Optimization and Operations Research in Mitigation of a Pandemic

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Contents

Operations Research: An interdisciplinary-science-based and data-driven strategy/policy/decision making process/system under complex/uncertain/dynamic environments.

- Inventory and Risk Pooling of Medical Equipment/Resources in a Pandemic
- New Norm: Operation/Optimization helps to maintain Social Distancing
- Indoor GPS and Tracking by Sensor Network for Contact-Tracing
- Dynamic and Equitable Region Partitioning for Hospital/Health-Care Services
- Efficient Public Good Allocating under Tight Capacity Restriction via Market Equilibrium Mechanisms/Platforms

Inventory and Risk Pooling of Medical Equipment/Resources in a Pandemic
Managing Uncertainty

The War of Ventilators: Inventory Management in Uncertain Environments – Decentralization vs. Centralization

I want 30,000

You ask too much!

Governors Fight Back Against Coronavirus Chaos: 'It's Like Betting on eBay With 50 Other States'
(New York Times, March 31, 2020)

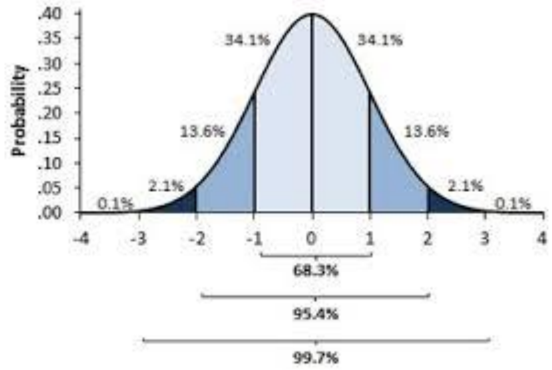
There is a "bizarre situation" in which every state buys its own ventilators, pitting them against each other in bidding war. The federal government, Fema, should have been the purchasing agent: buy everything and then allocate by need to the states. However, Fema gets involved and starts bidding and even drives up the price.

Beat Uncertainty Through Safety Stock: How Many to Order

The order quantity q^* satisfies:

$$\text{Prob}(D \leq q^*) = 0.99$$

$$\Rightarrow \quad F(q^*) = 0.99$$



Consider $D \sim N(10000, 9000^2)$

The needed quantity of ventilators is

$$q^* = \mu + \boxed{F_{norm}^{-1}(0.99) * \sigma}$$

Safety Stock

$$= 10000 + 2.3263 * 9000$$

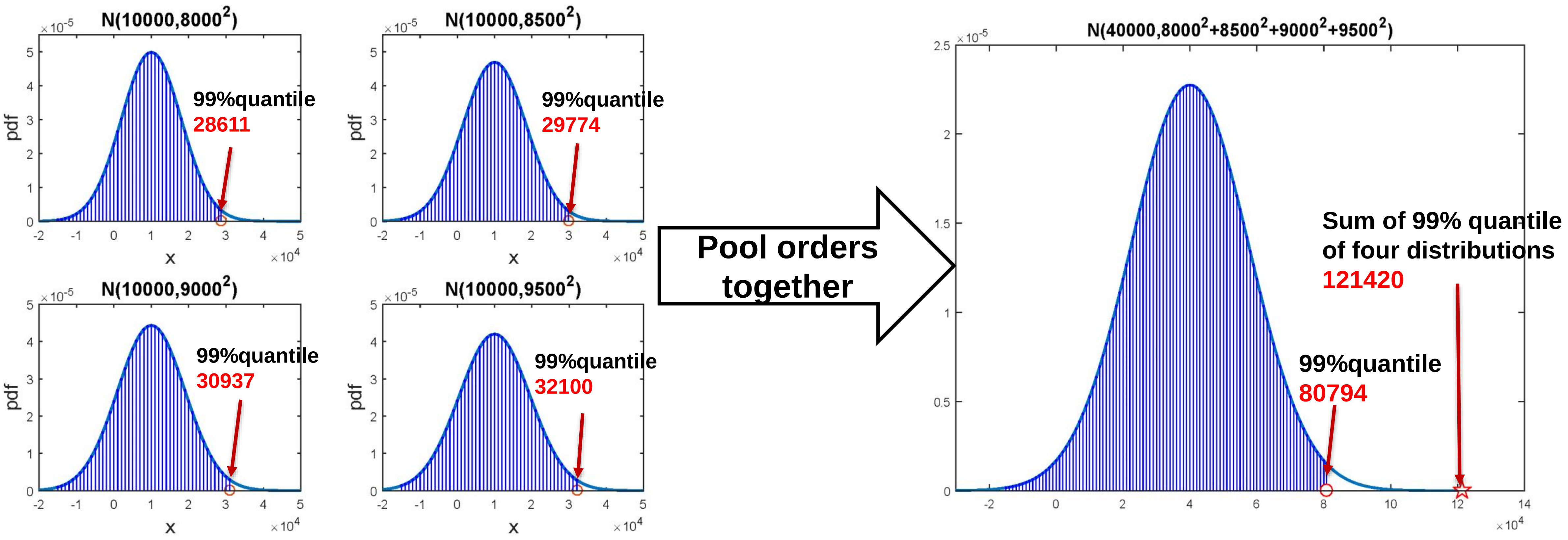
$$= 30937$$

Inventory Network Management: Risk Pooling via Centralization

How many ventilators to order for COVID-19?

- The number of patients is random and follows a certain distribution $F(\cdot)$
- Desired guarantee level : 99%

Inventory Network Management:
Centralization vs. Decentralization

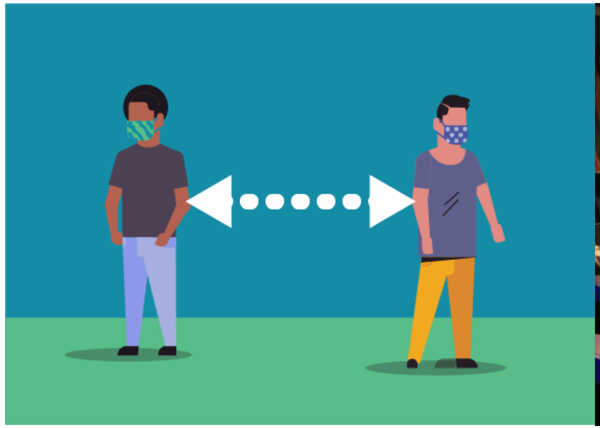


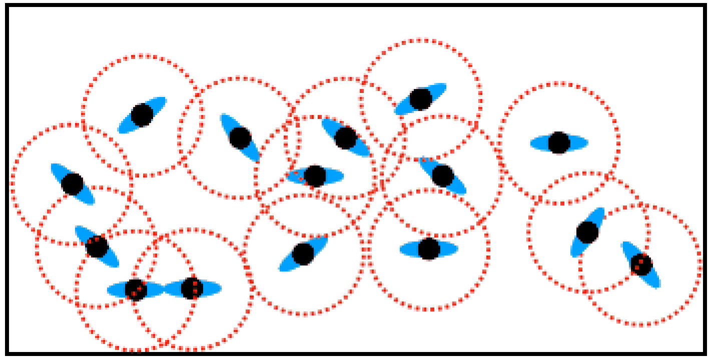
***Even though they may be correlated, the variance can still be reduced due to time delays…**

New York and 6 Other States Form Consortium to Buy Vital COVID-19 Supplies to Fight ‘Totally Inefficient’ Process *(Time, MAY 3, 2020)*

New Norm: Operation/Optimization helps to maintain Social Distancing
Discrete Optimization Solution

Social Distancing: Mathematical Implication and Solution





Accommodate max# of people in finite space

with sufficient distance from each other

Math Representation of SD

$$||x_i - x_j||_2 \geq 6$$

nonconvex in position variables x.

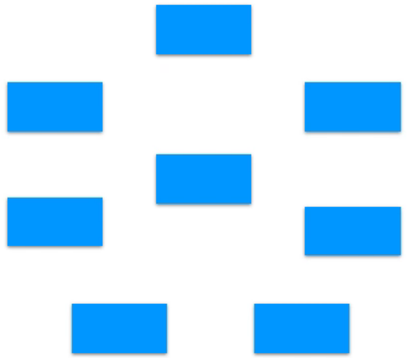
Two Scenarios

People accommodated Discretely

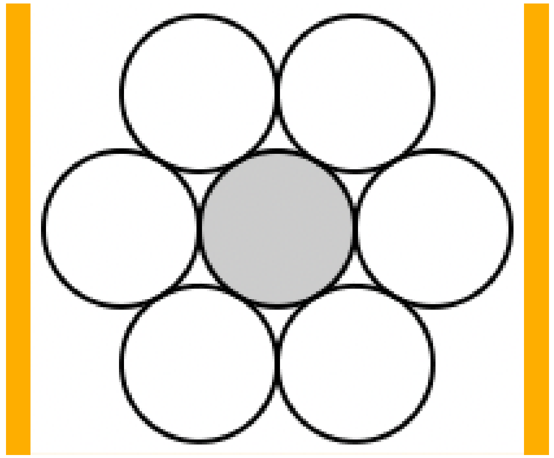
- Indoors: theater, restaurant, school, etc.
- Combinatorial/discrete optimization
- The Max-Independent Set Model

People accommodated Continuously

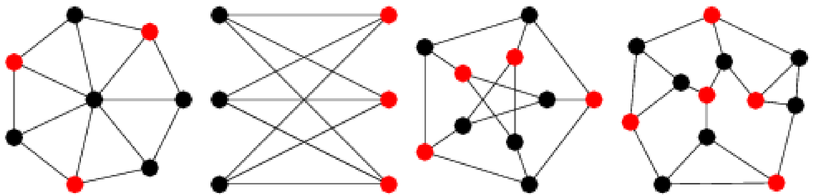
- Outdoors: beach, square, etc.
- Non-convex continuous optimization
- The Kissing Problem



Tables at a restaurant



Max-Independent Set Problem



Given seats in a theater, find an arrangement of maximum seats such that no two seats are within the unsafe distance

Given a graph G, find a subset of vertices of maximum cardinality such that no two vertices in the subset are directly connected

IP Formulation of Max-independent Set Problem

$$\begin{aligned} \max_{x} \quad & \textstyle\sum_i x_i \\ \text{s.t.} \quad & x_i + x_j \leq 1 \quad (i,j) \in E \\ & x_i \in \{0,1\} \quad \forall i \end{aligned}$$

Whether a vertex appears in the set

Max-independent set problem is NP Hard, but approximation is possible on planar graphs

(Chiba, Norishige , T. Nishizeki , and N. Saito, 1982)

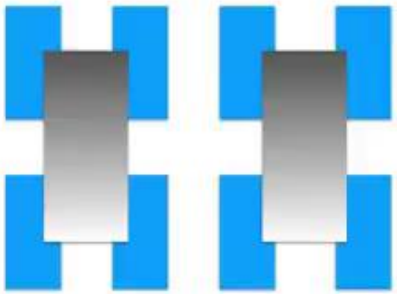
Convex relaxation can be applied to find upper bound

(Lovasz, and L, 1979)

New Extension: Humanized Arrangement?

What if we allow families/friends to sit together ?

- Potentially more people accommodated
- Independent Set fails to capture the extension
- Max-Independent Set Problem with Clusters
- Can be formulated as 0-1 integer programming



Seats at a restaurant

IP Formulation for Seat Assignment

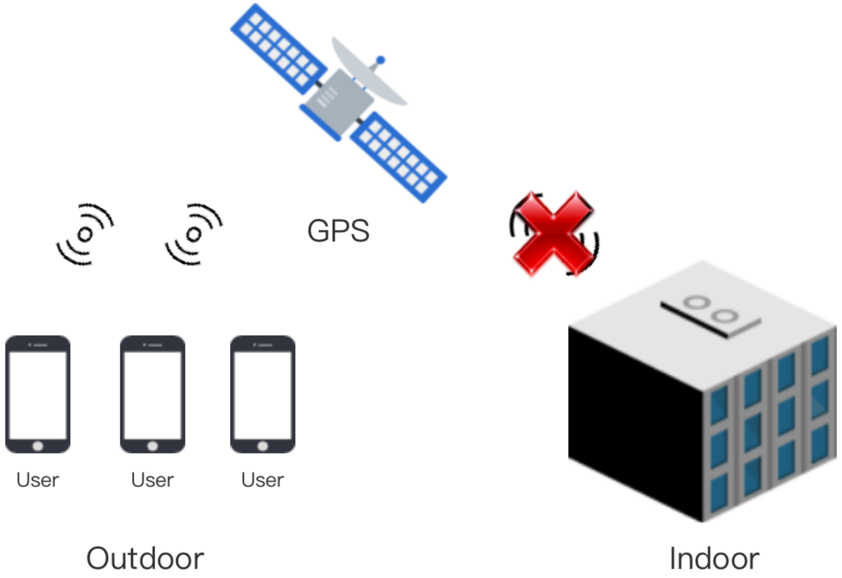
$$
\begin{aligned}
\max_{\mathbf{x}} \quad & \sum_{i=1}^{m} \sum_{p=1}^{n} x_{ip} \\
\text{subject to} \quad & \\
& \sum_{i=1}^{m} x_{ip} \leqslant 1 && \forall p \\
& \sum_{p=1}^{n} x_{ip} \leqslant 1 && \forall i \\
& x_{ip} + x_{jq} \leqslant 1 && \forall \text{ strangers } i, j, \text{ close seats } p, q \\
& x_{ip} \in \{0, 1\} && \forall i, p
\end{aligned}
$$

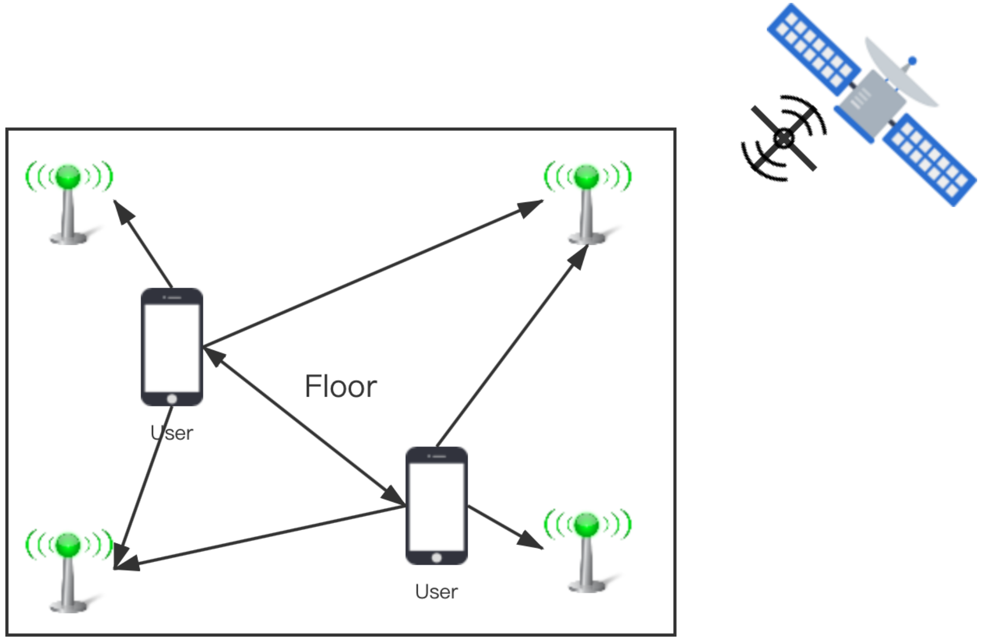
Whether person *i* is assigned to seat *p*

Indoor GPS and Tracking of Sensor Network for Contact-Tracing
Semidefinite Optimization Solution

Indoor Trajectory Tracking Sensor Localization Problem

Identify trajectory during pandemic





Outdoors: Using GPS

Indoors: Using Indoor Signal Anchors

Sensor Network Localization (SNL)

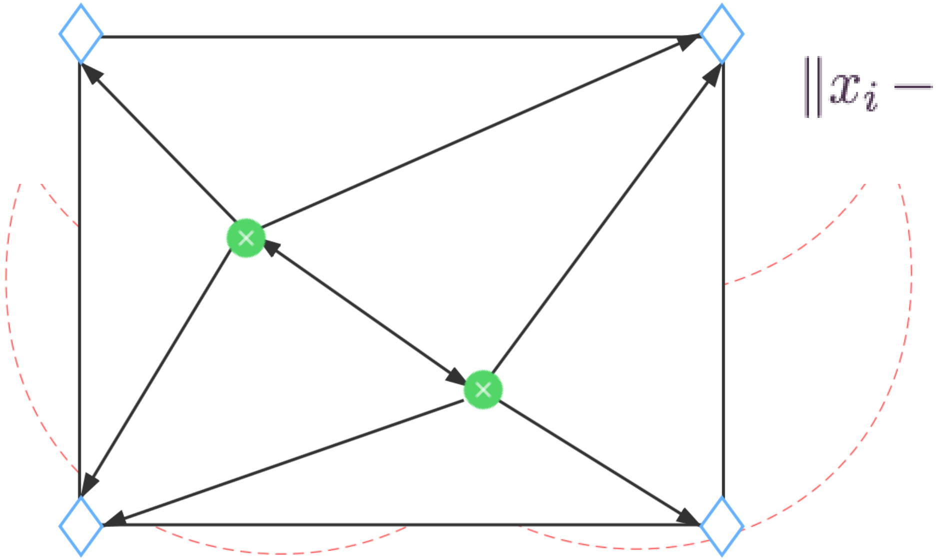
Given m anchor points $a_1, \ldots, a_m \in \mathbb{R}^d$ whose locations are known and n sensors points $x_1, \ldots,$ $x_n \in \mathbb{R}^d$ whoses locations we wish to determine. Furthermore, we are given the Euclidean distance $\bar{d}_{ij}$ between a_k and x_j for some k, j and d_{ij} between x_i and x_j for some i, j. The Sensor Localization Problem is to find a realization of $x_1, \ldots, x_n$ such that

Distance between anchor and sensor

$$\|a_k - x_j\|^2 = \bar{d}_{kj}^2, \bar{d}_{kj} \text{ specified}$$

Distance between sensor and sensor

$$\|x_i - x_j\|^2 = d_{ij}^2, d_{ij} \text{ specified}$$



- Hard to track even for $d = 1$
- Can be formulated and relaxed as SDP feasibility problem (Biswas and Y 2004; So and Y, 2007)

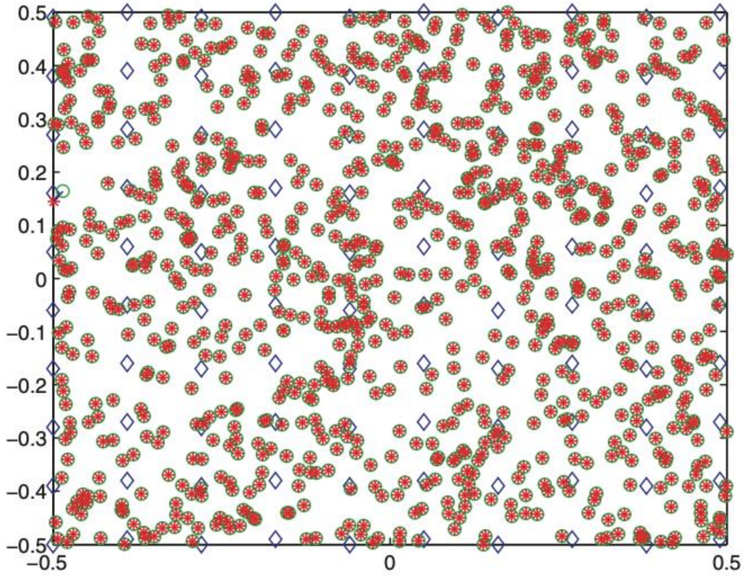
SNL Solution by Semidefinite Programming

Let $X = [x_1, \ldots, x_n] \in \mathbb{R}^{d \times n}$

$$\begin{array}{lll} \min\limits_{X,Y} & 0 & \\ \text{s.t.} & e_{ij}^\top Y e_{ij} = d_{ij}^2 & , \bar{d}_{kj} \text{ specified} \\ & (a_k; e_j)^\top \begin{pmatrix} I_d & X \\ X^\top & Y \end{pmatrix} (a_k; e_j) = \bar{d}_{kj}^2 & , d_{ij} \text{ specified} \\ & Y = X^\top X & \end{array}$$

Relax $Y \succeq X^\top X$ and let $Z = \begin{pmatrix} I_d & X \\ X^\top & Y \end{pmatrix} \succeq 0$

$$\begin{array}{lll} \min\limits_{Z} & 0 & \\ \text{s.t.} & Z_{1:d,1:d} = I_d & \\ & (\mathbf{0}; e_{ij})(\mathbf{0}; e_{ij})^\top \bullet Z = d_{ij}^2 & , d_{ij} \text{ specified} \\ & (a_k; e_j)(a_k; e_j)^\top \bullet Z = \bar{d}_{kj}^2 & , \bar{d}_{kj} \text{ specified} \\ & Z \succeq 0 & \end{array}$$

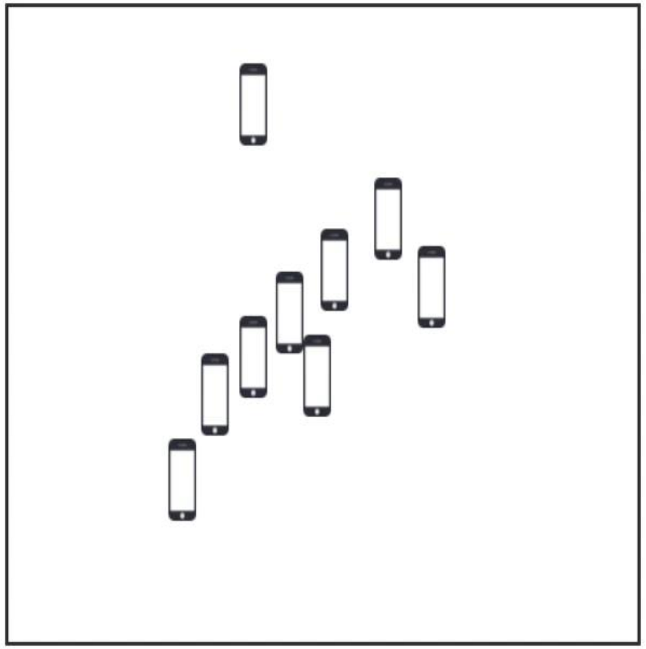


- Relaxation is tight for uniquely localizable graph
- Solution could be slow
- But can be acceleration by edge-based SDP (Wang et al. 2008)

Real-Time Sensor Network Tracking (SNT)

(Naber & Ye 2020, Wang & Ding 2008)

- Work under milder conditions
- A real-time version of sensor localization problem
- Retrieve moving trajectory and predict
- A combination of ESDP for tracking and Gradient Method for error minimization



Edge-based Relaxation

$$
\begin{array}{lll}
\min\limits_{Z} & 0 & \\
\text{s.t.} & Z_{(1,2)} = I & \\
& (\mathbf{0}; e_{ij})(\mathbf{0}; e_{ij})^\top \bullet Z = d_{ij}^2 & , d_{ij} \text{ specified} \\
& (a_k; e_j)(a_k; e_j)^\top \bullet Z = \bar{d}_{kj}^2 & , \bar{d}_{kj} \text{ specified} \\
& Z_{(1,2,i,j)} \succeq 0 & d_{ij} \text{ specified}
\end{array}
$$

Objects move subject to linear differential equation $\frac{dX(t)}{dt} = AX(t) + Zt + C$

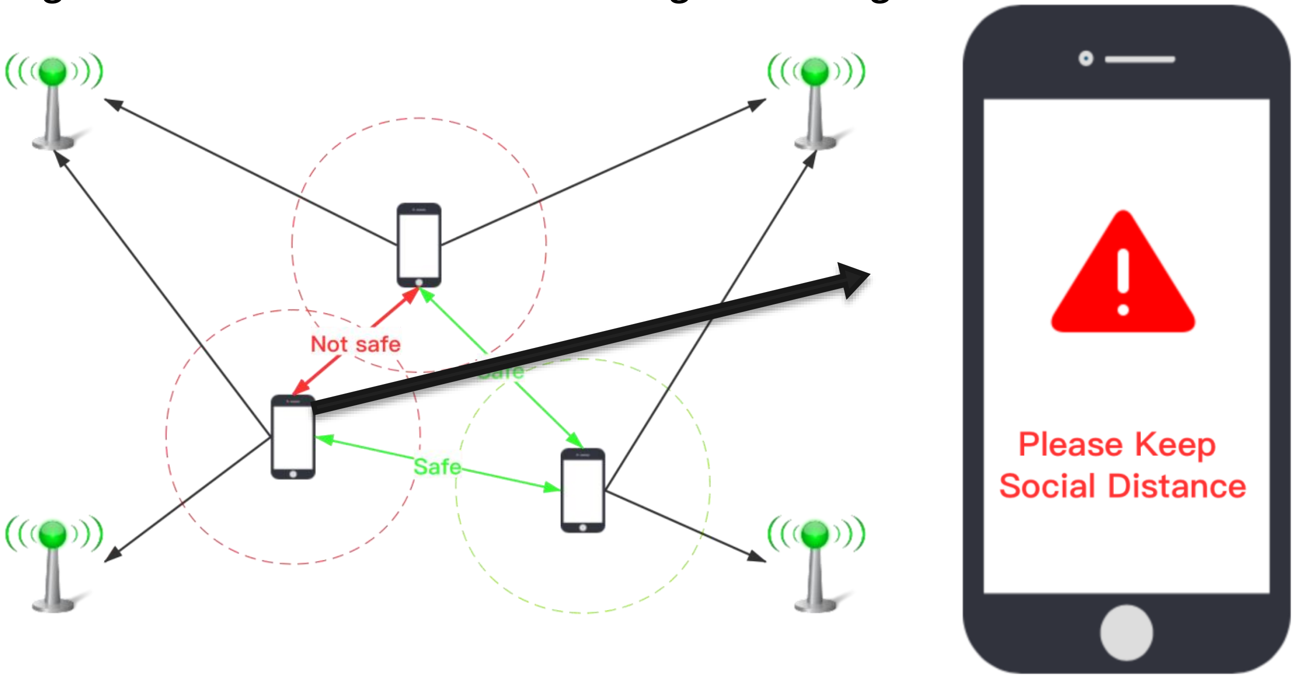
$$
\begin{array}{lll}
\min_\gamma & \sum_{i=1}^n \eta_i \gamma_i^2 & \\
\text{s.t.} & \gamma_i \geq \frac{X(t_i) - X(t_{i-1})}{t_i - t_{i-1}} - AX(t_i) - Zt_i - C & , \forall i \\
& \gamma_i \geq -\frac{X(t_i) - X(t_{i-1})}{t_i - t_{i-1}} + AX(t_i) + Zt_i + C & , \forall i \\
& A, C, Z \in \Lambda &
\end{array}
$$

A least-squares problem

Simple Distance Checking and Enforcing I

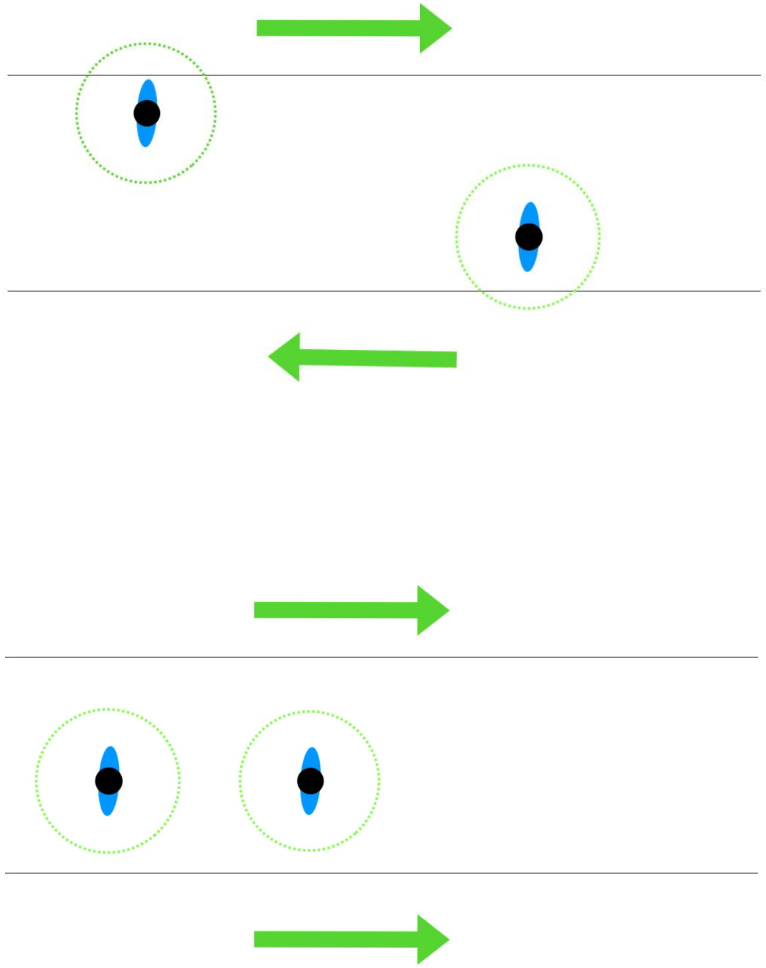
Primitive distancing enforcement

High-tech Solution for Distancing Alarming



Simple Distance Checking and Enforcing II

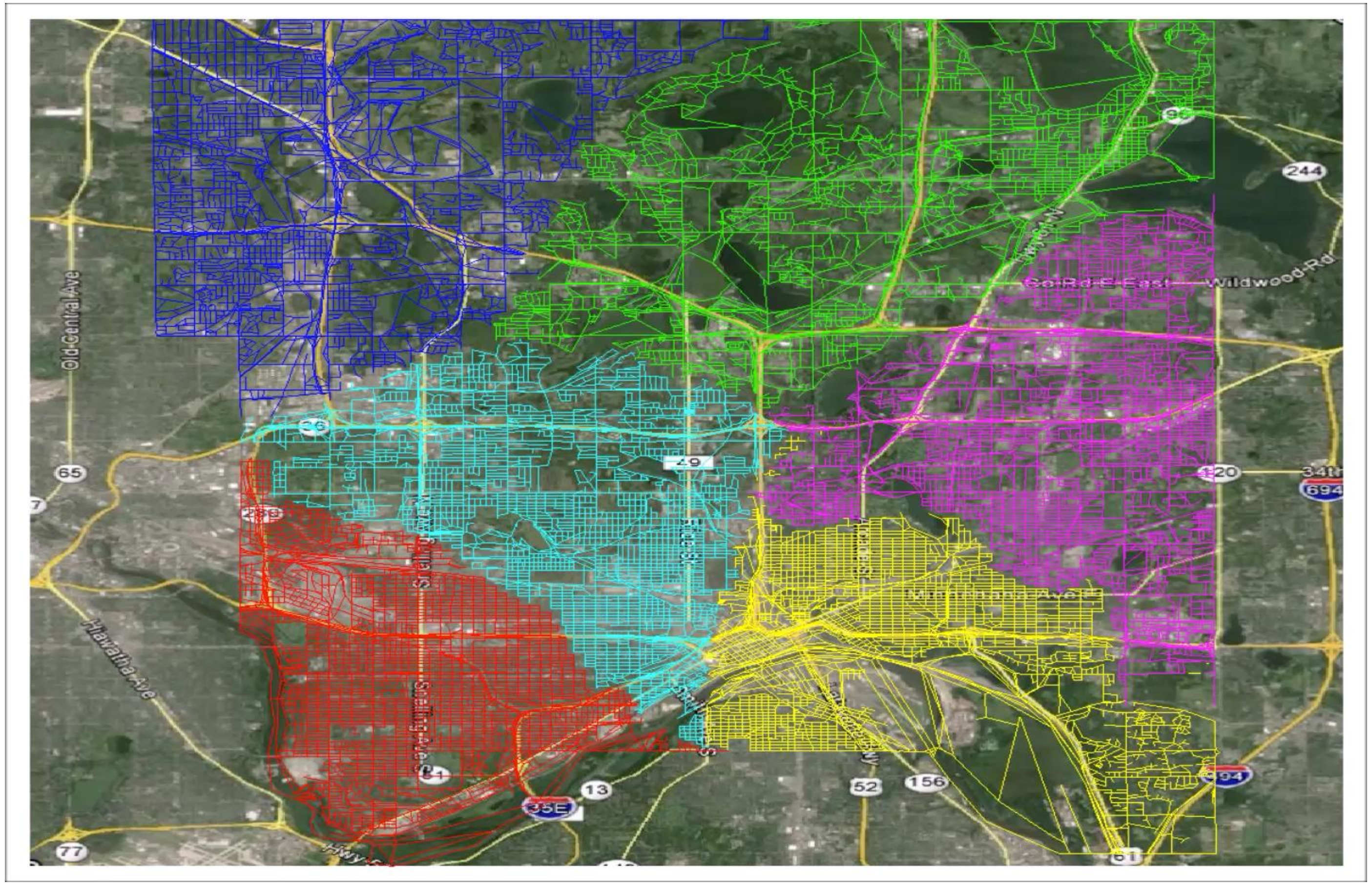
One-Way or Two-Way for pedestrian environments?



Dynamic and Equitable Region Partitioning for Hospital/Health-Care Services Computational Geometry Solution

Dynamic Hospital Service Region Partitioning

Computational Geometry Theory/Algorithm



Input Data:

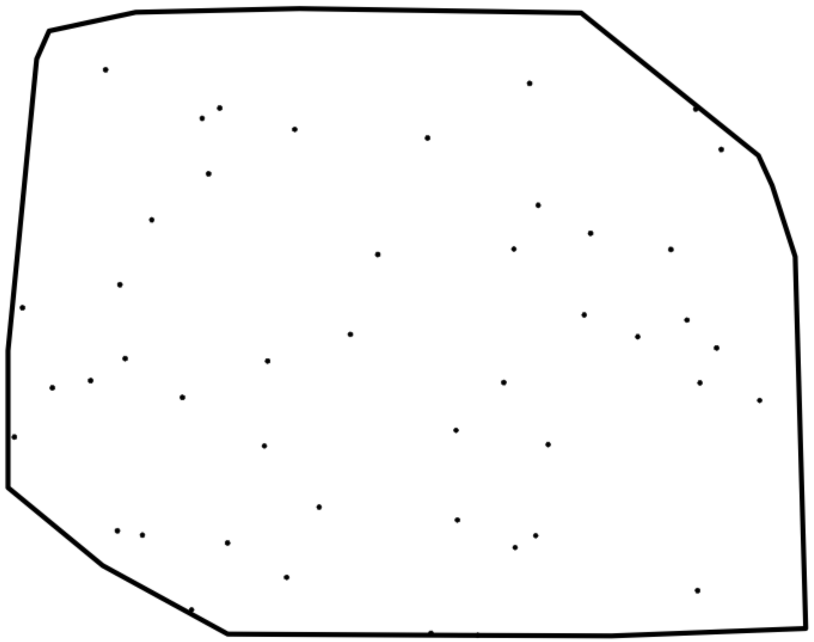
- Hospital location and capacity
- Pandemic density distribution prediction

Planning:

Partition the city into multiple regions such that

- Each region has a hospital nearby
- Each hospital will not be overrun
- Can be easily adjusted by input data change

Plane-Geometry Problem Statement and Theorem



n points are scattered inside a convex polygon *P* (in 2D) with *m* vertices. Does there exist a partition of *P* into *n* sub-regions satisfying the following:

Each sub-region is a convex polygon

– Travel convenience

Each sub-region contains one point

– Service center for the region

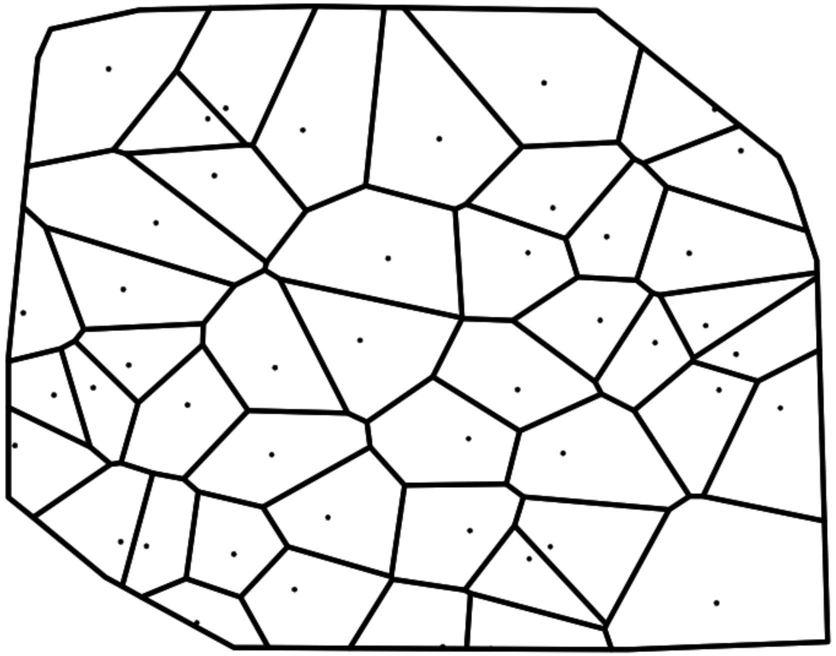
All sub-regions have equal area

– Load balance

Theorem Not only does such an equitable partition always exist, but also we can find it exactly in running time O(Nn log N), where N = m + n.

Armbruster, Carlsson and Y, 2012

Related Problem: Voronoi Diagram

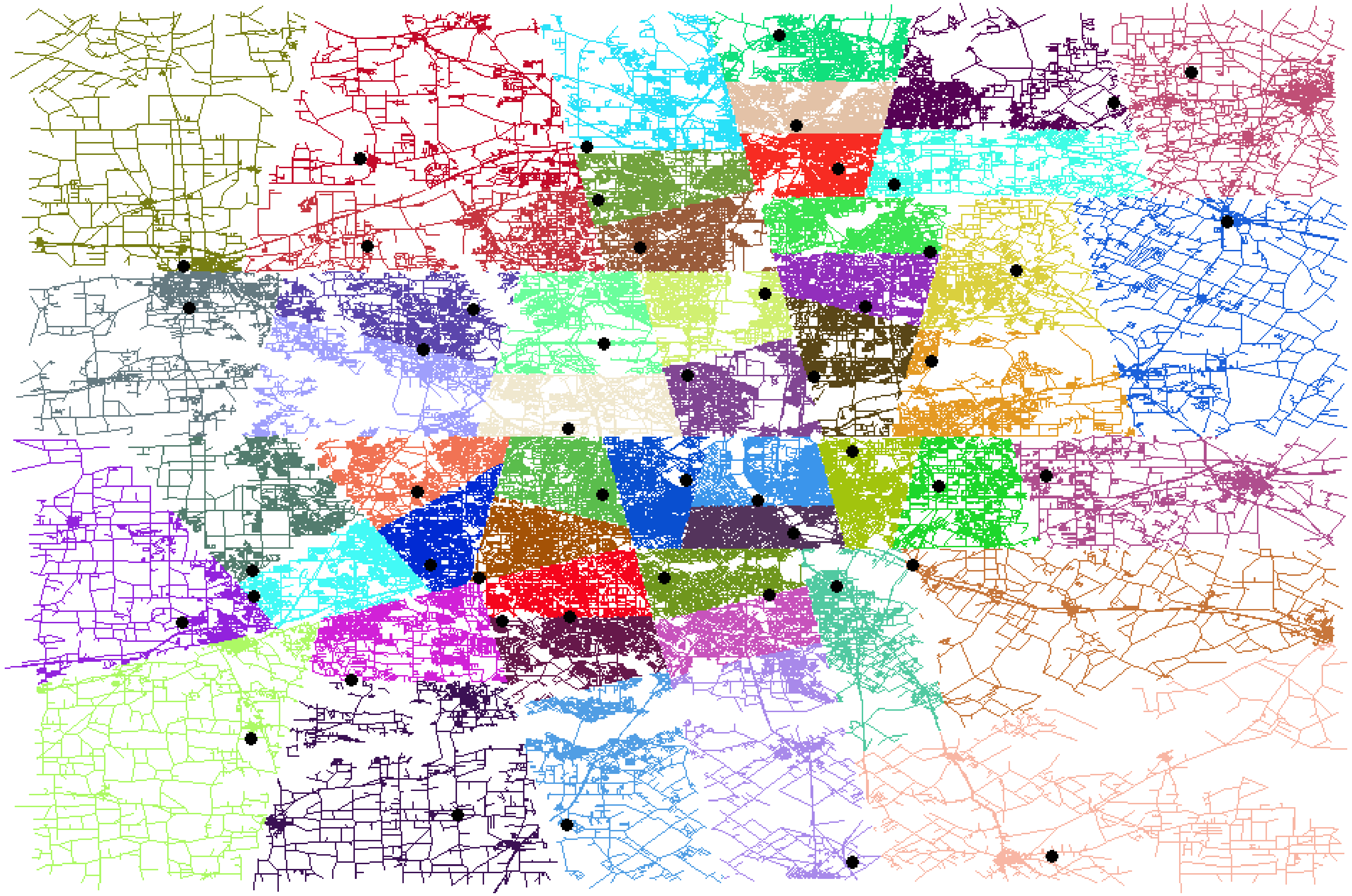


Voronoi Diagram: draw a middle perpendicular line between every two points.

The partition satisfies the first two properties (each sub-region is convex and contains one point), but the sub-regions have different areas.

In practice one can adjust the boundary to achieve the third property

Equitable Partition with Nonuniform Density Partition



Efficient Public Good Allocating under Tight Capacity Restriction via Market Equilibrium Mechanisms/Platforms

The Need for Efficient Public Good Allocation-Market Equilibrium Model

Either open: An overcrowded open beach

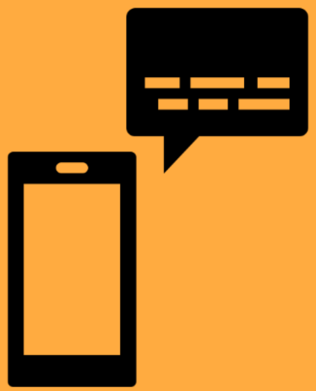
Or closed: a completely empty beach generating no value to society

An overview of our Solution

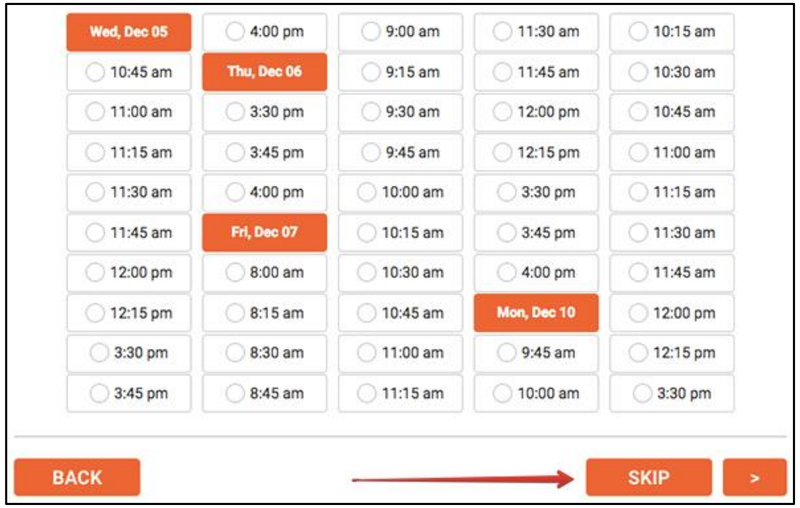
Step 1: Create Schedule for Time of Use of Public Good, e.g. Beach

Step 2: Prices assigned for different times of use

Step 3: Transfer Electronic Coupons



Step 4: Users Purchase time of use permits



Enforcement: Park rangers/ entrance booths to check for permits

Pathway to Setting Fisher Prices

- **Addressing Step 2: How can we set appropriate prices for different times of use given that the mechanism does not have complete information on everyone's utilities or constraints?**

Phase 1: Initialize Prices with some prior knowledge of utilities and constraints

Phase 2: Mechanism Learns Utilities and Constraints of Consumers

Phase 3: Iteratively Update Prices based on learnt information of consumer behavior

Through our proposed pricing scheme that generalizes the Fisher Market framework, customers will purchase permits in a “controlled” manner resulting in neither overcrowded or underused public resources

Ideal Outcome

To achieve an Intermediate between the two extreme scenarios, open or closed, through "Time of Use" goods

Create different time zones and people purchase permits to use the beach at one time-zone so that the population density on beach can be upper limited

The Fisher-Market with Budget and Physical Constraints
Centralized vs Decentralized Decision Making

- In the social optimization problem

  Choose: $\lambda_i = \sum_t r_{it}$

- **Main Result 1: KKT Equivalence of Social and Individual Optimization Problems**

  The dual of capacity constraints is the equilibrium price vector

- How do we obtain the perturbation parameter

  Fixed-Point Iterations

Individual Optimization Problem:

$$\max_{\mathbf{x}_i} \sum_j u_{ij} x_{ij}$$
$$\text{s.t.} \quad \mathbf{p}^T \mathbf{x}_i \leq w_i$$
$$A_t^T \mathbf{x}_i \leq 1, \forall t \in T_i$$
$$\mathbf{x}_i \geq \mathbf{0}$$

$$\Updownarrow$$

Social Optimization Problem:

Budget Perturbation

$$\max_{\mathbf{x}_i} \sum_i (w_i + \lambda_i) \log(\sum_j u_{ij} x_{ij})$$
$$\text{s.t.} \quad \sum_i x_{ij} \leq \bar{s}_j, \forall j \in [M]$$
$$A_t^T \mathbf{x}_i \leq 1, \forall t \in T_i, \forall i \in [N]$$
$$x_{ij} \geq 0, \forall i, j$$

r_{it}: Dual Variable of Physical Constraint

Jalota, Qi, Pavone and Y, 2020

Grouping and “Time of Use” for School/Class?

- Create pods/clusters. Children/students form groups/pools of about 12 and stay together all day, avoiding contact with other pods. **Test every group as ONE example for possible infection (if positive then test individuals in the group) before schools open.**

- Partial days. Keep classed smaller by splitting the children into groups. For example, one group comes to school one day and the other group the next day. The rest of the work is done at home. The split could also be morning/afternoon.

- Split classes into either Zoom or In-Person. Also create pods for university in-person classes for course discussions and team projects.

Summaries for Mitigation of a Pandemic and Economy Reopening

- **Strategies: The goal is not to eradicate virus but to reduce the ultimate fatality rate by frequent sample testing…**
- **Policies: Differentiate high and low risk groups…**
- **Operations: Centralized and mobilized task-force, inventory management of materials/equipment, Efficient public spaces/goods allocation for social distancing…**
- **Methodologies: Based on Sciences/Technologies such as Mathematical Optimization, Computer Monte Carlo Simulation, Statistical and Machine Learning, Mechanism Designs …**
- **Reopen Economy: Low-risk groups return to work, Lean and agile supply-chain, Algorithm-based machine decisioning and robotics…**