Errata for Linear and Nonlinear Programming, 3rd and/or 4th Edition

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Chapter 1. Introduction

1. **p.5, line -7**: 'exploited' is spelled 'explioted' (reported by Alex Lemon)

2. **p.5, line -7**: 'Today linear programming software packages are capable of automatically identifying sparse structure within the input data and take advantage of this sparsity in numerical computation.' This sentence lacks parallelism between 'identifying' and 'take advantage.' One solution is 'Today linear programming software packages are capable of automatically identifying sparse structure within the input data and taking advantage of this sparsity to perform numerical computations efficiently.' (reported by Alex Lemon)

3. **p.6, line -3**: 'A picture is worth a thousand words.' This quote seems to be falsely attributed to Confucius. (reported by Alex Lemon)

4. **p.7, line 6**: 'because the method actually converges to an exact solution a finite number of steps.' Missing the word 'in.' (reported by Alex Lemon)

5. **p.7, line 14**: 'it is meaningful to consider the speed of converge.' This should read 'convergence.' (reported by Alex Lemon)

6. **p.7, line 21**: 'There are in fact two aspects of convergence rate theory.' A hyphen is missing. Also in this paragraph, 'polynomial time algorithms' should be hyphenated. A hyphen is also missing in 'convergence rate theory' at the beginning of the next paragraph. (reported by Alex Lemon)

Chapter 2. Basic Properties of Linear Programs

1. **p.13, Example 4, line 1**: 'A second approach for converting to standard form when x_1 is unconstrained in sign is to eliminate, x_1 together with one of the constraint equations.' There should not be a comma after 'eliminate.' (reported by Alex Lemon)

2. **p.15, Example 2, line 3**: 'Associated with the shipping of a unit of product from origin i to destination j is a unit shipping cost c_{ij}.' I think c_{ij} would be accurately described as 'a per-unit shipping cost' or simply 'a shipping cost.' (reported by Alex Lemon)

3. **p.17, Example 4, line 3**: The objective function in Example 4 does not seem to include the cost of purchasing goods; perhaps it should be

$$\sum_{i=1}^{n} (p_i \cdot s_i - p_i \cdot u_i - r \cdot x_i)$$

   Otherwise, the problem is always unbounded: the solution $s = t \cdot \mathbf{1}, u = x = 0, z = C \cdot \mathbf{1}$ achieves an objective value of $t \cdot \sum_{i=1}^{n} p_i \to \infty$ as $t \to \infty$. (reported by Alex Lemon and Jae Park)

4. **p.17, Example 4**: p_i is not defined (reported by Alex Lemon)

5. **p.17, Example 4**: It is not explained that z_i is a slack variable. (reported by Alex Lemon)

6. **p.17, Example 5**: This is actually a linear classifier, not a support vector machine. (reported by Alex Lemon)

7. **p.17, Example 5**: There is no objective function (reported by Alex Lemon)

8. **p.17, Example 5**: Even when viewed as a feasibility problem, the problem is not well-posed because there are infinitely many separating hyperplanes (reported by Alex Lemon)

9. **p.18, line 2**: 'An order by the $j-$th participant consists of an vector...' should be either 'consists of an $m-$vector' or 'consists of a vector.' (reported by Alex Lemon)

10. **p.18, line 7**: The text seems to suggest that q_j and x are integers; it should be made clear that these quantities are real numbers, which matches the formulas. (reported by Alex Lemon)

11. **p.20, line 7**: 'There is ambiguity associated with a degenerate basic solution, however, since the zero-valued basic and nonbasic variables can be interchanged.' This statement is not technically correct. Suppose

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \quad \text{and} \quad b = \begin{bmatrix} 1 \\ 0 \end{bmatrix}.$$

If we choose our basis to be $B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, then the basic solution is $x_b = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$. However, we cannot exchange the zero-valued basic variable x_2 with the nonbasic variable x_3 since $B = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$ is not a basis. (reported by Alex Lemon)

12. **p.25, Example 2, line -1**: The first basic solution is $(2, -1, 0)$, not $(2, 1, 0)$. (reported by Alex Lemon and Jae Park)

13. **p.26, line -1**: The notation $z = -7/2$ is clearer than $z = -3(1/2)$. (reported by Alex Lemon)

14. **p.27, Fig. 2.5**: The notation z = -7/2 is clearer than $z = -3\frac{1}{2}$. (reported by Alex Lemon)

Chapter 3. The Simplex Method

1. **p.33, line -2**: 'understand' should be 'understands' (first word of the penultimate line) (this is not necessarily a mistake: it could be an elliptical phrase - Alex). (reported by Alex Lemon)

2. **p.34, Equation (3)**: The subscripts of the b_i on the right side of (3) should not be bold (compare with the right side of (1)). (reported by Alex Lemon)

3. **p.34, Equation (4)**: The term $y_{2,m+2}x_{m+2}$ is awkwardly aligned. (reported by Alex Lemon)

4. **p.34, Equation (4)**: The last line does not have the term $y_{m,m+2}x_{m+2}$ which breaks parallelism. (reported by Alex Lemon)

5. **p.35, line 4**: The spacing seems inconsistent in the first equation. Consider

$$x_1 = y_{10}, \quad x_2 = y_{20}, \quad \cdots, \quad x_m = y_{m0}, \quad x_{m+1} = 0, \quad \cdots, \quad x_n = 0$$

   or

$$x_1 = y_{10}, \quad x_2 = y_{20}, \cdots, x_m = y_{m0}, x_{m+1} = 0, \cdots, x_n = 0.$$

   (reported by Alex Lemon)

6. **p.35, Tableau**: In the tableau, may want to either completely fill out the third row, or omit it. (reported by Alex Lemon)

7. **p.35, Tableau**: In the tableau, write y_{1n}, whereas in (4), write $y_{1,n}$ (probably want to change (4)). (reported by Alex Lemon)

8. **p.35, Equation (5)**: I think the expression

$$y'_{ij} = y_{ij} - \frac{y_{iq}}{y_{pq}} y_{pj}$$

   is clearer than

$$y_{ij}? = y_{ij} - \frac{y_{pj}}{y_{pq}} y_{iq}$$

   because it reflects the fact that multiples of row p are added to the other rows in order to perform the elimination; the former expression is what appears in (11) below. (reported by Alex Lemon)

9. **p.36, line -5**: The whole section 'Second Interpretation' is somewhat confusing. Emphasis should be placed on the fact that y_{ij} is the coefficient of the $i-$th basic variable (that is, the variable that is basic for the $i-$th column) in the expansion of $\mathbf{a}_j$ in terms of the basis. Otherwise, it doesn't make sense that y'_{pj} is the coefficient of $\mathbf{a}_q$ in (10). (reported by Alex Lemon)

10. **p.37, Equation (7)**: Should have $\cdots$ instead of $\cdot$ in second, third and final rows of third column of (7). (reported by Alex Lemon)

11. **p.37, Equation (7)**: Either fill out the third row completely, or omit it. (reported by Alex Lemon)

12. **p.38, line 1**: 'Denoting the coefficients of the new tableau, which gives...' should be 'Denoting the coefficients of the new tableau, which give...'. (reported by Alex Lemon)

13. **p.40, line 1**: 'with the vector $\mathbf{a}_q$ replacing the vector $\mathbf{a}_p$, where p corresponds to...' (missing comma after $\mathbf{a}_p$). (reported by Alex Lemon)

14. **p.40, line 4**: 'can be regarded as the one which left the basis...' should be 'can be regard as the one that left the basis...' (restrictive versus nonrestrictive relative phrase). (reported by Alex Lemon)

15. **p.40, line 12**: 'In summary, we have deduced that, given a basic feasible solution...' (missing comma after 'deduced that') (reported by Alex Lemon)

16. **p.40, line -9**: 'which element in the $q-$th column to use as the pivot' ('the' missing before 'pivot') (reported by Alex Lemon)

17. **p.41, Example 3**: Mention that you are pivoting on $\mathbf{a}_1$. (reported by Alex Lemon)

18. **p.41, line -9**: 'Corresponding to the two interpretations of pivoting and extreme points developed algebraically...' (no comma after 'points'). (reported by Alex Lemon)

19. **p.42, line 2**: Should have a_i instead of a_1. (reported by Alex Lemon and Jae Park)

20. **p.43, First Tableau**: Should include $\cdots$ between zeros in the first, second, and the last rows and also consider including two $\ddots$ in the middle areas. (reported by Jae Park)

21. **p.48, First Tableau**: In the first tableau, the column of numbers below b should be $(2; 5; 6)$ instead of $(2; 5; 5)$. This correct column follows from the problem formulation given above, and is used to give the correct results in all subsequent tableaux. (reported by Alex Lemon and Jae Park)

22. **p.54, line -5**: The solution is $x_5 = 2$, whereas the text reads $x_5 - 2$. (reported by Alex Lemon

and Jae Park)

23. **p.59, First Tableau**: The second line of text refers to the 'the pivot indicated.' This should be variable 2, which would be indicated as in other examples with a circle around the number 5, just below y_1. (reported by Alex Lemon)

24. **p.59, Third Equation**: The two answers should be separated by a comma and a space. (reported by Jae Park)

25. **p.64, Second Equation**: The equation after 'This master problem has variables' needs a transpose on either the left or right side of the equation. (reported by Alex Lemon)

26. **p.65, Second Equation**: In the equation immediately preceding equation (49), r^* should be defined as the minimum over i of r_i^*, not the minimum over i of r_1^*. (reported by Alex Lemon and Jae Park)

27. **p.65, Equation (49)**: The term λ_{m+i} should not be bold faced. (reported by Alex Lemon)

28. **p.66, Example 2**: Example 2 should be labeled Example 1. (reported by Alex Lemon)

29. **p.66, line -1**: The constraint at the bottom of the page should be

$$\sum_{i=1}^{I} \alpha_i \mathbf{L}_i \mathbf{x}_I + \sum_{j=1}^{J} \beta_J \mathbf{L}_2 \mathbf{y}_j = \mathbf{b}_0$$

(note the subscript on $\mathbf{b}_0$). (reported by Alex Lemon)

30. **p.67, line 2**: The upper limit on the sum in the second equation should be J instead of j. (reported by Alex Lemon and Jae Park)

31. **p.67, line 5**: 'A starting basic feasible solution is $\mathbf{s} = \mathbf{b}$...' should be 'A starting basic feasible solution is $\mathbf{s} = \mathbf{b}_0$...' (reported by Alex Lemon)

32. **p.69, Second Tableau**: There is a '0.' in front of s_1 that should be removed. (reported by Alex Lemon)

33. **p.73, Problem 18**: The subscripts in (a) and (b) should be changed from $x_1 \geq 0$ to $x_i \geq 0$. (reported by Jae Park)

34. **p.73, Problem 21**: Consider adding $\ddots$(diagonal dots) between ones and between zeros and $\vdots$(vertical dots) between zeros. (reported by Jae Park)

35. **p.74, Problem 22**: The subscript in the last row should be changed from $x_1 \geq 0$ to $x_i \geq 0$. (reported by Jae Park)

36. **p.75, Problem 26**: The subscript in the last row should be changed from $x_1 \geq 0$ to $x_i \geq 0$. (reported by Jae Park)

37. **[4th ed] p.94 Problem 30(b)**: Problem 30(b): Exercise 4 $\rightarrow$ Exercise 29.

Chapter 4. Duality

1. **p.79, Equation (1)**: The right side of the first inequality constraint of the dual should read $\lambda^T \mathbf{A} \leq \mathbf{c}^T$ instead of $\lambda^T \mathbf{A} \leq c^T$ (note that $\mathbf{c}$ is bold in the correction) (reported by Alex Lemon)

2. **p.79, Equation (1)**: The 0 in the bottom constraint of the primal and dual problems should be bold. (reported by Alex Lemon)

3. **p.79, line -4**: Below equation (1), we should have '$\mathbf{b}$ is an m-dimensional vector' instead of '$\mathbf{b}$ is an n-dimensional vector'. (reported by Alex Lemon)

4. **p.80, line -14**: Remove the line in
$$\left[\begin{array}{c} \mathbf{A} \\ \hline -\mathbf{A} \end{array}\right]$$
(the line is dashed in the actual text). (reported by Alex Lemon)

5. **p.80, line -12**: In the unnumbered equation after 'the corresponding dual is,' we want to maximize the objective rather than minimize it. (reported by Alex Lemon and Jae Park)

6. **p.82, Equation (4)**: We want to maximize rather than minimize (reported by Alex Lemon and Jae Park)

7. **p.83, Fig. 4.1**: The figure is not valid in the sense that the values of the dual and primal criteria must meet in the middle because otherwise there is no feasible or optimal solution for both. And if there were no solution, then at least one of the sets must be empty. (reported by Alex Lemon)

8. **p.87, Fig. 4.2 and Fig. 4.3**: The slope of the vector $\mathbf{b}$ is not consistent with the other vectors. (reported by Alex Lemon)

9. **p.87, Fig 4.3**: The distances of the orthogonal lines from the origin are not consistent; for example, the line with normal vector a_4 is farther away from the origin than the line with normal vector a_2 (the distances should 6 and 12, respectively). (reported by Alex Lemon)

10. **p.91, Eq 12c**: Remove the unnecessary gap between $m+1$ and $m+2$. (reported by Jae Park)

11. **p.92, Example**: The subscript should be changed from x_i to x_1 in the first constraint. (reported by Jae Park)

12. **p.94, Eq 18**: The inequality in the constraint should hold for $i \in P$. (reported by Jae Park)

13. **p.104, Exercise 6**: We don't use **b** vector here. Instead, we should mention that '**x** and **c** be n-vectors'. (reported by Jae Park)

Chapter 5. Interior-Point Methods

1. **p.119, Example 1, line 6**: 'the unit cube is also defined by the inequalities $x_i \geq 0, (1 - x_i)^d \geq 0$ with $d > 1$' needs the additional condition that d must be odd. (reported by Alex Lemon)

2. **p.119, Example 1, line 10**: 'Also, the additional of redundant inequalities can also change the location...' should be 'The addition of redundant inequalities can change the location...' (drop the first 'also,' and change 'additional' to 'addition') (reported by Alex Lemon)

3. **p.120, line 13**: 'We introduce the notion...' should be 'We introduce the notation...'. (reported by Alex Lemon)

4. **p.123, First Equation**: We should have $y_1 + s_1 = -1$ instead of $y_1 + s_1 = 1$. (reported by Alex Lemon)

5. **p.140, Exercise 6**: The definition of Z^* should be corrected from $Z^* = \{j : x_j^* > 0\}$ to $Z^* = \{j : x_j^* = 0\}$. (reported by Jae Park)

6. **p.128, Example 3** Besides $y_1 \leq -1$ and $y_2 \leq 0$, there are two more (redundant) constraints: $y_1 \leq 0$ and $y_2 \leq 0$ in the problem.

Chapter 6. Transportation and Network Flow Problems

1. **p.154, Example 1**: We are using Example 1 from Section 6.1, not 5.1. (reported by Jae Park)

2. **p.162, Table 6.1**: One more column for arc $(4, 3)$ should be added. (reported by Jae Park)

3. **p.169, Equation (15)**: 'minimize f' should be changed to 'maximize f'. (reported by Jae Park)

4. **p.169, Equation (15)**: All six summations sum up to $j = m$, not n. (reported by Jae Park)

5. **p.169, Equation (15)**: The constraint $0 \leq x_{ij}$ in Equation (15) should be dropped or Equation (17), (18), and (19) should be changed. (reported by Jae Park)

6. **p.176, Problem 6**: The summation in the third row should range from $i = 1$ to m, not n. (reported by Jae Park)

7. **p.178, Problem 5(b) or 4th ed. p.94 Problem 30(b)**: 'upper triangular' $\rightarrow$ 'lower triangular'. (reported by Jae Park)

Chapter 8. Basic Descent Methods

1. **p.245**: The proof for Lemma 1 was omitted. (reported by Jae Park)

2. **p.245, Theorem 1**: $\beta^2 \to \beta$ in the first inequality. (reported by Jae Park)

3. **p.245**: Inequality (26) should be the one below the current one. (reported by Jae Park)

4. **p.246, line 3**: Omitted transpose $(x_{k+1} - x_k) \to (x_{k+1} - x_k)^T$. (reported by Jae Park)

5. **p.246, the line after Equation (30)**: 'holts for' $\to$ 'holds for'. (reported by Jae Park)

Chapter 9. Conjugate Direction Methods

1. **p.275, Equation (32)**: Missing parenthesis $E(x_{k+1} \rightarrow E(x_{k+1})$. (reported by Jae Park)

2. **p.282, Problem 9**: Missing brackets. (reported by Jae Park)

Chapter 10. Quasi-Newton Methods

1. **p.286, Line (-2)**: $\mathbf{B}_k \to B_k$. (reported by Jae Park)

2. **p.287, Line 7**: $\mathbf{B}_k \to B_k$. (reported by Jae Park)

3. **p.287, Second Equation**: $(\mathbf{p}_k^T \mathbf{P}_k)^2 \to (\mathbf{p}_k^T \mathbf{p}_k)^2$. (reported by Jae Park)

4. **p.294, Second to the Last Equation**: there are multiple typos here; and the correct one is

$$H_{k+1}^{BFGS} = H_k + \left(1 + \frac{q_k^T H_k q_k}{p_k^T q_k}\right) \frac{p_k p_k^T}{p_k^T q_k} - \frac{p_k q_k^T H_k + H_k q_k p_k^T}{p_k^T q_k}$$

   (reported by Jae Park and Ademir Ribeiro)

5. **pp.300-301**: $F_k \to F$. (reported by Jae Park)

6. **p.301, Line 3**: 'because $H_k F = F^{1/2} R_k F^{1/2}$' $\to$ 'because $H_k F = F^{-1/2} R_k F^{1/2}$'. (reported by Jae Park)

Chapter 11. Constrained Minimization Conditions

1. **p.324, Figure 11.2(c)**: Missing subscript $\nabla h(x^*)^T \rightarrow \nabla h_2(x^*)^T$. (reported by Jae Park)

2. **p.344, Equation 40**: $\mu^{T_1} \rightarrow \mu^T$. (reported by Jae Park)

Chapter 14. Dual and Cutting Plane Methods

1. **p.438, Equation 4**: 'maximize' → 'minimize'. (reported by Jae Park)

2. **p.442, Equation 13**: 'minimum' → 'min'. (reported by Jae Park)

Appendix A. Mathematical Review

1. **p.507, line -6**: Do we ever use the notation

$$\arg\min_{x \in S} \{f(x)\}$$

in analogy with the notation for the minimum over a set? (reported by Alex Lemon)

2. **p.508, line 20**: An $m \times n$ matrix all of whose elements are zero is called a *zero* matrix and denoted **0**. **A** square matrix...' The **A** should not be bold. (reported by Alex Lemon)

3. **p.508, line 21**: 'A square matrix (a matrix with $m = n$) whose elements $a_{ij} = 0$ for $i \neq j$, and $a_{ii} = 1$ for $i = 1, 2, \ldots, n$ is said to be an identity matrix...'. Perhaps this should read 'whose elements are $a_{ij} = 0$'. (reported by Alex Lemon)

4. **p.510, line -14**: In enumerating the properties of symmetric matrices, 'iii)' is used to list a property and '(iii)' is used to reference a property. (reported by Alex Lemon)

5. **p.511, line 2**: 'Similarly, we define positive semidefinite, negative definite and negative semidefinite if $x^T Ax \geq 0$, < 0 or ≤ 0 for all x.' This sentence lacks parallelism. Perhaps con- sider 'Similarly, we define A to be positive semidefinite, negative definite or negative semidefinite respectively if $x^T Ax \geq 0$, < 0 or ≥ 0 for all x.' (reported by Alex Lemon)

6. **p.511, line 12**: For this we use Q as·above and define.' There should not be a '·' between 'as' and 'above'. (reported by Alex Lemon)

7. **p.511, Section A.5, line 1**: There is $\{x_k\}_k^\infty = 0$ instead of $\{x_k\}_{k=0}^\infty$. Additionally, the sentence is somewhat convoluted. I would prefer 'denoted $\{x_k\}_{k=0}^\infty$ (or simply $\{x_k\}$ if the index set is understood), is said to converge...' (reported by Alex Lemon)

8. **p.511, Section A.5, line 4**: Do we really write limit $x_k = x$? The standard notation is $\lim x_k = x$. The latter notation is used in the book on page 218. (reported by Alex Lemon)

9. **p.511, Section A.5, line 7**: 'such that $\{x_k\}_{k \in \mathcal{K}}$ is convergent to x.' This is unnecessarily awkward. Why not just say 'such that $\{x_k\}_{k \in \mathcal{K}}$ converges to x?' (reported by Alex Lemon)

10. **p.512, Section A.5, line 12**: In standard mathematical usage, a sphere is

$$S(x, \epsilon) = \{y \in \mathbb{R}^n : \|x - y\| = \epsilon\}$$

and a ball is

$$B(x, \epsilon) = \{y \in \mathbb{R}^n : \|x - y\| < \epsilon\}$$

(reported by Alex Lemon)

11. **p.514, Theorem, -1**: In conclusion (iii) of the implicit function theorem,

$$h_i(\phi_i(\hat{x}), \phi_2(\hat{x}), \ldots, \phi_m(\hat{x}), \hat{x}) = 0$$

should be

$$h_i(\phi_1(\hat{x}), \phi_2(\hat{x}), \ldots, \phi_m(\hat{x}), \hat{x}) = 0.$$

The difference lies in the subscript on the first argument of h_i. (reported by Alex Lemon)

Appendix B. Convex Sets

1. **p.515, line 10**: 'shows the certain familiar set operations preserve convexity' should be 'shows that certain familiar set operations preserve convexity'. (reported by Alex Lemon)

2. **p.521, line 9**: The definition given for relative interior does not appear to be correct. Consider the set

$$S = \{x \in \mathbf{R}^3 : \|x\|_\infty \leq 1 \text{ and } x_3 = 0\}$$

The set S is closed, and its interior is empty. Therefore, the boundary of S is given by S, so the definition of relative interior given in the book says that the relative interior of S is the empty set. However, it is clear that we want our definition of relative interior to give

$$\text{relint}(S) = \{x \in \mathbf{R}^3 : \|x\|_\infty \leq 1 \text{ and } x_3 = 0\}.$$

(reported by Alex Lemon)