Retirement Income Analysis
with scenario matrices

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6. Inflation-Protected Investments

Riskless and Risky Investments

Directly or indirectly, most sources of retirement income depend on the return from some sorts of investment. To understand the range of likely incomes from such sources, we need to construct matrices of possible future *investment returns*. In this and the next chapter, we shall do so in the most parsimonious manner possible, with a single *riskless* investment and a single *risky* one. While this may seem hopelessly oversimplified, financial economic theory provides a possible rationale (or rationalization) for doing so. Moreover, it will be less limiting that one might at first think, since it is possible to combine these two prototypical investments in a myriad of ways, and introduce other types of investments that can be used to analyze particular strategies, as will be seen in later chapters.

Recall from the discussion of inflation that our focus is on *real,* not *nominal* income. Hence it is important that our returns be stated initially in real terms, and that the riskless asset provide payments with predictable purchasing power, not those with fixed nominal monetary values. For these reasons, we will generate matrices of asset returns stated in real terms. Of course, our cost of living matrices can be used to convert real values to nominal, or vice-versa. We will do so frequently when analyzing alternative retirement income strategies.

This chapter deals with the first of our two investments – a riskless real asset; the next deals with our key risky alternative.

Riskless Real Returns

We can presume that from the early days of commerce, people made agreements to repay loans in terms of goods and services. That said, most governments appear to prefer to issue the majority of their bonds with promised payments stated in units of currency. However, such bonds are difficult or expensive to sell when there is considerable uncertainty about changes in the cost of living. In a fascinating paper titled "The Invention of Inflation-indexed Bonds in Early America" published in 2003, Robert Shiller documented a precursor of bonds of this type now issued by governments around the world. He writes that the bonds in question were " … issued by the Commonwealth of Massachusetts in 1780 during the Revolutionary War. These bonds were invented to deal with severe wartime inflation and with angry discontent among soldiers in the U.S. Army with the decline in purchasing power of their pay."

The bond certificates stated the terms succinctly:

> "Both principal and interest to be paid in the then current money of said state, in a greater or less sum, according as five bushels of corn, sixty-eight pounds and four-seventh parts of a pound of beef, ten pounds of sheeps wool, and sixteen pounds of sole leather shall then cost, ..."

The remainder of the sentence reveals the motivation for issuing the bonds, indicating that the cost of the basket at the time of issuance was

> "... thirty-two times and a half what the same quantities of the same articles would cost ... in the year of our Lord one thousand seven hundred and seventy-seven..."

After experiencing 3200% inflation of the price of a basket of these four goods in three years, it is no wonder that bond buyers were eager to be guaranteed a constant amount of purchasing power for the bonds' interest and principal.

After this early period of runaway inflation, monetary policy in the United States evolved, and price changes were less dramatic most of the time. This was also true, with some notable exceptions, in a number of other countries. At least partially for this reason, few inflation-indexed bonds were issued by governments with good credit until the latter part of the twentieth century. In 1981, the United Kingdom issued its first "Index-linked Gilts" (colloquially called *linkers*), with principal and interest payments based on changes in the U.K. General Index of Retail Prices (RPI). In 1997, the United States introduced Treasury Inflation-Protected Securities (TIPS), indexed to the U.S. Consumer Price Index. TIPS are backed by the "full faith and credit" of the United States Government, and their coupon and principal payments are generally considered very safe.

With some exceptions, TIPS follow the approach of the early Massachusetts bonds. Each issue has a maturity date *T,* a principal amount P_T (for example, $1,000) and a annual coupon rate *r* (for example 0.25 of 1%), used to determine an amount paid at six-month intervals up to maturity. At time *t*, when a coupon payment is due, the amount paid will be based on the coupon rate times an *accrued principal* value. At maturity, the amount paid will equal the accrued principal at that time. Importantly, at any time, the accrued principal will equal the larger of (1) the initial principal value and (2) the initial principal value times the ratio of the Consumer's Price Index for all Urban Consumers (CPI-U) at the time divided by its level at the time of issue. Absent deflation, all payments will thus be constant in real (purchasing power) terms. However, if at the time of any payment the CPI-U is below its level at the time of issuance, the real value of the payment will be greater than the guaranteed amount.

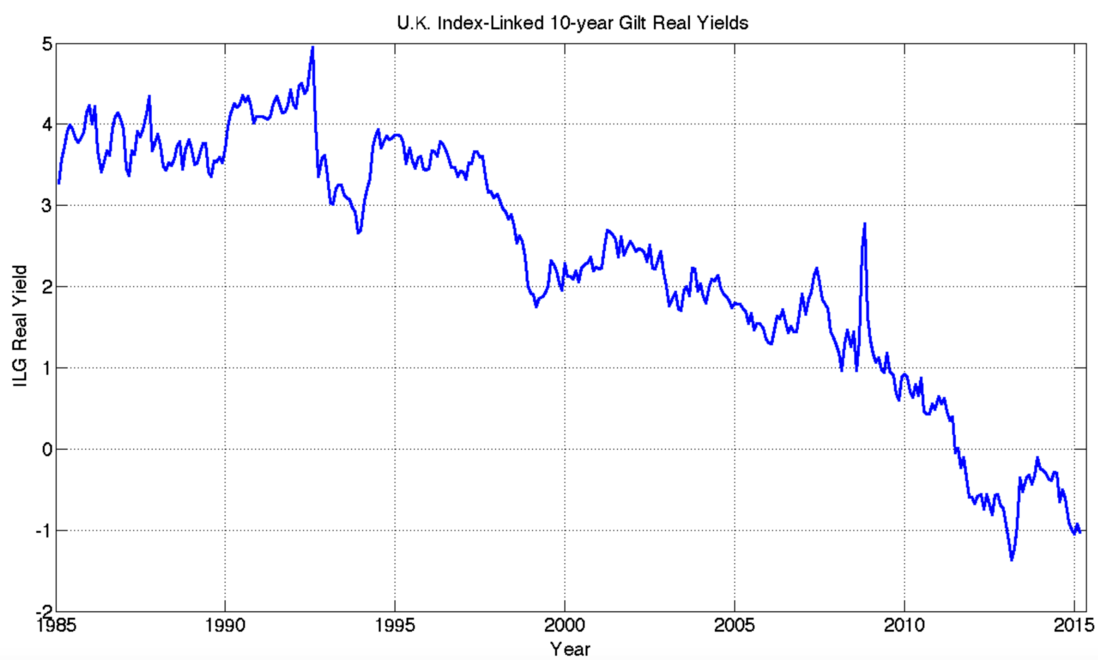
When a new set of TIPS securities is issued, the Treasury Department conducts a complex type of auction to determine the initial price and coupon value. Typically, the coupon rate and maturity are set beforehand in the hopes that the initial price will be close to $1,000 per bond, but this may not turn out to be the case. After the initial auction, the price of a TIPS issue will be set in the market, and may vary substantially.

TIPS may be purchased from the U.S. Treasury and held by the Treasury Department or elsewhere. They may be bought or sold prior to maturity through a bank, broker or dealer. Unlike stocks, the price paid or received for a bond (including a TIPS certificate) on the secondary market is determined by the broker or dealer, leading to the possibility of a substantial gap between the purchase cost and sales proceeds.

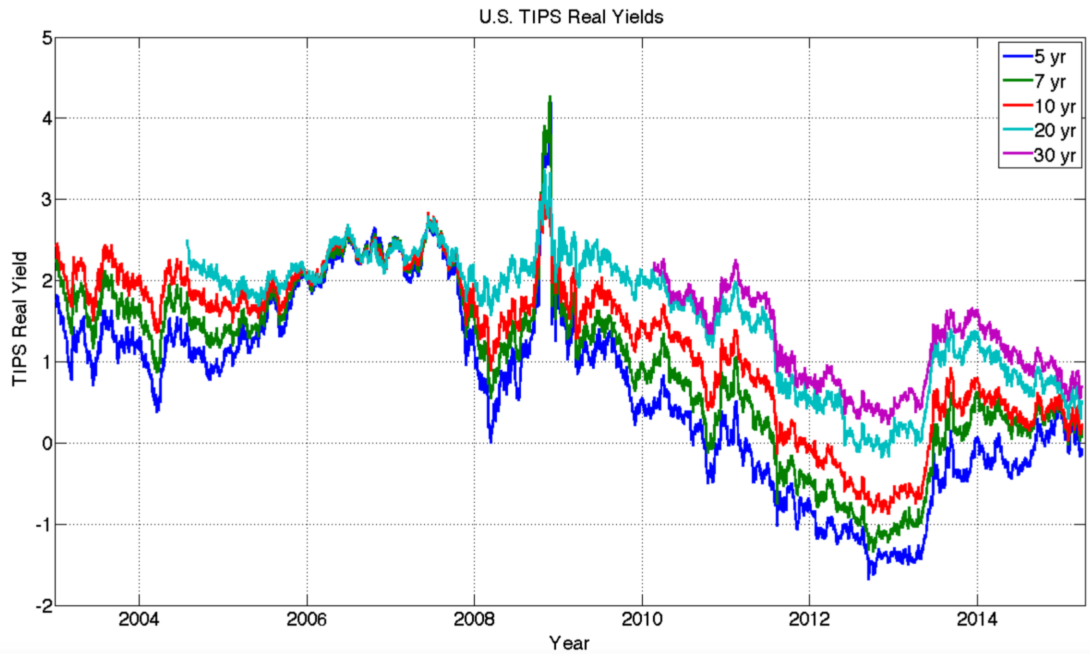
TIPS Yields To Maturity

As with any bond, the yield based on the current market price of a TIPS issue can differ substantially from the initial amount. To better reflect prospects for such a security, market analysts compute a *yield-to-maturity* on the assumption that there will be no further inflation. Basically, this is the discount rate that makes the present discounted value of all future coupon payments and the principal payment equal to the current price of the bond.

The figure below shows the real yields to maturity for index-linked gilts in the United Kingdom from 1985 through early 2015. The secular decline is dramatic. Until the late 1990's such instruments provided from slightly under 3% yield to maturity to almost 5%. But then yields started falling, reaching zero in the latter part of 2011. Thereafter, "linkers" were priced to provide a negative real yield!



The experience for U.S. TIPS yields was similar, as shown in the next figure.

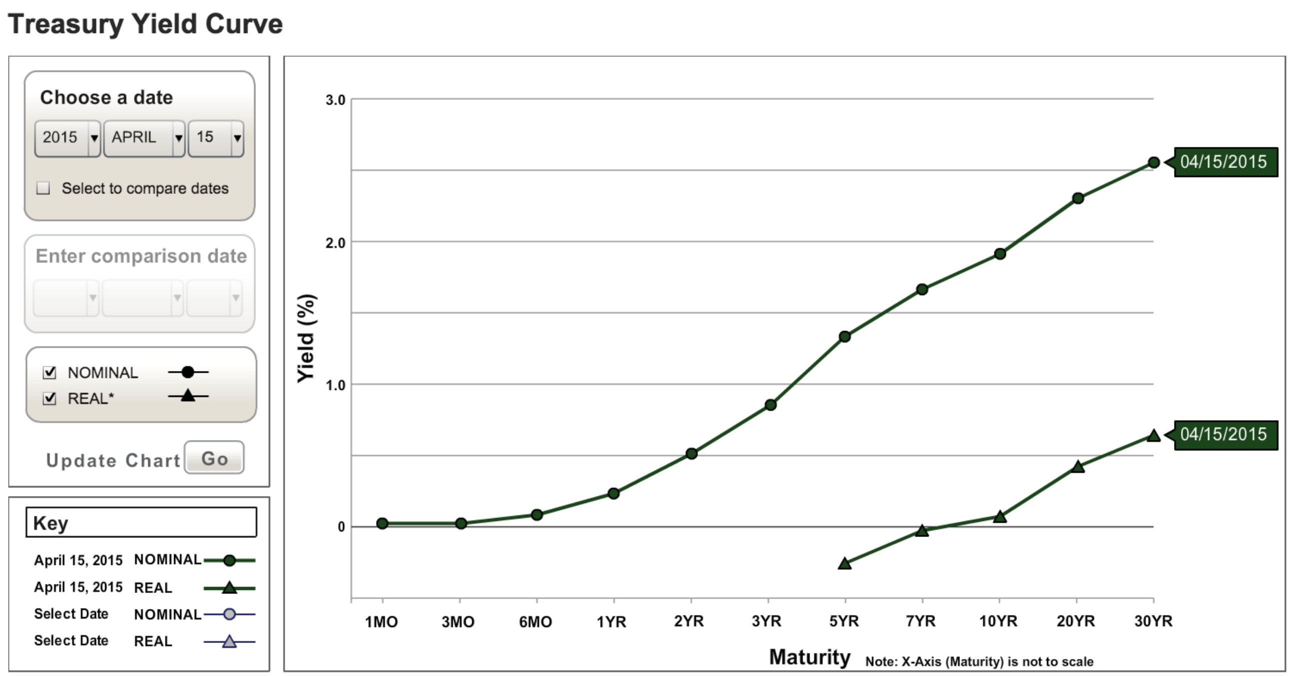


Real yields for 10-year TIPS moved into negative territory toward the end of 2011. They then rebounded to small but positive values, falling back to virtually zero in early 2015.

This graph also shows yields to maturity for TIPS with different remaining lives at each date (based on a curve relating yield to maturity fitted to the yields from all TIPS). As can be seen, at any given time, longer maturities had greater yields to maturity, although the differences across maturities were relatively small in the latter months.

Implied Future Inflation Rates

It can be instructive to compare the yields on TIPS with “regular” treasury bonds that promise payments fixed in nominal terms. The Treasury department maintains a web site that makes it simple to do so. The figure below was obtained from the site in April, 2015.



Note that both curves are upward-sloping. We will have more to say about this later. Here we focus on the spread between the two curves.

Consider the yields on the two instruments with 30-year maturities. Traditional treasury bonds offered a yield to maturity of roughly 2.5% per year while inflation-protected bonds with the same maturity were priced to yield roughly 0.6%. Some analysts consider the the difference (here, 1.9%) as “the market's” estimate of expected inflation over the next 30 years. In this case it is remarkably close to the Federal Reserve's goal of 2%. Of course, actual inflation is likely to differ. Moreover, the spread between these yields is undoubtedly influenced to an extent by other factors, including risks . But the calculation is often performed, and the results can provide useful information.

TIPS Mutual Funds and ETFs

While it is possible to purchase TIPS directly, many investors choose instead to invest in a fund that holds TIPS of different maturities.

An advantage for direct investment is the ability to create a *ladder* of such securities, with different maturities that will pay desired amounts at each of a number of future dates. When yields were higher it was more difficult to accomplish this, but clever people on Wall Street (seeking a chance to make a profit) made it easier by purchasing TIPS, then using them to back new instruments, each of which paid a given real amount on a single date. Such securities are called (cutely) *Separate Trading of Registered Interest and Principal of Securities*, or STRIPS – in this case TIPS STRIPS. Of course when coupon payments are small relative to the final principal payment, the need for such STRIPS (and any extra cost to be paid to the "strippers") may be substantiality diminished.

In April 2015, there were 39 different issues of TIPS outstanding, with remaining lives from less than one month to almost 30 years. Barclays Capital, which computes and publishes data on the returns of a great many different securities, publishes daily values of *Barclays US Government Inflation-Linked Bond Index* that reflects changes in the value of all outstanding TIPS with maturities greater than or equal to one year. And a number of financial institutions offer *index funds* designed to replicate, as closely as possible, the returns on this index.

It is important to insure that returns of an index fund track those of its underlying index, with expenses that are as small as possible. In 2015, two funds met these criteria well – the Vanguard Inflation-Protected Securities Fund and the Schwab U.S. TIPS Exchange Traded Fund.

Vanguard offers two versions of its TIPS fund. *Investor shares* are available for those with less than $50,000 invested; *Admiral shares* for those with more. Annual expenses forfund management are 0.20% per year (20 basis points, or 20 cents per $100 invested) for Investor shares (ticker symbol:VIPSX) , and half that for the Admiral shares (ticker symbol: VAIPX). The fund returns have tracked those of its chosen index (the Barclays Index) extremely well: through early 2015, variations in the return of the index explained 99% of the variation in the returns of the fund (that is, the R-squared value was 0.99). While the fund's holdings varied slightly from those of the index, the differences were very small.

The Vanguard fund is a so-called *open-end* mutual fund. Investors can purchase shares from the fund or sell shares back to it for redemption on a daily basis. This requires the fund to buy or sell underlying TIPS securities as needed, to cover differences between purchases and redemptions, thereby incurring some costs, although they tend to be minor.

The Schwab fund (ticker symbol: SCHP) differs in structure. It holds a set of securities designed to replicate the same Barclays index, and issues tradable shares representing proportional interests in the portfolio. The ETF shares can be traded on exchanges, like any share of common stock. From time to time, the fund may purchase more TIPS, issuing new shares; it may also redeem existing shares, giving a combination of its TIPS shares in return, (although such transactions are generally only made with large financial institutions). While the market prices of the shares of an ETF can differ from the current value of its underlying security holdings, such differences tend to be very small. The Schwab ETF returns also mirror those of the Barclays TIPS index well. The fund's expense ratio (0.07% or 7 basis points per year) is even lower than that of the Vanguard Admiral fund, but investors have to pay commissions to purchase or sell shares in the ETF on the open market. However, such commissions can be very small indeed and should be incurred infrequently.

There are other TIPS funds, but the expense ratios of those available in early 2015 were considerably greater. This may not seem important, but it is. For example, consider a fund that costs 0.20% (20 basis points) per year. Compared with the value of shares held, this not much greater than 0.10% – 10 cents out of every $100. But it must be paid each year. Thus it should be compared with the likely amount spent each year, which might be, say, $5.00 per $100 of initial investment. The added cost would thus be 10 cents out of $5.00 each year, or 2% less to spend. Expenses for many mutual funds that hold equities (stocks) are even more dramatic. A typical equity index fund may have an annual expense ratio of 0.10% or less of assets, while an actively-managed fund might charge 1.10%. If the sustainable withdrawal rate per $100 of value is $5.00, the added cost for active management is $1.00 out of $5.00, or 20% per year! And, as we will see in the next chapter, the average actively-managed equity fund is likely to perform no better than a passively-managed index fund *before costs*. The moral is that costs matter very much indeed.

If the mix of maturities represented in the Barclays index is appropriate for an investor, it may make good sense to pay the relatively low expenses charged by an efficient index mutual fund or ETF. Such funds can buy and sell TIPS at wholesale prices, provide needed tax information, and so on. In mid-2015, SCHP or VAIPX appeared to be the best choices for such a role.

On the other hand, the timing of the cash flows from the portfolio held by a TIPS fund or ETF may not be ideal for a particular investor. TIPS mutual funds and ETFs typically hold all available securities, with maturities from less than one year to as much as 30 years. As indicated earlier, in bond parlance these funds provide a *ladder* of bond cash flows. A common measure of the timing of cash flows from a bond or bond fund is its *duration* – a weighted average of the future times at which cash would be received, using weights based on the present values of the cash flows. In mid-2015, the average duration of the Vanguard broad TIPS funds was 8.1 years. Roughly, this indicates that the the overall value of each fund would react to a parallel shift in the TIPS yield structure in a magnitude similar to that of a TIPS Strip with a maturity of 8.1 years.

Investors interested in shorter-term portfolios can invest in Vanguard's Short-Term Inflation-Protected Securities Index Fund, which holds only TIPS with remaining maturities of less than five years. As do the other Vanguard TIPS funds, investor shares have a fee of 20 basis points, and Admiral Shares a fee of 10 basis points, but for this fund Admiral Shares require only a minimum of $10,000 invested. In mid-2015, each had a duration of 2.5 years.

For many, investment in one or two of funds with different mixes of maturities should suffice. But some may wish to purchase individual TIPS securities to better match likely horizons despite the increased effort, bookkeeping, tax accounting, etc..

Forward Interest Rates

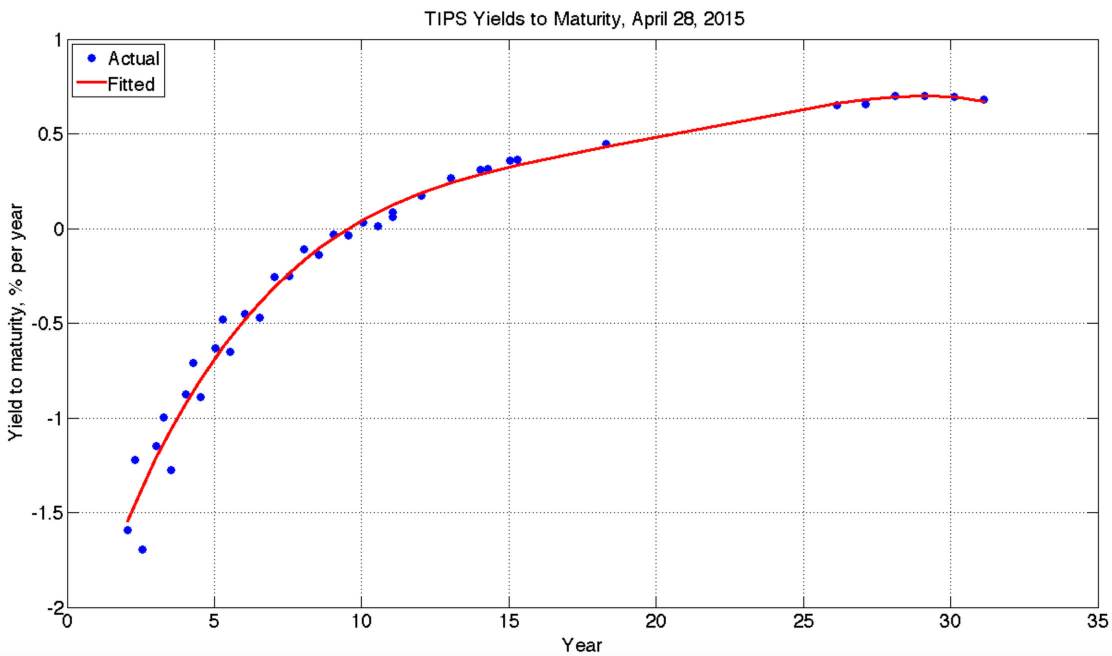
As the prior figures showed, it is typical for the yields-to-maturity of longer-term TIPS to exceed those of shorter-term ones. At most (but not all) times, real yield curves are upward-sloping.

To better understand this phenomenon, it is important to distinguish between *spot* interest rates and *forward* interest rates.

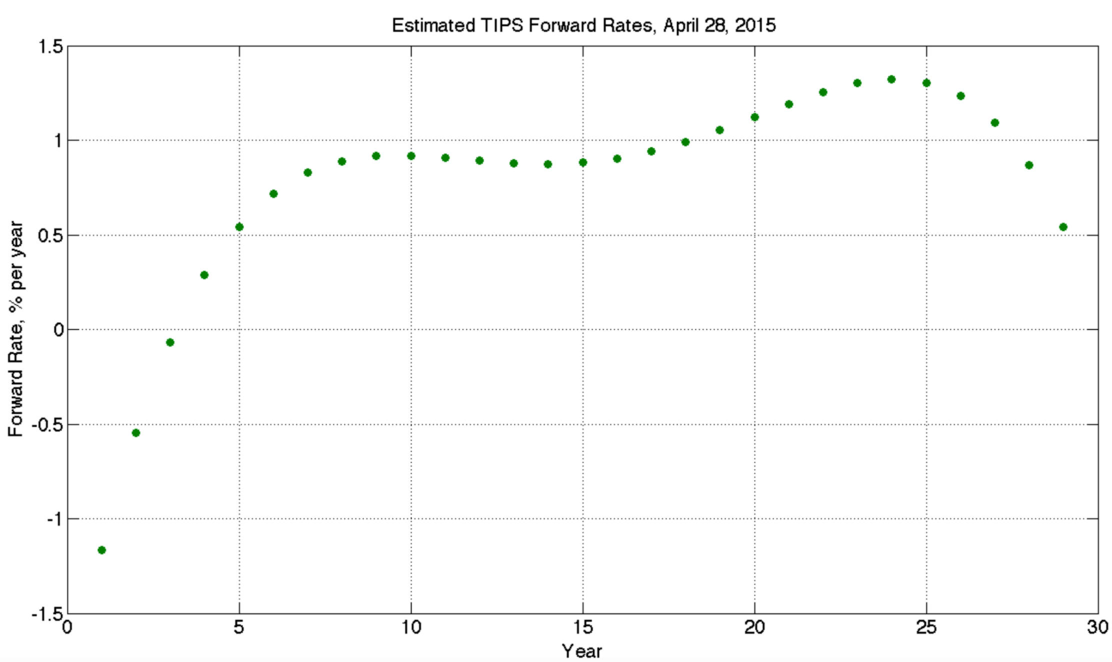
Let's say that the real yield to maturity on a 1-year TIPS STRIP is 1% per year while the yield to maturity on a 2-year TIPS STRIP is 2% per year. Assuming that there will be no deflation in the next two years, this means that $1 invested in the 1-year maturity will grow to $1.01in a year, while $1 invested in the 2-year maturity will grow to $1.02 in two years. In bond-speak, the 1-year spot rate is 1% and the 2-year spot rate is 2%. Both are known today and are certain. An investment of $1,000 in a one-year strip would provide $1,000*1.01 = $1,010 in a year, while an investment of $1,000 in a two-year strip would provide $1,000*(1.02*1.02) = $1040.40 in two years

Now imagine that you could buy a 2-year strip for $1,000 and simultaneously *short sell* a 1-year strip for $1,000. The latter procedure would involve borrowing the instrument, pledging your 2-year strip as collateral and promising to pay back the principal value of the 10year strip ($1,010) in a year. Your cash inflows and outflows would net to $0 today, -$1,010.00 in a year and +$1,040.40 in two years. Note that these values would be known today and, absent a U.S. Government default, subject to no uncertainty In effect, you would have locked in a one-year investment a year in advance at an interest rate of 3.01%, since 1040.40/1010.00 = 1.0301. In bond-speak, the *one-year forward rate* for money one year hence is 3.01%. Note that this is considerably higher than the two-year spot rate of 2%, since the latter is a function of the first year's spot rate and the second year's forward rate.

The next two figures illustrate the differences. The first plots the yields to maturity of all the TIPS outstanding on April 28, 2015, along with a 4-th degree polynomial curve (chosen by experimentation) fitted to the data.



The next figure shows the associated forward rates for each year, based on the fitted curve.



This tells a different story. Only the first three rates are negative and the majority are close to 1% per year. But what is the moral of the story? Is the forward rate for a future date a good estimate of the likely spot rate at that time? The answer is, at best, “maybe”. Forward rates for traditional non-indexed government bonds have been shown to larger than the subsequent spot rates more than half the time. Due to the major secular changes in TIPS rates since their relatively recent introduction, it is difficult to say whether this bias is likely to be germane for current or future TIPS yields. Moreover, interest rates in general were in uncharted territory in early 2015, with banks in some countries offering loans with negative nominal interest rates (“give us your money and we'll give you back *less* at a future date”) – an unusual situation indeed.

Current and Future Riskless Real Returns

Scenario matrices are about the future, not the past. Our proximate goal is to create matrices of annual and cumulative future real returns provided by inflation-indexed government bonds in different scenarios. One possible approach would produce a matrix of riskless rates, with each row containing the current 1-year spot rate in the first column and the current forward rate for each year in the corresponding subsequent column. Such rates would be riskless, in the sense that every row (scenario) would be the same. But future spot and future forward rates would almost certainly be different, thus our matrix of current forward rates would at best, provide estimates for such rates.

Instead, we take the simplest possible approach, assuming that every forward rate equals the current spot rate, so that the term structure is "flat". This assumption was clearly at odds with the facts in early 2015 – a time of historically low real and marginal interest rates in many countries.

The goal of this book is to illustrate ways in which scenario matrices can be used to investigate key aspects of the provision of retirement income. To do so we build *models* that abstract from many of the details of the "real world", choosing instead to focus on the central elements. With luck, our models may pass the test attributed to Alfred Einstein: "Everything should be made as simple as possible, but not simpler."

We will discuss some of the issues associated with ignoring varying real interest rates in later chapters. Suffice it to say here that we choose to keep things simple by assuming that real rates equal for every maturity and do not change over time.

Riskless Real Return Scenario Matrices

Given this simplifying assumption, it is a simple matter to create scenario matrices for riskless real returns. Our default assumption is that there is a constant real riskless rate of 1% per year, so that $1 at the beginning of the year will grow to an amount at the end of the year that can purchase goods and services that would have cost $1.01 at the initial prices. The program statements are straightforward. First we add the following to the *market_Create* function:

```
% risk-free real investments
   market.rf    = 1.01;  % risk-free real return
```

Of course, one can change this parameter for any given case in the corresponding script (although we will retain this assumption for all our examples).

Next, we add the following to the *market_process* function:

```
% compute risk-free real returns matrix
    market.rfsM = market.rf * ones( nrows, ncols );

% compute cumulative risk-free real returns matrix
   m = cumprod( market.rfsM , 2 );
   market.cumRfsM = [ ones(nrows,1) m(:,1:ncols-1) ];
```

The first statement creates a matrix of the desired size with the same entry in every cell. The next two statements provide a matrix of the compounded values, indicating the ratio of the value of a riskless investment at the beginning of each year to the value invested at the beginning of the first year. It does so by first cumulating the product of the entries "horizontally" along the second dimension, as we did for the matrix of cumulative inflation values, and then changing from end-of-year values by appending all but the last column of results to a vector of ones, as we also did for the inflation values..

Having constructed riskless real return matrices, we now turn to investments that are clearly risky and must be treated as such. For reasons that will become clear in the next chapter, we focus on an investment termed the *market portfolio*.