Interior-Point Algorithms: Final Project Report
NSF Grant DDM-9207347

Principal Investigator: Yinyu Ye

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1 PART I—Project ID Information

See NSF Form 98A.

2 PART II—Summary of Completed Project

This document is the final project report on an optimization project funded by NSF Grant DDM-9207347. This project is to develop and analyze recent interior-point algorithms, a seminal development in optimization, for solving large-scale linear, quadratic, convex, and global optimization problems that arise in production management and resource allocation. These algorithms have the potential to be faster than the traditional methods currently in use and could solve problems never solved before. It is a continuation of my previous NSF projects, and it is motivated by the demand to resolve several theoretical and practical issues concerning interior-point algorithms. I also do research on implementation and application to solving problems in production scheduling and network planning. My research topics include the following.

1. Initialization and Termination: Develop efficient initialization and termination techniques to generate an initial point, and obtain an optimal basic solution or detect infeasibility.

2. Complexity Theory: Study complexity issues for linear programming (LP) with data given by real numbers, analyze "condition" numbers that determine the degree of difficulty of LP problems, and "precondition" some problems and develop strongly polynomial interior-point algorithms.

3. Asymptotical and avarage analysis of interior-point algorithms: Design and prove super-linearly and quadratically convergent interior-point algorithms, analyze them for solving randomly generated LP problems, and to explain the excellent practical behavior of these algorithms.

4. Decomposition and Column Generation: Develop various decomposition and column generation schemes for solving large-scale or infinite-dimensional practical optimization problems.

5. Nonlinear and Global Optimization: Extend LP techniques to solving nonlinear and global optimization problems arising from engineering design and control.

6. Implementation and Application: Implement the state of the art interior-point algorithm, and apply the algorithm to solving real-world problems.

The project resulted in 21 publications in several major journals of operations research and management sciences. The MCI Real-Time-Restoration project, where my students and I did joint research work, had been put into operation. The educational beneficiaries of the project are the students at the University of Iowa. Several Ph.D. students are partially supported by this grant and they are actively involved in the project. Some results are part of their Ph.D. theses and published in top journals.

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3 PART III—Technical Information

3.1 Introduction

This document is the final report on an operations research project funded by NSF Grant DDM-9207347. The project principally considers the interior-point algorithms for large-scale linear programming (LP), quadratic programming (QP), linear complementarity problem (LCP), and nonlinear programming (NP) problems arisen in production management and manufacturing. The discoveries and advances resulting from the project include the following.

- Twenty one new journal publications, three new conference papers, and five new working papers have been resulted from the project: four in *Mathematical Programming*, three in *Mathematics of Operations Research*, three in *SIAM J. on Optimization*, three in *Annals Operations Research*, two in *Operations Research Letters*, and the rest of them in *ACM Symposium on Theory of Computing*, *Operations Research*, *Management Science*, *J. of Complexity*, *JOTA*, etc. These papers are listed in Section 5, where [J*] for journal publication, [C*] for conference papers, and [W*] for working paper.
- A computational optimization laboratory (COL) has been established in my department at Iowa. We have had two post-doctor associates, one visiting Ph.D. student fully supported by the Danish government, and two visiting students, supported by the Brazil government, and two our own Ph.D. students in Management Science.
- Two implementation codes have been developed and installed in several major universities. One solves linear programming and the other solves convex optimization.
- The MCI Real-Time-Restoration project, where my students and I did joint research work with MCI, has been implemented into operation. MCI has reported a significant improvement to its service.

In general, progress in the area of developing efficient algorithms for solving large-scale optimization problems will be of great importance in improving the efficiency of manufacturing systems, communication networks, aircraft routing, multi-commodity-flow operations, and resources planning. Strengthening research in this area will definitely contribute to the national interest in industrial competitiveness and scientific knowledge.

I briefly summarize several results from this project. In what follows, the references [•] are listed in Section 6.

3.2 Progress in Initialization and Termination

There four major developments in the initialization and termination of interior-point algorithms.

First, Todd, Mizuno and myself [J18][J17] developed a homogeneous and self-dual LP algorithm. Our algorithm possesses the following features: 1) It solves the linear programming problem without any regularity assumption concerning the existence of optimal, feasible, or interior feasible solutions; 2) It can start at any positive primal-dual pair, feasible or infeasible, near the central ray of the positive orthant (cone), and it does not use any big M penalty parameter or lower bound; 3) Each iteration solves a system of linear equations whose dimension is almost the same as that solved in the standard (primal-dual) interior-point algorithms; 4) If the LP problem has a solution, the algorithm generates a sequence that approaches feasibility and optimality simultaneously, or if the problem is infeasible or unbounded, the algorithm will correctly detect infeasibility for at least one of the primal and dual problems; 5) It retains the best complexity $O(\sqrt{n}L)$-iteration bound. This development resolved a key remaining issues concerning interior-point algorithms for LP. Later, in [J9] Xu and I generalized the simplified homogeneous and self-dual LP algorithm. In particular, the searching direction is obtained by solving a Newton equation system without infeasibility residual terms on its right hand side. This will eliminate any accumulate numerical error during the iterative process.

Second, Andersen and I [J2][W3] presented an extension of a homogeneous self-dual algorithm to solving the monotone complementarity problem (MCP). The algorithm does not need to use any "big-M" parameter or two-phase method, and it generates either a solution converging towards feasibility and complementarity simultaneously or a certificate proving infeasibility. Moreover, if the MCP is polynomially solvable with an interior feasible starting point, then it can be polynomially solved without using or knowing such information at all. To our knowledge, this is the first interior-point and infeasible-starting algorithm for solving the MCP that possesses these desired features. Preliminary computational results are presented.

Third, in exact arithmetic the simplex method applied to a particular linear programming problem instance either shows that it is infeasible, shows that its dual is infeasible, or generates optimal solutions to both problems. Interior-point methods do not provide such clear-cut information. In [W5] Todd and I provide general tools (extensions of the Farkas Lemma) for concluding that a problem or its dual is likely (in a certain well-defined sense) to be infeasible, and apply them to develop stopping rules for a generic infeasible-interior-point method and for the homogeneous self-dual algorithm for linear programming. These rules allow precise conclusions to be drawn about the linear programming problem and its dual: either near-optimal solutions are produced, or we obtain "certificates" that all optimal solutions, or all feasible solutions to the primal or dual, must have large norm. Our rules thus allow more definitive interpretation of the output of such an algorithm than previous termination criteria. We give bounds on the number of iterations required before these rules apply. Our tools may also be useful for other iterative methods for linear programming.

Fourth, in paper [J3] Andersen and I propose an approach to combine linear programming (LP) interior-point and simplex pivoting algorithms, which is related to the work of Bixby and Saltzman. We present a strongly polynomial procedure in any iteration of an interior-point algorithm to produce a basis. We show that this basis will eventually become optimal after a finite number of interior-point iterations. Our combined method naturally handles the case where the interior-point algorithm does not provide an exact optimal or feasible solution. In fact, our method allows either cross over to the simplex algorithm or to execute the two algorithms alternately. Furthermore, if LP data are integers, the total number of pivoting steps to generate an optimal basis can be bounded above by $O(n\sqrt{n}L)$, where L is the data size and n is the number of variables in the standard primal form. Later, I'll report partial computational results for this combined approach.

3.3 Progress in Complexity Theory

Consider a linear programming (LP) problem in the standard form:

$$\begin{array}{lll} (LP) & \text{minimize} & c^T x \\ & \text{subject to} & Ax = b, \; x \geq 0, \end{array}$$

where $c \in R^n$, $A \in R^{m \times n}$ and $b \in R^m$ are given, $x \in R^n$, and ^{T} denotes transpose.

Based on our previous work [J10][J11][W1], Vavasis and I [J1] recently developed a **prima-dual** "layered-step" interior point (LIP) algorithm for linear programming. This algorithm follows the central path, either with short steps or with a new type of step called a "layered least squares" (LLS) step. The algorithm returns the exact global minimum after a finite number of steps—in particular, after $O(n^{1.5}|B||N|\chi(A))$ iterations, where $\chi(A)$ is a function of the coefficient matrix, and sets B and N represent the strictly complementary partition of the LP solutions; thereby $|B| + |N| = n$. Note that this bound is independent of b and c.

The LLS steps can be thought of as accelerating a path-following interior point method whenever near-degeneracies occur. One consequence of the new method is a new characterization of the central path: we show that it composed of at most $|B| * |N|$ alternating straight and curved segments. This is the first such result for LP with data given by real numbers. If the LIP algorithm is applied to integer data, we get as another corollary a new proof of a well-known theorem by Tardos that linear programming can be solved in strongly polynomial time provided that A contains small-integer entries.

My student Hung and I [J8] dealt with path-following algorithms, which generate a sequence of points within certain neighborhoods of a central-path. They prevent iterates from prematurely getting too close to the boundary of the feasible region. Depending on their norm used, these neighborhoods have different sizes. A paradox is that among all existing (infeasible or feasible) path-following algorithms, the theoretical iteration complexity, $O(\sqrt{n}L)$, of small-neighborhood algorithms is significantly better than the complexity, $O(nL)$, of wide-neighborhood algorithms, while in practice wide-neighborhood algorithms outperform small-neighborhood ones by a big margin. Here again, n is the number of LP variables and L is the LP data length. In this paper, we present an $O(n^{\frac{n+1}{2n}}L)$-iteration (infeasible) primal-dual high-order algorithm that uses wide neighborhoods. Note that this iteration bound is asymptotical $O(\sqrt{n}L)$, i.e., the best bound for small-neighborhood algorithms, as n increases. Jansen, Roos, Terlaky and I [J4] later this high-order scheme also works for other algorithms and complementary problems.

Todd and I [J6] also proved a complexity lower bound for interior-point LP algorithms. We have shown that there is an LP problem that almost all polynomial interior-point algorithms need $\Omega(n^{1/3})$ iterations to solve it. Another lower bound analysis is given by Ji and I [J19].

3.4 Progress in Asymptotical and Avarage Analysis

Quadratic convergence for general LP, assuming neither convergence of the iteration sequence nor nondegeneracy, was first established by Ye, Güler, Tapia and Zhang. The algorithm was based on the predictor-corrector algorithm of Mizuno, Todd and Ye. A stronger result was developed later where Luo and I [C2] showed how to construct an $O(\sqrt{n}L)$ algorithm whose order of convergence exactly equals 2. These results are not trivial since the Jacobian matrix of the KKT equations for LP tends towards singular in solving degenerate LP problems.

While almost every result concerning interior-point algorithms for LP has been immediately extended to the monotone linear complementarity problem (LCP) or convex quadratic programming (QP), the extension of the quadratic convergence result seems very hard. The main reason is a fundamental difference between LP and LCP: LCP (or QP) may not have a strict complementarity solution but LP always has, and the existence of a strict complementarity solution plays an important role in the local convergence analysis. This fact presents certain challenges to developing an efficient termination criterion or superlinearly convergent algorithm for LCP or QP.

But this problem has been completely resolved by a recent work of Anstreicher and myself, where we proved a quadratically convergent predictor-corrector algorithm for the LCP possessing a strictly complementary solution. Moreover, we proved it is impossible for the available polynomial interior-point algorithms to achieve superlinear convergence for the LCP that does not possess a strictly complementary solution.

Based on our earlier work, Anstreicher, Ji, Potra and myself [P21] have developed a rigorous average analysis of interior-point algorithms. We have shown that the expected number of iterations required to solve a random LP problem is bounded above by $O(\sqrt{n}\log(n))$, where n is the number of variables in the standard LP form:

$$\min \quad c^T x, \quad s.t. \quad Ax = b, \quad x \geq 0.$$

3.5 Progress in Decomposition and Column Generation

Methods, which allow for column generation—the addition of columns or constraints to the system only when needed—permit a great deal of flexibility for solving optimization problems in which the number of columns is either very large or not explicitly known. Thus, we need to develop interior-point algorithms that allow decomposition or column generation and are implementable with practical efficiency. This will advance the practical solvability for a large class of optimization problems, such as semi-infinite programming, convex nonlinear programming, and combinatorial optimization.

Several approaches for column generation or decomposition were proposed and developed. One method, the analytic center cutting plane method was developed and analyzed by several people, including myself, to solve general convex problems defined by a separation oracle. The oracle is called at the analytic center of a polytope, which contains a solution set and is given by the intersection of the linear inequalities previously generated from the oracle. If the center is not in the solution set, separating hyperplanes will be placed through the center to shrink the containing polytope. Its complexity or convergence result has been established in solving general convex problems by Goffin, Luo and me [J13][C3], where we proved that the method is a pseudo-polynomial algorithm for solving general convex optimization problems.

However, the complexity bound holds only when one cutting plane is placed in each step. Thus, the question remains open when multiple cuts are added. Moreover, adding multiple cuts actually is a key to practical effectiveness and it presents theoretical difficulties in analyzing other cutting plane methods. In paper [W6], I overcome this difficulty and show that the analytic center cutting plane algorithm, with multiple cuts added in each step, still is a fully polynomial approximation algorithm.

3.6 Progress in Nonlinear and Global Optimization

First, I have extended the LP homogeneous and self-dual algorithm to the monotone linear complementarity problem which is a quadratic optimization problem [J2]. The problem was an interesting open question. Thus, we can solve QP or detect infeasibility using interior-point algorithm without the big-M technique. More recently, Andersen and I [W3] have extended the algorithm to solving the monotone nonlinear complementarity problem, which includes general convex optimization problems.

Second, I have developed an algorithm to solving the von Neumann economic growth problem, which is a quasi-convex multi-fractional optimization problem [J15]. In this paper, I established the first polynomial algorithm for solving this problem.

Third, my colleagues Kortanek, Xu and I [W4] developed some theoretical and computational results for solving the posynomial geometric programming (GP) problem using an infeasible primal-dual interior-point algorithm. We applied the Newton method for a perturbed KKT system arising from the dual GP problem and develop effective techniques in choosing iterate direction and step length. Our computational results indicate that the algorithm, coupled with these new techniques, leads to an efficient and stable implementation for solving GP programs. Competitive numerical results are presented for 40 or so of the most challenging posynomial geometric programming problems in the field, where for a significant number of these we see similar numerical performance to linear programming. Potra and I [J5] also locally superlinear converegent interior-point algorithms for nonlinear monotone complementary problems.

In Xue and I [W2] the problem of minimizing a sum of Euclidean norms is studied. This problem is convex but not everywhere differentiable. By transforming the problem into a standard convex programming problem in conic form, we show that an ϵ-optimal solution can be computed efficiently using interior-point algorithms. As applications to this problem, polynomial time algorithms are derived for the Euclidean single facility location problem, the Euclidean multi-facility location problem, and the shortest network under a given tree topology. In particular, by solving the Newton equation in linear time using *Gaussian elimination on leaves of a tree*, we present an algorithm which computes an ϵ-optimal solution to the shortest network under a given full Steiner topology interconnecting N regular points, in $O(N\sqrt{N}(\log(\bar{c}/\epsilon)+\log N))$ arithmetic operations where $\bar{c}$ is the largest pair wise distance among the given points. Previous best known result on this problem is a graphical algorithm which requires $O(N^2)$ arithmetic operations under certain conditions

More recently, I [W1] present a potential reduction algorithm to approximate a Karush-Kuhn-Tucker (KKT) point of general quadratic programming. We show that the algorithm is a fully polynomial-time approximation scheme, and its running-time dependency on accuracy $\epsilon \in (0,1)$ is $O((\frac{1}{\epsilon})\log(\frac{1}{\epsilon})\log(\log(\frac{1}{\epsilon})))$, compared to the previously best-known result $O((\frac{1}{\epsilon})^2)$. Furthermore, the limit of the KKT point satisfies the second-order necessary optimality condition of being a local minimizer. The result is based on my earlier work on solving a ball-constrained quadratic minimization [J20].

3.7 Progress in Implementation and Application

First, in [J12]], Xu, Hung and I have developed a simplification and generalization of the recent Ye-Todd-Mizuno homogeneous and self-dual linear programming (LP) algorithm [P18]. The simplified and generalized algorithm solves the linear programming problem without any regularity assumption concerning the existence of optimal, feasible, or interior feasible solutions. It also detects LP infeasibility based on a provable criterion. (It was a major problematical issue to detect infeasibility for practical interior-point methods. Some theoretical results hold only for feasible cases. Other approaches to detect infeasibility are somewhat difficult to implement in practice.) Our preliminary implementation with a simple predictor and corrector technique results in an efficient computer code in practice.

Starting simply from $x=e$, $y=0$, $z=e$ (slack of dual) and using the default values for program parameters, our code successfully solves or detects infeasibility of all encountered test problems in NETLIB. We compare our computational results in solving NETLIB feasible problems, with a comparison to OB1.60, the state of the art implementation of interior-point algorithms. Among 91 feasible problems, our code performs better than OB1 in cpu time for 63 problems. Among 28 infeasible problems, OB1 has difficulty to detect them, see Table 1.

Second, I present our computational results of the approach that combines interior-point and simplex pivoting algorithms [J3]. The whole source code is written in ANSI C and is compiled on a HP 715/75 workstation using the HP C compiler and +O3 optimization option. All computational results are reported on the same workstation. All timing results are obtained with the C procedure

Problem Summary					HLF		OB1.60		
Name	Rows	Cols	Nonzeros	BR	it	cpu	it	cpu	status*
bgdbg1	349	407	1485	B	6	.21			0
bgetam	401	688	2489	B	7	1.07	7	1.22	3
bgindy	2672	10116	75019		10	83.20	6	70.64	3
bgprtr	21	34	90		6	.02	9	.07	3
box1	232	261	912	B	5	.13	3	.15	1
chemcom	289	720	2190	B	5	.30	6	.56	3
cplex2	225	221	1059	B	44	1.44	47	1.81	1
ex72a	198	215	682	B	6	.12	10	.26	1
ex73a	194	211	668	B	6	.08	7	.19	5
forest6	67	95	270	B	6	.02	11	.10	3
galenet	9	8	16	B	4	.00			0
gosh	3793	10733	97257	B	23	224.83	17	107.13	3
gran	2569	2520	20151	B	17	167.30			0
greenbea**	2505	5405	35159	B	18	23.46			0
itest2	10	4	17		4	.04			0
itest6	12	8	23		5	.03			0
klein1	55	54	696		15	.25	54	.97	3
klein2	478	54	4585		16	54.81	38	67.21	3
klein3	995	88	12107		17	630.52	93	1548.14	3
mondou2	313	604	1623	B	8	.34	4	.27	3
pang	362	460	2666	B	24	1.80	33	3.04	5
pilot4i	411	1000	5145	B	15	3.76			0
qual	324	464	1714	B	32	1.70	59	4.22	5
reactor	319	637	2995	B	15	1.28			0
refinery	324	464	1694	B	13	.71	14	1.07	3
vol1	324	464	1714	B	30	1.63	42	3.14	3
woodinfe	36	89	209	B	10	.07			0

Table 1: Performance on Solving NETLIB Infeasible Problems (B-W).

* OB1 termination status:

```
0 crush(presolve) detects that model is infeasible.
1 optimal.
3 model appears to be dual unbounded. (or primal infeasible.)
5 numerical difficulties encountered. unable to reach solution with desired accuracy.
```

Name	ROWS	COLS
CRE-A	3517	4067
CRE-B	9649	72447
CRE-C	3069	3678
CRE-D	8927	69980
KEN-07	2427	3602
KEN-11	14695	21349
KEN-13	28633	42659
OSA-07	1119	23949
OSA-14	2338	52460
OSA-30	4351	100024
PDS-02	2954	7535
PDS-06	9882	28655
PDS-10	16559	48763
Sum	108120	479168

Table 2: The Kennington problems.

Name	RGAP	ITE	PBI	DBI	TOPT	TCRO	RATIO
CRE-A	9.7E-11	30	4.9E-14	5.8E-11	19.83	1.98	0.10
CRE-B	4.4E-15	45	1.2E-13	5.8E-11	289.36	19.07	0.07
CRE-C	4.9E-09	29	2.3E-13	8.7E-11	16.89	1.56	0.09
CRE-D	6.0E-11	45	1.1E-13	2.9E-11	245.94	13.13	0.05
KEN-07	9.9E-11	16	0.0E+00	7.3E-12	5.48	0.36	0.07
KEN-11	3.9E-12	21	0.0E+00	4.5E-12	65.31	6.04	0.09
KEN-13	1.5E-11	25	0.0E+00	3.8E-12	200.17	29.96	0.15
OSA-07	3.7E-09	31	0.0E+00	6.8E-13	53.72	0.74	0.01
OSA-14	6.1E-10	18	0.0E+00	1.6E-12	78.34	2.19	0.03
OSA-30	3.2E-11	26	5.7E-14	2.0E-12	210.23	10.38	0.05
PDS-02	1.8E-11	27	0.0E+00	0.0E+00	21.52	1.76	0.08
PDS-06	2.0E-14	38	0.0E+00	0.0E+00	455.78	28.83	0.06
PDS-10	2.5E-13	44	0.0E+00	0.0E+00	2564.14	84.37	0.03
Sum		395			4226.71	200.37	0.05

Table 3: Termination results for the Kennington problems.

"clock" which measures the time in number of cpu sec.

The NETLIB problems are relative small and may not really represent todays large-scale LP problems. Therefore, we evaluate the cross-over procedure for the so-called Kennington problems. In Table 2, we have shown some problem related statistics. The columns ROWS and COLS show the numbers of rows and columns of the constraint matrix in each of the 13 problems.

In Table 3 we have shown the computational results for solving the Kennington problems. The column RGAP shows the relative duality gap of the interior-point solution and ITE shows the number of interior-point iterations, when the cross-over starts. PBI and DBI are the maximal primal and dual bound infeasibilities, respectively. The columns TOPT and TCRO show the total time spent in optimizing and in doing cross-over, respectively. TCRO is included in TOPT. Finally, RATIO shows the ratio TCRO/TOPT.

We see that in all the cases we obtain an optimal basic solution to the original problem with very low PBI and DBI. Usually PBI and DBI are below 10^{-10}, which is a very good accuracy. Often the feasibility measures are much better. It can be seen that all the problems are solved in total 4227 cpu. sec. and out of these are 200 spent in the cross-over procedure. In other words, on average we obtain the optimal basic solution in less than 5% of the interior-point time.

Third, Andersen and I [W3] report computational results to demonstrate effectiveness of the homogeneous algorithm for linearly constraint convex programming problems. All our code is

TITLE	ROWS	COLS	STATUS	ITER	TIME	RGAP	PFEAS1	DFEAS1
25FV47	821	1876	OPTIMAL	18	14.8	4.9E-10	2.0E-13	1.0E-15
DEGEN3	1503	2604	OPTIMAL	14	86.1	8.7E-09	1.0E-09	1.0E-10
GREENBEA	2392	5495	OPTIMAL	27	49.0	6.1E-09	1.0E-09	7.0E-13
KLEIN2	477	531	INFEASIBLE	21	21.8	1.0E+00	2.0E-02	6.0E-15
B1M100T1	0	10000	OPTIMAL	10	19.4	2.4E-10	4.0E-16	2.0E-13
B1M200T1	0	40000	OPTIMAL	10	107.9	9.1E-10	4.0E-16	5.0E-13
B1M300T1	0	90000	OPTIMAL	11	346.4	3.3E-10	7.0E-16	2.0E-12
B2M100T1	0	10000	OPTIMAL	13	20.9	5.9E-09	0.0E+00	5.0E-14
B2M200T1	0	40000	OPTIMAL	12	113.5	8.6E-13	0.0E+00	6.0E-13
B2M300T1	0	90000	OPTIMAL	10	287.8	9.9E-10	0.0E+00	2.0E-14
B3M100T20	0	10000	OPTIMAL	11	21.9	7.7E-11	6.0E-16	1.0E-11
B3M200T20	0	40000	OPTIMAL	10	108.0	4.2E-09	2.0E-16	3.0E-09
B3M300T20	0	90000	OPTIMAL	14	440.1	2.6E-09	8.0E-17	5.0E-09
PILOT	1441	4693	OPTIMAL	33	254.9	9.3E-09	2.0E-08	4.0E-11
VOL1	323	459	INFEASIBLE	29	2.6	1.0E+00	3.0E-01	5.0E-12
WOOD1P	244	2595	OPTIMAL	22	58.0	2.9E-10	9.0E-10	2.0E-15

Table 4: Quadratic test problems.

written in ANSI C. The test is performed on a HP 715/75 UNIX workstation. All timing results are obtained using the C procedure "clock".

The test problem is formed by adding a quadratic term $.5x^TQx$ to the objective function of some of the NETLIB problems. The Q matrix has the form

$$\begin{bmatrix} 2 & -1 & & \\ -1 & 2 & & \\ & & 2 & -1 \\ & & -1 & 2 \end{bmatrix}.$$

Q is clearly positive definite because it is strictly diagonal dominant. Moreover, the Q matrix is block diagonal. We select the feasible problems 25FV47, DEGEN3, GREENBEA, PILOT and WOOD1P and two (primal) infeasible problems KLEIN2 and VOL1 from the NETLIB LP problems.

In addition we use some box-constrained QP problems arising from the so-called obstacle and elastic-plastic torsion problems. The name of these problems all start with the letter B. The B2 problems are slightly modified. The original problems contain some very large upper bounds. These bounds are always redundant and therefore we have removed them in B2, which leads to a reduction in the number of iterations by approximately 50%. Our results are presented in Table 4.

The columns in Table 4 shows the number constraints, the number variables, the solution status, the number of iterations, the solution time in seconds, $rgap$, $pfeas_1$ and $dfeas_1$. We see that all of the feasible problems are solved to the required precision and two infeasible problems are correctly detected. As expected, the algorithm uses very few iterations to solve these problems. In summary our algorithm seems to work well for solving quadratic problems.

For applications to real-word problems, my colleagues and I [J14] have also applied an optimization algorithm to solving a popular production management problem: structured uncapacitated facility location problem. We developed a strongly polynomial algorithm with the currently best complexity bound. My student Bosch, Woodworth and I [J7] successively apllied an interior-point algorithm to solving quantile regression with smoothing splines.

My students and I have been supported by an MCI grant on research for a Real-Time Restoration project. This project is to improve the restoration process for the MCI network during emergence situation. We have applied mathematical modeling and optimization techniques to reroute communication traffic as quickly and efficiently as possible. The project has been implemented and put into operation in 1993. The result? According to *MCI Network National*, February, 1994, "What used to take just more than an hour to reroute traffic during a catastrophic transmission outage, now only takes minutes with the implementation of Real-Time Restoration."

4 Education and Human Resources

Some beneficiaries, educationally, of the project are the Ph.D. students at the University of Iowa. We have supported several Ph.D. students at UI from this NSF grant. Some results mentioned above are part of their Ph.D. theses.

In particular, my student Hung is now an assistant professor in Donghai University of Taiwan, Ron Bosch is now a post-doctor at Harvard University, and Erling Andersen is an assistant professor in Odense University of Denmark.

Another outgrowth of our research has been a new seminar, meeting once a week, among graduate students and faculty members from several departments. I have also brought some of the new research topics and results to an optimization and mathematical programming course at the graduate level. I am writing a tex book "Interior-Point Algorithms: Theory and Analysis."

5 Publications and Technical Reports

5.1 New Journal Papers

[J1] S. Vavasis and Y. Ye, "A primal-dual interior-point method whose running time depends only on the constraint matrix," Technical Report 93-1391, Department of Computer Science, Cornell University (1993). to appear in *Mathematical Programming.*

[J2] Y. Ye, "On homogeneous and self-dual algorithm for LCP," Working Paper, Department of Management Sciences, The University of Iowa (July, 1994), to appear in *Mathematical Programming.*

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5.2 New Conference Papers

[C1] "An accelerated interior-point method whose running time depends only on A," (S. Vavasis and Y. Ye), *Proc. of the Twenty-Sixth ACM Symposium on Theory of Computing* (1994) 512-521.
[C2] "A genuine quadratically convergent polynomial interior point algorithm for linear programming," (Z.-Q. Luo and Y. Ye) in Ding-Zhu Du and Jie Sun, eds., *Advances in Optimization and Approximation* (Kluwer Academic Publishers, Boston, 1994).
[C3] "On the complexity of a column generation algorithm for convex or quasiconvex feasibility problems," (J. Goffin, Z. Luo and Y. Ye), in W. Hager, D. Hearn and P. Pardalos eds., *Large Scale Optimization: State of the Art* (Kluwer Academic Publishers, Boston, 1994) pp. 182-191.

5.3 New Working Papers

[W1] "On the complexity of approximating a KKT point of quadratic programming," Working Paper, Department of Management Sciences, The University of Iowa (November, 1995).
[W2] "Efficient algorithms for minimizing a sum of Euclidean norms with applications," (G. Xue and Y. Ye), Working Paper, Department of Management Sciences, The University of Iowa (July, 1995).

[W3] "On a homogeneous algorithm for the monotone complementarity problem," (E. Andersen and Y. Ye), Working Paper, Department of Management Sciences, The University of Iowa (May, 1995).
[W4] "An infeasible interior-point algorithm for solving primal and dual geometric programs," (K. O. Kortanek, X. Xu, and Y. Ye), Working Paper, Department of Management Sciences, The University of Iowa (November, 1994).
[W5] "Approximate Farkas lemmas and stopping rules for iterative infeasible-point algorithms for linear programming," (M. J. Todd and Y. Ye), Working Paper, Department of Management Sciences, The University of Iowa (September, 1994).
[W6] "Complexity analysis of the analytic center cutting plane method that uses multiple cuts," Working Paper, Department of Management Sciences, The University of Iowa (September, 1994).