rates for ages from 51 through 100 for males and females in the U.S..


As can be seen, on average, mortality rates decreased by $1 \%$ or slightly more for those under 85 , by considerably less for those over 85 , and actually increased slightly for centenarians.

The following figure shows a somewhat similar pattern for those in other countries.


Outside the U.S., mortalities seem to have improved more for those in the 60 to 80 age groups than for their younger or older fellow citizens.

The net effect of mortality rate changes that are age-dependent as well as time-dependent is to alter the relationship between such rates and age as time passed. The following diagram, for U.S. Males illustrates the point.

Fitted Mortality Rates, U.S. Males, 1959, 1964 and 2009


The passage of time led to lower lines due to the general reduction in mortalities but the greater progress in reducing mortality for those of younger ages led to an increase in the slope of the relationship.

Similar relationships held for U.S. females and for both males and females in our non-U.S. Country aggregates.

## Computing Mortality Ratios

One might imagine that it would be tedious to compute mortality ratios and their averages. But matrix operations make it quite simple. For example, assume that $\mathbf{m 1}$ is a mortality table with $\mathbf{r}$ rows and $\mathbf{c}$ columns. Then one can create a new matrix, $\mathbf{m 2}$, of the same size in which each column is the same as that for the initial year using matrix multiplication:

$$
\mathrm{m} 2=\mathrm{m} 1(:, 1)^{*} \text { ones }(1, c)
$$

Next, obtain a matrix, $\mathbf{m 3}$, of the ratios of each entry in the original matrix to the corresponding entry in the first row using element-wise matrix division:

$$
\mathrm{m} 3=\mathrm{m} 2 . / \mathrm{m} 1
$$

Finally, obtain the mean ratios by year (m4) and by age (m5):

```
m4 = mean(m3)
m5 = mean(m3')
```

This provides an example of the reasons it pays to learn matrix operations. The statements are concise, powerful, and not error-prone. Moreover, they provide the needed information very quickly. The premise of this book is that one should think about and analyze strategies for providing future retirement income in matrix terms. This example shows that matrices and the tools for processing them are also useful for studying historic data.

## Projecting Future Mortality Rates

The past may be prologue, as the famous saying holds; or maybe not. But for constructing scenarios of possible future outcomes, we need predictions of future mortality rates. Our analyses of history revealed some relationships that clearly can help inform future projections. The remainder of this chapter shows how two organizations in the United States create sets of forecasted mortality rates.

## The Society of Actuaries Mortality Tables

During and following the second World War, industrial companies and other employers in the United States adopted plans that provided workers with post-retirement annuity payments based on their earnings and years of service. These were called defined-benefit (DB) pension plans because the amounts paid after retirement (benefits) were defined by formulas included in employment agreements. In the latter part of the twentieth century, such plans began to be replaced by agreements in which the employer and employee contributed to savings plans owned by the employee, which could be invested to provide funds that could finance income to be received after retirement. These are called defined contribution (DC) plans since the amounts contributed are defined in employment contracts. In the second decade of the twentyfirst century, DC plans predominate among employers in the private sector. In the public sectors (especially among state and local governments) DB plans are still popular, there is also movement towards either DC or mixtures of DB and DC plans. But a number of major employers still offer DB plans.

In the U.S., federal tax laws allow employers to deduct as labor expense the reasonable cost of pre-funding accrued obligations for post-retirement benefits. Moreover, non-governmental employers are required to make premium payments to a quasi-governmental organization which insures that at least some of such obligations will be paid to beneficiaries if the employer becomes bankrupt. Moreover, such premiums depend in part on the extent to which a DB plan is funded. Both the Internal Revenue Service (the government taxation agency) and the Pension Benefit Guarantee Corporation (the insuring agency) require that calculations of the present values of accrued future post-retirement benefits be based on appropriate mortality tables. And for some years, such tables have been provided by the United States Society of Actuaries (SOA).

For much of the early part of the twenty-first century, the relevant SOA tables were the RP2000 Mortality Table and either the Scale AA Mortality Improvement scale or the later Scale BB table. In 2014, the Society of Actuaries released the RP-2014 Mortality Tables and the MP2014 projection scale with the recommendation that "they should be considered as replacements for the current mortality basis ...".

Since the SOA tables are specifically designed for analysis of retirement income, they warrant our attention.

## The RP-2014 Mortality Tables

The RP-2014 mortality tables provide estimated mortality rates for the year 2014, based on the mortality experience of 38 private employer defined benefit retirement plans. The final dataset reflected "approximately 10.5 million life-years of exposure and more than 220,000 deaths", according to the SOA RP-2014 Mortality Tables Report (which is the source for most of the information in this section). We will concentrate on the tables for "Healthy Annuitants" which include the populations of "Healthy Retirees" (not in disability status at the time of retirement) and "Beneficiaries" (those older than 17 who are receiving benefits earned by deceased relatives).

The report provides tables for two sub-populations based on "collar type" (using somewhat dated terminology). All participants in a plan were considered "Blue Collar" if at least 70 percent of the participants were either paid on a per-hour basis or were members of labor unions. Otherwise all the participants were considered "White Collar". This reflected practice in traditional industrial firms such as automotive manufacturers which had separate retirement plans for (1) salaried workers who were usually not union members and (2) hourly workers who were often members of unions. The report also provides tables for all workers, no matter their collar type. And of course, there are separate tables for males and females in each of the three categories.

The tables were based on raw mortality rates from plan experience for the years from 2004 through 2008. These were then projected using the scale MP-2014 mortality improvement rates described later in this chapter. The projected rates were subsequently graduated by fitting smoothing functions and then extended to extreme (very old or very young) ages using "a variety of actuarial techniques). The final result was a set of gender-specific tables with a base year of 2014. The SOA committee that produced the tables stated that ".. as of the release date the ... tables .. represent the most current and complete benchmarks of U.S. Private pension plan mortality experience, and the Committee recommends consideration of their use for the measurement of private pension plan obligations, effective immediately."

The report notes that "... raw mortality rates usually include some random fluctuations that can mask the underlying 'true' mortality rates... (thus) 'the final set of raw rates were graduated to produce smooth tables that reflect underlying mortality patterns." Interestingly, the mortalities were also "amount-weighted" using the annual retirement benefits, putting more weight on the experience of those with higher benefits. For the ages for which there were fewer than 10 deaths (over 104 for males and 106 for females) rates were assumed to equal 0.500000 for males from aged 111 through 119 and females aged 113 through 119. Mortality rates at age 120 for both males and females were assumed to equal 1.00000, ruling out the possibility that anyone could live to exceed that age.

The figure below shows the resulting mortality rates for males and females, regardless of "collar". The general pattern is similar to that shown earlier for the broader groups in the human mortality database in 2010. As before, males were subject to greater mortality rates at all but the greatest ages, with the average ratio of male mortality to female mortality equal to 0.745 .

The rates at which mortality increases with age are very similar to that found by Gompertz. The ratio of mortality at age 100 to that at age 50 implies an annual (compounded) ratio of 1.091 for males and 1.096 for females. While the curvatures of the plots show that the increases are not constant across ages, the average change from year to year is remarkably close to the classic estimate of $9 \%$.


The following figures compare the RP-2014 mortality estimates from age 50 through 100 with the actual mortality rates in 2010 from the $H M D$ database.


Clearly, the pension plan annuitants had lower mortalities. The mean ratios of RP2014 rates to HMD rates were 0.738 for males and 0.796 for females. Some of this could have been due to progression in mortality rates between 2010 and 2014, but that would likely account for a cumulative decline of $4 \%$, not more than $20 \%$. Healthy pension plan annuitants almost certainly experience lower mortality rates than the general population.

The next two figures show the differences in mortalities for blue and white collar populations.



It is clear that blue collar employees comprise a larger part of the total populations at the earlier retirement ages than at later ages. Moreover, the results for both groups combined are more reflective of mortality rates for white collar employees, likely due in part to their higher salaries and thus greater weights in the averages.

Overall, the mean ratios of white collar to blue collar mortality rates between ages 50 and 100 inclusive are 0.734 for males and 0.800 for females. Based on past experience, salaried workers are expected to live longer than hourly workers, with the difference more pronounced for males than for females.

A caveat is in order here. Since the RP-2014 results are based on the experience of employees who have annuity payments in their later lives, there may be another factor leading to lower rates of mortality. In many private defined benefit pension plans, workers are given the option of receiving a lump-sum payment at retirement in lieu of some or all of their earned future retirement payments. It is possible that those with expectations of earlier mortality may be more likely to choose lump sum payments and thus have less or no weight in the RP-2014 samples. This sort of adverse selection could lead to mortality rates for annuitants that are lower than for the general populations of workers in the participating pension plans. Moreover, it is possible that those with annuity payments take better care of themselves and thus live longer than those who have to survive on a single lump sum payment, a characteristic termed moral hazard. We shall have more to say about these aspects in the next chapter.

## The MP-2014 Mortality Improvement Tables

In 2014, the U.S. Society of Actuaries issued a report recommending a set of projected mortality improvements for future years, termed the MP-2014 Mortality Improvement Scale, which we will refer to as simply "MP-2014". The key elements are contained in two tables one for males, the other for females. In each table, the rows represent ages and the columns future years. Every age from 21 through 114 is covered, along with a preliminary row for ages less than or equal to 20 and a final row for ages of 115 and over. Future years from 2015 through 2026 are covered in separate columns. There is also a final column for 2027 and thereafter. Each element indicates the projected improvement rate for a given age (row) and year (column), expressed as a proportion. For example, if the projected improvement for a given age in year $t$ is 0.02 , mortality in year $t$ is projected to equal $0.98(1-0.02)$ times the mortality in year $t-1$.

This "two-dimensional" approach to mortality improvement projection reflects the assumption that at until 2027, mortalities may change by different proportions every year, but thereafter the improvement rates for each age will remain constant. That said, the Society of Actuaries Mortality Improvement Scale MP-2014 Report, which is the source of most of the material in this section, "... anticipates that (an) updating process be performed at least triennially ...(with) a new scale released in 2017 ... called Scale MP-2017". And so on.

Some additional details are needed to fully understand the process used to create the tables. Since the mortality experience used to produce the underlying RP-2014 tables involved years "centering" on 2006, projections were actually made from that year forward, so the RP-2014 mortality rates are themselves based on 7 years' improvement factors. The choice of the final set of improvement factors, reflected an assumption that such factors would converge to a set of "final rates" over a twenty year period.

Since the SOA Committee did not have annual data for historic mortality rates, it utilized a database for the mortalities of people in the Social Security system's "area population", which includes some U.S. Citizens living in other countries. Mortality rates were smoothed in the hope of better revealing underlying differences and trends (this author's characterization). The smoothed mortality rates were then used to produce (smoothed) mortality improvement rates. The results were summarized in two-dimensional heat maps, each of which shows the magnitudes of entries in a table with a corresponding color from a color map, which maps values into different colors. The figures below show maps for males and females, for ages from 35 to 95 (on the vertical axis) and years from 1951 through 2030 (on the horizontal axis). Smoothed historic values are plotted to the left of the dashed vertical white line (for 2007); projected values for 2014 lie on the solid vertical white line, and projected values for the years 2027 through 2030 are in the four columns on the right side. Also shown is a diagonal line showing the smoothed historic rates (before 2007) and projected rates (after 2007) for the cohort of people born in 1935 and hence of age 35 in 1970. (In these diagrams ages increase from the bottom of the diagram, so cohort data fall along upward-sloping diagonal lines).

MP-2014 Heat Map for Males


MP-2014 Heat Map for Females


The SOA committee indicated that it ".. continues to find heat maps extremely helpful in the identification of various types of historical mortality improvement trends in the United States", noting that "... vertical patterns of unusually high or low mortality improvement indicate past 'period' effects, while diagonal patterns of unusually high or low improvement indicate year-ofbirth 'cohort' effects." In contrast, "By their very structures, one-dimensional "age only" scales are unable to project period and cohort effects, which have been the predominant features of U.S. Mortality improvement experience over the last 50 years."

The figure below shows values from the MP-2014 tables for 2015, 2021 and 2027+. As can be seen, the projected improvements from 2014 to 2015 differ greatly, with mortalities for ages between 70 and 90 predicted to improve considerably more than all but those for people in their early 50 's. Subsequent rate improvements decrease each year, reaching the assumed long-term rates in 2027 of $1 \%$ per year for males and females under 85, with lower rates for those under 115 and no mortality improvements thereafter for the truly old over 115.


The SOA report points out that there have been "... relatively high levels of mortality improvement for the so-called 'silent generation', born between 1925 and 1942 (and) ... low levels of mortality improvement for the 'baby boom' generation (especially for males born around 1950 and females born around 1955)." But as the figure shows, such differences are expected to diminish as the projected long-term rates approach.

The figure reflects a decision by the committee to utilize a simple average of horizontal and diagonal interpolations, which "... produced an appropriate balance of anticipated age/period and cohort effects". While the both can be observed in this diagram, the cohort effects do not seem to be particularly pronounced. Simple linear interpolations between the 2015 and 2027 improvement rates give intermediate estimates that correlate well with the MP-2014 values for both males and females the correlation coefficients are 0.95 .

## United States Social Security Mortality Tables

The procedures used by the U.S. Social Security Administration (SSA) to estimate current and future mortality rates are complex. According to the Office of Chief Actuary's Note Number 150, January 2013 (from which quotations in this section are taken) every four years, a technical panel is selected "... from the nation's most knowledgeable demographers, economists and actuaries". And intensive review from such technical panels "has influenced the evolution of the projection methods", and the projections developed by the Office ".. have been subject to annual full scope audit by a major independent accounting firm."

The SSA bases its mortality estimates for those 65 and over on historical death rates of individuals enrolled in Medicare and eligible for Social Security benefits, using data from the National Center for Health Statistics. Estimates for those under 65 are based on State death reports and census population data.

Current SSA mortality estimates are based on assumptions about future mortality that incorporate both age-specific and cause-specific extrapolation. In particular, five causes of mortality are considered: cardiovascular disease, cancer, violence, respiratory disease and "other". Three sets of predicted mortality rates are created - alternative II, which is termed the Trustee's best estimate - is used here .

The SSA mortality estimates in this chapter were provided by the Social Security Administration in 2013 for research at the Stanford University Institute for Economic Policy Research and are used with permission from the Institute.

We begin with a comparison of the mortality estimates for ages 50 through 119 (the last age included in the data set) for males. The SSA estimates are shown in the figure below, along with those for the total private pension population from RP-2014 tables shown earlier.


As can be seen, the general patterns are similar, with two major differences. First, the SSA mortality rates continue to increase after age 110 , while those for the private sector remain constant at 0.500 (jumping to 1.0 at age 120, which is not shown). Second, the SSA mortalities are higher, with an average ratio of 1.27 to 1 . This shows once more the difference between the mortality rates of the general population and those who receive pensions from private corporations.

## T

he following figure provides the same information for females. Again, mortality is greater for the general population, but the ratio is somewhat smaller, at 1.21 to 1 .


## Social Security Mortality Improvement Projections

As indicated earlier, future mortality improvements projected by Social Security vary by both age and year. And improvement rates for each sex continue to change from year to year, rather than converging to an "ultimate" set of such rates that is assumed to continue after a specified year (such as 2027 in the case of the MP-2014 projections).

The following figure shows the projected Social Security improvement rates for males and females for three selected years (2015, 2027 and 2075). In all three years, rates differ across ages. And over time, there seems to be a convergence towards rates of roughly $1 \%$ for ages from roughly 50 to $60,0.70 \%$ for ages from 55 to 80 , and $0.50 \%$ for those over 85 .


Qualitatively, these projections differ from those made by the Society of Actuaries for private pension plans, But quantitatively, they are not wildly different. This may not be surprising, since although very different methods were utilized, each projection was made after analyzing historic data.

This concludes our exploration of two key sets of mortality projections for U.S. Citizens. In the next chapter we will use some of these results to provide our first scenario matrix.

